Einstaklingsverkefni 2 (e. Individual Assignment 2)

T-117-STR1, Strjál stærðfræði I, 2024-3

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Skilafrestur (e. Deadline): 10.09.2024

Hér er Einstaklingsverkefni 2. Skilafrestur er þriðjudaginn 10.september 2024 kl. 23:59*. Þetta eru ein af 5 einstaklingsskilum. Þau gilda alls 20% af lokaeinkunn, en lægstu einkunn er sleppt.

Mjög mikilvægt er að nemendur noti þetta skjal, fylli inn sínar lausnir á viðeigandi staði og skili útfylltu skjali á Gradescope sem pdf. Bæði er leyfilegt að prenta út skjalið, fylla inn handvirkt og skanna það svo aftur inn (**eða** nota þetta LaTeX sniðmát og fylla inn í það). **Ekki verður farið yfir verkefni sem ekki nota þetta skjal (eða LaTeX sniðmátið), og fyrir slík verkefni fæst 0 í einkunn.**

*nemendur á Austurlandi skila miðvikudaginn 11.september 2023 kl.23:59 og skilafrestur þeirra í Canvas/Gradescope er stilltur miðað við það

English version:

("This is the second individual assignment. The deadline is Tuesday, September 10th, 2024, at 23:59*. Students hand in solutions on pdf on Gradescope. This is one of 5 individual assignments. All in all, their weight is 20% of the final grade, but the lowest grade is dropped.")

Students must use this document, fill in their solutions in the designated spaces, and return the completed document to Gradescope as a pdf. You are allowed to print the document, fill it in writing, and scan it, (**or** use this LaTeX template and fill it in). **Assignments solutions that do not use this document (or the LaTeX template) will not be reviewed and will receive a grade of 0.**

*Students in the east of Iceland hand in on Wednesday, September 11th, 2024, at 23:59, and their deadline is set in Canvas/Gradescope accordingly

Skiladæmi (e. Hand-in problems) :

Dæmi 1 (e. Problem 1) (20%)

Látum n vera heila tölu. Sannið eftirfarandi reglu með **beinni sönnun**.
("Let n be an integer. Give a **direct proof** of the following rule.")

Regla: Ef n er oddatala, þá er $3n$ oddatala.
("Rule: If n is odd, then $3n$ is odd.")

Svar við Dæmi 1 (e. Answer to Problem 1)

We will use the definition of an odd integer as an 2k+1, where k is an integer. We will now define n as an odd integer and insert it into the formula 3n to get 2l + 1 where l is an integer

3n =
$3\times(2k+1) = 2l+1$
$= 6k+3 = 2l+1$
$= 6k+2+1 = 2l+1$
$= 2(3k+1)+1 = 2l+1$

If we mark 3k + 1 as l we can then insert that into the formula for an odd integer. We can see that the integer is not a multiplication of 2, therefor the rule if n is odd then 3n is odd is true.

l = 3k + 1
2(3k+1) + 1 = 2l + 1

Dæmi 2 (e. Problem 2) (20%)

Notið **sönnun með mótskilyrðingu** til að sanna eftirfarandi regluna.
("Use **proof by contraposition** to prove the following rule.")

> Regla: Ef n er heiltala og $5n-1$ er slétt tala þá er n oddatala.
> ("Rule: If n is an integer and $5n-1$ is even, then n is odd.")

Svar við Dæmi 2 (e. Answer to Problem 2)

Let's assume that the conclusion of the statement is false, "If n is an integer, and 5n-1 is even then n is odd", consequently will be "if n is even then 5n-1 is odd" Then we use 2k for the defenition of an even number and insert it into the formula instead of n.

$5\times(2k)-1$
$=10k-1$
$=2\times(5k)-1$

Since we know that the multiplication of 2 integers is even and adding or subtracting 1 will make it odd, we have proven that if n is even then 5n-1 is odd.
This is a negation of the premise of the theorem. Because the negation of the conclusion of the conditional statement implies that the hypothesis is false, the original conditional statement is true. We have proved that if 5n-1 is even then n is odd.

Dæmi 3 (e. Problem 3) (16%)

Sannið seinni gleypiregluna (e.absorption law) í töflu 1 í kafla 2.2 í bókinni, þ.e. sýnið að ef A og B eru mengi, þá gildir að $A \cap (A \cup B) = A$. Notið mengja rithátt (e. set builder notation), skilgreiningar á snið og sammengjum$*$ og rökfræði jafngildi (töflu 6 í kafla 1) til verksins.
("Prove the second absorption law from Table 1 in chapter 2.2, i.e. by showing that if A and B are sets, then $A \cap (A \cup B) = A$. Use set builder notation, definitions of union and intersection$*$ and logical equivalences (table 6, chapter 1).

Hint: Í fyrirlestri 3.1 þá var sannað með mengja rithátt (e. set builder notation), skilgreiningu á fyllimengi og rökfræði jafngildi að $\overline{(\overline{A})} = A$.
("**Hint:** In lecture 3.1 it was proved with set builder notation, definition of complement law and logical equivalences that $\overline{(\overline{A})} = A$.")

