

Game theory

(Sections 17.5-17.6)



Game theory

- **Game theory** deals with systems of interacting agents where the outcome for an agent depends on the actions of all the other agents
 - Applied in sociology, politics, economics, biology, and, of course, AI
- **Agent design:** determining the best strategy for a rational agent in a given game
- **Mechanism design:** how to set the rules of the game to ensure a desirable outcome

Modelling behaviour

Game theory in practice

Computing: Software that models human behaviour can make forecasts, outfox rivals and transform negotiations

Sep 3rd 2011 | from the print edition

The
Economist



<http://www.economist.com/node/21527025>

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FACEBOOK DOESN'T MAKE AS MUCH MONEY AS IT COULD—ON PURPOSE



YOU CAN THINK of John Hegeman as Facebook's chief economist. He spends his days thinking about the economics of Facebook advertising.

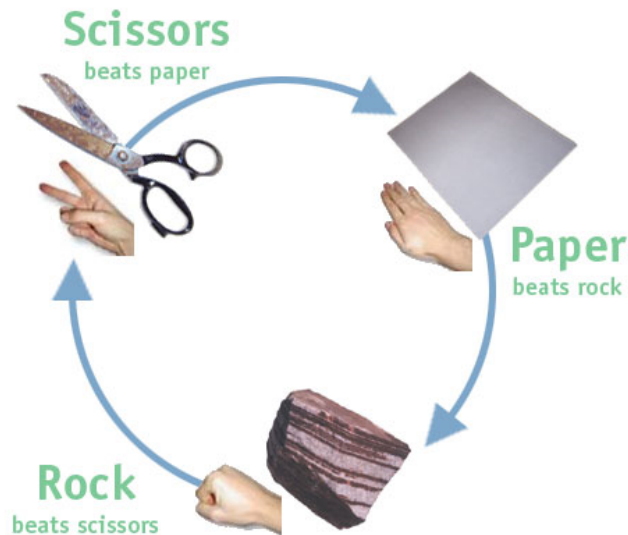
That's an enormous thing. Facebook pulled in \$4.04 billion in the second quarter of this year. And the overall economy of Facebook advertising, as Hegeman describes it, is far larger. Advertising, you see, is very much a part of everything else on the world's largest social network. Hegeman doesn't just think about ads. He thinks about how ads fit with the rest of Facebook.

When he joined Facebook in 2007, after getting a master's in economics at Stanford University, Hegeman helped build the online auction that drives the company's advertising system. Auctions are the standard way that online services accept ads from advertisers and place them on web pages and inside smartphone apps. That's what Google uses with [AdWords](#), the system that serves up all those ads when you look for stuff on the company's Internet search engine. Advertisers bid (in dollars) for placement on the results page when you key in a particular word or group of words. But in building Facebook's advertising system, Hegeman and team took online auctions in a new direction.

<http://www.wired.com/2015/09/facebook-doesnt-make-much-money-couldon-purpose/>

Simultaneous single-move games







- Players must choose their actions at the same time, without knowing what the others will do
 - Form of partial observability



Player 2

Normal form representation:

Player 1

			
	0,0	1,-1	-1,1
	-1,1	0,0	1,-1
	1,-1	-1,1	0,0

Payoff matrix

(Player 1's utility is listed first)

Is this a zero-sum game?

Prisoner's dilemma

- Two criminals have been arrested and the police visit them separately
- If one player testifies against the other and the other refuses, the one who testified goes free and the one who refused gets a 10-year sentence
- If both players testify against each other, they each get a 5-year sentence
- If both refuse to testify, they each get a 1-year sentence

	Alice: Testify	Alice: Refuse
Bob: Testify		
Bob: Refuse		

Prisoner's dilemma

- Alice's reasoning:
 - Suppose Bob testifies. Then I get 5 years if I testify and 10 years if I refuse. So I should testify.
 - Suppose Bob refuses. Then I go free if I testify, and get 1 year if I refuse. So I should testify.
- **Dominant strategy:** A strategy whose outcome is better for the player regardless of the strategy chosen by the other player

