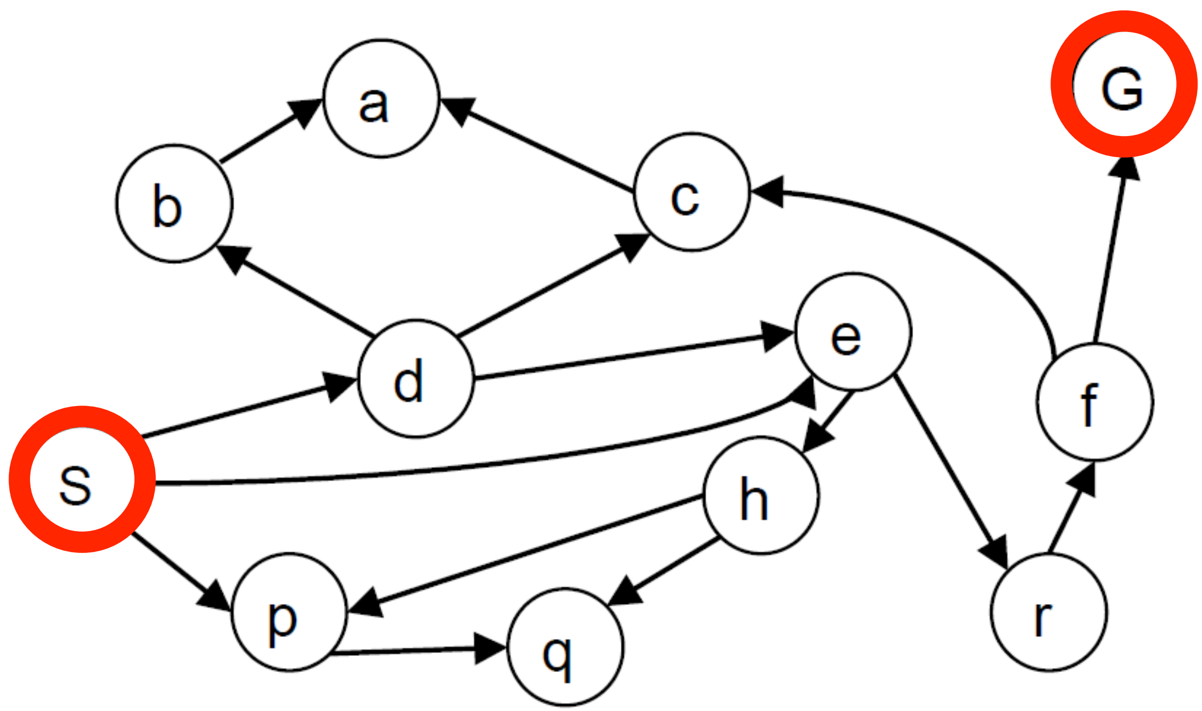
Uninformed search strategies

- A **search strategy** is defined by picking the order of node expansion
- **Uninformed** search strategies use only the information available in the problem definition
  - Breadth-first search
  - Depth-first search
  - Iterative deepening search
  - Uniform-cost search

Breadth-first search

- Expand shallowest unexpanded node
- Implementation: *frontier* is a FIFO queue

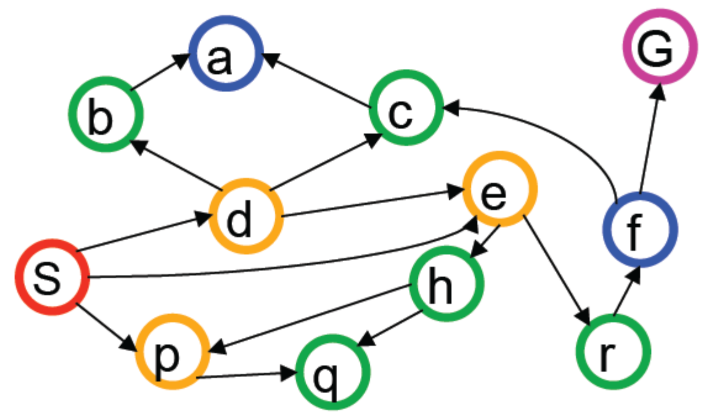
Example state space graph for a tiny search problem