$*$Skilgreiningar á sammengi og sniðmengi ("Definitions of union and intersection"):

$$A \cup B = \{x | x \in A \vee x \in B\}$$

$$A \cap B = \{x | x \in A \wedge x \in B\}$$

Svar við Dæmi 3 (e. Answer to Problem 3)

$A \cap (A \cup B) = \{x | x \in A \wedge (x \in A \vee x \in B)\}$ By definition of union and intersection
$= (A \cap A) \cup (A \cap B) = \{x | (x \in A \wedge x \in A) \vee (x \in A \wedge x \in B)\}$ By Distributive Law
$= A \cup (A \cap B) = \{x | x \in A \vee (x \in A \wedge x \in B)\}$ By Idempotent Law
$= (A \cap U) \cup (A \cap B) = \{x | (x \in A \wedge T) \vee (x \in A \wedge x \in B)\}$ By Identity Law
$= A \cap (U \cup B) = \{x | x \in A \wedge (T \vee x \in B)\}$ By Distributive Law
$= A \cap (B \cup U) = \{x | x \in A \wedge (x \in B \vee T)\}$ By Commutative Law
$= A \cap U = \{x | x \in A \wedge T\}$ By Domination Law
$= A = \{x \in A\}$ By Identity Law

We can therfore conclude that $A \cap (A \cup B) =$ A and have proven the Absorption Law

Dæmi 4 (e. Problem 4) (14%+10%)

Könnun meðal 500 sjónvarpsáhorfenda gefur eftirfarandi upplýsingar. Af þeim horfa 285 á fótbolta, 195 á handbolta og 115 á körfubolta. Það horfa 70 á fótbolta og handbolta, 45 á fótbolta og körfubolta og 42 á handbolta og körfubolta. Það eru 50 sem horfa ekki á neina af þessum íþróttagreinum.
("A survey of 500 television viewers provides the following information. Of them 285 watch soccer, 195 watch handball and 115 watch basketball. Also, 70 watch soccer and handball, 45 watch soccer and basketball and 42 watch in handball and basketball. There are 50 viewers who do not watch any of these sports.")

a) Hve margir þátttakendur í könnuninni horfa á allar íþróttagreinarnar? ("ow manu watch all three sports?")

b) Hversu margir horfa á fótbolta en hvorki handbolta né körfubolta? ("How many watch soccer but neither handball nor basketball?")

Sýnið alla útreikninga í svarkassanum, og setjið að auki lokasvörin á línurnar fyrir neðan. ("Show all calculations in the answer box, and additionally put your final answers on the lines below.")

Svör við Dæmi 4 (e. Answers to Problem 4)

Part A

F = Soccer H = Handball K = Basketball

$|F| = 285$ $|H| = 195$ $|K| = 115$ $|F \cap H| = 70$ $|F \cap K| = 45$ $|H \cap K| = 42$ $U = 500$

$\overline{|F \cup H \cup K|} = 50$

$|F \cup H \cup K| = U - \overline{|F \cup H \cup K|} = 500 - 50$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$|F \cup H \cup K| = |F| + |H| + |K| - |F \cap H| - |F \cap K| - |H \cap K| + |F \cap H \cap K|$

$450 = 285 + 195 + 115 - 70 - 45 - 42 + |F \cap H \cap K|$

$450 = 438 + |F \cap H \cap K|$

$|F \cap H \cap K| = 12$

$PartB$

Answer = |F| - $(|F \cap H| - |F \cap H \cap K|) - (|F \cap K| - |F \cap H \cap K|) - |F \cap H \cap K|$

$Answer = 285 - (70 - 12) - (45 - 12) - 12$

$Answer = 182$

a) 12

b) 182

Dæmi 5 (e. Problem 5) (20%)

Gefin er í kennslubókinni tafla með mengjareglum, tafla 1 í kafla 2.2. Notið mengjareglurnar í þessari töflu til að sýna fram á eftirfarandi formúlu. Vísið í eina mengjareglu í hverju einasta skrefi.

("There is a table with set identities in the textbook, table 1 in chapter 2.2. Use the set identities in that table to show the following. Show clearly which set identity you are using in each step")

$$B \cup \overline{(\overline{A} \cap B)} = U$$

Svar við Dæmi 5 (e. Answer to Problem 5)

$B \cup \overline{(\overline{A} \cap B)}$

$= B \cup (\overline{(\overline{A})} \cup \overline{B})$ By De Morgans Law

$= B \cup (A \cup \overline{B})$ By Complementation law

$= B \cup (\overline{B} \cup A)$ By Commutative law

$= (B \cup \overline{B}) \cup A$ By Associative law

$= U \cup A$ By Complement law

$= A \cup U$ By Commutative Law

$=$ U By Domination Law

We can therefor, conclude that $B \cup \overline{(\overline{A} \cap B)} =$ U is true