Satslogik

Patrik Jansson

$p_1 = a \wedge \neg a = F$

$p_2 = a \vee \neg a = T$

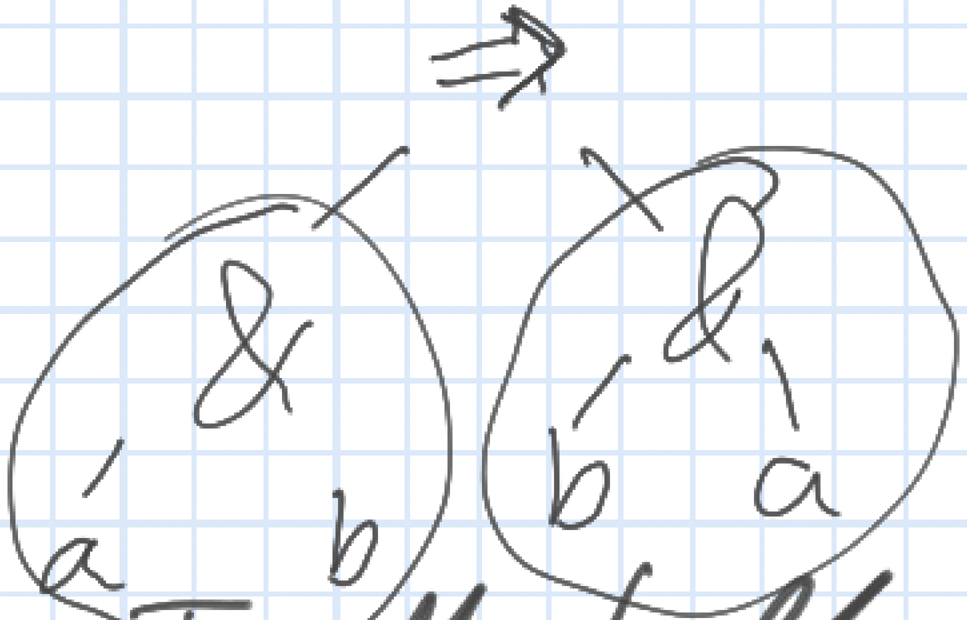
$p_3 = a \Rightarrow b$

$p_4 = (a \wedge b) \Rightarrow (b \wedge a) = T$

F	F	F	T	F	F	F
F	F	T	T	F	F	T
T	F	F	T	T	F	F
T	T	T	T	T	T	T
1	2	3	4	5	6	7

Syntax

False	$\bot$	F	nullary
True	$\top$	T	nullary
Not	$\neg$	~	unary
And	$\wedge$	&	binary
Or	$\vee$	|	binary
Implies	$\supset$	$\Rightarrow$	binary



Truth table

a	$\Rightarrow$	b
F	T	F
F	T	T
T	F	F
T	T	T

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$p_1 = a \wedge \neg a$
$p_2 = a \vee \neg a$
$p_3 = a \Rightarrow b$
$p_4 = (a \wedge b) \Rightarrow (b \wedge a)$

type Tab = Name → $\mathbb{B}$

$p_1, p_2, p_3, p_4 :: Prop$
$p_1 = And\ (Name\ \texttt{"a"})\ (Not\ (Name\ \texttt{"a"}))$
$p_2 = Or\ \ (Name\ \texttt{"a"})\ (Not\ (Name\ \texttt{"a"}))$
$p_3 = Implies\ (Name\ \texttt{"a"})\ (Name\ \texttt{"b"})$
$p_4 = Implies\ (And\ a\ b)\ \ (And\ b\ a)$
  where $a = Name\ \texttt{"a"}; b = Name\ \texttt{"b"}$

data *Prop* = *Con* *Bool*
  | *Not* *Prop*
  | *And* *Prop* *Prop*
  | *Or* *Prop* *Prop*
  | *Implies* *Prop* *Prop*
  | *Name* *Name*
type *Name* = *String*

syntax → semantik

eval : Prop → Tab → $\mathbb{B}$
eval (Name n) t = t n

(Name → $\mathbb{B}$) → $\mathbb{B}$

DSL → δσλ
DSLsofMath

Bevis

$a : A$ "a är ett bevis av A"

```
data Either p q where
  Left  :: p -> Either p q
  Right :: q -> Either p q
```

$$\frac{a : A \qquad b : B}{(a,b) : A \wedge B}\ \text{AndIntro}$$

och &

$$\frac{a : A}{\text{Left } a : A \vee B}\ \text{OrIntroL} \qquad \frac{b : B}{\text{Right } b : A \vee B}\ \text{OrIntroR}$$

eller |

Bevis

$a : A$ "a är ett bevis av A"

$$\frac{a : A \qquad b : B}{(a,b) : A \wedge B}\ \text{AndIntro}$$

$$\frac{p : A \wedge B}{\mathit{fst}\ p : A} \qquad \frac{p : A \wedge B}{\mathit{snd}\ p : B}$$

