

Velkommen til DMA 2022!

Plan for forelæsningen i dag:

- Plan for ugen.
- Introduktion af MA.
- Pseudokode: hvad og hvorfor?

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Plan for forelæsningen i dag:

- Plan for ugen.
- Introduktion af MA.
- Pseudokode: hvad og hvorfor?

Plan for resten af dagen:

- Kl. 10.15–12: Øvelsestimer
- Derefter: Selvstudium.

Plan for ugen

- Mandag: Pseudokode.
- Tirsdag: Formelle rammer for DMA, motivation og baggrund for algoritmik, arrays, RAM-modellen, køretidsanalyse.
- Fredag: Rekursive algoritmer, mere køretidsanalyse.

Kort introduktion

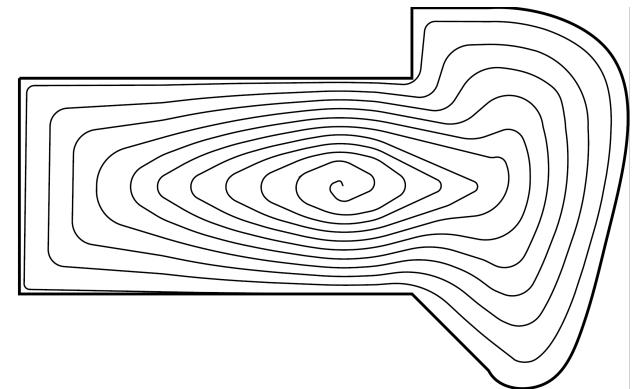
Mikkel Abrahamsen, lektor, DIKU

- BSc i mat.
- MSc og PhD på DIKU

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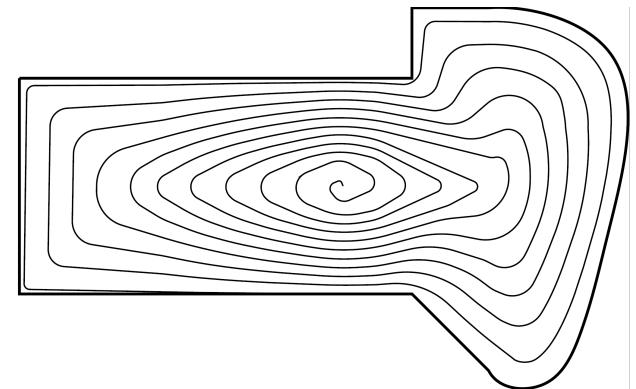
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3shape ▶



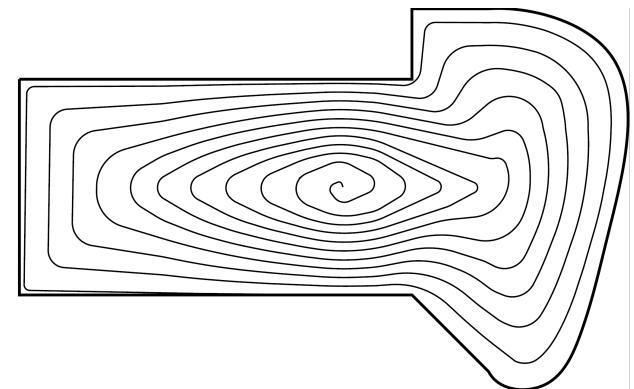
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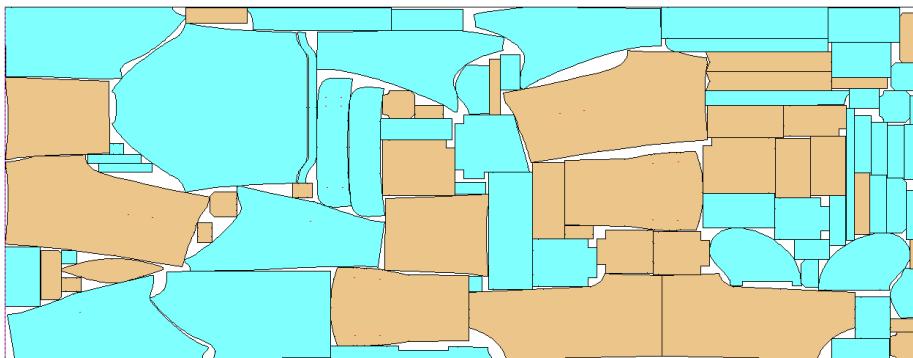
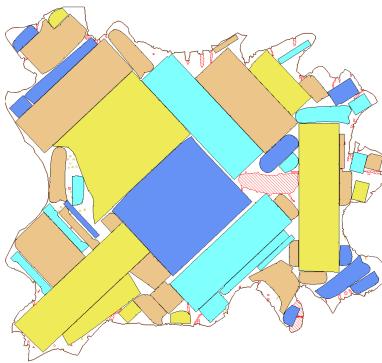
3shape ▶



