# Brug for algoritmikere

## Danmark vil mangle 22.000 it-professionelle i 2030

Manglen på it-folk fortsætter så langt øjet rækker, og vil være næsten tre gange så høj om ti år

## Ny FN-måling: Danmark er fortsat verdensmestre i offentlig digitalisering

12-07-2020 | Strategier

FN har netop offentliggjort E-Government Survey 2020, og Danmark ligger for anden gang i træk på en flot første plads. I 2018 rykkede Danmark fra en tidligere 9. plads til første pladsen.



# DIKU i verdensklasse!

## CSRankings: Computer Science Rankings

CSRankings is a metrics-based ranking of top computer science institutions around the world. **Click on a triangle** (▶) to expand areas or institutions. **Click on a name** to go to a faculty member's home page. **Click on a chart icon** (the after a name or institution) to see the distribution of their publication areas as a bar chart. **Click on a Google Scholar icon** () to see publications, and **click on the DBLP logo** () to go to a DBLP entry. *Applying to grad school? Read this first.* **Do you find CSrankings useful? Sponsor CSrankings on GitHub.**

Rank institutions in the world by publications from 2012 to 2022

**All Areas** [off | on]

**AI** [off | on]

- ▶ Artificial intelligence
- ▶ Computer vision
- ▶ Machine learning & data mining
- ▶ Natural language processing
- ▶ The Web & information retrieval

**Systems** [off | on]

- ▶ Computer architecture
- ▶ Computer networks
- ▶ Computer security
- ▶ Databases
- ▶ Design automation
- ▶ Embedded & real-time systems
- ▶ High-performance computing
- ▶ Mobile computing
- ▶ Measurement & perf. analysis
- ▶ Operating systems
- ▶ Programming languages
- ▶ Software engineering

**Theory** [off | on]

- ▶ Algorithms & complexity ☑
- ▶ Cryptography
- ▶ Logic & verification

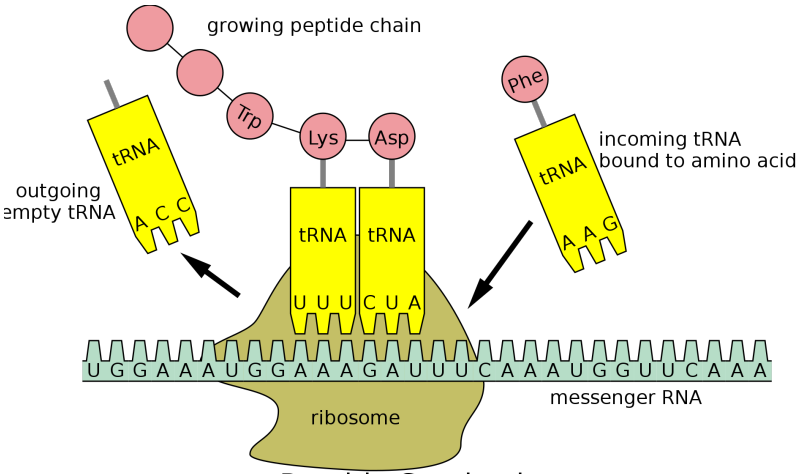
**Interdisciplinary Areas** [off | on]

- ▶ Comp. bio & bioinformatics
- ▶ Computer graphics
- ▶ Economics & computation
- ▶ Human-computer interaction
- ▶ Robotics
- ▶ Visualization

| # | Institution | Count | Faculty |
|---|---|---|---|
| 1 | ▶ Carnegie Mellon University | 94.0 | 19 |
| 2 | ▶ Massachusetts Institute of Technology | 81.3 | 20 |
| 3 | ▶ Tel Aviv University | 61.6 | 20 |
| 4 | ▶ Stanford University | 54.2 | 12 |
| 5 | ▶ University of Michigan | 50.3 | 12 |
| 6 | ▶ Princeton University | 47.7 | 11 |
| 7 | ▶ Columbia University | 46.1 | 12 |
| 8 | ▶ Univ. of Illinois at Urbana-Champaign | 40.8 | 9 |
| 9 | ▶ University of Washington | 39.7 | 11 |
| 10 | ▶ Univ. of California - San Diego | 37.1 | 11 |
| 11 | ▶ Weizmann Institute of Science | 35.2 | 10 |
| 12 | ▶ University of Copenhagen | 35.0 | 8 |
| 13 | ▶ Univ. of California - Berkeley | 34.1 | 14 |
| 14 | ▶ EPFL | 32.9 | 12 |
| 15 | ▶ University of Waterloo | 32.6 | 16 |
| 16 | ▶ University of Warsaw | 28.0 | 17 |
| 17 | ▶ University of Pennsylvania | 27.9 | 10 |
| 18 | ▶ Bar-Ilan University | 27.6 | 8 |
| 19 | ▶ Cornell University | 25.3 | 15 |
| 20 | ▶ Rutgers University | 25.1 | 9 |
| 21 | ▶ Hebrew University of Jerusalem | 24.8 | 15 |
| 22 | ▶ University of Texas at Austin | 24.2 | 10 |
| 23 | ▶ Univ. of California - Los Angeles | 24.0 | 4 |

# Algoritmer før (moderne) computere

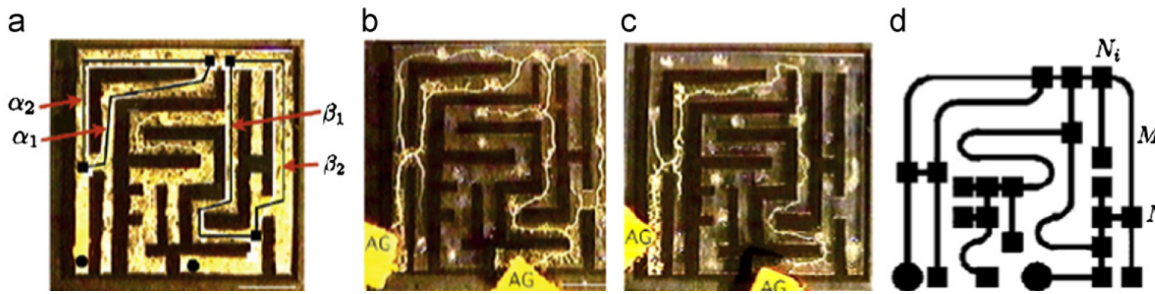
"Before there were computers, there were algorithms. But now that there are computers, there are even more algorithms, and algorithms lie at the heart of computing."



Peptide Synthesis

**A Biological Solution to a Fundamental Distributed Computing Problem**

Yehuda Afek,[1]* Noga Alon,[1,2]* Omer Barad,[3]* Eran Hornstein,[3] Naama Barkai,[3]† Ziv Bar-Joseph[4]†




Physarum can compute shortest paths ☆

Vincenzo Bonifaci [a,*,1], Kurt Mehlhorn [b], Girish Varma [c,1]

[a] *Istituto di Analisi dei Sistemi ed Informatica "Antonio Ruberti" – CNR, Rome, Italy*
[b] *MPI for Informatics, Saarbrücken, Germany*
[c] *Tata Institute of Fundamental Research, Mumbai, India*

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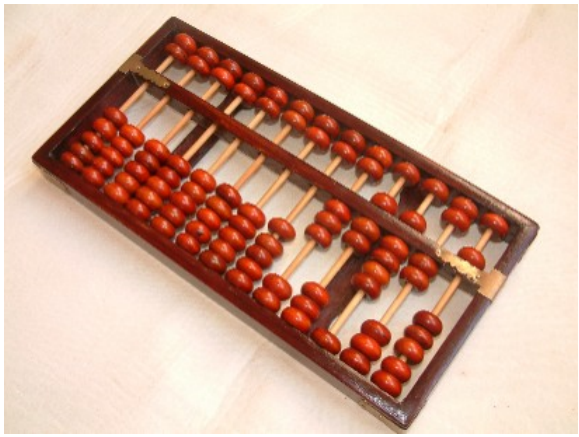


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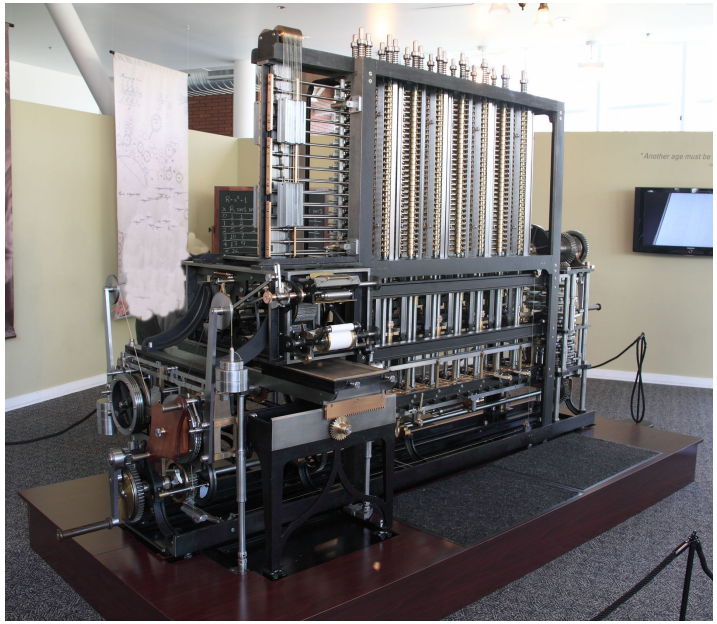

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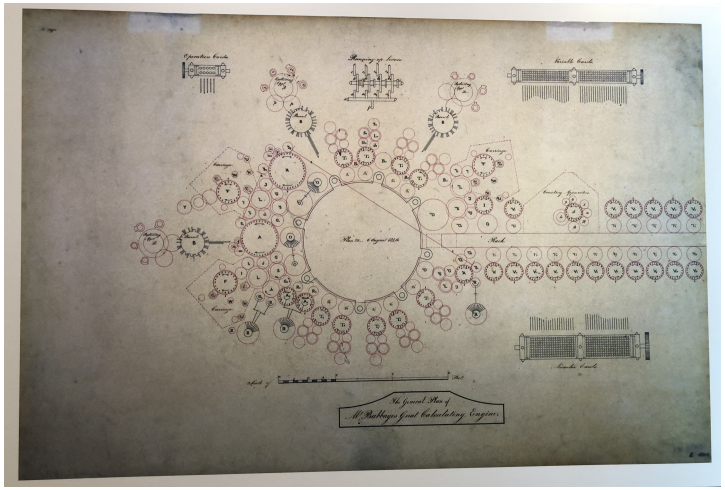

Charles Babbage: The Analytical Engine

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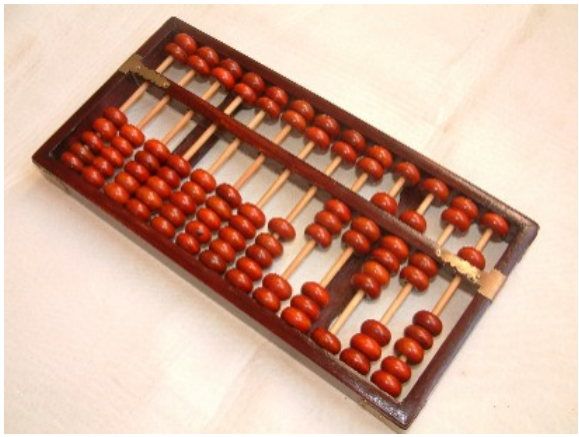

