



Einstaklingsverkefni 3 (e. Individual Assignment 3)

T-117-STR1, Strjál stærðfræði I, 2024-3

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Skilafrestur (e. Deadline): 01.10.2024

Hér er Einstaklingsverkefni 3. Skilafrestur er þriðjudaginn 1. október 2024 kl. 23:59*. Þetta eru ein af 5 einstaklingsskilum. Þau gilda alls 20% af loka-einkunn, en lægstu einkunn er sleppt.

Mjög mikilvægt er að nemendur noti þetta skjal, fylli inn sínar lausnir á viðeigandi staði og skili útfylltu skjali á Gradescope sem pdf. Bæði er leyfilegt að prenta út skjalið, fylla inn handvirk og skanna það svo aftur inn (eða nota þetta L^AT_EX sniðmát og fylla inn í það). **Ekki verður farið yfir verkefni sem ekki nota þetta skjal (eða L^AT_EX sniðmátið), og fyrir slík verkefni fæst 0 í einkunn.**

*nemendur á Austurlandi skila miðvikudaginn 2. október 2024 kl. 23:59 og skilafrestur þeirra í Canvas/Gradescope er stilltur miðað við það

English version:

("This is the third individual assignment. The deadline is Tuesday, October 1st, 2024, at 23:59*. Students hand in solutions on pdf on Gradescope. This is one of 5 individual assignments. All in all, their weight is 20% of the final grade, but the lowest grade is dropped.")

Students must use this document, fill in their solutions in the designated spaces, and return the completed document to Gradescope as a pdf. You are allowed to print the document, fill it in writing, and scan it, (or use this L^AT_EX template and fill it in). **Assignments solutions that do not use this document (or the L^AT_EX template) will not be reviewed and will receive a grade of 0.**

*Students in the east of Iceland hand in on Wednesday, October 2nd, 2024, at 23:59, and their deadline is set in Canvas/Gradescope accordingly

Skiladæmi (e. Hand-in problems) :

Dæmi 1 (e. Problem 1) (16%)

Finnið margfeldið af AB ef ("Find the product AB if")

$$A = \begin{bmatrix} 1 & 0 \\ -5 & 4 \\ -7 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 3 & -3 & 5 \\ 0 & 2 & -2 & 4 & -4 \end{bmatrix}$$

Sýnið allar milliniðurstöður í útreikningum. Setjið alla útreikninga í efri kassann, og setjið lokasvar fylkið í neðri kassann hér fyrir neðan. ("Show all intermediate steps in the computation. Put all of your calculations in the first box below, and your final answer matrix in the second box on this page.")

Útreikningar fyrir dæmi 1 (e. Calculations for problem 1)

$$\begin{aligned} AB &= \begin{bmatrix} 1 \times 1 + 0 \times 0 & 1 \times -1 + 0 \times 2 & 1 \times 3 + 0 \times -2 & 1 \times -3 + 0 \times 4 & 1 \times 5 + 0 \times -4 \\ -5 \times 1 + 4 \times 0 & -5 \times -1 + 4 \times 2 & -5 \times 3 + 4 \times -2 & -5 \times -3 + 4 \times 4 & -5 \times 5 + 4 \times -4 \\ -7 \times 1 + -3 \times 0 & -7 \times -1 + -3 \times 2 & -7 \times 3 + -3 \times -2 & -7 \times -3 + -3 \times 4 & -7 \times 5 + -3 \times -4 \end{bmatrix} \\ &= \begin{bmatrix} 1 + 0 & -1 + 0 & 3 + 0 & -3 + 0 & 5 + 0 \\ -5 + 0 & 5 + 8 & -15 + -8 & 15 + 16 & -25 + -16 \\ -7 + 0 & 7 + -6 & -21 + 6 & 21 + -12 & -35 + 12 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 3 & -3 & 5 \\ -5 & 13 & -23 & 31 & -41 \\ -7 & 1 & -15 & 9 & -23 \end{bmatrix} \end{aligned}$$

☐ Ég þarf meira pláss fyrir útreikninga ("I need more space for my calculations")

Lokasvar fyrir dæmi 1 (e. Final answer for problem 1)

$$AB = \begin{bmatrix} 1 & -1 & 3 & -3 & 5 \\ -5 & 13 & -23 & 31 & -41 \\ -7 & 1 & -15 & 9 & -23 \end{bmatrix}$$

☐ Ég þarf meira pláss fyrir svarið ("I need more space for my answer")

Dæmi 2 (e. Problem 2) (16%)

Ef A og B eru bæði $n \times m$ fylki með $AB = BA = I_n$, þá er B kallað andhverfa A (fylkið B er þarna ótvírætt ákvarðað, einstakt) og A er þá sagt vera andhverfanlegt. Rithátturinn $B = A^{-1}$ táknar að B sé andhverfa A . ("If A and B are both $n \times m$ matrices with $AB = BA = I_n$, then B is called the inverse of A (such a matrix B is unique) and A is said to be invertible. The notation $B = A^{-1}$ denotes that B is the inverse of A .")

