Tyngde opgave i 30131 (2025)

Jordens Tyngdefelt

Deadline 4/12 kl 15
1 stk rapport med appendix i Pdf format indleveres i Learn

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Rapport:
Grupper af 3 personer
Word limit: 1500
Al kode skal placeres i Appendix
Nøglefigurer placeres i hovedrapport. Ekstra figurer i Appendix

1 stk rapport per gruppe med appendix i Pdf format oploades til Learn

Opgaven består af följgende:

1) **Jordens tyngdefelt: Part A+B (Opgaver stillet på engelsk men kan besvares på dansk)**

Alle spørgsmål som SKAL adresseres er mærket med et "bullet point".

EARTH'S GRAVITY FIELD

This assignment is concerned with the gravity field of the Earth and modelling of geophysical structures. It addresses chapters 3 and 4 in the 3rd edition of "Fundamentals of Geophysics" (corresponding to chapter 2 in the 2nd edition) and is structured into two parts:

A. Loading, Manipulating and Gridding of Spatially Distributed Data
B. Anomaly Enhancement and Modelling Gravity Anomalies

Table 1 Files for the assignment.

File name	File type
db.dat	ASCII data file
Gauss2Dconv_v1.py	Python function
Helping_Python_script_Tyngde.ipynb	Jupyter Notebook with hints to the assignment. Primarily adressing Python hints

Python module recommendations:
Numpy, matplotlib, cartopy, scipy

Along with this document, you are handed the files listed in Table . **Make sure you have these files available**. They will be introduced as we go through the assignment.

PART A – EARTH'S GRAVITY FIELD

Loading, Manipulating and Gridding of Spatially Distributed Data

The first part of the assignment introduces the observations along with some tools to manipulate the data. From a set of spatially distributed gravity observations, you are asked to derive gravity anomalies and structure the data into a grid format, which is convenient for further analysis.

Question 1: Loading the Data File

The spatially distributed gravity observations are contained within the file ***db.dat***. You can open the file in any text editor (ASCII format) and inspect the information. Each row in the file represents a gravity observation and has the format

Statno [.] Latitude [deg] Longitude [deg] Height [m] Gravity [m/s²]

The first task is to load the data into Python and visualize the data.

- Plot the scattered (lon,lat) points with a colorbar corresponding to the gravity value
- Plot the scattered (lon,lat) points with a colorbar corresponding to the height value
- Comment on your figures: Do they look reasonable? What is the magnitude of the values? How do they correlate? What is the source for the gravity signal? What are the long- and the short wavelength trends describing?

Question 2: From Gravity to Gravity Anomalies

The normal gravity field, γ, refers to the gravity field of a rotational ellipsoid with an associated mass and rotational velocity. In other words, using a mathematical model of the Earth, we can compute a reference gravity value. On the surface of the ellipsoid, the normal gravity acceleration, γ_0, can be computed as

$$\gamma_0(\phi) = \gamma_e \frac{1 + k\sin^2(\phi)}{\sqrt{1 - e^2\sin^2(\phi)}} \qquad (1)$$

where

$\gamma_e = 9.780\ 326\ 7715\ \mathrm{m/s^2}$ is normal gravity on the equator,

ϕ is latitude,

$k = 0.001\ 931\ 851\ 353$ is a constant related to the "gravity flattening", and

$e^2 = 0.006\ 694\ 380\ 022\ 90$ is the first eccentricity squared of the geometrical ellipsoid

This can be further upward continued to measurement level using the series expansion

$$\gamma(\phi, h) = \gamma_0(\phi)\left\{1 - \frac{2}{a}(1 + f + m - 2f\sin^2(\phi))\,h + \frac{3}{a^2}\,h^2\right\} \qquad (2)$$

where

[Type here]

$a = 6\ 378\ 137\ \mathrm{m}$ is the ellipsoidal semi-major axis,

$f = 0.003\ 352\ 810\ 681\ 18$ is the ellipsoidal flattening, and

$m = 3.449\ 786\ 003\ 055\ 778\ \cdot\ 10^{-15}$

Gravity observations, g, can then be expressed relative to the normal gravity field, γ. This quantity is known as the gravity anomaly

$$\Delta g = g - \gamma \tag{3}$$

The gravity anomaly, Δg, is usually expressed in units of $\mathrm{mGal} = 10^5\ \mathrm{m/s^2}$, which represents the order of magnitude that separates gravity observations from the mathematical model.