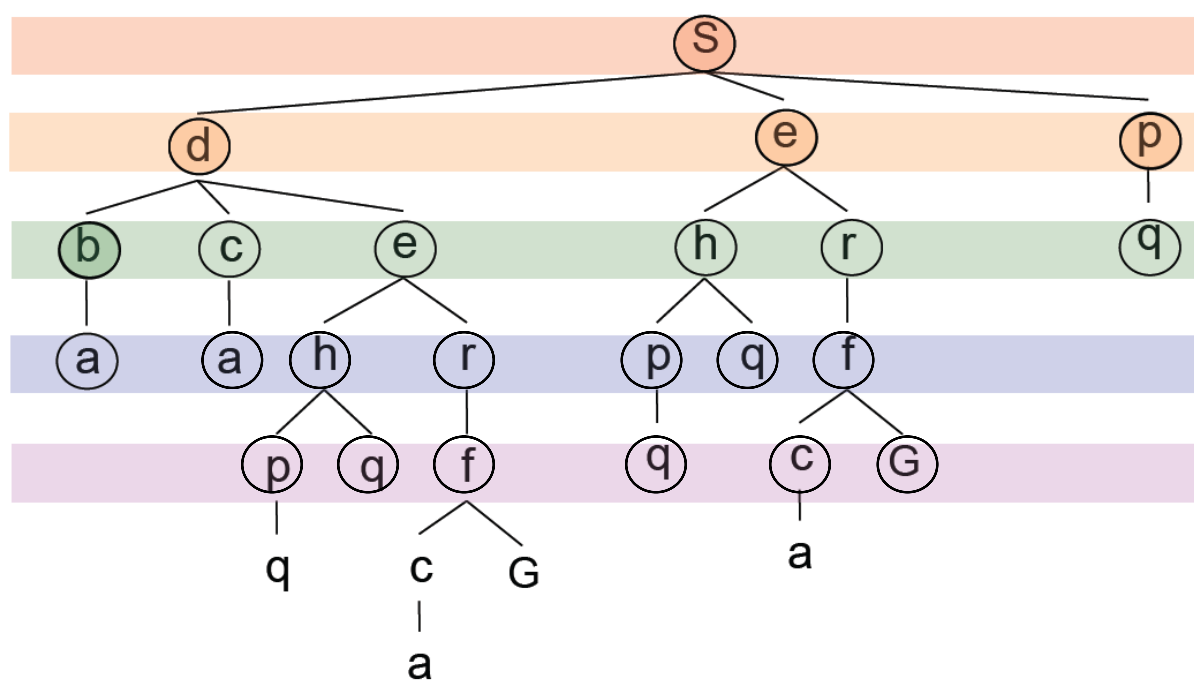


Example from P. Abbeel and D. Klein

Breadth-first search

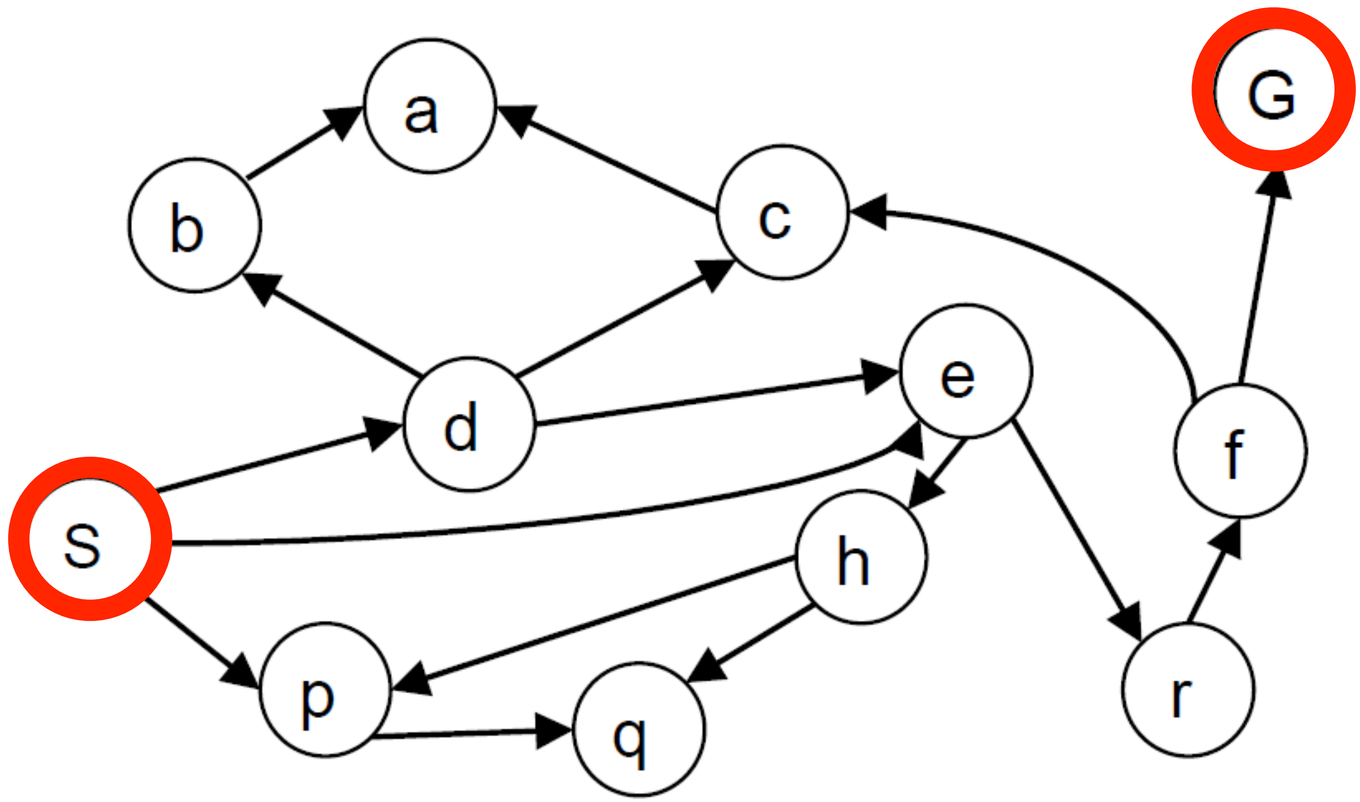
- Expansion order:
(S,d,e,p,b,c,e,h,r,q,a,a,
h,r,p,q,f,p,q,f,q,c,G)