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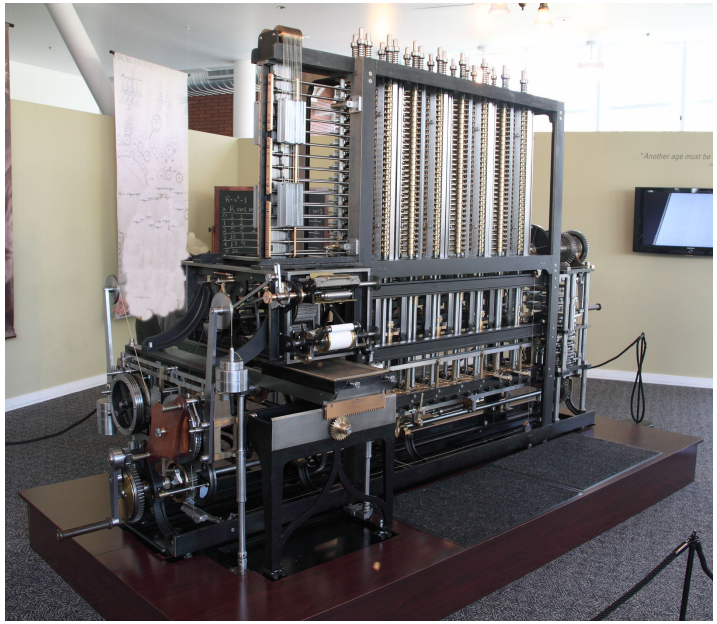

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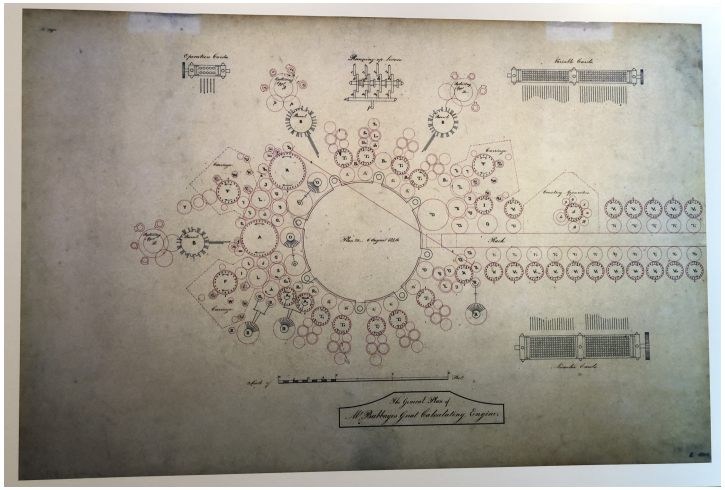

Charles Babbage: The Analytical Engine

```
 1 9 7 3 4
+5 4 2 8 3
---------
```

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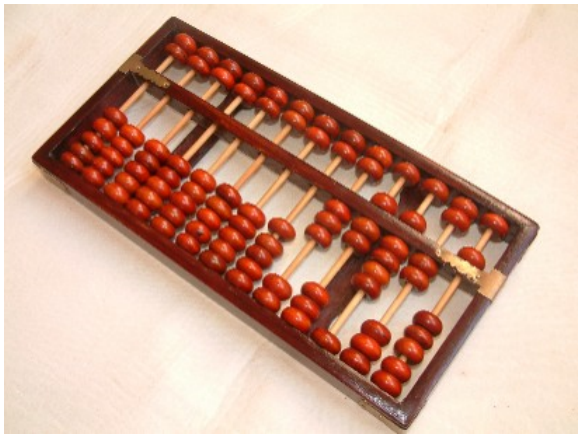

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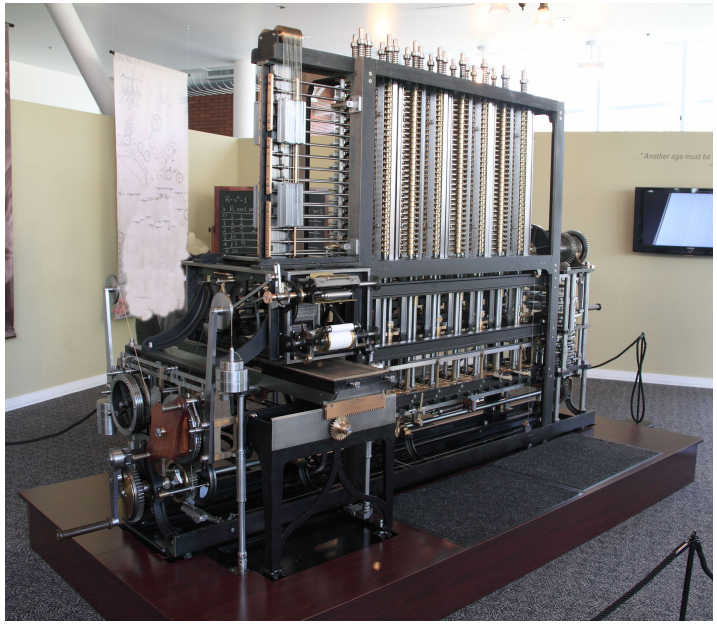

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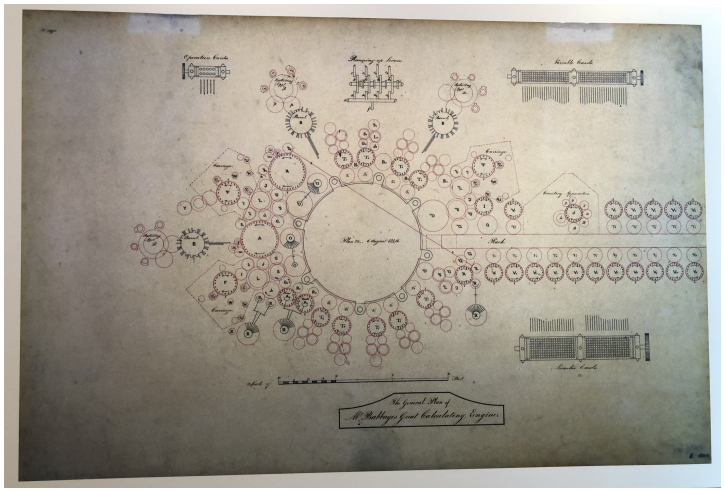

Charles Babbage: The Analytical Engine

```
 1 9 7 3 4
+5 4 2 8 3
-----------
           7
```

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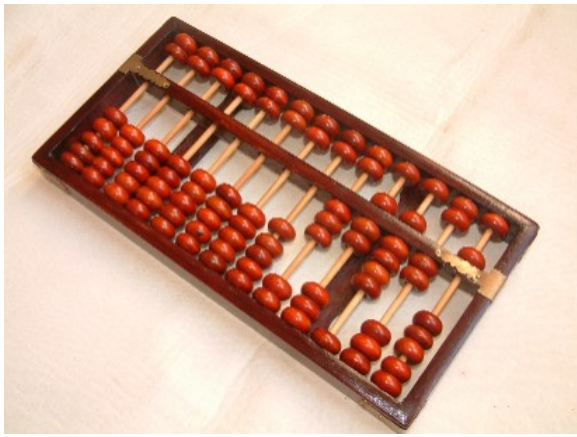

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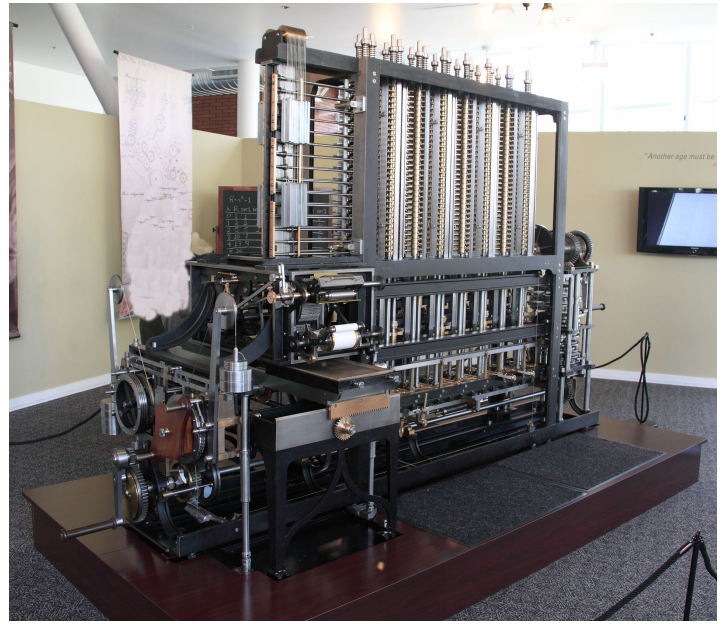

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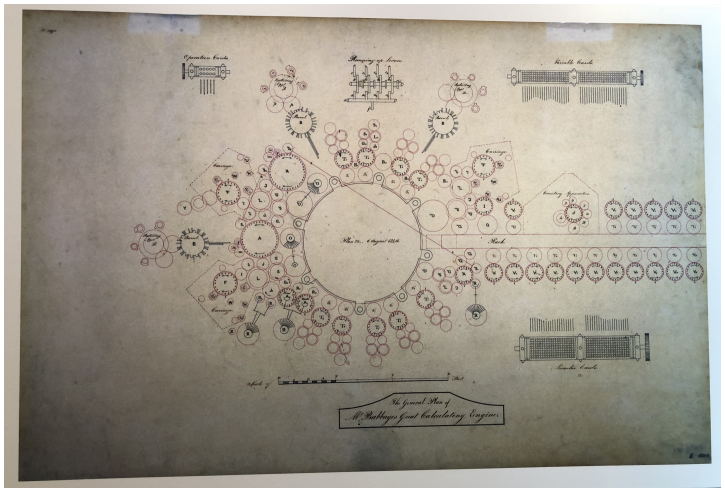

Charles Babbage: The Analytical Engine

$$\begin{array}{r} \phantom{+}{\scriptstyle 1}\phantom{0000} \\ \phantom{+}19734 \\ +54283 \\ \hline \phantom{+}17 \end{array}$$

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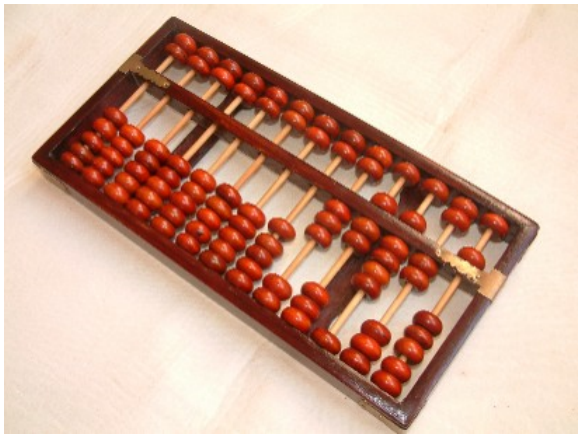

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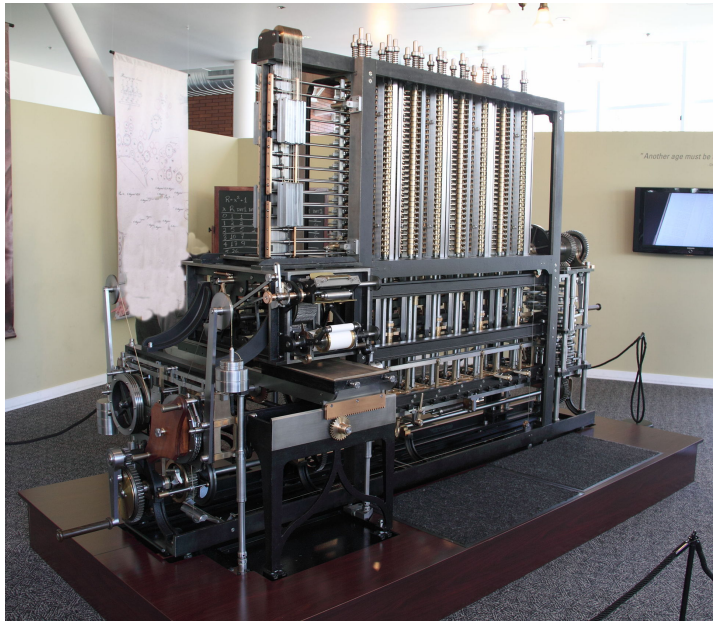