- Forskning: algoritmisk geometri
- Anvendelser inden for computergrafik, computer-aided design/manufacturing, geografiske informationssystemer, mv.



Pakningsproblemer: Eksempel på algoritmisk geometri



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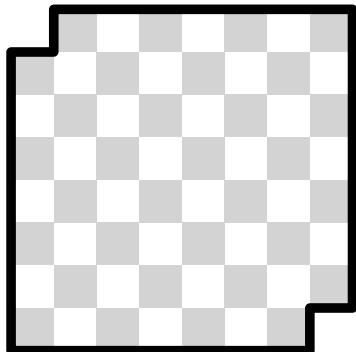
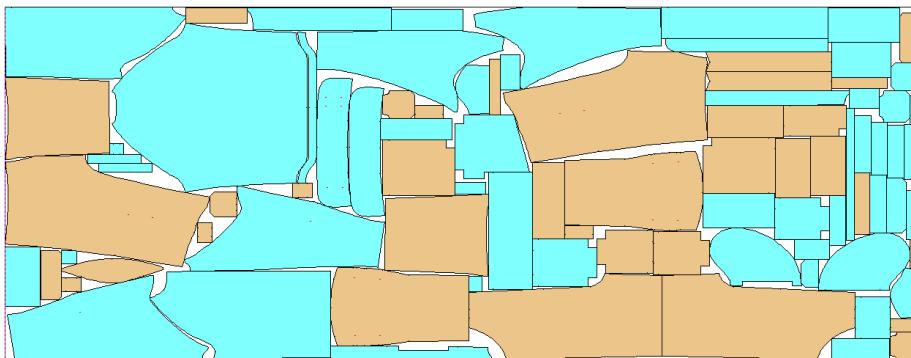
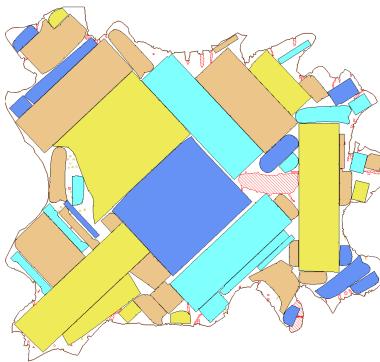
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Mathematicians Scrutinize the Challenge of Efficient Christmas Cookie Cutting

Optimally cutting lots of complex shapes from a sheet of dough is even harder than many researchers suspected.