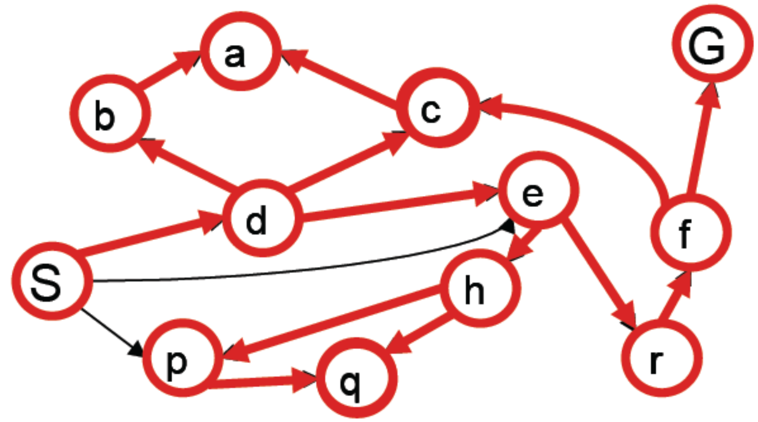
Depth-first search

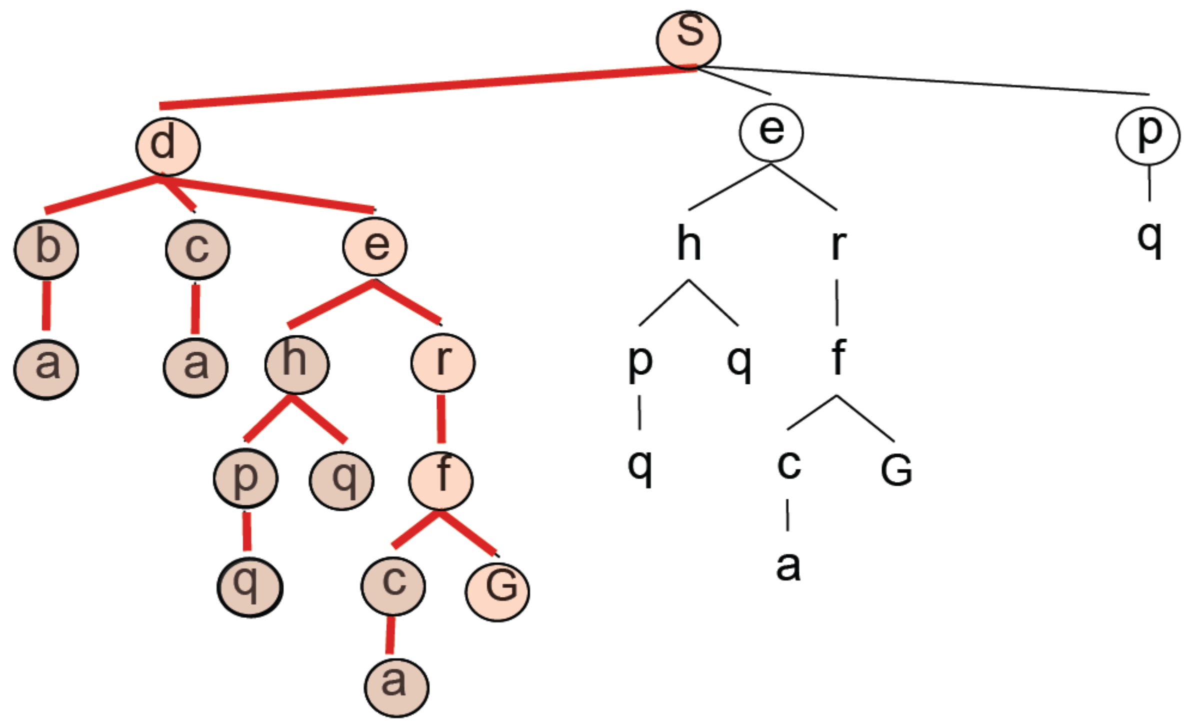
- Expand deepest unexpanded node
- Implementation: *frontier* is a LIFO queue