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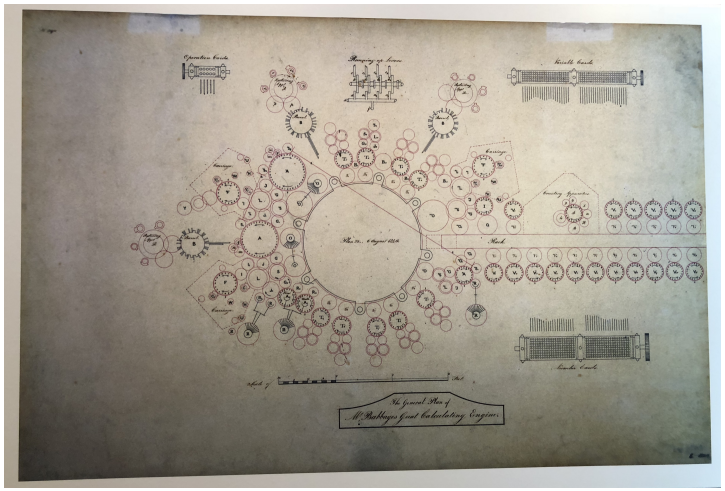

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```
  1 1
  1 9 7 3 4
+5 4 2 8 3
-----------
    0 1 7
```

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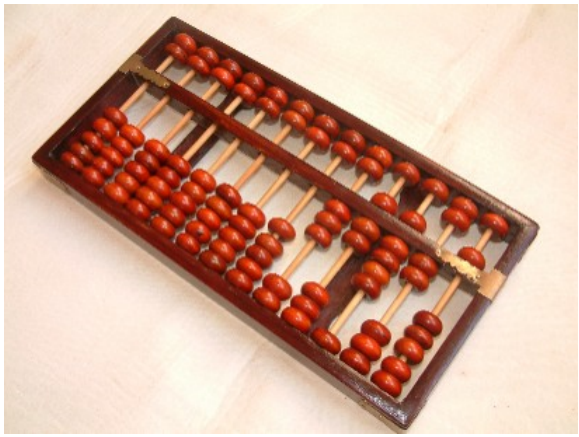

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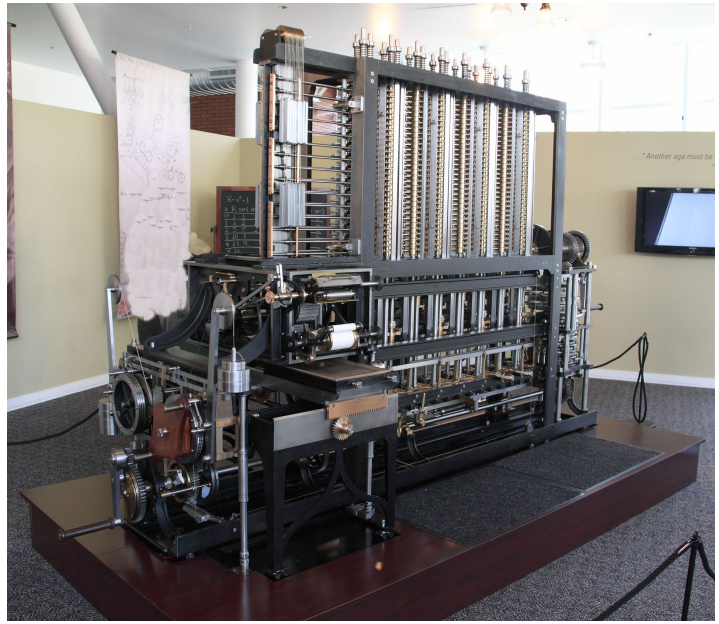

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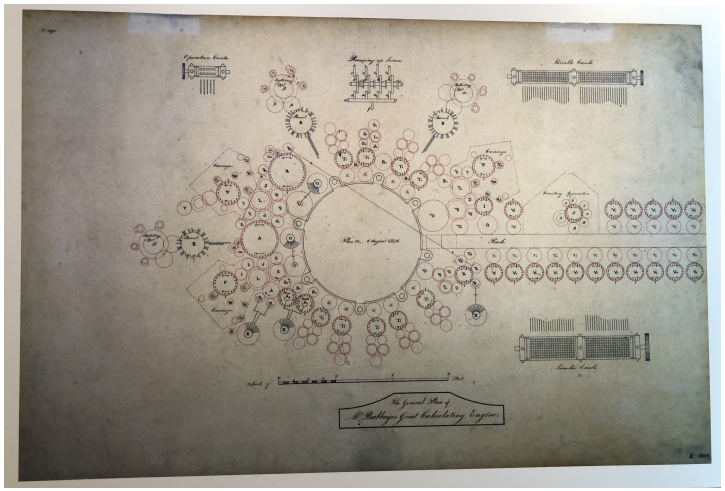

Charles Babbage: The Analytical Engine

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ 1\ \ \ \ \ } \\ 1\ 9\ 7\ 3\ 4 \\ +5\ 4\ 2\ 8\ 3 \\ \hline 4\ 0\ 1\ 7 \end{array}$$

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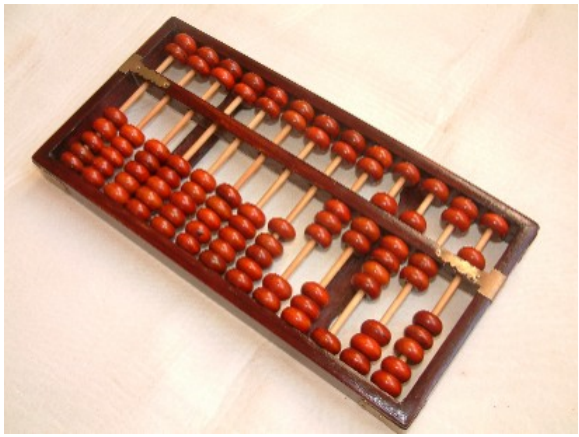

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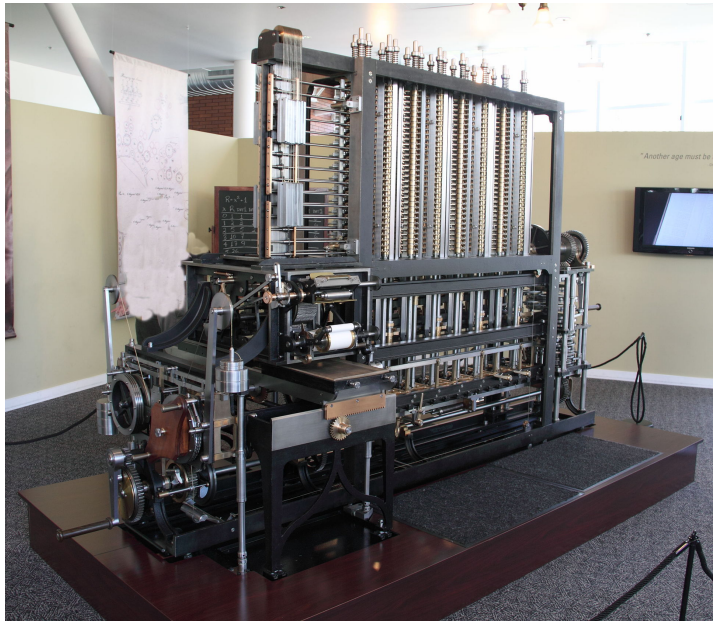

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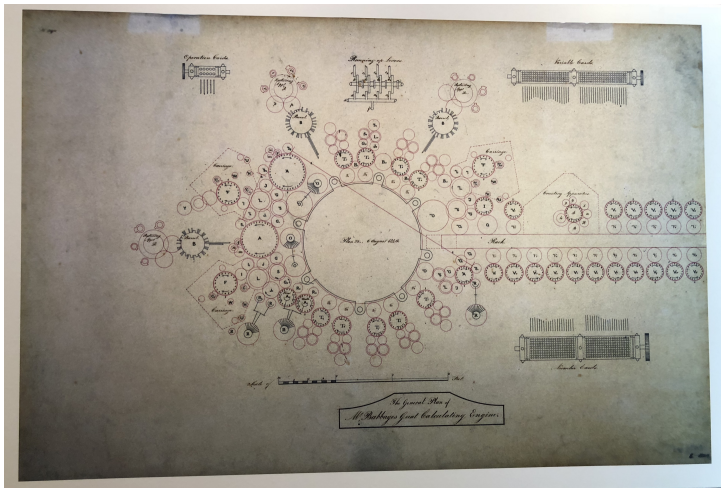
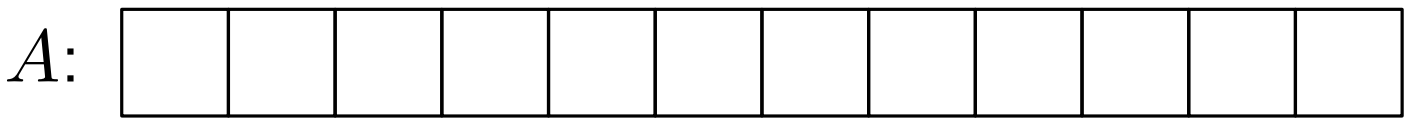


Charles Babbage: The Analytical Engine