Sýnið að: ("Show that:")

$$\begin{pmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 3 \end{pmatrix} \text{ sé andhverfa ("is the inverse of")} \begin{pmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{pmatrix}$$

Sýnið allar milliniðurstöður í útreikningum. ("Show all intermediate steps in the computation.")

Svar við dæmi 2 (e. Answer to problem 2)

$$\begin{aligned} A &= \begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix} \quad B (A^{-1}) = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 3 \end{bmatrix} \\ AB &= \begin{bmatrix} 7 \times 2 + -8 \times 1 + 5 \times -1 & 7 \times 3 + -8 \times 2 + 5 \times -1 & 7 \times -1 + -8 \times 1 + 5 \times 3 \\ -4 \times 2 + 5 \times 1 + -3 \times -1 & -4 \times 3 + 5 \times 2 + -3 \times -1 & -4 \times -1 + 5 \times 1 + -3 \times 3 \\ 1 \times 2 + -1 \times 1 + 1 \times -1 & 1 \times 3 + -1 \times 2 + 1 \times -1 & 1 \times -1 + -1 \times 1 + 1 \times 3 \end{bmatrix} \\ &= \begin{bmatrix} 14 + -8 + -5 & 21 + -16 + -5 & -7 + -8 + 15 \\ -8 + 5 + 3 & -12 + 10 + 3 & 4 + 5 + -9 \\ 2 + -1 + -1 & 3 + -2 + -1 & -1 + -1 + 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ BA &= \begin{bmatrix} 2 \times 7 + 3 \times -4 + -1 \times 1 & 2 \times -8 + 3 \times 5 + -1 \times -1 & 2 \times 5 + 3 \times -3 + -1 \times 1 \\ 1 \times 7 + 2 \times -4 + 1 \times 1 & 1 \times -8 + 2 \times 5 + 1 \times -1 & 1 \times 5 + 2 \times -3 + 1 \times 1 \\ -1 \times 7 + -1 \times -4 + 3 \times 1 & -1 \times -8 + -1 \times 5 + 3 \times -1 & -1 \times 5 + -1 \times -3 + 3 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 14 + -12 + -1 & -16 + 15 + 1 & 10 + -9 + -1 \\ 7 + -8 + 1 & -8 + 10 + -1 & 5 + -6 + 1 \\ -7 + 4 + 3 & 8 + -5 + -3 & -5 + 3 + 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

We have now proven that $AB = BA = I_3$
and that B is the inverse of A

□ Ég þarf meira pláss fyrir útreikninga ("I need more space for my calculations")

Dæmi 3 (e. Problem 3) (6%+6%+8%)

Látum ("Let") $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ og ("and") $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

a) Finnið ("Find") $A \wedge B$.

b) Finnið ("Find") $A \vee B$.

c) Finnið ("Find") $A \odot B$.

Sýnið allar milliniðurstöður í útreikningum. ("Show all intermediate steps in the computation.")

Svör við dæmi 3 (e. Answers to problem 3)

$$A \wedge B \equiv \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \wedge \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 \wedge 0 & 1 \wedge 1 \\ 0 \wedge 1 & 0 \wedge 0 \end{bmatrix} \equiv \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

a)

☐ Ég þarf meira pláss fyrir útreikninga ("I need more space for my calculations")

$$A \vee B \equiv \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 \vee 0 & 1 \vee 1 \\ 0 \vee 1 & 0 \vee 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

b)

☐ Ég þarf meira pláss fyrir útreikninga ("I need more space for my calculations")

$$A \odot B \equiv \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \equiv \begin{bmatrix} (1 \wedge 0) \vee (1 \vee 1) & (1 \wedge 1) \vee (1 \wedge 0) \\ (0 \wedge 0) \vee (0 \vee 1) & (0 \wedge 1) \vee (0 \wedge 0) \end{bmatrix} \equiv \begin{bmatrix} 0 \vee 1 & 1 \vee 0 \\ 0 \vee 0 & 0 \vee 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

c)

☐ Ég þarf meira pláss fyrir útreikninga ("I need more space for my calculations")

Dæmi 4 (e. Problem 4) (5%+5%+5%+5%)

Finnið þessi gildi. Sýnið útreikninga. ("Evaluate these quantities. Show calculations")

- a) $-19 \bmod 7$ b) $352 \bmod 19$ c) $-115 \bmod 3$ d) $98 \bmod 10$

Útreikningar fyrir dæmi 4 (e. Calculations for problem 4)

a)

$$\begin{aligned} p &= -19 \bmod 7 & q &= -19 \operatorname{div} 7 & \lfloor \frac{-19}{7} \rfloor &= -3 & -19 &= 7q + p \\ -19 &= 7 \times -3 + p & -19 &= -21 + p & p &= 2 \end{aligned}$$