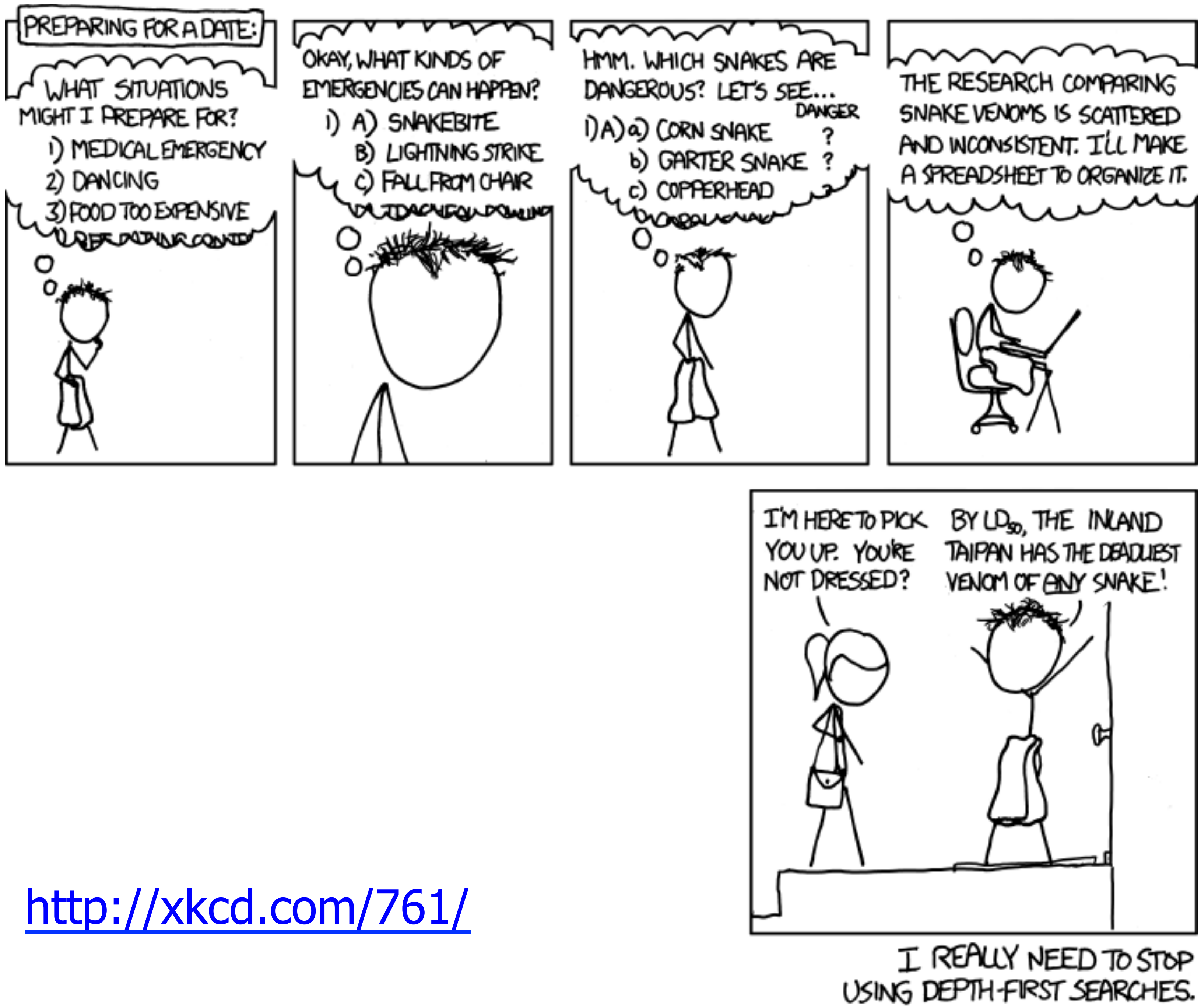


Depth-first search

- Expansion order: (d,b,a,c,a,e,h,p,q,q, r,f,c,a,G)







http://xkcd.com/761/

Analysis of search strategies

- Strategies are evaluated along the following criteria:
  - **Completeness:** does it always find a solution if one exists?
  - **Optimality:** does it always find a least-cost solution?
  - **Time complexity:** number of nodes generated
  - **Space complexity:** maximum number of nodes in memory
- Time and space complexity are measured in terms of
  - ***b****:* maximum branching factor of the search tree
  - ***d****:* depth of the optimal solution
  - ***m***: maximum length of any path in the state space (may be infinite)

Properties of breadth-first search

- **Complete?**

  Yes (if branching factor b is finite)
- **Optimal?**

  Yes – if cost = 1 per step
- **Time?**

  Number of nodes in a b-ary tree of depth d: $O(b^d)$

  (d is the depth of the optimal solution)
- **Space?**

  $O(b^d)$

- Space is the bigger problem (more than time)

Properties of depth-first search

- **Complete?**
  Fails in infinite-depth spaces, spaces with loops
  Modify to avoid repeated states along path
  → complete in finite spaces
- **Optimal?**
  No – returns the first solution it finds
- **Time?**
  Could be the time to reach a solution at maximum depth m: $O(b^m)$
  Terrible if m is much larger than d
  But if there are lots of solutions, may be much faster than BFS
- **Space?**
  $O(bm)$, i.e., linear space!

Iterative deepening search

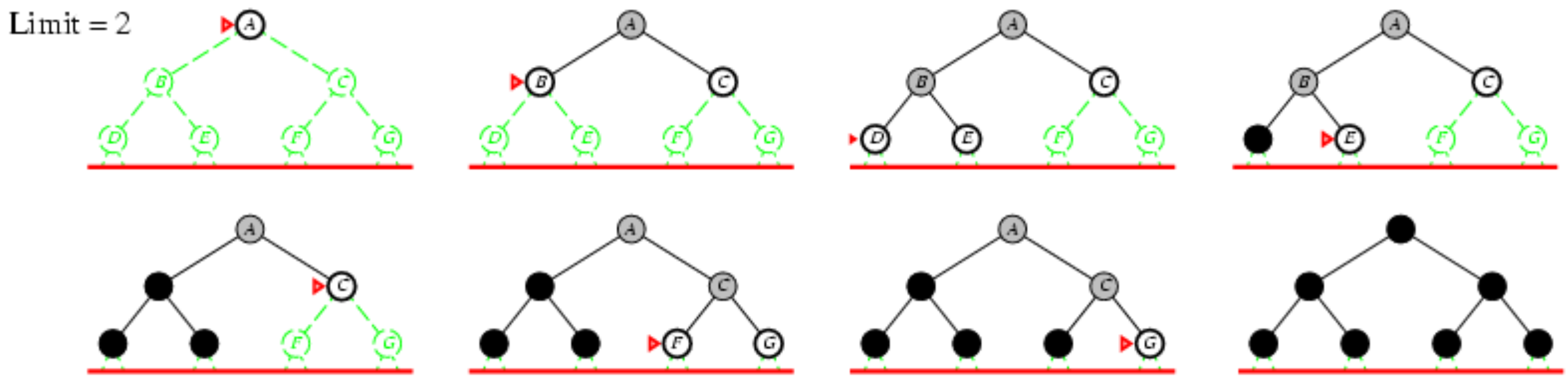
- Use DFS as a subroutine
  1. Check the root
  2. Do a DFS searching for a path of length 1
  3. If there is no path of length 1, do a DFS searching for a path of length 2
  4. If there is no path of length 2, do a DFS searching for a path of length 3…