```
  1  1  1
  1  9  7  3  4
+5  4  2  8  3
-----------------
  7  4  0  1  7
=================
```

# Arrays (tabeller)



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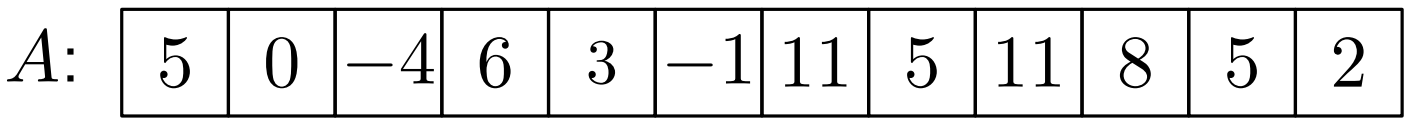

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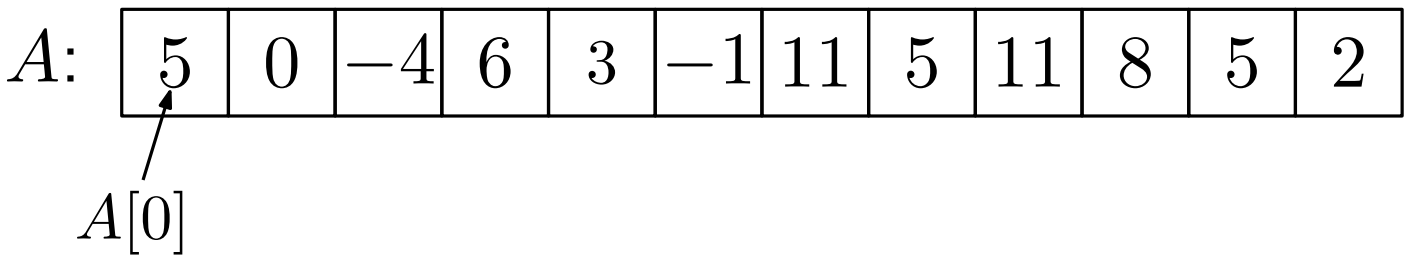

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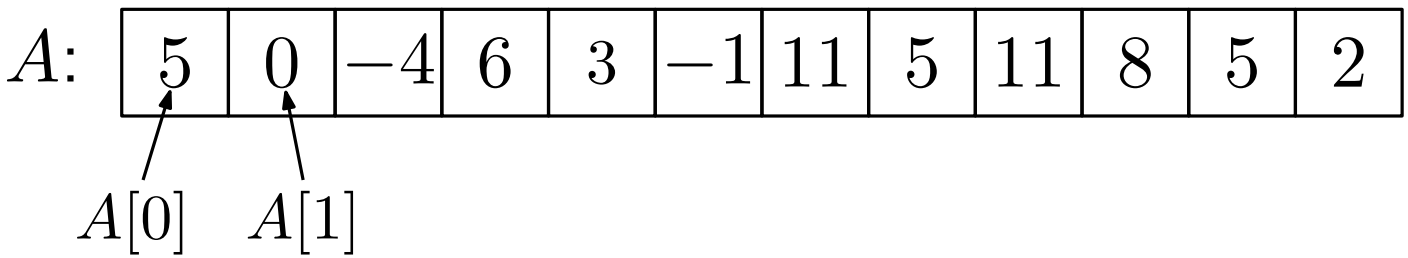

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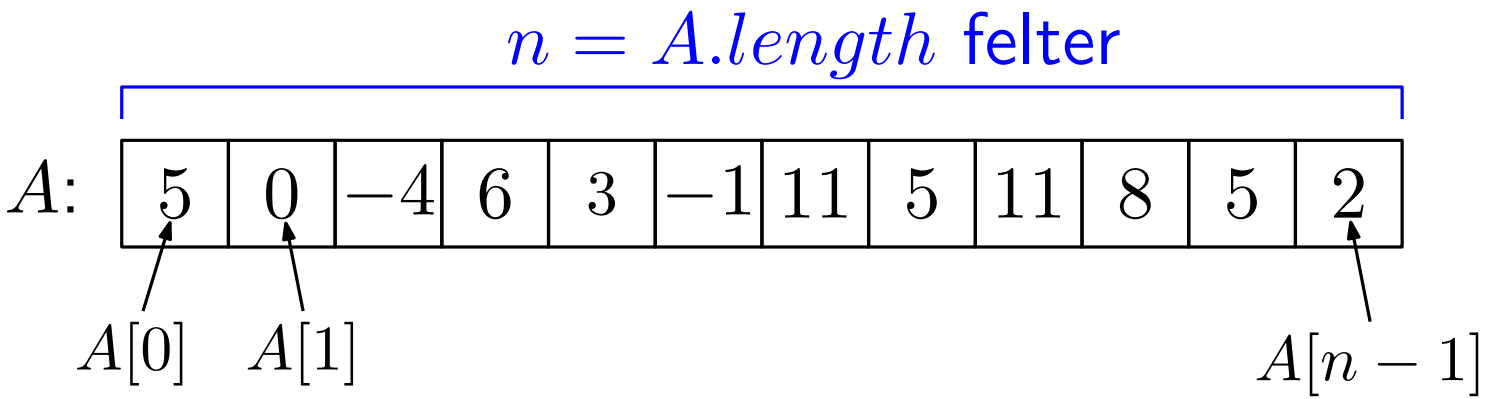

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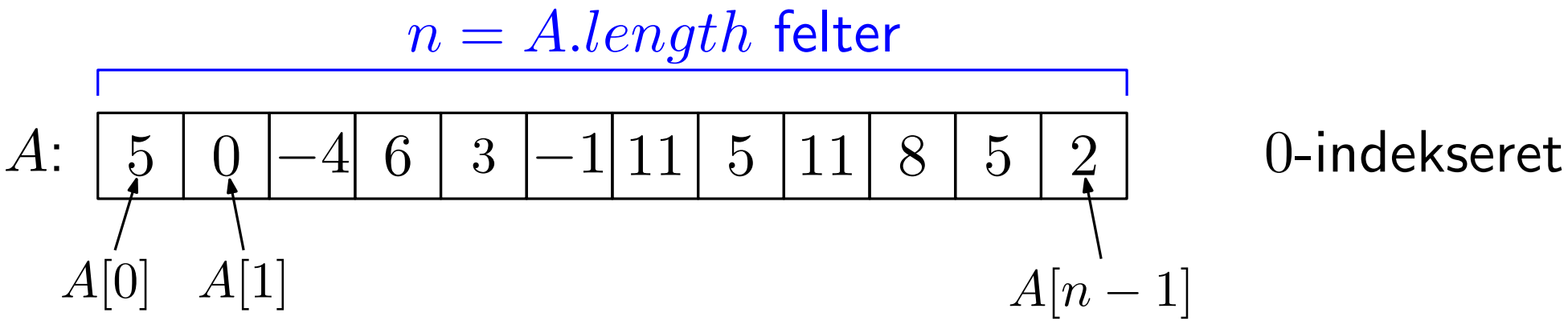


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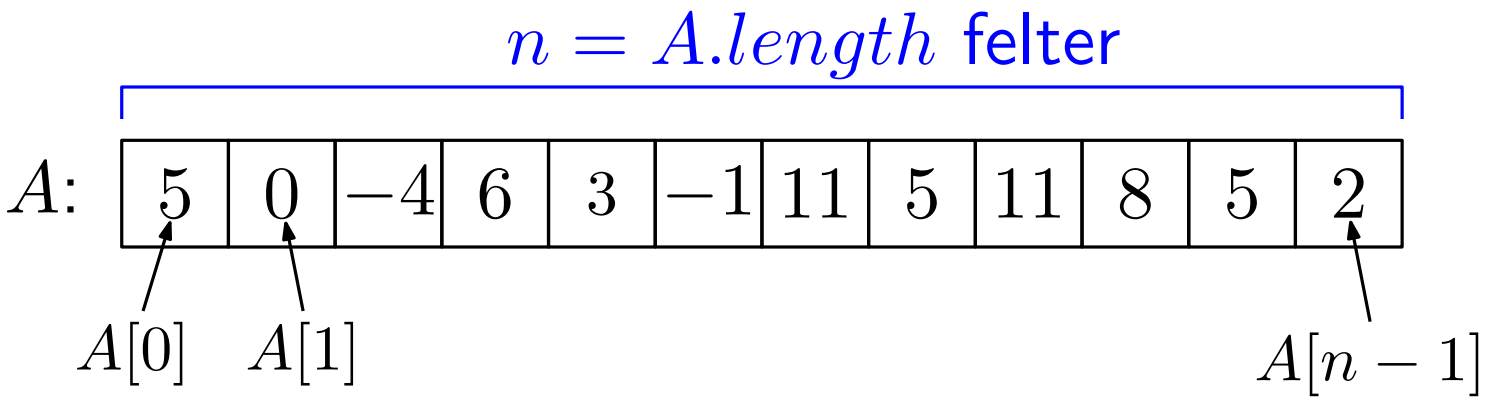


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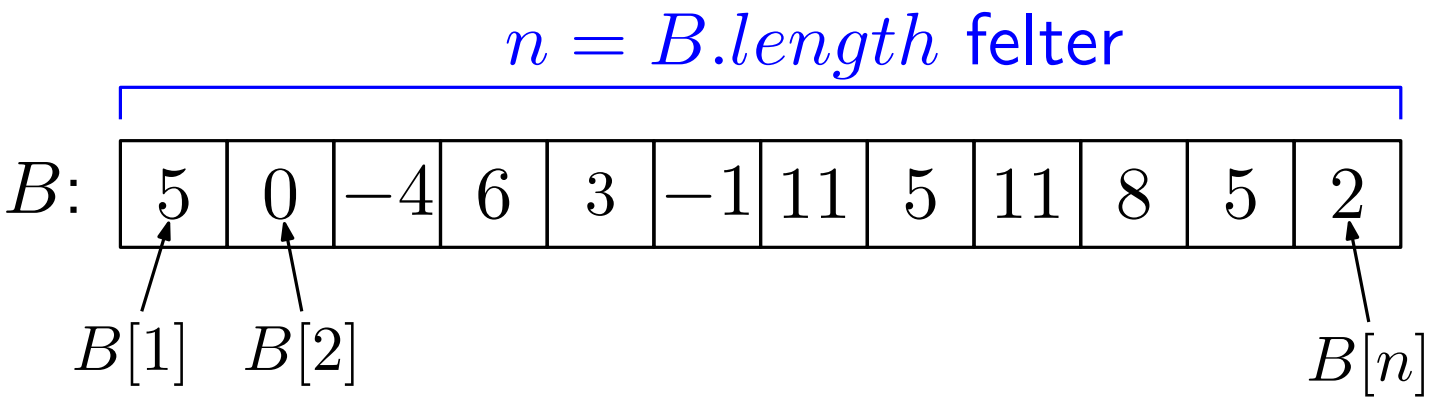
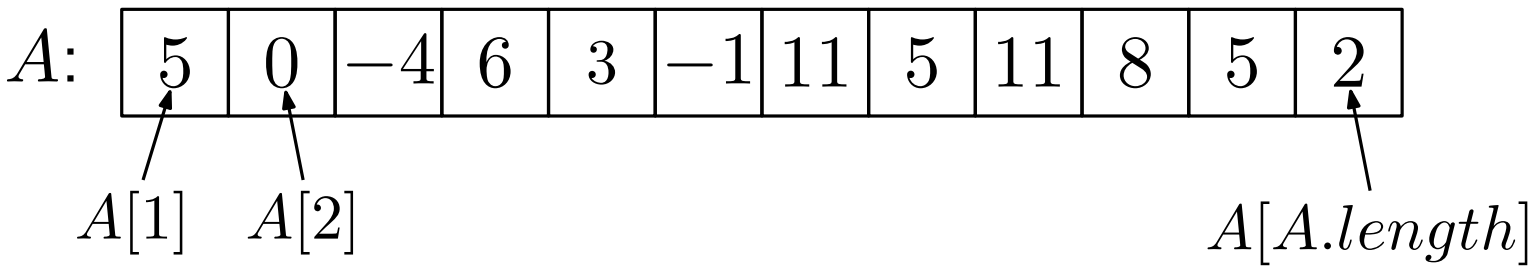


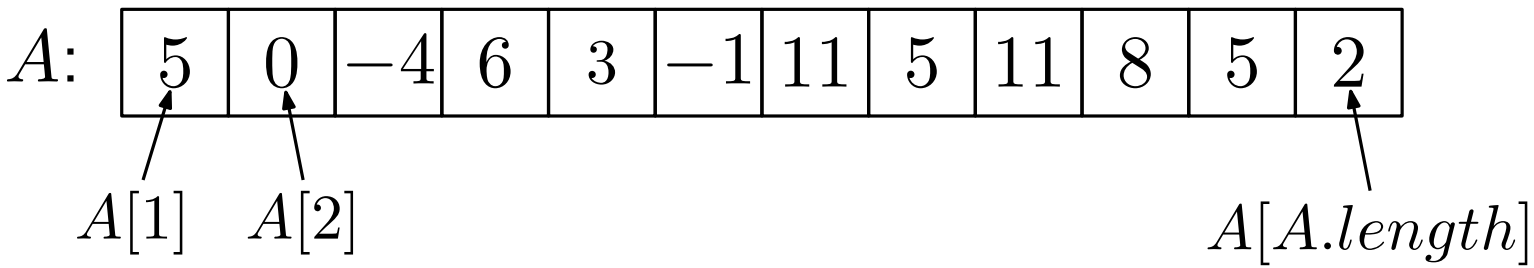

1-indekseret

# Maximum i array




```
findMax1(A)
    max = 1
    for i = 1 to A.length
        if A[i] > A[max]
            max = i
    return max
```

# Maximum i array




```
findMax1(A)
    max = 1
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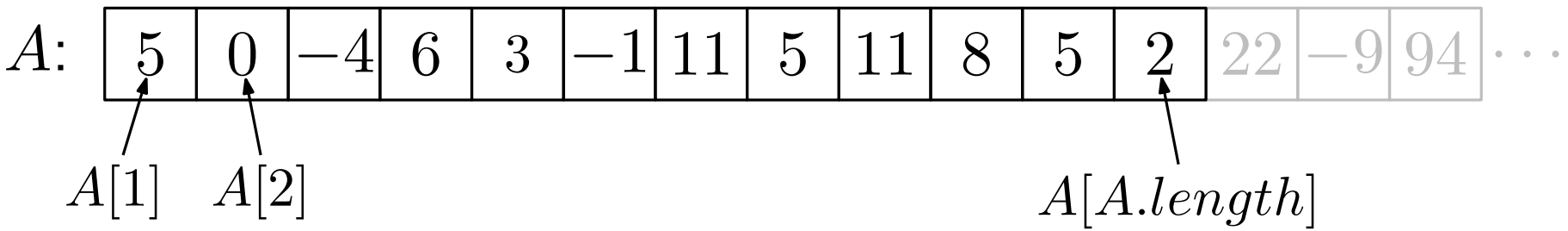
        if A[i] > A[max]
            max = i
    return max
```

```
findMax2(A, m)
    max = 1
    for i = 1 to m
        if A[i] > A[max]
            max = i
    return max
```

# Maximum i array

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# Maksimum i array

**Input:** Array $A$ af størrelse $n$, dvs. $A[0 : n-1]$.
**Output:** Et index $i$, $0 \leq i < n$, så $A[i] = \max\{A[0], \ldots, A[n-1]\}$.

# Maksimum i array

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**Output:** Et index $i$, $0 \leq i < n$, så $A[i] = \max\{A[0], \ldots, A[n-1]\}$.

**Algoritme:**

```
findMax(A, n)
    max = 0
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let findMax (A: int[]) (n: int) =
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findMax($A$, $n$)
  $max = 0$
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Ligesom rigtig kode, men uden besværet.
Skrevet til at blive læst af *mennesker*, ikke en computer.
Meget mere præcis end rent sproglig beskrivelse.