Pakningsproblemer: Eksempel på algoritmisk geometri



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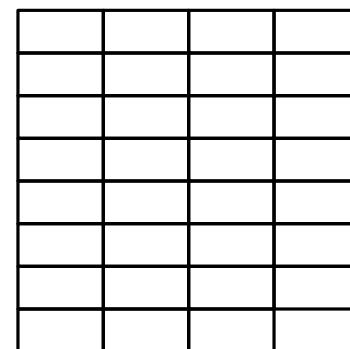
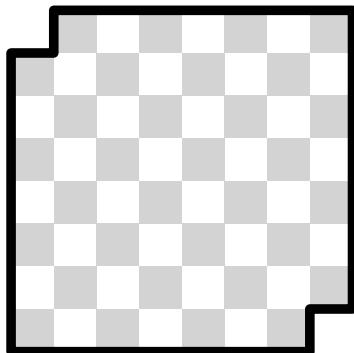
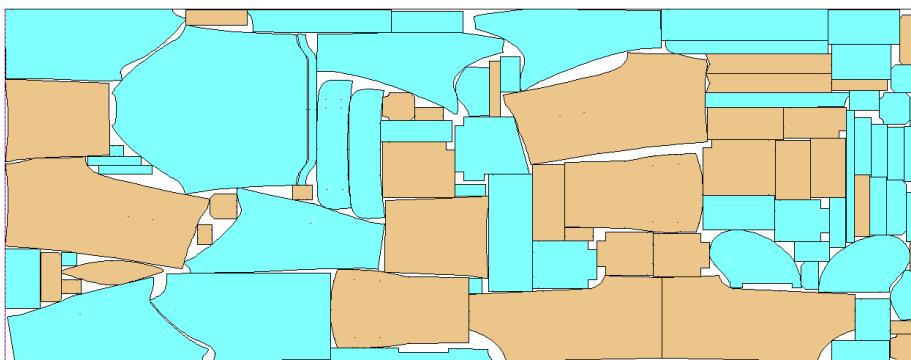
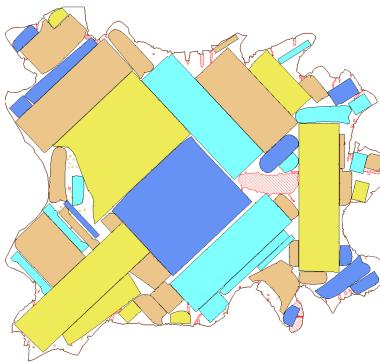
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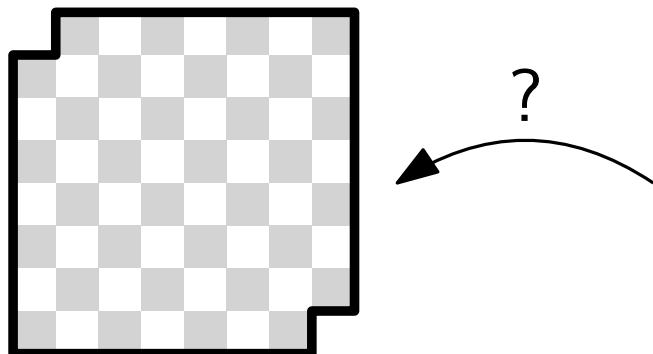
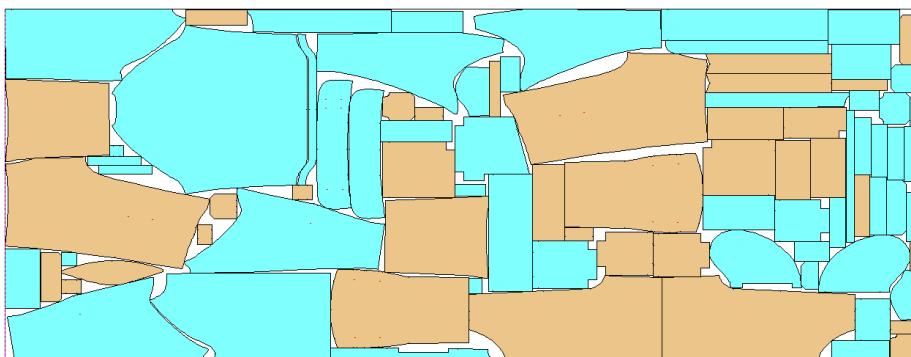