Iterative deepening search

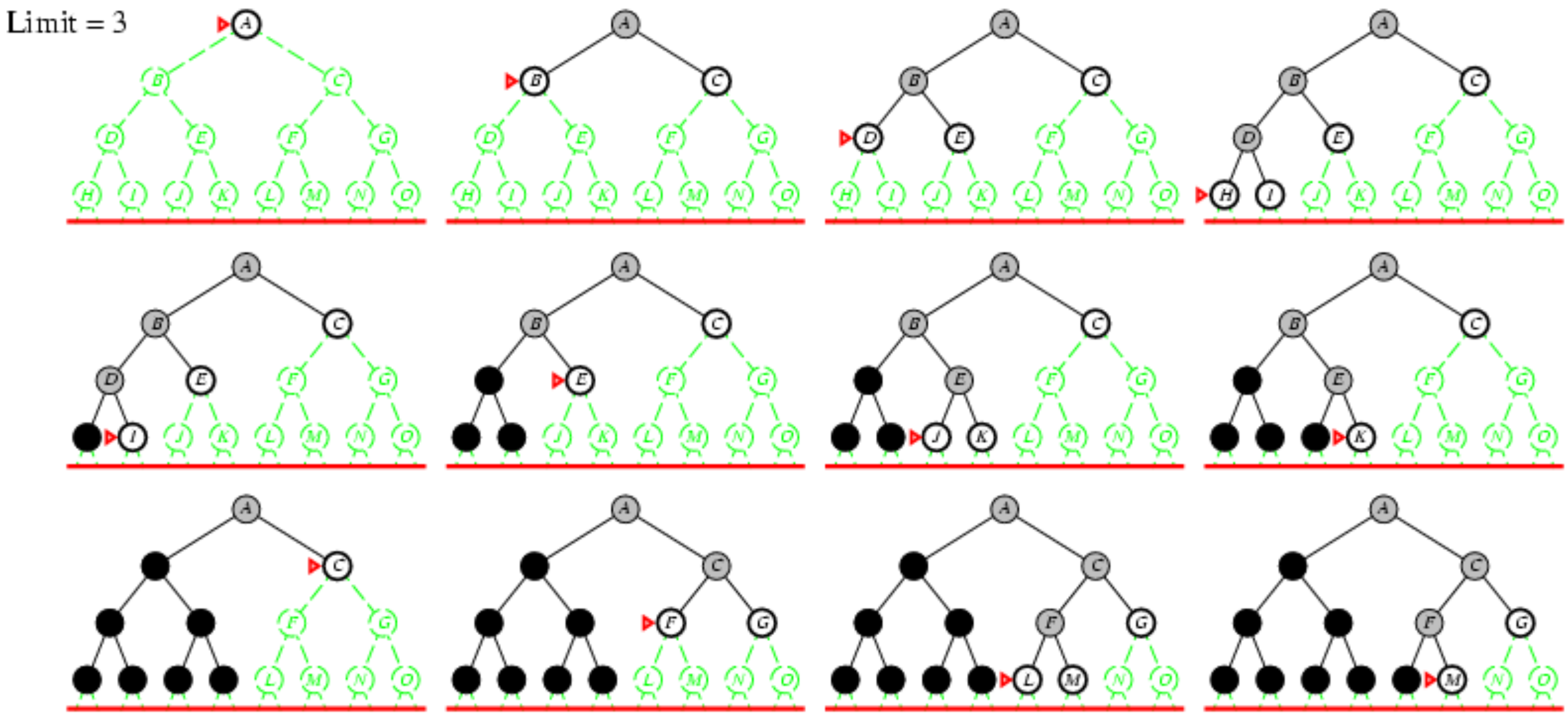
Limit = 0 

Iterative deepening search

Iterative deepening search



Iterative deepening search



Properties of iterative deepening search

- **Complete?**

  Yes

- **Optimal?**

  Yes, if step cost = 1