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# Tidskompleksitet: Hvilken maskine?


- Forskellige instruktioner.
- Forskellige hastigheder.
- Der kommer en ny model i morgen.

# Løsning: RAM-modellen!

- Abstrakt matematisk model af en computer.
- Simplere end alle virkelige computere.
- Skal aldrig opdateres.
- Alligevel realistisk.

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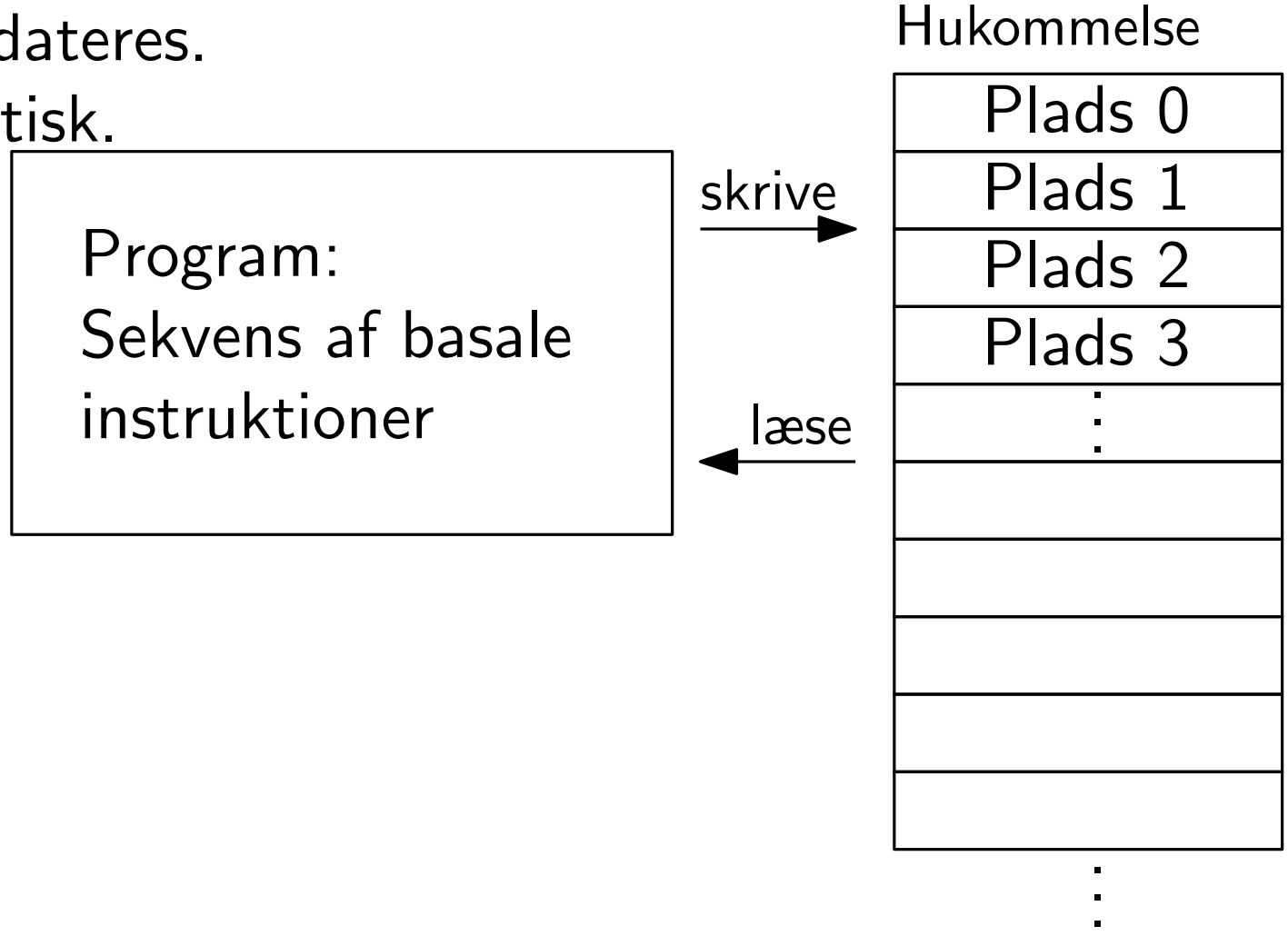

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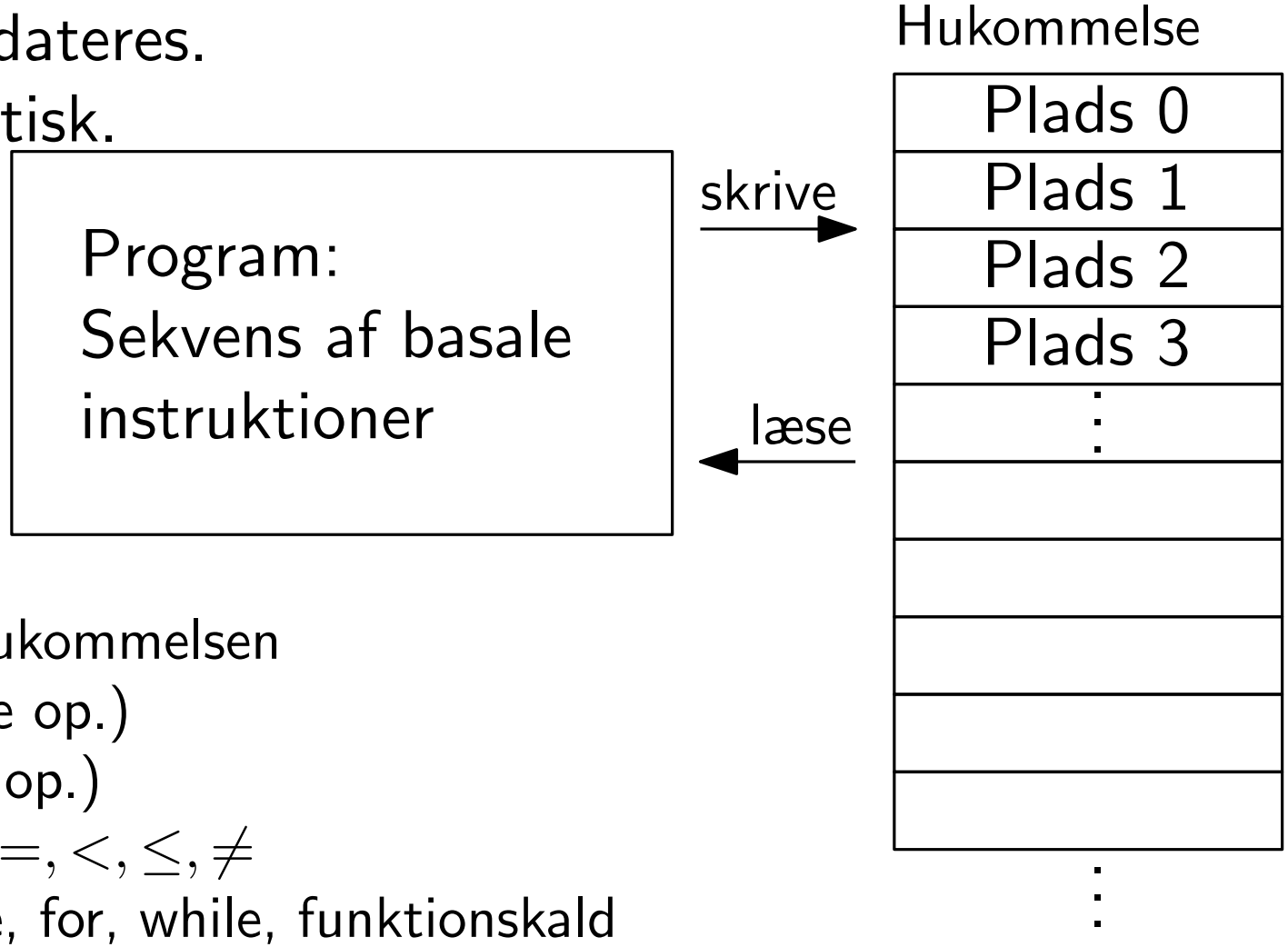


Basale instruktioner:
Læse fra/skrive til hukommelsen
$+, -, \cdot, /$ (aritmetiske op.)
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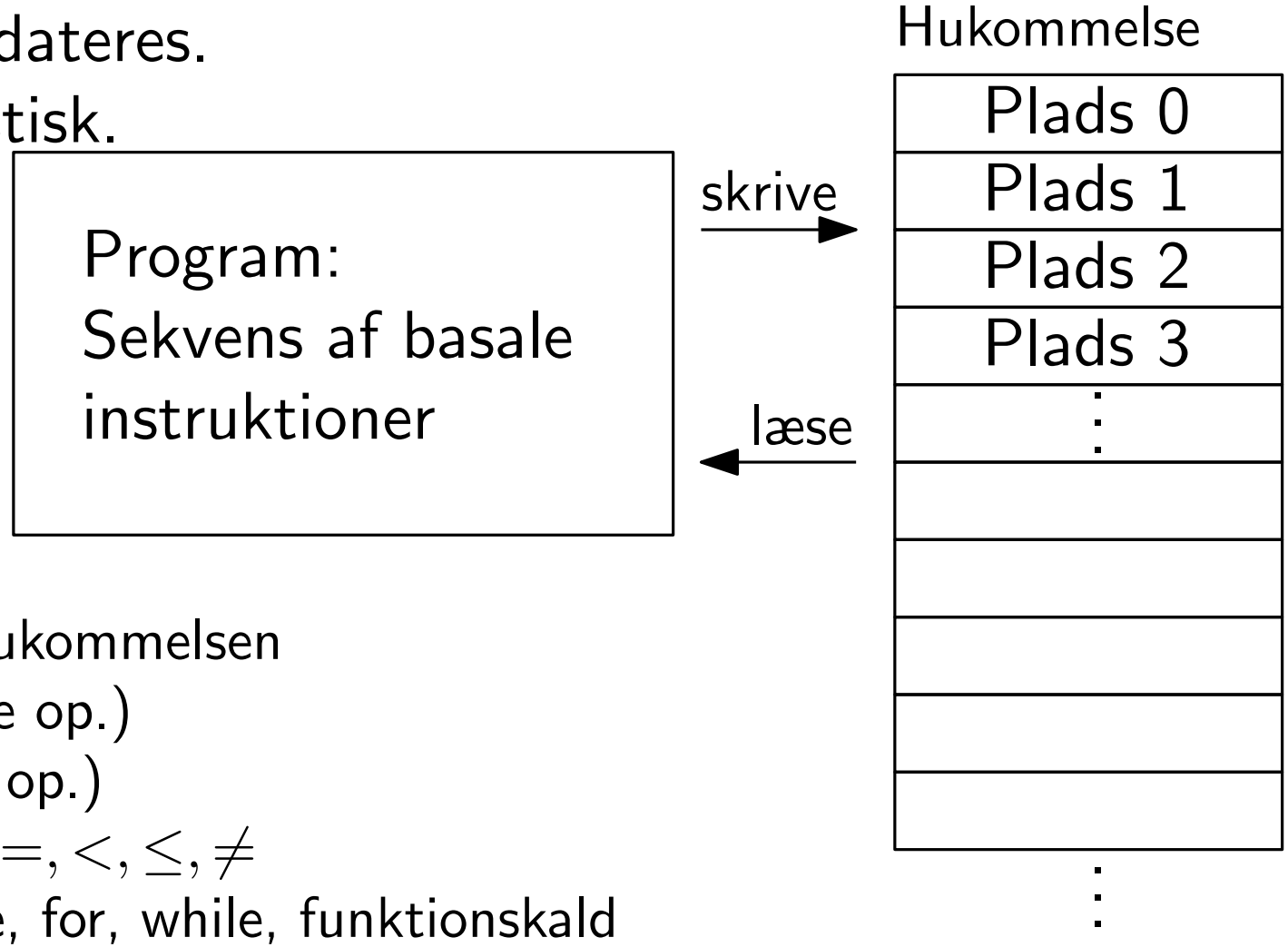