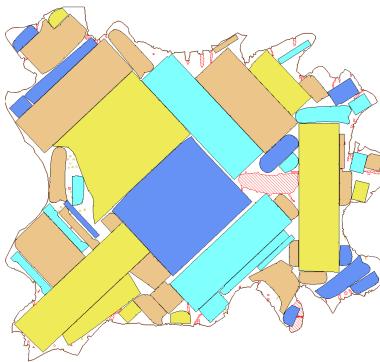
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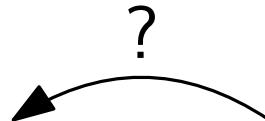
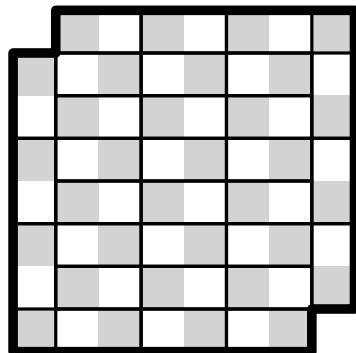
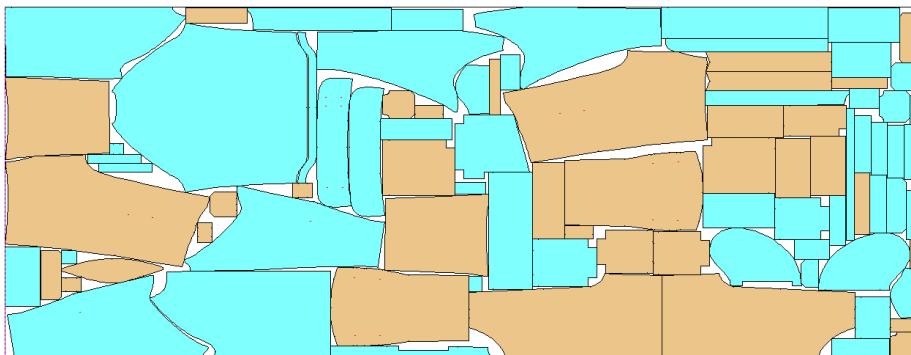
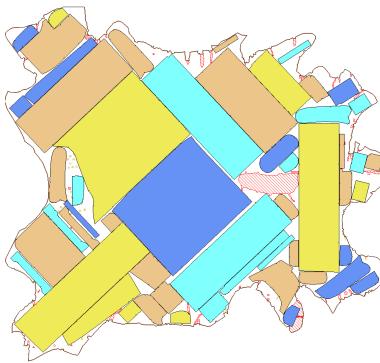
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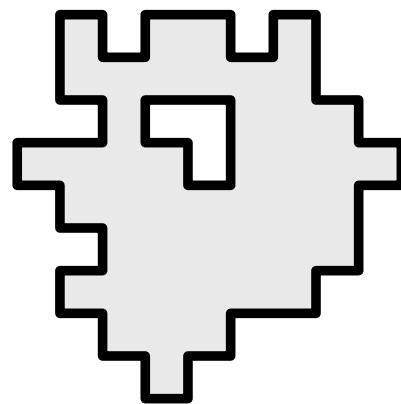
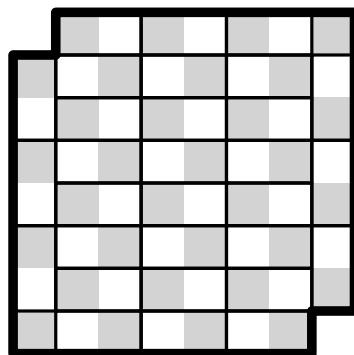