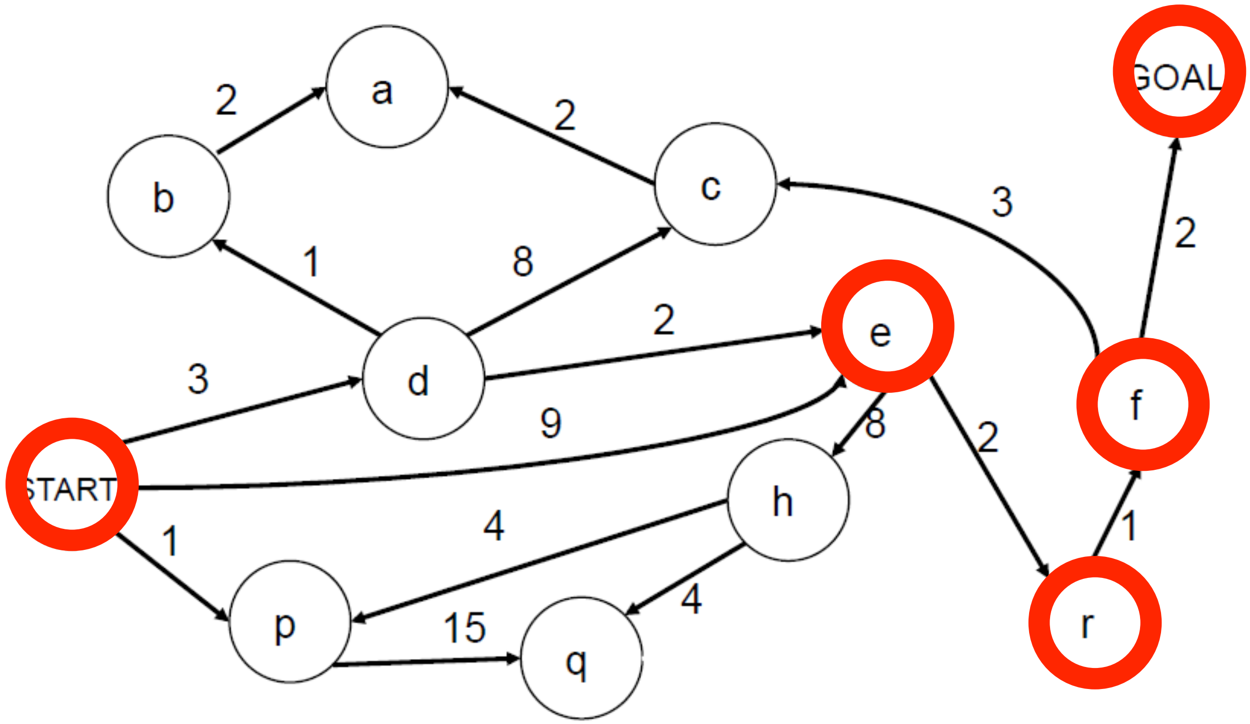
- **Time?**

  $(d+1)b^0 + d\, b^1 + (d-1)b^2 + \ldots + b^d$

- **Space?**

  $O(bd)$

Search with varying step costs

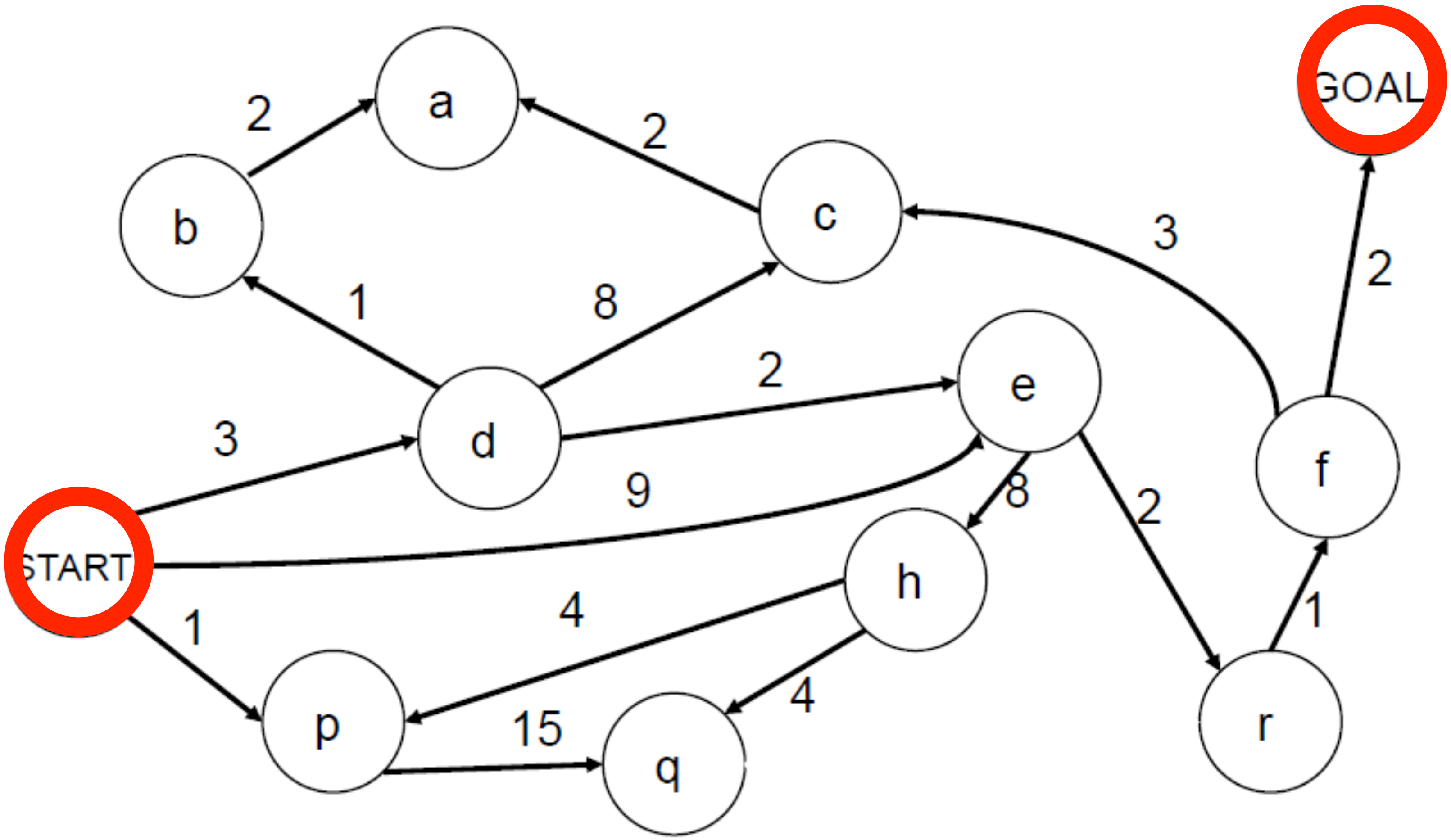


- BFS finds the path with the fewest steps, but does not always find the cheapest path