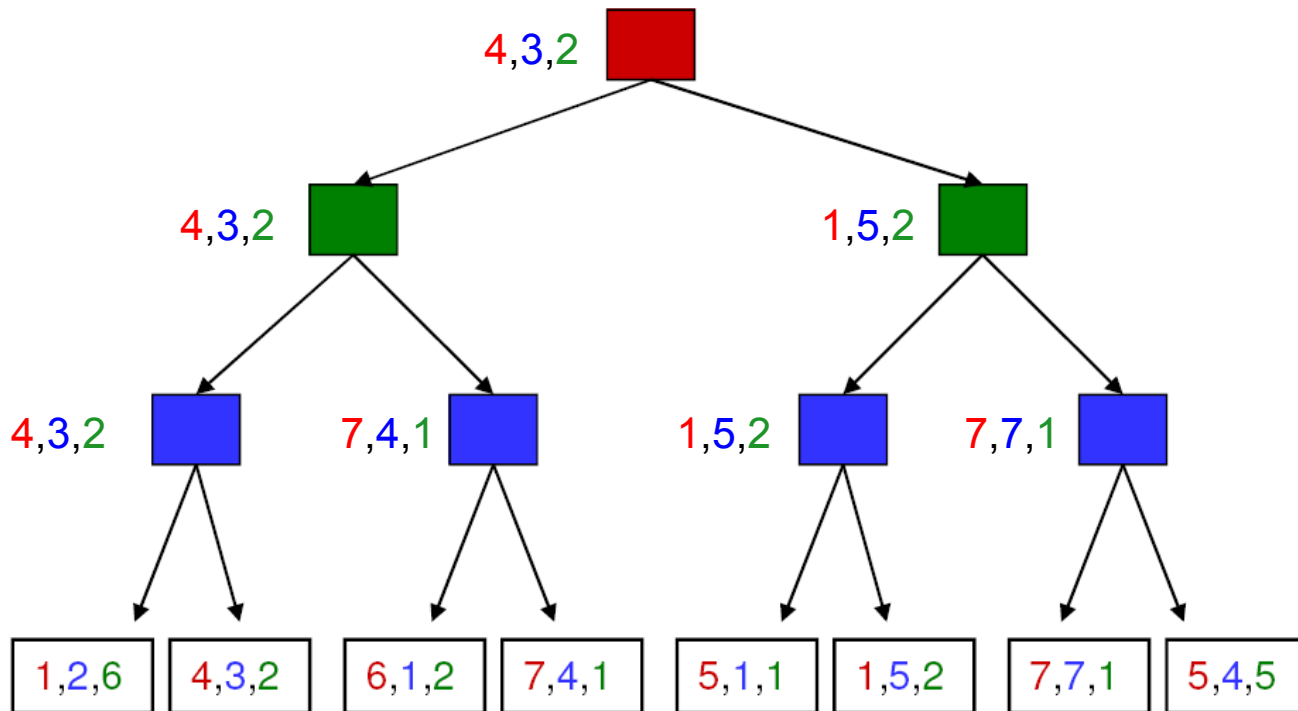
	Alice: Testify	Alice: Refuse
Bob: Testify	-5,-5	-10,0
Bob: Refuse	0,-10	-1,-1

Prisoner's dilemma

- **Nash equilibrium:** A pair of strategies such that no player can get a bigger payoff by switching strategies, provided the other player sticks with the same strategy
 - (Testify, testify) is a *dominant strategy equilibrium*
- **Pareto optimal outcome:** It is impossible to make one of the players better off without making another one worse off
- In a non-zero-sum game, a Nash equilibrium is not necessarily Pareto optimal!

