

**DMA WEEK 3**  
**EXAMPLE FROM TUESDAY'S LECTURE**

In the following, LN(3) stands for lecture notes of week 3.

Consider the following two functions

$$f(n) := 2^n - n^{100}$$

$$g(n) := n^5.$$

Let us use the rules of Theorem 11 in LN(3) to determine which of these grows faster. We break the problem down in the following steps:

- We begin by determining the growth rate of  $f(n)$ . Note that

$$f(n) = 1 \cdot 2^n + (-1) \cdot n^{100}.$$

By (R6), we know that  $n^{100}$  is  $o(2^n)$ . By (R3), this implies that  $f(n)$  is  $\Theta(2^n)$ , i. e. that

$$(*) \quad f \text{ grows at the same rate as } 2^n.$$

- Next, we use (R2) to conclude that  $g$  is  $\Theta(n^5)$  i. e. that

$$(**) \quad g \text{ grows at the same rate as } n^5.$$

- Finally, we combine (\*), (\*\*) and (R6) to conclude that  $g$  is  $o(f)$ , i. e.

$$\underline{f \text{ grows faster than } g}.$$