Uniform-cost search

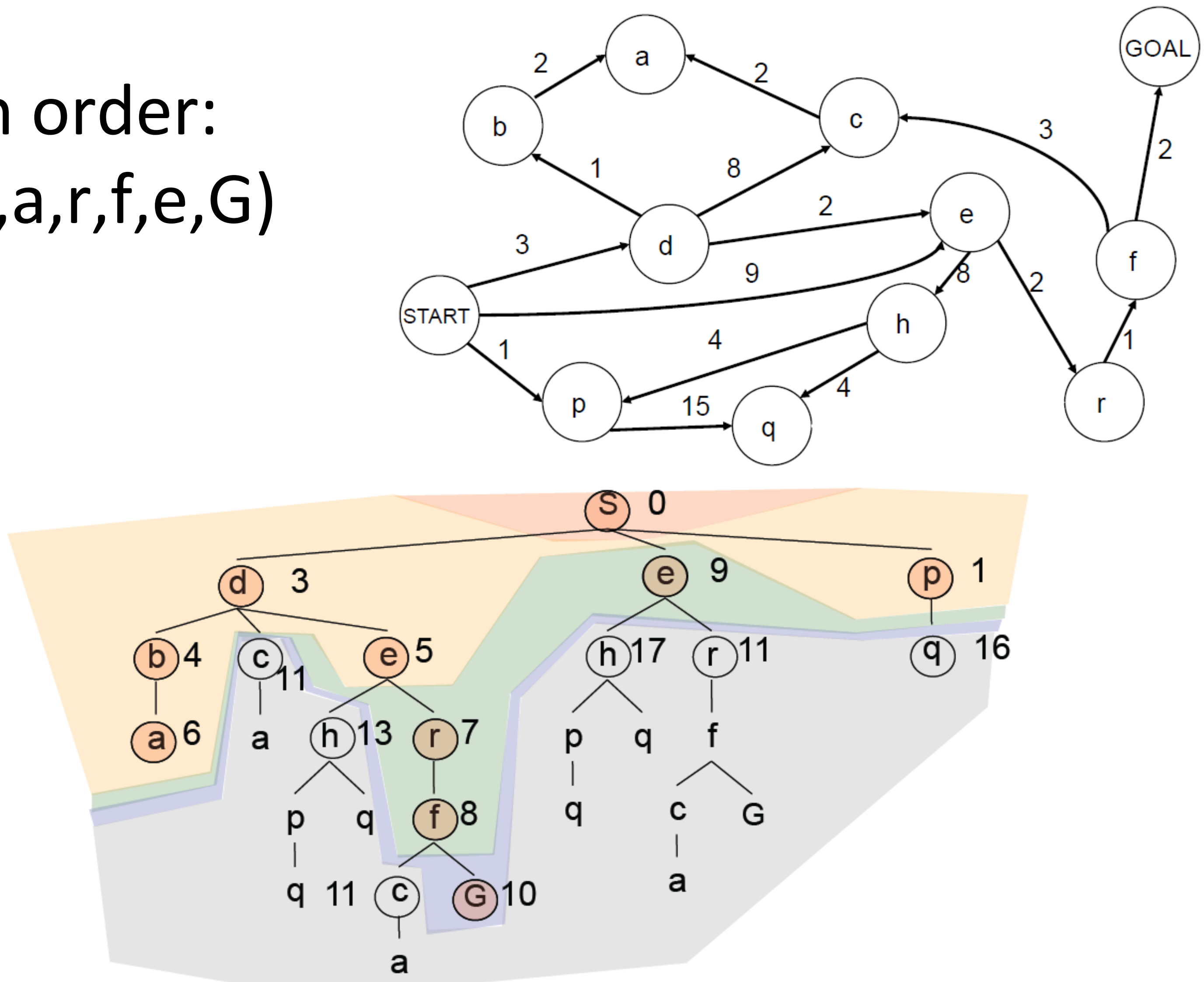
- For each frontier node, save the total cost of the path from the initial state to that node
- Expand the frontier node with the lowest path cost
- Implementation: *frontier* is a priority queue ordered by path cost
- Equivalent to breadth-first if step costs all equal
- Equivalent to Dijkstra's algorithm in general