	Alice: Testify	Alice: Refuse
Bob: Testify	-5,-5	-10,0
Bob: Refuse	0,-10	-1,-1

Recall: Multi-player, non-zero-sum game



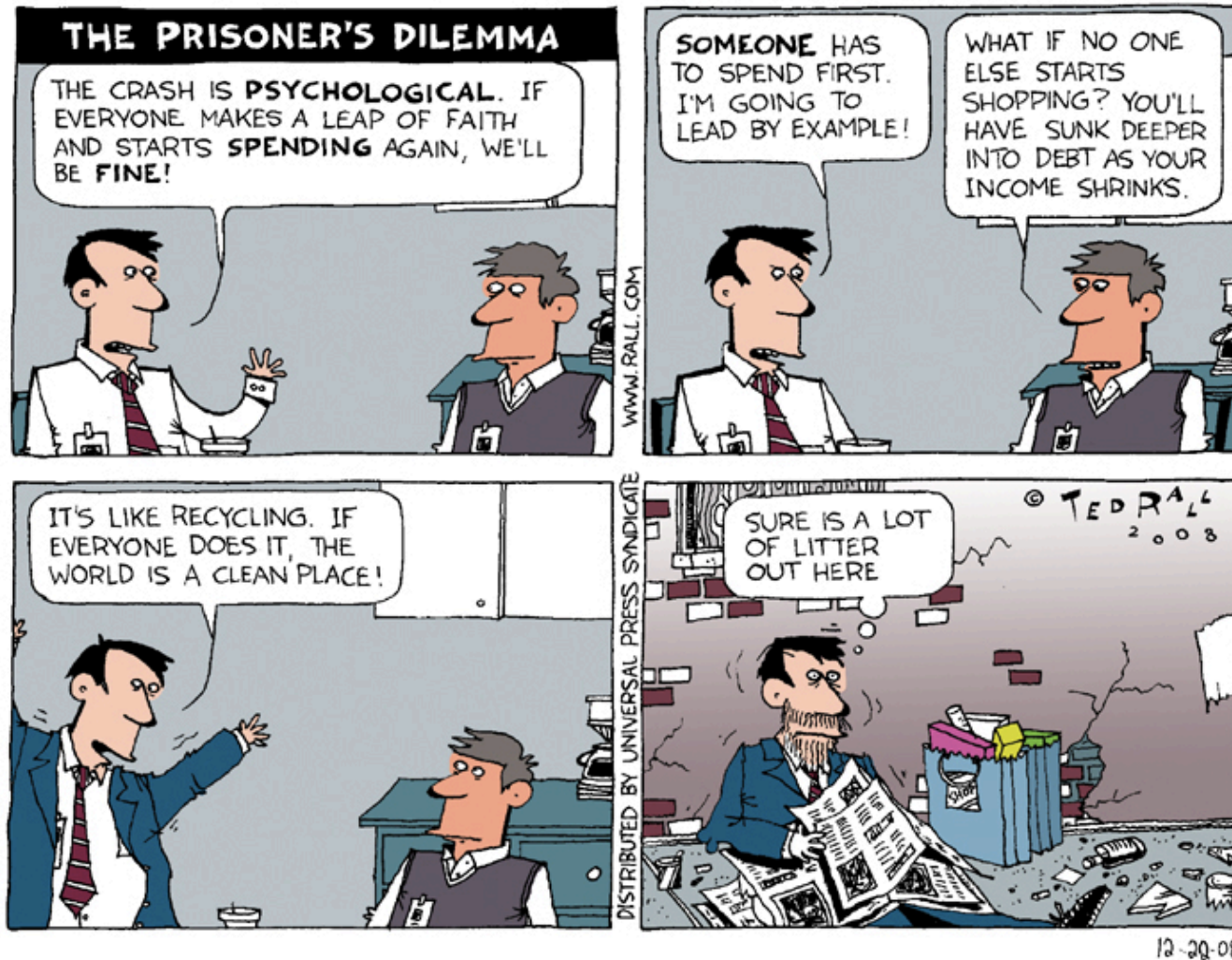
Prisoner's dilemma in real life

- Price war
- Arms race
- Steroid use
- Diner's dilemma
- Collective action in politics

	Defect	Cooperate
Defect	Lose – lose	Lose big – win big
Cooperate	Win big – lose big	Win – win