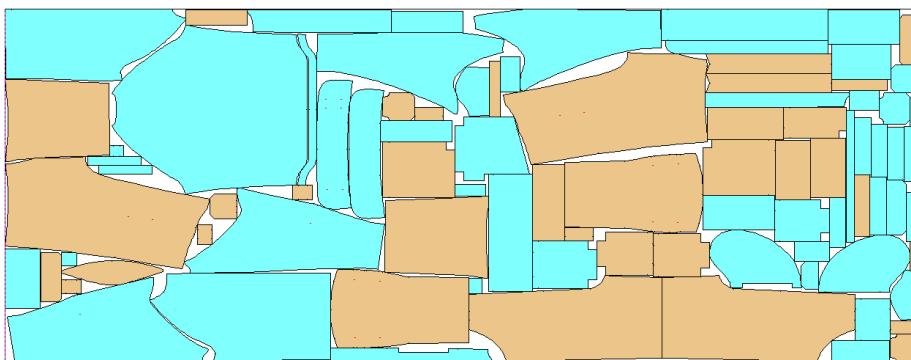
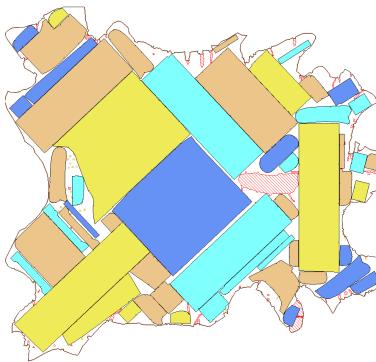
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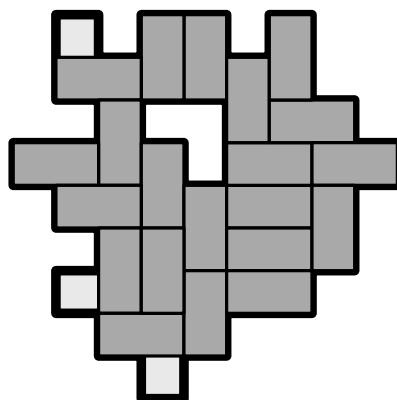
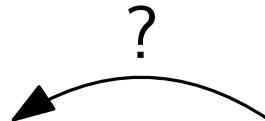
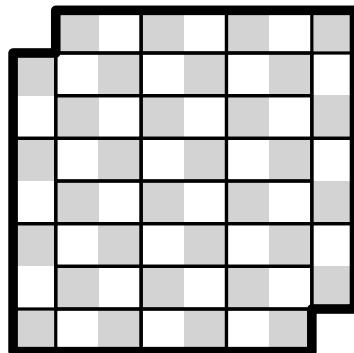
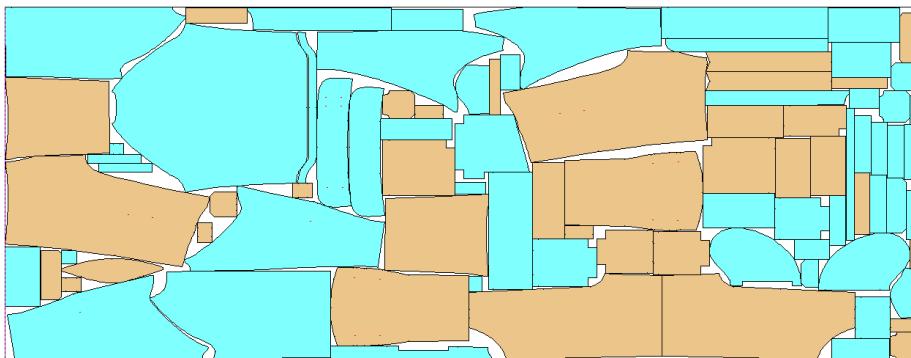
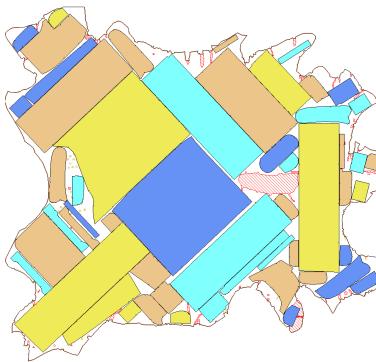
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Pseudokode: hvad og hvorfor?