☐ Ég þarf meira pláss fyrir útreikninga ("I need more space for my calculations")

b)

$$\begin{aligned} p &= 352 \bmod 19 & q &= 352 \operatorname{div} 19 & \lfloor \frac{352}{19} \rfloor &= 18 & 352 &= 19q + p \\ 352 &= 19 \times 18 + p & 352 &= 342 + p & p &= 10 \end{aligned}$$

☐ Ég þarf meira pláss fyrir útreikninga ("I need more space for my calculations")

c)

$$\begin{aligned} p &= -115 \bmod 3 & q &= -115 \operatorname{div} 3 & \lfloor \frac{-115}{3} \rfloor &= -39 & -115 &= 3q + p \\ -115 &= 3 \times -39 + p & -115 &= -117 + p & p &= 2 \end{aligned}$$

☐ Ég þarf meira pláss fyrir útreikninga ("I need more space for my calculations")

d)

$$\begin{aligned} p &= 98 \bmod 10 & q &= 98 \operatorname{div} 10 & \lfloor \frac{98}{10} \rfloor &= 9 & 98 &= 10q + p \\ 98 &= 10 \times 9 + p & 98 &= 90 + p & p &= 8 \end{aligned}$$

☐ Ég þarf meira pláss fyrir útreikninga ("I need more space for my calculations")

Lokasvör fyrir dæmi 4 (e. Final answers for problem 4)

- a) 2 b) 10 c) 2 d) 8

Dæmi 5 (e. Problem 5) (12%)

Skráið niður fimm tölur sem tilheyra menginu $\{a | a \equiv 5 \bmod 15\}$ (þ.e. eru jafngildar 5 modulo 15). Sýnið alla útreikninga. ("List five integers that are in the set $\{a | a \equiv 5 \bmod 15\}$ (that is, are congruent to 5 modulo 15). Show all calculations.")

Svar við dæmi 5 (e. Answer to problem 5)

$$x \times 15 + 5$$

$$0 \times 15 + 5 = 5 \quad 1 \times 15 + 5 = 20 \quad 2 \times 15 + 5 = 35 \quad 3 \times 15 + 5 = 50 \quad 4 \times 15 + 5 = 65$$

$$\{a | a \equiv 5 \bmod 15\} = \{\dots, 5, 20, 35, 50, 65, \dots\}$$

☐ Ég þarf meira pláss fyrir útreikninga ("I need more space for my calculations")

Dæmi 6 (e. Problem 6) (8%+8%)

Sýnið skref fyrir skref hvernig þið finnið eftirfarandi með reikniriti Evklíðs. ("Show step by step how you find the following with the Euclidian Algorithm.")

a) $\gcd(235, 477)$

b) $\gcd(1529, 14039)$

Útreikningar fyrir dæmi 6 (e. Calculations for problem 6)

$$\gcd(235, 477)$$

$$\lfloor \frac{477}{235} \rfloor = 2 \quad 477 - 235 \times 2 = 7 \quad 477 = 235 \times 2 + 7$$

$$\lfloor \frac{235}{7} \rfloor = 33 \quad 235 - 7 \times 33 = 4 \quad 235 = 7 \times 33 + 4$$

$$\lfloor \frac{7}{4} \rfloor = 1 \quad 7 - 4 \times 1 = 3 \quad 7 = 4 \times 1 + 3$$

$$\lfloor \frac{4}{3} \rfloor = 1 \quad 4 - 3 \times 1 = 1 \quad 4 = 3 \times 1 + 1$$

$$\lfloor \frac{3}{1} \rfloor = 3 \quad 3 - 1 \times 3 = 0 \quad 3 = 1 \times 3 + 0$$

$$\gcd(235, 477) = 1$$

a)

☐ Ég þarf meira pláss fyrir útreikninga ("I need more space for my calculations")

$$\gcd(1529, 14039)$$

$$\lfloor \frac{14039}{1529} \rfloor = 9 \quad 14039 - 1529 \times 9 = 278 \quad 14039 = 1529 \times 9 + 278$$

$$\lfloor \frac{1529}{278} \rfloor = 5 \quad 1529 - 278 \times 5 = 139 \quad 1529 = 278 \times 5 + 139$$

$$\lfloor \frac{278}{139} \rfloor = 2 \quad 278 - 139 \times 2 = 0 \quad 278 = 139 \times 2 + 0$$

$$\gcd(1529, 14039) = 139$$

b)

☐ Ég þarf meira pláss fyrir útreikninga ("I need more space for my calculations")

Lokasvör fyrir dæmi 6 (e. Final answers to problem 6)

a) $\underline{\hspace{2cm} 1 \hspace{2cm}}$

b) $\underline{\hspace{2cm} 139 \hspace{2cm}}$