$f : A \Rightarrow B$

$f : A \rightarrow B$

```haskell
data Either p q where
  Left  :: p -> Either p q
  Right :: q -> Either p q
```

$$\frac{a : A}{\mathit{Left}\ a : A \vee B}\ \text{OrIntroL} \qquad \frac{b : B}{\mathit{Right}\ b : A \vee B}\ \text{OrIntroR}$$

$$\frac{e : A \vee B \qquad f : A \Rightarrow C \qquad g : B \Rightarrow C}{\mathit{orElim}\ e\ f\ g : C}$$

$\mathit{orElim}\ (\mathit{Left}\ a)\ f\ g = f\ a$

$\mathit{orElim}\ (\mathit{Right}\ b)\ f\ g = g\ b$

Mängdlära (Pure set theory)

Empty : M

Sing : $M \to M$

Union : $M \to M \to M$

Snitt : $M \to M \to M$

$\emptyset = \{\ \} =$ tom mängd

$\{x\} =$ mängd med bara x

$A \cup B$, $A \cap B$

card : $M \to N$

Elem : $M \to M \to Prop$

$|\emptyset| = 0$

$x \in \{x\}$

$|\{x\}| = 1$

$|A \cup B| \geq |A|$

Mängdlära (Pure set theory)

(for all x)

Empty : M

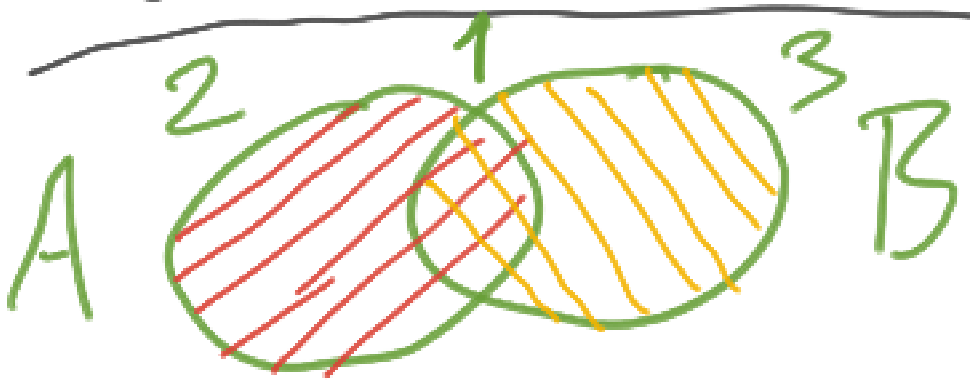
Sing : $M \to M$

Union : $M \to M \to M$

Snitt : $M \to M \to M$

Elem : $M \to M \to$ Prop

card : $M \to \mathbb{N}$



Union

eller

$$\boxed{x \in \{x\}}$$

$$\boxed{(x \in A \cup B) \Leftrightarrow (x \in A) \vee (x \in B)}$$

$$\boxed{(x \in A \cap B) \Leftrightarrow (x \in A) \wedge (x \in B)}$$

och

$$|\emptyset| = 0$$

$$|\{x\}| = 1$$

$$|A| + |B| = |A \cup B| + |A \cap B|$$

$$2 + 3 = 4 + 1$$

Mängdlära (Pure set theory)

$$m_2 \cup m_1 = \{m_1\} \cup \{m_0\}$$

$$t_{01} = \{m_1, m_0\}$$

$$|t_{01}| = 2$$

$$\exists u.\, |u| = 3$$

t_{12}

t_{02}

$$x \cup y = y \cup x \qquad x \cup x = x$$

$$x \cup m_0 = x \cup \emptyset = x$$

$\cup$	m_0	m_1	m_2	t_{01}	
m_0	m_0				
m_1	m_1	m_1			
m_2	m_2	t_{01}	m_2		
t_{01}	t_{01}			t_{01}	
m_3	m_3				m_3
X	X				

Mängdlära (Pure set theory)

$m_0 = \emptyset$ $\quad |m_0| = 0$

$m_1 = \{m_0\}$ $\quad |m_1| = 1$

$m_2 = \{m_1\}$ $\quad |m_2| = 1$

$m_3 = \{m_2\}$ $\quad |m_3| = 1$

$\vdots \qquad \vdots$

$$t_{01} = \{m_0, m_1\} = \{m_0\} \cup \{m_1\} = m_1 \cup m_2$$

$\cup$	m_0	m_1	m_2
m_0	m_0		
m_1		m_1	
m_2			m_2