Problem 2

Setup $A\bar{v} = \bar{b}$, $v_{i+(j-1)m} = u_{i,j}$, $mn \times mn$ matrix

Inner points: $\frac{u_{i+1,j} - u_{i-1,j}}{h^2} + \frac{u_{i,j+1} + u_{i,j-1}}{k^2} - \left(\frac{2}{h^2} + \frac{2}{k^2} + \frac{2H}{K\delta}\right) u_{i,j} = 0$

$$\frac{v_{i+1+(j-1)m} - v_{i-1+(j-1)m}}{h^2} + \frac{v_{i+jm} + v_{i+(j-2)m}}{k^2} - \left(\frac{2}{h^2} + \frac{2}{k^2} + \frac{2H}{K\delta}\right) v_{i+(j-1)m} = 0$$

$$-\left(\frac{2}{h^2} + \frac{2}{k^2} + \frac{2H}{K\delta}\right) v_{i+(j-1)m} + \frac{1}{h^2} v_{i+1+(j-1)m} - \frac{1}{h^2} v_{i-1+(j-1)m} + \frac{1}{k^2} v_{i+jm} + \frac{1}{k^2} v_{i+(j-2)m} = 0$$

eq_1 : $A(t,t) = -\left(\frac{2}{h^2} + \frac{2}{k^2} + \frac{2H}{K\delta}\right)$
eq_2 : $A(t,t+1) = \frac{1}{h^2}$
eq_3 : $A(t,t-1) = -\frac{1}{h^2}$
eq_4 : $A(t,t+m) = \frac{1}{k^2}$
eq_5 : $A(t,t-m) = \frac{1}{k^2}$
$b(t) = 0$

$\boxed{t = i + (j-1)m}$

for $2 \le i \le m-1$
$2 \le j \le n-1$

Bottom: $0 = \left(\frac{2kH}{K} - 3\right) u_{i,1} + 4u_{i,2} - u_{i,3}$

$0 = \left(\frac{2kH}{K} - 3\right) v_i + 4v_{i+m} - v_{i+2m}$

$\boxed{t = i}$

for $j=1$, $2 \le i \le m-1$

eq_1 : $A(t,t) = \frac{2kH}{K} - 3$
eq_2 : $A(t,t+m) = 4$
eq_3 : $A(t,t+2m) = -1$
$b(t) = 0$

Top: $0 = \left(\frac{2kH}{K} - 3\right) u_{i,n} + 4u_{i,n-1} - u_{i,n-2}$

$0 = \left(\frac{2kH}{K} - 3\right) v_{i+(n-1)m} + 4v_{i+(n-2)m} - v_{i+(n-3)m}$

$\boxed{t = i + (n-1)m}$

for $j=n$ $2 \le i \le m-1$

eq_1 : $A(t,t) = \frac{2kH}{K}$
eq_2 : $A(t,t-m) = 4$
eq_3 : $A(t,t-2m) = -1$
$b(t) = 0$

Left: $0 = \left(\frac{2hH}{K} - 3\right) u_{1,j} + 4u_{2,j} - u_{3,j}$

$0 = \left(\frac{2hH}{K} - 3\right) V_{1+(j-1)m} + 4V_{2+(j-1)m} - V_{3+(j-1)m}$

$eq_1: A(t,t) = \frac{2hH}{K} - 3$
$eq_2: A(t,t+1) = 4$
$eq_3: A(t,t+2) = -1$
$b(t) = 0$

$\boxed{t = 1 + (j-1)m}$

for $i = 1$, $L < j \leq n$

Right: $0 = \left(\frac{2hH}{K} - 3\right) u_{m,j} + 4u_{m-1,j} - u_{m-2,j}$

$0 = \left(\frac{2hH}{K} - 3\right) V_{jm} + 4V_{-1+jm} - V_{-2+jm}$

$eq_1: A(t,t) = \frac{2hH}{K} - 3$
$eq_2: A(t,t-1) = 4$
$eq_3: A(t,t-2) = -1$
$b(t) = 0$

$\boxed{t = m + (j-1)m = jm}$

for $i = m$, $1 \leq j \leq n$

Heat left: $-\frac{2hP}{LSK} = -3u_{1,j} + 4u_{2,j} - u_{3,j}$

$-\frac{2hP}{LSK} = -3V_{1+(j-1)m} + 4V_{2+(j-1)m} - V_{3+(j-1)m}$

$eq_1: A(t,t) = -3$
$eq_2: A(t,t+1) = 4$
$eq_3: A(t,t+2) = -1$
$b(t) = -\frac{2hP}{LSK}$

$\boxed{t = 1 + (j-1)m}$

for $i = 1$, $1 \leq j \leq L$