http://en.wikipedia.org/wiki/Prisoner's_dilemma

Prisoner's Dilemma in cartoons



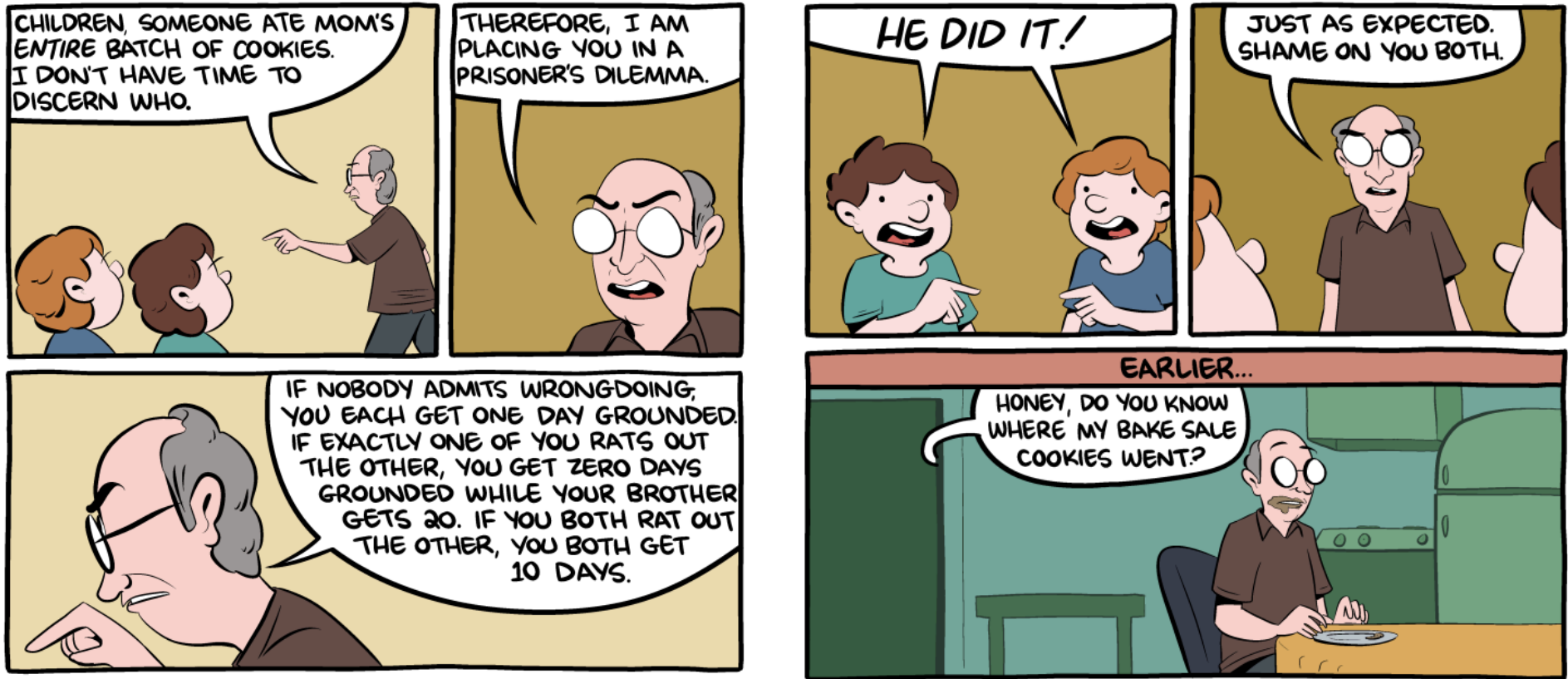
<http://rall.com/comic/cartoon-for-december-22-2008>

Prisoner's Dilemma in cartoons



<https://xkcd.com/1016/>

Prisoner's Dilemma in cartoons



smbc-comics.com

<http://www.smbc-comics.com/comic/parenting-game-theory>

Is there any way out of Prisoner's Dilemma?

- Superrationality
 - Assume that the answer to a symmetric problem will be the same for both players
 - Maximize the payoff to each player while considering only identical strategies
 - Not a conventional model in game theory
- Repeated games
 - If the number of rounds is fixed and known in advance, the equilibrium strategy is still to defect
 - If the number of rounds is unknown, cooperation may become an equilibrium strategy
 - Tournaments of repeated Prisoner's Dilemma

Review: Game theory

- Normal form representation of a game
- Dominant strategies
- Nash equilibria
- Pareto optimal outcomes
- Prisoner's dilemma



Stag hunt



	Hunter 1: Stag	Hunter 1: Hare
Hunter 2: Stag	2,2	1,0
Hunter 2: Hare	0,1	1,1

- Is there a dominant strategy for either player?
- Is there a Nash equilibrium?
 - (Stag, stag) and (hare, hare)
- Model for cooperative activity

Prisoner's dilemma vs. stag hunt

Prisoner's dilemma

	Cooperate	Defect
Cooperate	Win – win	Win big – lose big
Defect	Lose big – win big	Lose – lose

Players can gain by defecting unilaterally

Stag hunt

	Cooperate	Defect
Cooperate	Win big – win big	Win – lose
Defect	Lose – win	Win – win