- Overskuelig og præcis beskrivelse af algoritmer.
- Skrevet til mennesker, ikke til computere.
- Ligner rigtig kode, men uden besværet.
- Formål: at beskrive og formidle algoritmer mest effektivt.

Eksempler på simpel pseudokode

```
Example1( $n$ )
```

```
     $a = 5$ 
```

```
     $b = a + n$ 
```

```
    return  $b$ 
```

Eksempler på simpel pseudokode

```
Example1( $n$ )
```

```
     $a = 5$ 
```

```
     $b = a + n$ 
```

```
    return  $b$ 
```

Kald Example1(3).

Eksempler på simpel pseudokode

```
Example1( $n$ )
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     $a = 5$ 
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     $b = a + n$ 
```

```
    return  $b$ 
```

Kald Example1(3).

Kald Example1(7).

Eksempler på simpel pseudokode

Example1(n)

$a = 5$

$b = a + n$

return b

Kald Example1(3).

Kald Example1(7).

Example2()

$x = 7$

$x = x + 1$

$y = x + 2$

return y

Eksempler på simpel pseudokode

Example1(n)

$a = 5$

$b = a + n$

return b

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Example2()

$x = 7$

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Eksempler på simpel pseudokode

Example1(n)

$a = 5$

$b = a + n$

return b

Kald Example1(3).

Kald Example1(7).

Example2()

$x = 7$

$x = x + 1$

$y = x + 2$

return y

Kald Example2().

Example3(n)

$a = \text{Example1}(2n)$

$b = \text{Example2}()$

return $a + b$

Kald Example3(4).

Eksempler på simpel pseudokode

Example1(n)

$a = 5$

$b = a + n$

return b

Kald Example1(3).

Kald Example1(7).

Example3(n)

$a = \text{Example1}(2n)$

$b = \text{Example2}()$

return $a + b$

Kald Example3(4).

Example2()

$x = 7$

$x = x + 1$

$y = x + 2$

return y

Kald Example2().

Example3a(n)

return Example1(2 n)+Example2()

Eksempler (if-else)

Example1(n)

$a = 5$

if $n \geq 4$

 return a

else

 return n

Eksempler (if-else)

Example1(n)

```
a = 5
if n ≥ 4
    return a
else
    return n
```

Kald Example1(3).

Eksempler (if-else)

```
Example1( $n$ )
```

```
     $a = 5$ 
```

```
    if  $n \geq 4$ 
```

```
        return  $a$ 
```

```
    else
```

```
        return  $n$ 
```

Kald Example1(3).

Kald Example1(7).

Eksempler (if-else)

Example1(n)

```
a = 5  
if n ≥ 4  
    return a  
else  
    return n
```

Kald Example1(3).

Kald Example1(7).

Example2(n)

```
x = 7  
x = x + 1  
if n == 3 or n is even  
    x = x + 1  
    n = n + 1  
x = x + n  
return x
```

Eksempler (if-else)

Example1(n)

```
a = 5  
if n ≥ 4  
    return a  
else  
    return n
```

Kald Example1(3).

Kald Example1(7).

Example2(n)

```
x = 7  
x = x + 1  
if n == 3 or n is even  
    x = x + 1  
    n = n + 1  
x = x + n  
return x
```

Kald Example2(5).

Eksempler (if-else)

Example1(n)

```
a = 5  
if n ≥ 4  
    return a  
else  
    return n
```

Kald Example1(3).

Kald Example1(7).

Example2(n)

```
x = 7  
x = x + 1  
if n == 3 or n is even  
    x = x + 1  
    n = n + 1  
x = x + n  
return x
```

Kald Example2(5).

Kald Example2(6).

Quiz

```
Quiz( $n$ )
```

```
     $a = 5$ 
```

```
    if  $2n \leq n^2$ 
```

```
        return  $a \cdot n^2$ 
```

```
    else
```

```
        return  $a^2/n$ 
```

Quiz

Quiz(n)

$a = 5$

if $2n \leq n^2$

 return $a \cdot n^2$

else

 return a^2/n

Hvad returneres når vi kalder Quiz(3)?