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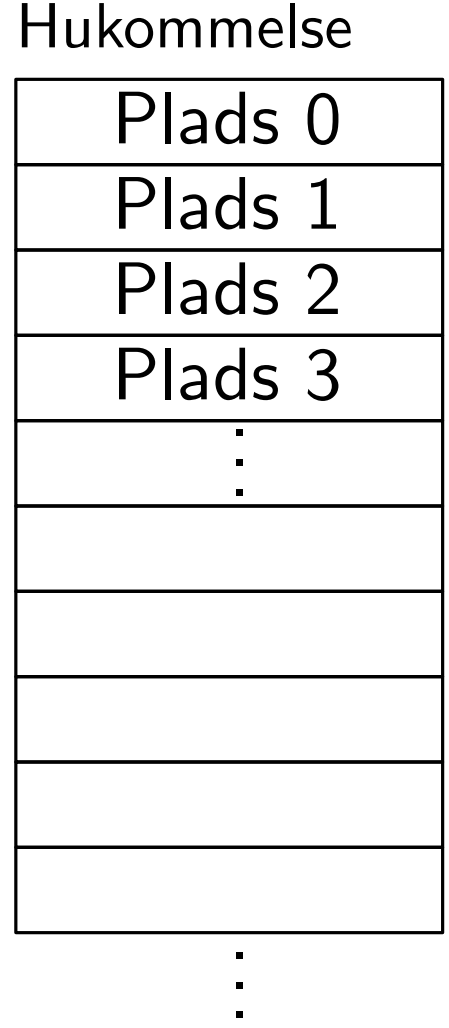
Program flow: if-else, for, while, funktionskald
Hver instruktion tager konstant tid.
Køretid = #instruktioner

# Ord/word

Hver plads kan indeholde et ord/word, fx:

- $w$ bits, konstant $w$.
- Et heltal mindre end en (stor) konstant $N$.
- Et tegn (bogstav/symbol).

Ikke lange tekster, mange tal eller vilkårligt store tal!
Disse må gemmes som flere ord.



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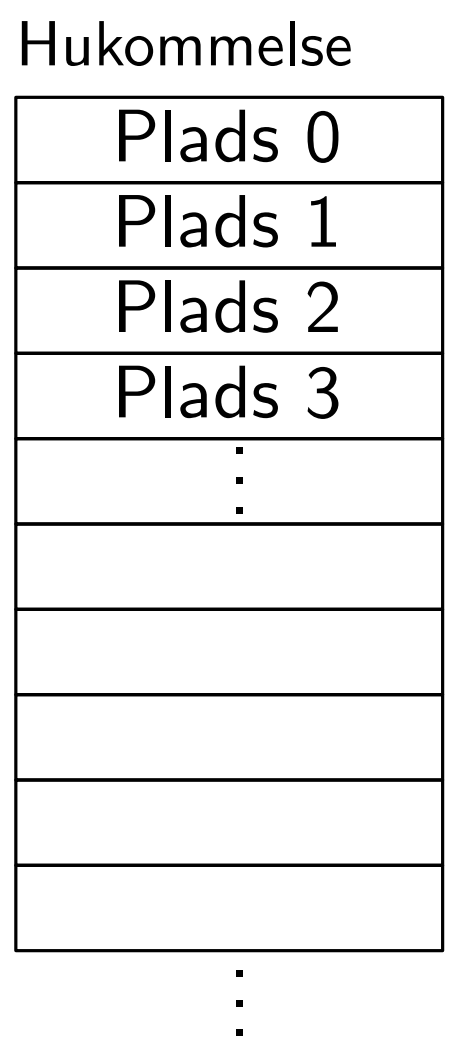

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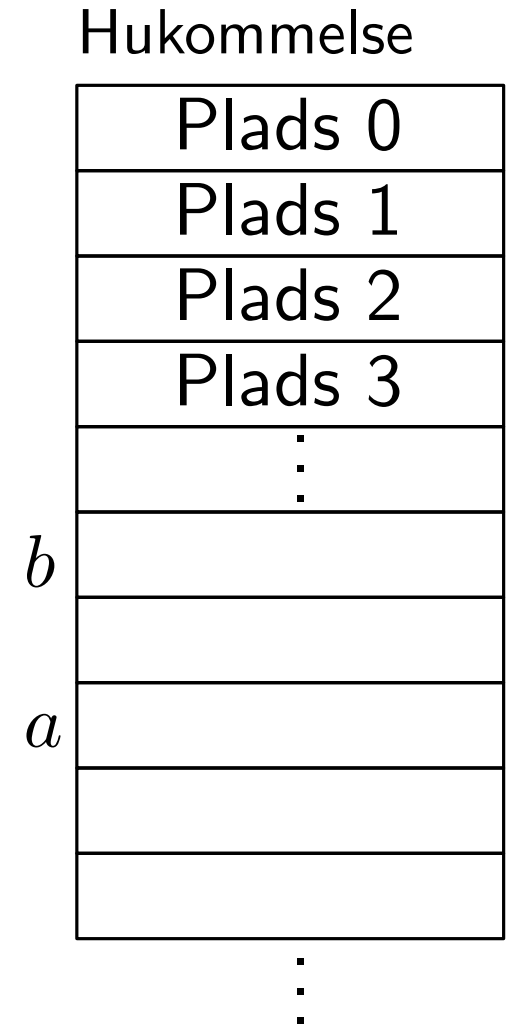

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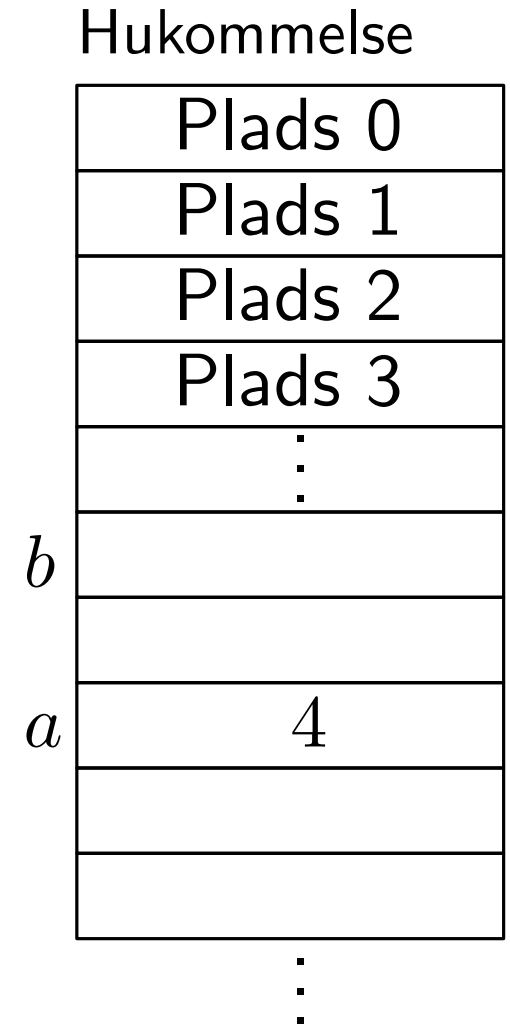

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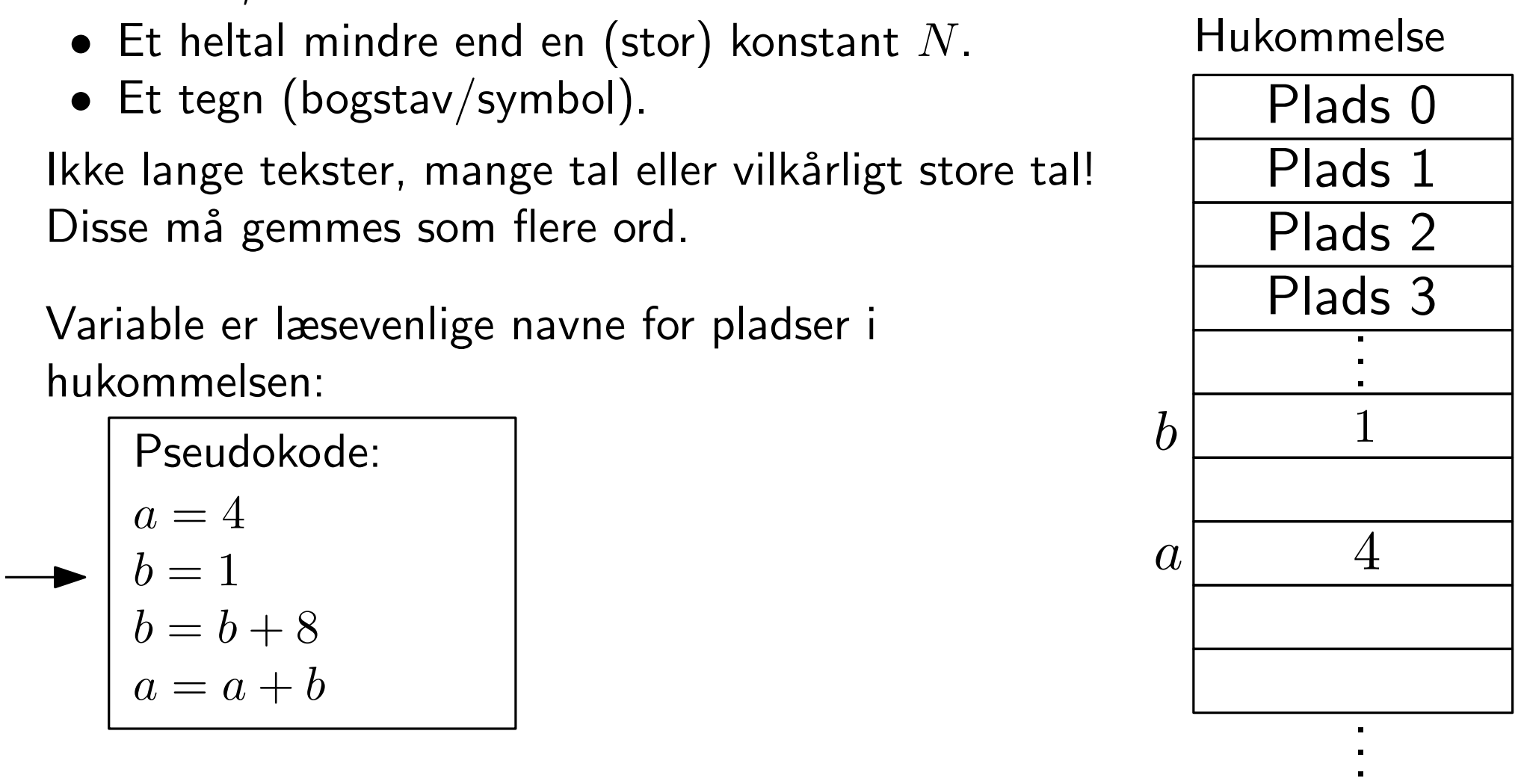

