

Problem 2

Setup $A\bar{u} = \bar{b}$, $V_{i+(j-1)m} = u_{i,j}$, $mn \times mn$ matrix

Inner points: $\frac{u_{i+1,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} + u_{i,j-1}}{k^2} - \left(\frac{2}{h^2} + \frac{2}{k^2} + \frac{2H}{K\delta}\right) u_{i,j} = 0$

$$\frac{V_{i+1+(j-1)m} + V_{i-1+(j-1)m}}{h^2} + \frac{V_{i+jm} + V_{i+(j-2)m}}{k^2} - \left(\frac{2}{h^2} + \frac{2}{k^2} + \frac{2H}{K\delta}\right) V_{i+(j-1)m} = 0$$

$$- \left(\frac{2}{h^2} + \frac{2}{k^2} + \frac{2H}{K\delta}\right) V_{i+(j-1)m} + \frac{1}{h^2} V_{i+1+(j-1)m} + \frac{1}{h^2} V_{i-1+(j-1)m} + \frac{1}{k^2} V_{i+jm} + \frac{1}{k^2} V_{i+(j-2)m} = 0$$

$$eq_1: A(t, t) = - \left(\frac{2}{h^2} + \frac{2}{k^2} + \frac{2H}{K\delta}\right)$$

$$eq_2: A(t, t+1) = \frac{1}{h^2}$$

$$eq_3: A(t, t-1) = \frac{1}{h^2}$$

$$eq_4: A(t, t+m) = \frac{1}{k^2}$$

$$eq_5: A(t, t-m) = \frac{1}{k^2}$$

$$b(t) = 0$$

$$t = i + (j-1)m$$

$$\text{for } \begin{matrix} 2 \leq i \leq m-1 \\ 2 \leq j \leq n-1 \end{matrix}$$

Bottom: $0 = \left(\frac{2kH}{K} - 3\right) u_{i,1} + 4u_{i,2} - u_{i,3}$

$$0 = \left(\frac{2kH}{K} - 3\right) V_i + 4V_{i+m} - V_{i+2m}$$

$$t = i$$

$$\text{for } j=1, 2 \leq i \leq m-1$$

$$eq_1: A(t, t) = \frac{2kH}{K} - 3$$

$$eq_2: A(t, t+m) = 4$$

$$eq_3: A(t, t+2m) = -1$$

$$b(t) = 0$$

Top: $0 = \left(\frac{2kH}{K} - 3\right) u_{i,n} + 4u_{i,n-1} - u_{i,n-2}$

$$0 = \left(\frac{2kH}{K} - 3\right) V_{i+(n-1)m} + 4V_{i+(n-2)m} - V_{i+(n-3)m}$$

$$t = i + (n-1)m$$

$$eq_1: A(t, t) = \frac{2kH}{K}$$

$$eq_2: A(t, t-m) = 4$$

$$eq_3: A(t, t-2m) = -1$$

$$b(t) = 0$$

$$\text{for } j=n, 2 \leq i \leq m-1$$

Left: $0 = \left(\frac{2hH}{K} - 3\right) u_{1,j} + 4u_{2,j} - u_{3,j}$

$$0 = \left(\frac{2hH}{K} - 3\right) v_{1+(j-1)m} + 4v_{2+(j-1)m} - v_{3+(j-1)m}$$

$$\text{eq}_1: A(t, t) = \frac{2hH}{K} - 3$$

$$\text{eq}_2: A(t, t+1) = 4$$

$$\text{eq}_3: A(t, t+2) = -1$$

$$b(t) = 0$$

$$t = 1 + (j-1)m$$

for $i=1$, $L < j \leq n$

Right: $0 = \left(\frac{2hH}{K} - 3\right) u_{m,j} + 4u_{m-1,j} - u_{m-2,j}$

$$0 = \left(\frac{2hH}{K} - 3\right) v_{jm} + 4v_{j-1,m} - v_{j-2,m}$$

$$\text{eq}_1: A(t, t) = \frac{2hH}{K} - 3$$

$$\text{eq}_2: A(t, t-1) = 4$$

$$\text{eq}_3: A(t, t-2) = -1$$

$$b(t) = 0$$

$$t = m + (j-1)m = jm$$

for $i=m$, $1 \leq j \leq n$

Heat left: $-\frac{2hP}{L\delta K} = -3u_{1,j} + 4u_{2,j} - u_{3,j}$

$$-\frac{2hP}{L\delta K} = -3v_{1+(j-1)m} + 4v_{2+(j-1)m} - v_{3+(j-1)m}$$

$$\text{eq}_1: A(t, t) = -3$$

$$\text{eq}_2: A(t, t+1) = 4$$

$$\text{eq}_3: A(t, t+2) = -1$$

$$b(t) = -\frac{2hP}{L\delta K}$$

$$t = 1 + (j-1)m$$

for $i=1$, $1 \leq j \leq L$