(a) $25/3$

(b) 5

(c) 75

(d) 45

(e) $9/5$

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Løkker

```
Loop1( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    print 15
```

Løkker

```
Loop1( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    print 15
```

Kald Loop1(5)

Løkker

```
Loop1( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    print 15
```

Kald Loop1(5)

Output: 1 2 3 4 5 15

Løkker

```
Loop1( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    print 15
```

Kald Loop1(5)

Output: 1 2 3 4 5 15

```
Loop2( $n$ )
```

```
     $i = 1$ 
```

```
    while  $i \leq n$ 
```

```
        print  $i$ 
```

```
         $i = i + 1$ 
```

```
    print 15
```

Løkker

```
Loop1( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    print 15
```

Kald Loop1(5)

Output: 1 2 3 4 5 15

```
Loop2( $n$ )
```

```
     $i = 1$ 
```

```
    while  $i \leq n$ 
```

```
        print  $i$ 
```

```
         $i = i + 1$ 
```

```
    print 15
```

Kald Loop2(5)

Output: 1 2 3 4 5 15

Løkker

Loop1(n)

for i from 1 to n

 print i

 print 15

Kald Loop1(5)

Output: 1 2 3 4 5 15

Loop2(n)

$i = 1$

while $i \leq n$

 print i

$i = i + 1$

 print 15

Kald Loop2(5)

Output: 1 2 3 4 5 15

1: **Loop2**(n):

2: $i \leftarrow 1$;

3: **while** ($i \leq n$):

4: **print**(i);

5: $i \leftarrow i + 1$;

6: **print**(15);

Løkker

Loop1(n)

for i from 1 to n

 print i

 print 15

Kald Loop1(5)

Output: 1 2 3 4 5 15

PrimeSum(n)

$s = 0$

for i from 1 to n

 if i is a prime number

$s = s + i$

return(s)

Loop2(n)

$i = 1$

while $i \leq n$

 print i

$i = i + 1$

 print 15

Kald Loop2(5)

Output: 1 2 3 4 5 15

1: **Loop2(n):**

2: $i \leftarrow 1;$

3: **while** ($i \leq n$):

4: **print**(i);

5: $i \leftarrow i + 1;$

6: **print**(15);

Løkker

```
Loop1( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    print 15
```

Kald Loop1(5)

Output: 1 2 3 4 5 15

Pas på! Hvordan
udregnes det?

```
PrimeSum( $n$ )
```

```
     $s = 0$ 
```

```
    for  $i$  from 1 to  $n$ 
```

```
        if  $i$  is a prime number
```

```
             $s = s + i$ 
```

```
    return( $s$ )
```

```
Loop2( $n$ )
```

```
     $i = 1$ 
```

```
    while  $i \leq n$ 
```

```
        print  $i$ 
```

```
         $i = i + 1$ 
```

```
    print 15
```

Kald Loop2(5)

Output: 1 2 3 4 5 15

```
1: Loop2( $n$ ):
```

```
2:    $i \leftarrow 1;$ 
```

```
3:   while ( $i \leq n$ ):
```

```
4:     print( $i$ );
```

```
5:      $i \leftarrow i + 1;$ 
```

```
6:   print(15);
```

Quiz

```
Loop4( $n$ )
```

```
     $s = 0$ 
```

```
    for  $i$  from 1 to  $n$ 
```

```
         $s = s + i$ 
```

```
    print  $s$ 
```

Quiz

```
Loop4( $n$ )
```

```
     $s = 0$ 
```

```
    for  $i$  from 1 to  $n$ 
```

```
         $s = s + i$ 
```

```
    print  $s$ 
```

Hvad udskriver funktionen når man kalder Loop4(5)?

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- (a) 5 10 15 20 25
- (b) 1 3 6 10 15
- (c) 1 2 3 4 5
- (d) 0 1 3 6 10
- (e) 1 1 2 3 5

Pas på!

```
L(n)
  i = 5
  while i ≠ n
    print i
    i = i + 1
```

Hvad sker der hvis vi kalder L(10)?

Pas på!

```
L(n)
  i = 5
  while i ≠ n
    print i
    i = i + 1
```

Hvad sker der hvis vi kalder L(10)?

5 6 7 8 9

Pas på!

```
L(n)
  i = 5
  while i ≠ n
    print i
    i = i + 1
```

Hvad sker der hvis vi kalder L(10)?