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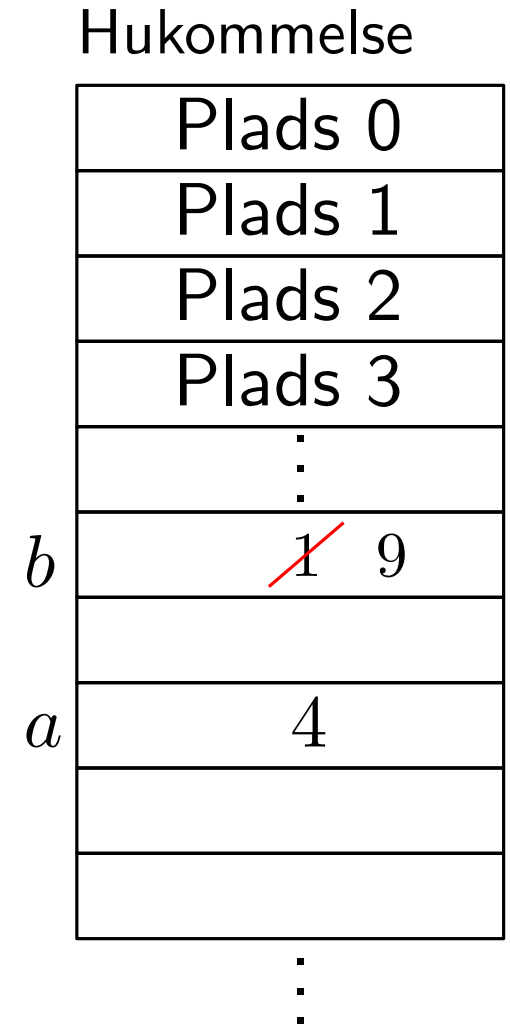

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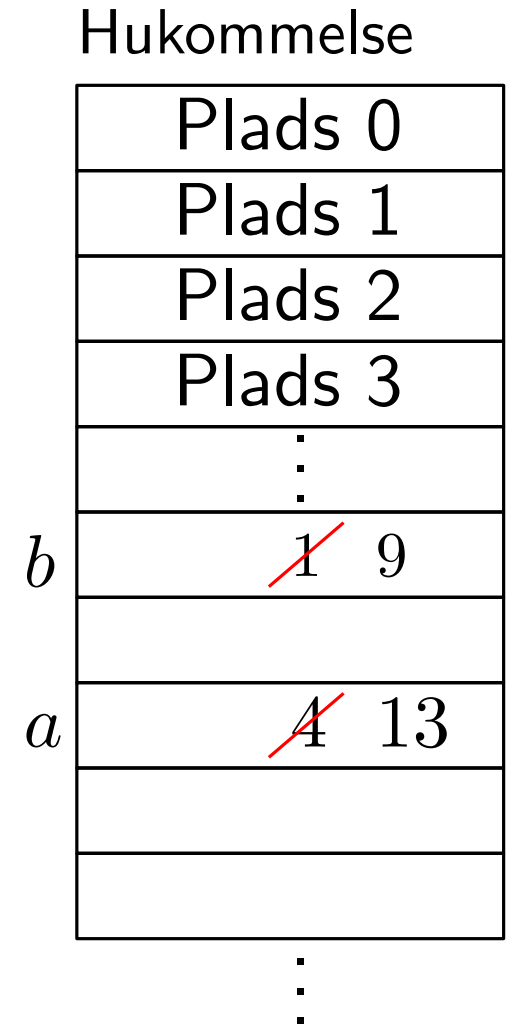

# Arrays/tabeller

Array: Sammenhængende del af hukommelsen.

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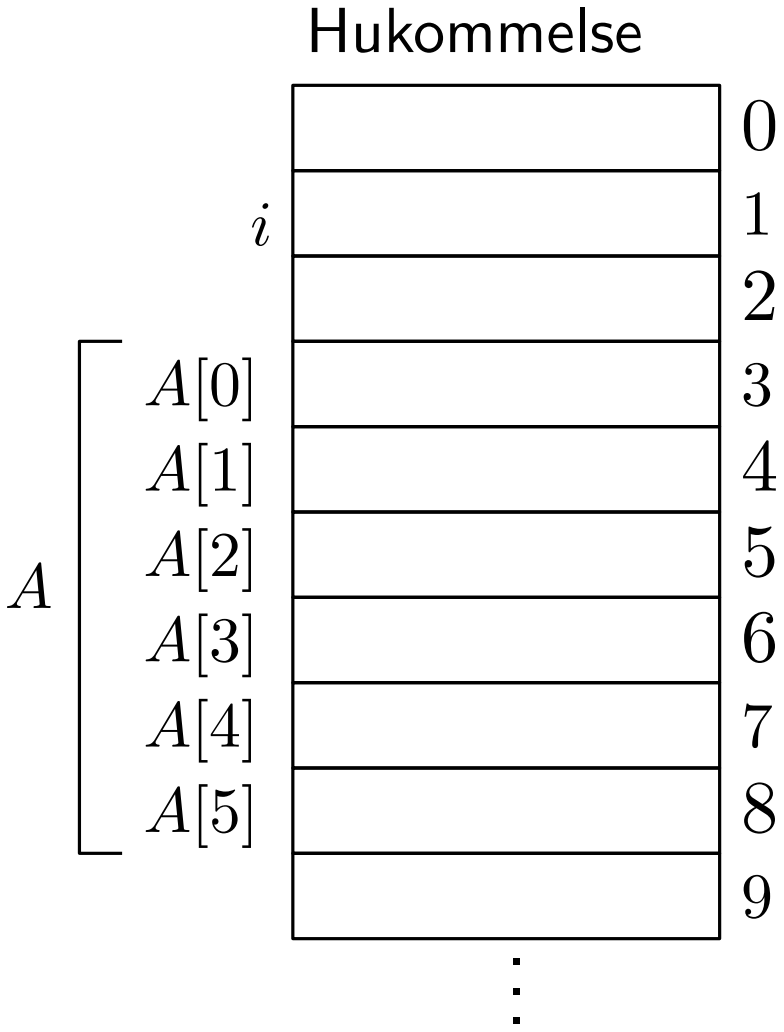

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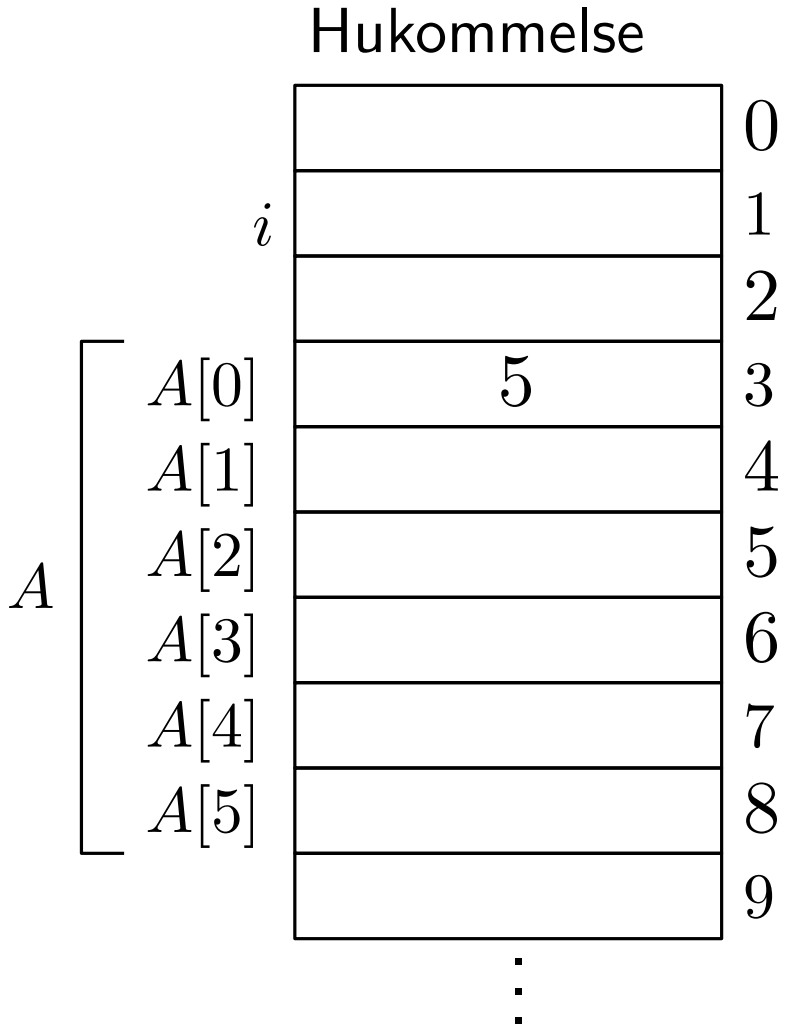

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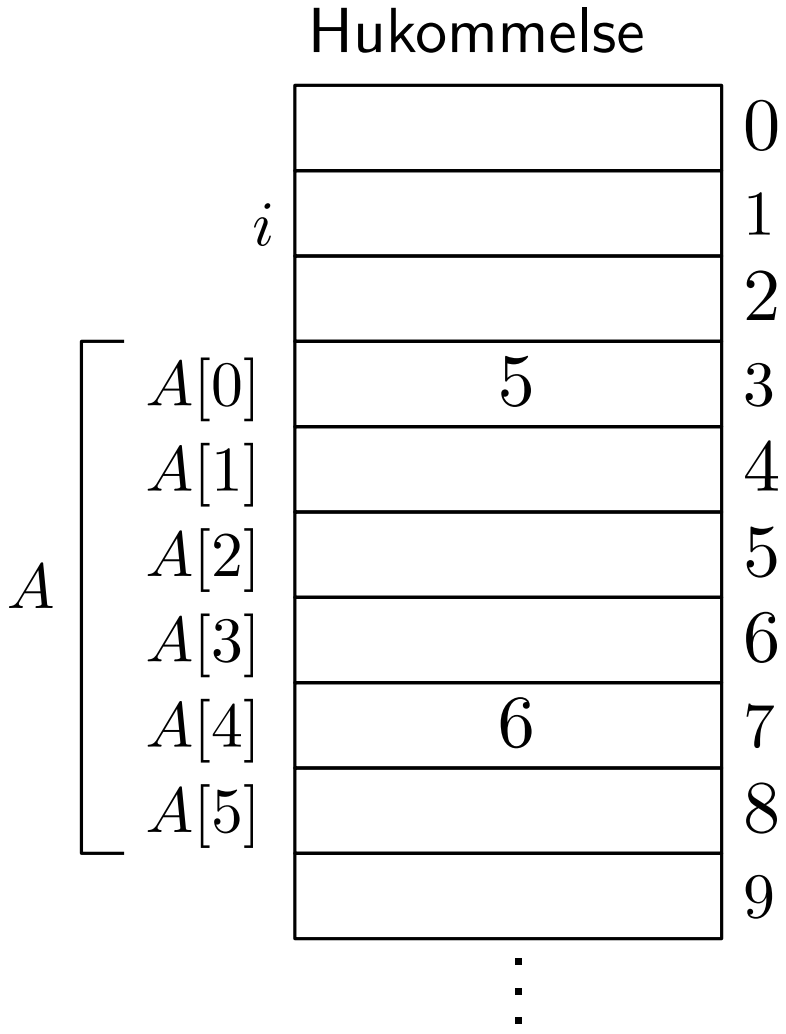