Players lose by defecting unilaterally

Game of Chicken



- Is there a dominant strategy for either player?
- Is there a Nash equilibrium?
(Straight, chicken) or (chicken, straight)
- *Anti-coordination* game: it is mutually beneficial for the two players to choose different strategies
 - Model of escalated conflict in humans and animals (hawk-dove game)
- How are the players to decide what to do?
 - Pre-commitment or threats
 - Different roles: the “hawk” is the territory owner and the “dove” is the intruder, or vice versa

http://en.wikipedia.org/wiki/Game_of_chicken

Game of Chicken in the movies



[Rebel without a cause](#)



[Footloose](#)

Mixed strategy equilibria



- **Mixed strategy**: a player chooses between the moves according to a probability distribution
- Suppose each player chooses S with probability $1/10$. Is that a Nash equilibrium?
- Consider payoffs to P1 while keeping P2's strategy fixed
 - The payoff of P1 choosing S is $(1/10)(-10) + (9/10)1 = -1/10$
 - The payoff of P1 choosing C is $(1/10)(-1) + (9/10)0 = -1/10$
 - Can P1 change their strategy to get a better payoff?
 - Same reasoning applies to P2

Finding mixed strategy equilibria

	P1: Choose S with prob. p	P1: Choose C with prob. $1-p$
P2: Choose S with prob. q	-10, -10	-1, 1
P2: Choose C with prob. $1-q$	1, -1	0, 0

- Expected payoffs for P1 given P2's strategy:
 P1 chooses S: $q(-10) + (1-q)1 = -11q + 1$
 P1 chooses C: $q(-1) + (1-q)0 = -q$
- In order for P2's strategy to be part of a Nash equilibrium, P1 has to be indifferent between its two actions:
 $-11q + 1 = -q$ or $q = 1/10$
 Similarly, $p = 1/10$

Existence of Nash equilibria

- Any game with a finite set of actions has at least one Nash equilibrium (which may be a mixed-strategy equilibrium)
- If a player has a dominant strategy, there exists a Nash equilibrium in which the player plays that strategy and the other player plays the *best response* to that strategy
- If both players have *strictly dominant* strategies, there exists a Nash equilibrium in which they play those strategies

Computing Nash equilibria

- For a two-player zero-sum game, simple linear programming problem
- For non-zero-sum games, the algorithm has worst-case running time that is exponential in the number of actions
- For more than two players, and for sequential games, things get pretty hairy

Nash equilibria and rational decisions

- If a game has a *unique* Nash equilibrium, it will be adopted if each player
 - is rational and the payoff matrix is accurate
 - doesn't make mistakes in execution
 - is capable of computing the Nash equilibrium
 - believes that a deviation in strategy on their part will not cause the other players to deviate
 - there is *common knowledge* that all players meet these conditions

http://en.wikipedia.org/wiki/Nash_equilibrium

Continuous actions:

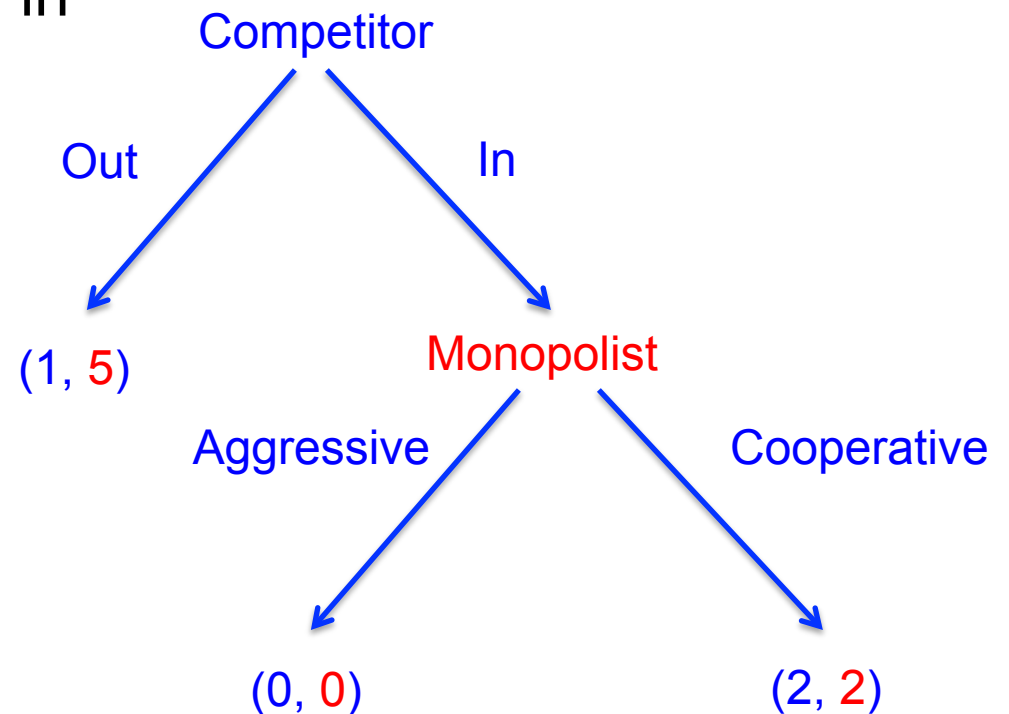
Ultimatum game

- Alice and Bob are given a sum of money S to divide
 - Alice picks A , the amount she wants to keep for herself
 - Bob picks B , the smallest amount of money he is willing to accept
 - If $S - A \geq B$, Alice gets A and Bob gets $S - A$
 - If $S - A < B$, both players get nothing
- What is the Nash equilibrium?
 - Alice offers Bob the smallest amount of money he will accept:
 $S - A = B$
 - Alice and Bob both want to keep the full amount: $A = S$, $B = S$
(both players get nothing)
- How would humans behave in this game?
 - If Bob perceives Alice's offer as unfair, Bob will be likely to refuse
 - Is this rational?
 - Maybe Bob gets some positive utility for “punishing” Alice?

http://en.wikipedia.org/wiki/Ultimatum_game

Sequential/repeated games and threats: Chain store paradox

- A monopolist has branches in 20 towns and faces 20 competitors successively
 - Threat: respond to “in” with “aggressive”



Mechanism design (inverse game theory)

- Assuming that agents pick rational strategies, how should we design the game to achieve a socially desirable outcome?
- We have multiple agents and a **center** that collects their choices and determines the outcome

Auctions

- Goals
 - Maximize revenue to the seller
 - Efficiency: make sure the buyer who values the goods the most gets them
 - Minimize transaction costs for buyer and sellers