Uniform-cost search example

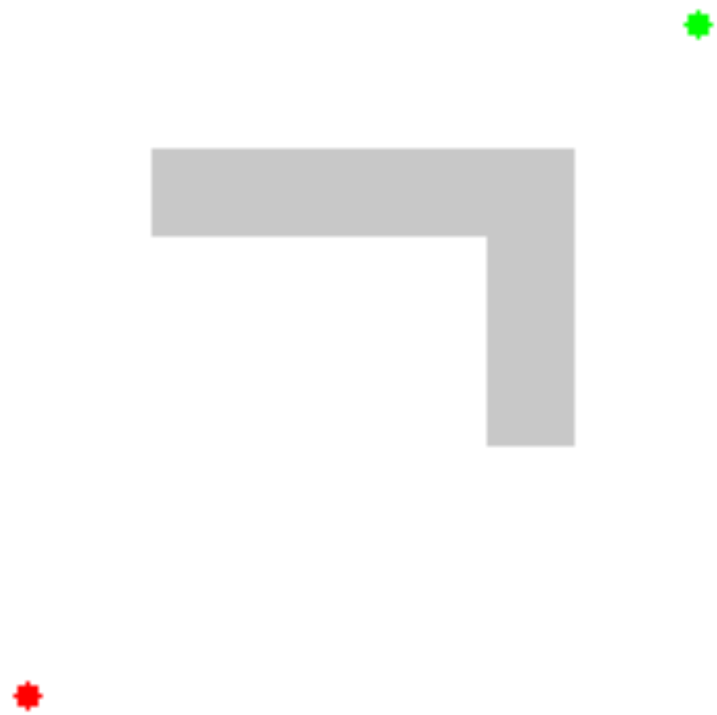


Uniform-cost search example

- Expansion order:
(S,p,d,b,e,a,r,f,e,G)



Another example of uniform-cost search



Source: [Wikipedia]

Properties of uniform-cost search

- **Complete?**

  Yes, if step cost is greater than some positive constant ε (we don't want infinite sequences of steps that have a finite total cost)

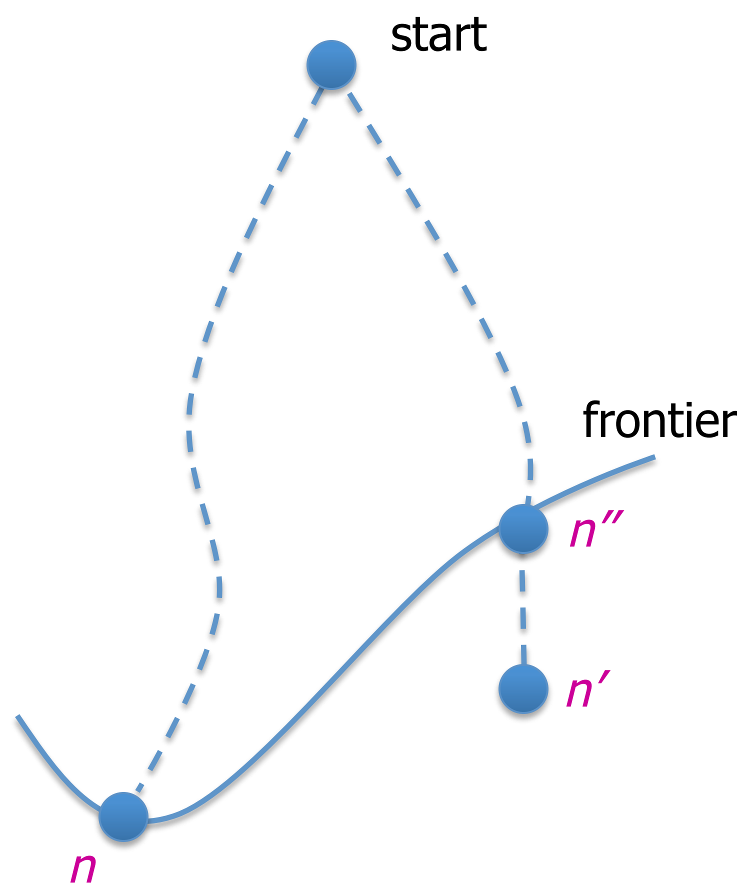
- **Optimal?**

  Yes

Optimality of uniform-cost search

- **Graph separation property**: every path from the initial state to an unexplored state has to pass through a state on the frontier
  - Proved inductively

- Optimality of UCS: proof by contradiction
  - Suppose UCS terminates at goal state n with path cost $g(n)$ but there exists another goal state n' with $g(n') < g(n)$
  - By the graph separation property, there must exist a node n'' on the frontier that is on the optimal path to n'
  - But because $g(n'') \leq g(n') < g(n)$, n'' should have been expanded first!



Properties of uniform-cost search

- **Complete?**

  Yes, if step cost is greater than some positive constant ε (we don't want infinite sequences of steps that have a finite total cost)

- **Optimal?**

  Yes – nodes expanded in increasing order of path cost

- **Time?**

  Number of nodes with path cost ≤ cost of optimal solution (C^*), $O(b^{C^*/\varepsilon})$

  This can be greater than $O(b^d)$: the search can explore long paths consisting of small steps before exploring shorter paths consisting of larger steps

- **Space?**

  $O(b^{C^*/\varepsilon})$

Review: Uninformed search strategies

Algorithm	Complete?	Optimal?	Time complexity	Space complexity
BFS	Yes	If all step costs are equal	$O(b^d)$	$O(b^d)$
DFS	No	No	$O(b^m)$	$O(bm)$
IDS	Yes	If all step costs are equal	$O(b^d)$	$O(bd)$
UCS	Yes	Yes	Number of nodes with $g(n) \leq C^*$	

b: maximum branching factor of the search tree
d: depth of the optimal solution
m: maximum length of any path in the state space
C*: cost of optimal solution
g(n): cost of path from start state to node n