- For each observation point, compute the normal gravity acceleration using Eq. (2) and derive the gravity anomaly using Eq. (3) in units of mGal
- Visualize/plot the data as before and include the figure in your report
- Comment on the figure: Does it look reasonable? What sources have been removed from the signal? Which sources of information remain? Etc.

Question 3: Gridding the Data

Having derived the gravity anomaly, Δg, at each point, it is convenient to structure the data in a regular mesh grid format. Start by forming a mesh grid with increments $d_{inc} = 1/60°$ in Python

- Interpolate the scattered gravity anomalies, Δg, onto the generated regular grid
- Visualize/plot the grid and include the figure in your report
- What are the benefits of using gridded data for analysis and what are the disadvantages?

Question 4: Bouguer Anomalies

The gravity anomalies we have derived so far, represent the anomalous gravity field with respect to a mathematical model that we denote the normal gravity field. In this mathematical model it is assumed that the Earth shape is a rotational ellipsoid with a homogenous mass distribution. The maps of gravity anomalies that we produced in the questions above, therefore show the deviations of the actual gravity field from the mathematical model. These anomalies are commonly known as free-air gravity anomalies.

Inspection of the figures of free-air gravity anomalies indicates that the majority of the signal originates from topography, which we have not included in the mathematical (normal) gravity model.

- How do we conclude that the free-air gravity anomalies mainly originate from topography?

A simple approach to compensate for topography is the Bouguer plate correction

$$A_B = 2\pi G\rho h = 0.000041888\ \frac{\mathrm{mGa}l}{\mathrm{m}}\ \rho h \tag{4}$$

which is applied to arrive at the *simple Bouguer anomaly*

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$$\Delta g_B = \Delta g - A_B \tag{5}$$

The term “simple” is used to distinguish the plate correction from more advanced topographic corrections resulting in what is commonly known as the *refined Bouguer anomaly or complete Bouguer anomaly*.

- Describe the Bouguer plate correction
- Compute the Bouguer plate correction using Eq. (4) and a density $\rho = 2670 kg/m$^3 (argue for this density); Derive the simple Bouguer anomaly for all (scattered) data points using Eq. (5)
- Compute the Bouguer plate correction using Eq. (4) and another suitable chosen density ρ (argue for the choice of this density); Derive the simple Bouguer anomaly for all (scattered) data points using Eq. (5)
- Grid the two Bouguer anomalies as in the previous question (similar to free-air anomalies)
- Visualize/plot the grids and include the figure in your report
- Comment on the figures: Do they look reasonable? Compare and explain the differences in the two Bouguer data sets? Which sources have been removed from the signal? Which sources of information remain? Etc.

This concludes part A of the assignment. In part B, you will need the two grids of Bouguer anomalies you have prepared in this section.

PART B - EARTH'S GRAVITY FIELD

Anomaly Enhancement and Gravity Modelling

Fourier methods represent a powerful tool for analyzing signal. Here, we will use to separate the gravity signal into short- and long-wavelength components. For this, you are handed the Python function `Gauss2Dconv_v1` which applies a 2D Gaussian filter to the grid of gravity anomalies. As a general rule of thumb, sources deeper below the surface of the Earth will generate gravity variations on a larger spatial scale than sources closer to the surface, which will generally result in signatures of smaller spatial scale. Fourier methods and spectral filtering can be used to remove sources of information that are not of interest, while enhancing features that are of interest.

In this part, we will analyze and model a high-density volcanic intrusion in central Jutland (Silkeborg Gravity High). The gravity signal from the intrusion should already be identifiable as a positive anomaly in the Bouguer gravity map created previously.

Question 5: Spectral Filtering

The first step is to enhance the signal of interest while removing signals that are not of interest, i.e. use spectral filtering. You can use the function `Gauss2Dconv_v1` to apply a 2D Gaussian filter to your map of Bouguer anomalies. The function requires a filter parameter in units of meters (sigma) which controls the degree of smoothing.

- Choose an initial value of sigma that you find reasonable. Apply spectral filtering to the grid of Bouguer anomalies
- Plot/visualize both the low- and high-wavenumber components of the signal (long and short wavelength)
- Adjust the value of sigma until to find a value that suitably enhances the Silkeborg Gravity High, while removing wavelength information that is not of interest (notice that this part is somewhat subjective)
- Is Silkeborg Gravity High a short or long wavelength feature – and why?
- List the final value of sigma and include the two figures (low- and high-pass) in your report
- Comment on the figures: Do they look reasonable? Why did you choose this value? What signal is present in one figure versus the other?