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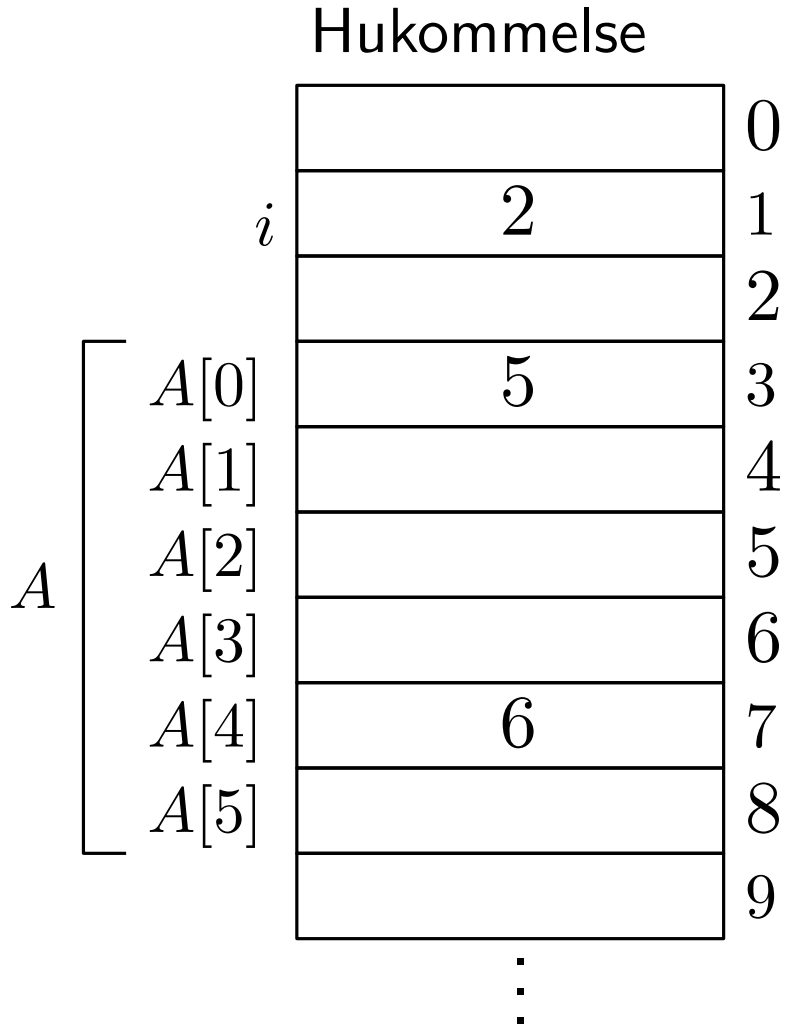

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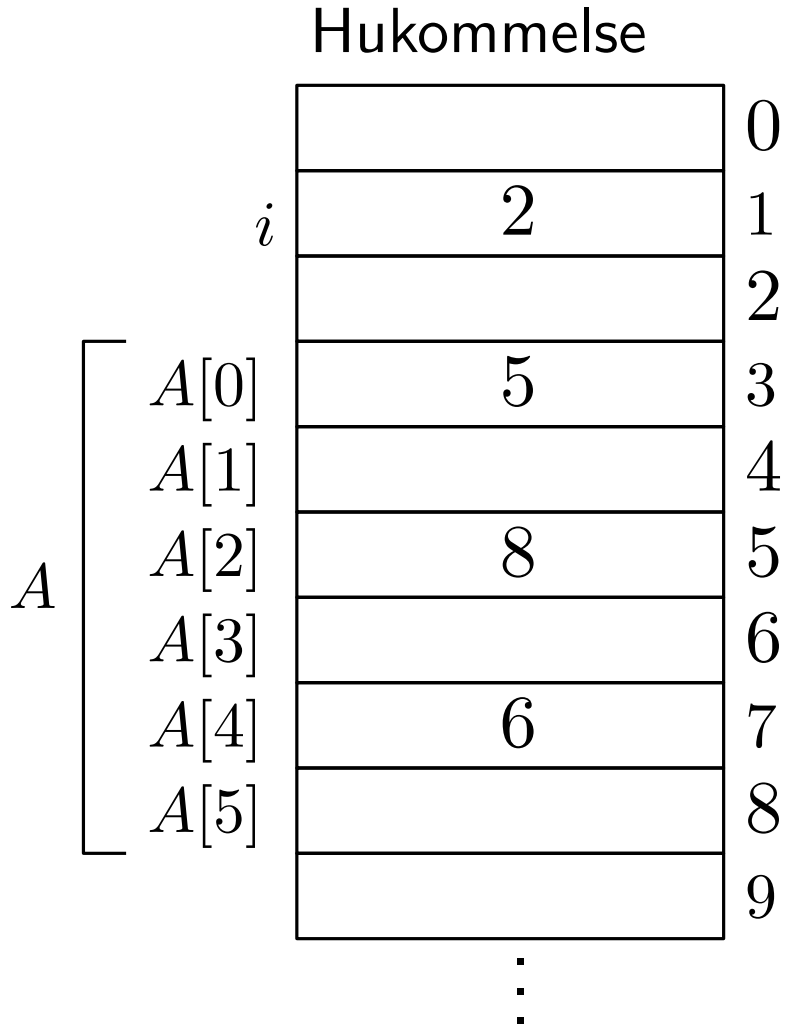

# Køretid/tidskompleksitet

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Tidskompleksitet i værste fald (worst-case complexity):
Maximal køretid for alle input af størrelse $n$.

# Eksempel

```
findMax(A, n)
    max = 0
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| skridt i linje | |
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| $c_1$ | små konstanter |
| $c_2$ | |
| $c_3$ | |
| $c_4$ | |
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skridt hver gang:

for

læse $i$

udregnet $i + 1$

gemme $i + 1$ som $i$

læse $n$

udregne $n - 1$

sammenligne ny $i$ med $n - 1$

# Eksempel

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| skridt i linje | | max. udførsler af linje |
|---|---|---|
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| $c_2$ | små konstanter | $n$ |
| $c_3$ | | $n$ |
| $c_4$ | | $n$ |
| $c_5$ | | $1$ |

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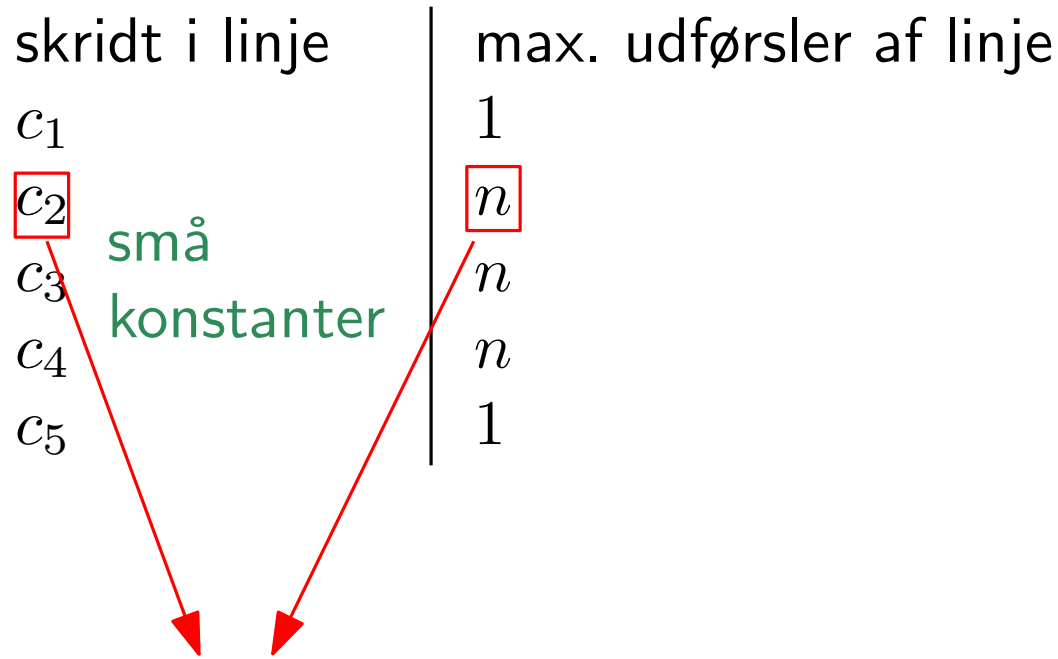


$T(n) =$ køretid $=$ skridt i alt $= c_1 + c_2 n + c_3 n + c_4 n + c_5 =$
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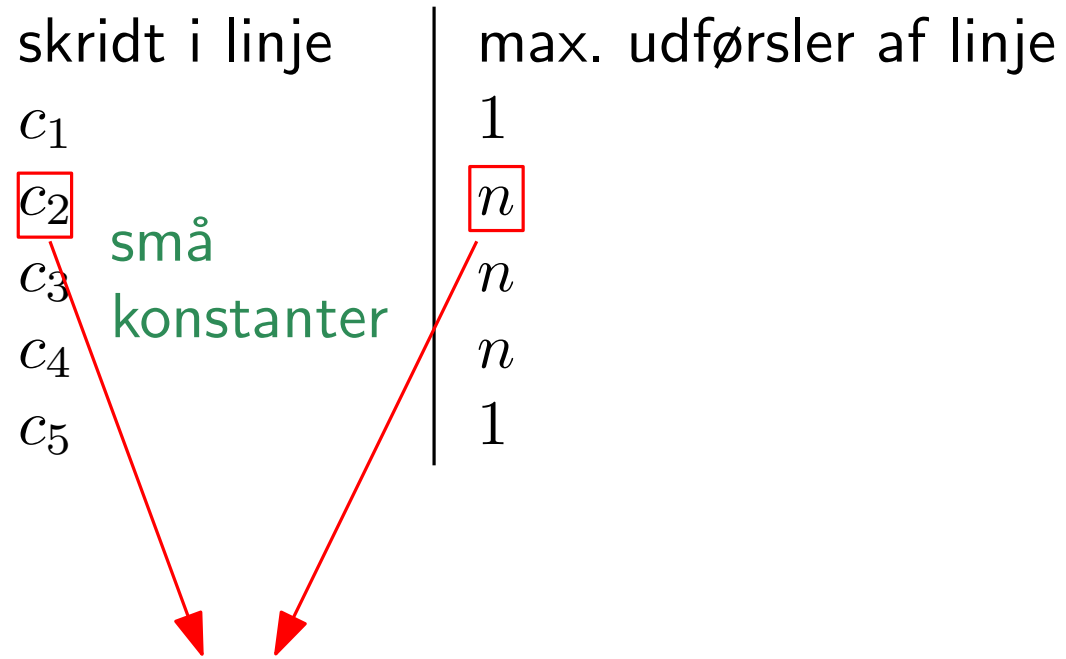


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Asymptotisk notation: Køretiden er lineær. Langsomt voksende led og konstanter ignoreres.

# Eksempel

```
doubleLoop(n)
    x = 0
    for i = 0 to n − 1
        x = x + 1
        for j = 0 to n − 1
            x = x + 1
    return x
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| $c_3$ | $n$ |
| $c_4$ | $n^2$ |
| $c_5$ | $n^2$ |
| $c_6$ | $1$ |

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|---|---|---|
| $x = 0$ | $c_1$ | 1 |
| for $i = 0$ to $n - 1$ | $c_2$ | $n$ |
| $x = x + 1$ | $c_3$ | $n$ |
| for $j = 0$ to $n - 1$ | $c_4$ | $n$ |
| $x = x + 1$ | $c_5$ | $n$ |
| return $x$ | $c_6$ | 1 |

$T(n) = c_1 + c_2n + c_3n + c_4n^2 + c_5n^2 + c_6 =$
$(c_2 + c_3 + c_4 + c_5)n + c_1 + c_6 = \Theta(n)$

# Quiz

```
Q(A, n) \\antag log_2 n er et heltal
    s = 0
    for i from 1 to n
        for j from 1 to log_2 n
            s = s + A[i] + A[j]
    return s
```

# Quiz

```
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    for i from 1 to n
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            s = s + A[i] + A[j]
    return s
```

Hvad er køretiden af Q$(A, n)$?

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(a) $\Theta(n)$

(b) $\Theta(n^2)$

(c) $\Theta(n + \log_2 n)$

(d) $\Theta(n \cdot \log_2 n)$

(e) $\Theta(n\sqrt{n})$