5 6 7 8 9

Hvad sker der hvis vi kalder L(3)?

Pas på!

```
L(n)
  i = 5
  while i ≠ n
    print i
    i = i + 1
```

Hvad sker der hvis vi kalder L(10)?

5 6 7 8 9

Hvad sker der hvis vi kalder L(3)?

5 6 7 8 9 10 11 12 ...

Uendelig løkke

Indrykning er vigtig

```
Loop1( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    for  $j$  from 1 to  $n$ 
```

```
        print  $j$ 
```

```
Loop2( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    for  $j$  from 1 to  $n$ 
```

```
        print  $j$ 
```

Indrykning er vigtig

```
Loop1( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    for  $j$  from 1 to  $n$ 
```

```
        print  $j$ 
```

```
Loop2( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    for  $j$  from 1 to  $n$ 
```

```
        print  $j$ 
```

Kald Loop1(3).

Indrykning er vigtig

```
Loop1( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    for  $j$  from 1 to  $n$ 
```

```
        print  $j$ 
```

```
Loop2( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    for  $j$  from 1 to  $n$ 
```

```
        print  $j$ 
```

Kald Loop1(3).

1, 2, 3, 1, 2, 3

Indrykning er vigtig

```
Loop1( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    for  $j$  from 1 to  $n$ 
```

```
        print  $j$ 
```

```
Loop2( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    for  $j$  from 1 to  $n$ 
```

```
        print  $j$ 
```

Kald Loop1(3).

1, 2, 3, 1, 2, 3

print(i) print(j)

Indrykning er vigtig

```
Loop1( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    for  $j$  from 1 to  $n$ 
```

```
        print  $j$ 
```

Kald Loop1(3).

1, 2, 3, 1, 2, 3

print(i) print(j)

```
Loop2( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    for  $j$  from 1 to  $n$ 
```

```
        print  $j$ 
```

Kald Loop1(3).

Indrykning er vigtig

```
Loop1( $n$ )
```

```
    for  $i$  from 1 to  $n$ 
```

```
        print  $i$ 
```

```
    for  $j$  from 1 to  $n$ 
```

```
        print  $j$ 
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Kald Loop1(3).

1, 1, 2, 3,

2, 1, 2, 3,

3, 1, 2, 3

Indrykning er vigtig

```
Loop1( $n$ )
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```
    for  $i$  from 1 to  $n$ 
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        print  $i$ 
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    for  $j$  from 1 to  $n$ 
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    for  $j$  from 1 to  $n$ 
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```
        print  $j$ 
```

Kald Loop1(3).

1, 1, 2, 3,

2, 1, 2, 3,

3, 1, 2, 3

print(i) print(j)

Quiz

Loop1(n)

$x = 0$

for i from 1 to n

$x = x + 1$

for j from 1 to n

$x = x + 1$

return x

Loop2(n)

$x = 0$

for i from 1 to n

$x = x + 1$

for j from 1 to n

$x = x + 1$

return x

Quiz

```
Loop1( $n$ )
```

```
     $x = 0$ 
```

```
    for  $i$  from 1 to  $n$ 
```

```
         $x = x + 1$ 
```

```
        for  $j$  from 1 to  $n$ 
```

```
             $x = x + 1$ 
```

```
    return  $x$ 
```

```
Loop2( $n$ )
```

```
     $x = 0$ 
```

```
    for  $i$  from 1 to  $n$ 
```

```
         $x = x + 1$ 
```

```
        for  $j$  from 1 to  $n$ 
```

```
             $x = x + 1$ 
```

```
    return  $x$ 
```

Hvad returneres når vi kalder Loop1(5) og Loop2(5)?

socrative.com → Student login.

Room name: ABRAHAMSEN3464

- (a) Loop1: 10, Loop2: 25
- (b) Loop1: 30, Loop2: 10
- (c) Loop1: 5, Loop2: 30
- (d) Loop1: 5, Loop2: 25
- (e) Loop1: 10, Loop2: 30