Question 6: Anomaly Enhancement

The density of volcanic intrusion rock is approximately $2900\ \mathrm{kg/m^3}$ and the average density of sediments in Jutland is 2000 $\mathrm{kg/m^3}$. Such a density contrast will result in local gravity anomaly in the gravity map. In this question, we are particularly interested in the volcanic intrusion located in the area between 56.0 to 56.5 degrees north and 9.0 to 10.0 degrees east. The intrusion generates a gravity anomaly called the Silkeborg Gravity High. To proceed, we will extract a subset of the grid, corresponding to the area of interest.

- Extract a segment of your filtered grid (either high or low pass – whatever you argue is best) corresponding to the specified area
- Visualize the filtered Bouguer anomalies in this area; Include the figure in your report.
- Comment on the figure: Does it look reasonable? What is the source of the signal?

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Question 7: Gravity Modelling

In this question, we will attempt to model the gravitational response of a simple geometrical form, namely a uniform sphere. The gravity response from such a sphere is

$$g_{sphere} = \frac{4}{3}\pi G\rho R^3 \frac{z}{(z^2 + x^2 + y^2)^{3/2}} \tag{6}$$

where

$G = 6.674\ 3015 \cdot 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg \cdot s}^2}$is the gravitational constant,

ρ is the density of the sphere,

R is the radius of the sphere,

z is the distance vertical distance from the center of the sphere, and

x, y are the horizontal distances from the center of the sphere

When concerned with gravity anomalies, the density contrast, $\Delta\rho$, with respect to the ambient material is used instead of the density itself. If we confine ourselves to a profile in two-dimensions, i.e. neglect y, the expression for the gravity anomaly thus becomes

$$\Delta g = \frac{4}{3}\pi G\Delta\rho R^3 \frac{z}{(z^2 + x^2)^{3/2}} \tag{7}$$

The peak value is reached at the horizontal position over/under the center of the sphere, i.e.

$$\Delta g_{max} = \frac{4}{3}\pi G\Delta\rho R^3 \frac{1}{z^2} \tag{8}$$

Another important value is the half-width-half-maximum distance, $x_{1/2}$, which occurs when $\Delta g = \frac{1}{2}\Delta g_{max}$. This distance is related to the depth of the sphere.

- Proof that half-width-half-maximum is satisfied when $x \approx 0.77z \implies z \approx 1.3\ x_{1/2}$

The shape of the anomaly indicates symmetry, allowing us to simplify the problem by extracting a single north-south profile.

- Define a north-south profile crossing through the center of the anomaly (approximately)
- Interpolate the filtered gravity anomalies onto the profile
- Convert the longitude to distance (in meters) along the line assuming the Earth is a sphere
- Plot/visualize the profile (gravity anomaly) as a function of distance
- Include the figure in your report

In order to more intuitively model the gravitational response, you should (subjectively) define a vertical line of symmetry and a horizontal zero reference line. Then shift the entire curve horizontally and vertically such that the "peak value" occurs at 0 distance along the vertical line and the profile flattens out to "0 mGal" along the horizontal line.

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- Once you have adjusted/shifted the curve, identify the peak of the curve, Δg_{max}.
- Plot the peak and half peak lines on top of the curve
- Include the figure in your report
- Identify the half-width-half-maximum distance, $x_{1/2}$, by inspecting your plot
- Compute the depth of the sphere as $z = 1.3\ x_{1/2}$
- Include the peak value, Δg_{max}, the half-width-half-maximum distance, $x_{1/2}$, and the depth, z, in your report

As mentioned previously, the density of volcanic rock is approximately $2900\ \mathrm{kg/m^3}$. The average density of marine sediments is 2000 $\mathrm{kg/m^3}$., leading to a density contrast of $\Delta\rho = 900\ \mathrm{kg/m^3}$. Having already estimated the depth, z, of the sphere, the radius, R, can be derived from Eq. (8).

- Estimate the radius, R, of the sphere from Eq. (8) using a density contrast of $\Delta\rho = 900\ \mathrm{kg/m^3}$ along with the depth, z, estimated before. Include the value in your report.

Having specified the depth, radius and density contrast of the sphere, we now have all the parameters to model the gravitational response using Eq. (7). Remember so assure you are using the correct units, i.e. be aware of mGal to m/s^2 conversion.

- Compute the gravitational response of your sphere along the profile using Eq. (7)
- Visualize/plot the computed response together with the observed response and confirm that they are similar. Include the figure in your report.