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$$x + 3 = 2$$
$$y * 3 = 2$$
$$z \hat{} 3 = 2$$
$$w \hat{} 2 = -1$$

$x : \mathbb{N} \to$ ingen lösning

$x : \mathbb{Z} \to x = -1$

$y : \mathbb{Z}$

$y : \mathbb{Q} \to y = \frac{2}{3}$

$z : \mathbb{Q}$

$z : \mathbb{R} \to z = \sqrt[3]{2}$

$w : \mathbb{R}$

$w : \mathbb{C} \to w = i = \sqrt{-1}$

DSL → δσλ

DSLsofMath

Definition of Complex Numbers

We begin by defining the symbol i, called **the imaginary unit,**[1] to have the property

$$i^2 = -1.$$

Thus, we could also call i the **square root of** -1 and denote it $\sqrt{-1}$. Of course, i is not a real number; no real number has a negative square.

$$IU = ImagUnit = \{ i, \ldots \}$$

Robert A. Adams • Christopher Essex

Calculus

A COMPLETE COURSE

$$i * i = -1$$

$$\mathbb{R}$$

$$* : \mathbb{R} \to \mathbb{R} \to \mathbb{R}$$

$$* : \mathbb{C} \to \mathbb{C} \to \mathbb{C}$$

$$* : IU \to IU \to IU \quad ?$$

$$(\mathbb{R} \cup IU) \to \ldots \quad ?$$

DEFINITION 1

A **complex number** is an expression of the form

$$a + bi \qquad \text{or} \qquad a + ib,$$

where a and b are *real numbers*, and i is the imaginary unit.

$bi + a$? $\quad ib + a$?

$a - bi$? $\quad bi - a$?

data CB = Plus_1 $\mathbb{R}$ $\mathbb{R}$ IU

| Plus_2 $\mathbb{R}$ IU $\mathbb{R}$

+ kommuterar? Ja för $\mathbb{R}$ ($\mathbb{Q}$, ...) ? för $\mathbb{C}$

Adams and Essex continue with examples:

> For example, $3+2i$, $\frac{7}{2}-\frac{2}{3}i$, $i\pi = 0+i\pi$ and $-3 = -3+0i$ are all complex numbers. The last of these examples shows that every real number can be regarded as a complex number.

1	$3+2i$	$\text{Plus}_1\ 3\ 2\ i$
2	$\frac{7}{2}-\frac{2}{3}i$	$\text{Plus}_1\ \frac{7}{2}\ (-\frac{2}{3})\ i$
3	$i\pi$	$\text{Plus}_2\ 0\ i\ \pi$
4	-3	$\text{Plus}_1\ (-3)\ 0\ i$

(We will normally use $a + bi$ unless b is a complicated expression, in which case we will write $a + ib$ instead. Either form is acceptable.)

$\pi =$!

$$\boxed{a + ib = a + bi}$$

It is often convenient to represent a complex number by a single letter; w and z are frequently used for this purpose. If a, b, x, and y are real numbers, and $w = a + bi$ and $z = x + yi$, then we can refer to the complex numbers w and z. Note that $w = z$ if and only if $a = x$ and $b = y$.

$$\text{Plus}_1 \stackrel{?}{=} \text{Plus}_2$$

$$bi + a = a + ib$$

$$= ib + a = a + bi$$