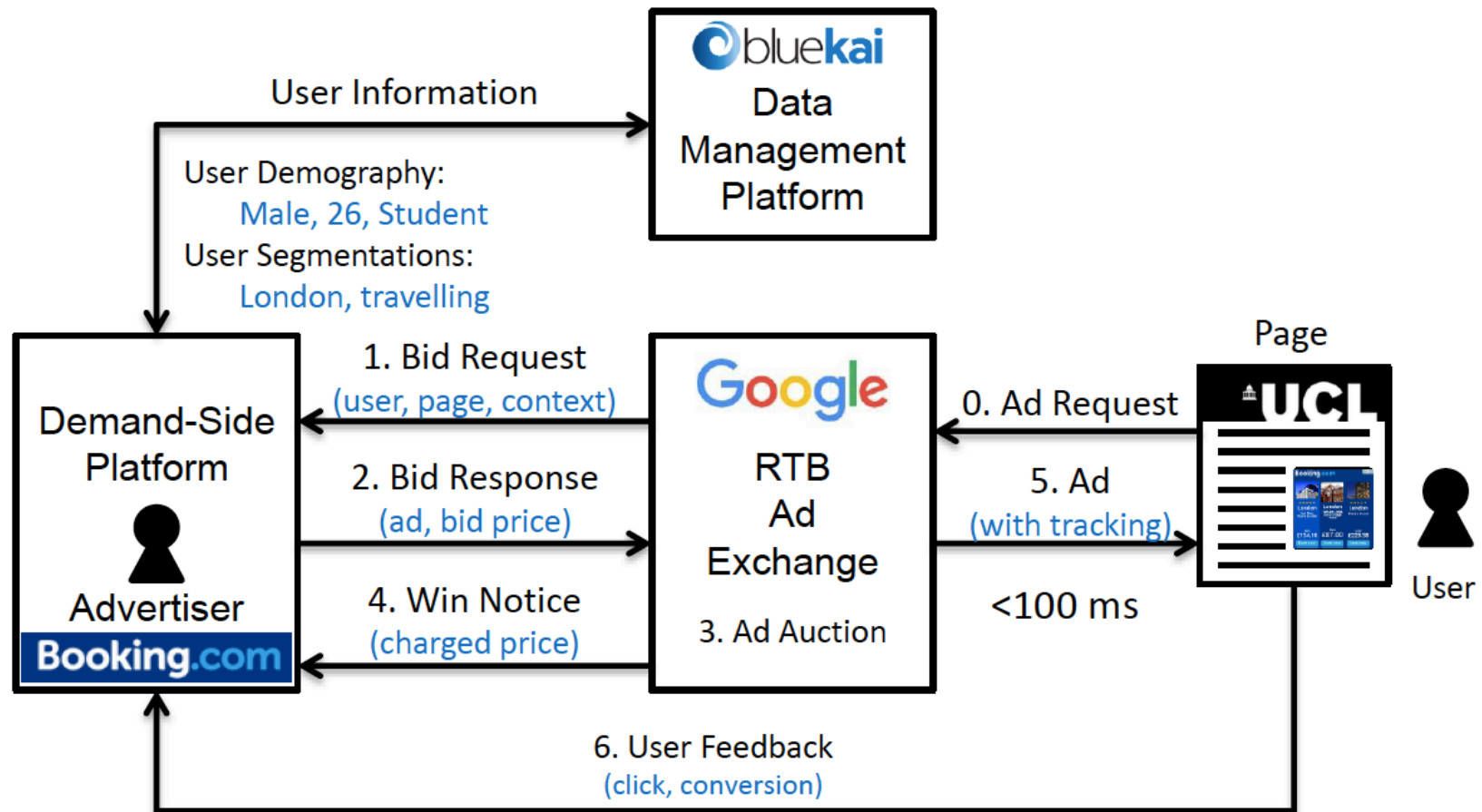
Ascending-bid auction

- What's the optimal strategy for a buyer?
 - Bid until the current bid value exceeds your *private value*
- Usually revenue-maximizing and efficient, unless the reserve price is set too low or too high
- Disadvantages
 - Collusion
 - Lack of competition
 - Has high communication costs

Sealed-bid auction

- Each buyer makes a single bid and communicates it to the auctioneer, but not to the other bidders
 - Simpler communication
 - More complicated decision-making: the strategy of a buyer depends on what they believe about the other buyers
 - Not necessarily efficient
- **Sealed-bid second-price auction:** the winner pays the price of the second-highest bid
 - Let V be your private value and B be the highest bid by any other buyer
 - If $V > B$, your optimal strategy is to bid above B – in particular, bid V
 - If $V < B$, your optimal strategy is to bid below B – in particular, bid V
 - Therefore, your dominant strategy is to bid V
 - This is a **truth revealing** mechanism

Application: Real-time bidding for Internet ads



Application: Real-time bidding for Internet ads

Buying advertising in Real Time - an Example

0.04 Sec – User ABC clicks on a URL and the publisher's content begins to load in browser

0.08 Sec – Publisher asks its ad server if an ad is available. If no ad, server asks Ad Exchange

0.10 Sec – Ad Exchange federates ad request to multiple demand side platforms(DSPs), the technology for buying media

0.12 Sec – Ad Exchange sends each DSP User ABC's anonymous profile website category and page ad safety information

0.125 Sec – Each DSP overlays advertiser targeting and budget rules, and applies third-party data

0.13 Sec – Each DSP algorithm evaluates and computes optimal bid for advertiser

0.14 Sec – Each DSP responds to Ad Exchange

0.18 Sec – Ad Exchange runs a second-price auction and selects winning bid from DSP responses

0.19 Sec – Ad Exchange sends price and ad from winning bid to publisher's ad server

0.23 Sec – Publisher's ad server tells browser which ad to display

0.31 Sec – Advertiser's ad server sends winning ad to browser

0.36 Sec – Browser displays web page including winning ad, and signals to winning DSP the ad was viewed

https://en.wikipedia.org/wiki/Real-time_bidding

Dollar auction

- A dollar bill is auctioned off to the highest bidder, but the second-highest bidder has to pay the amount of his last bid
 - Player 1 bids 1 cent
 - Player 2 bids 2 cents
 - ...
 - Player 2 bids 98 cents
 - Player 1 bids 99 cents
 - If Player 2 passes, he loses 98 cents, if he bids \$1, he might still come out even
 - So Player 2 bids \$1
 - Now, if Player 1 passes, he loses 99 cents, if he bids \$1.01, he only loses 1 cent
 - ...
- What went wrong?
 - When figuring out the expected utility of a bid, a rational player should take into account the future course of the game
- What if Player 1 starts by bidding 99 cents?

Dollar auction

- A dollar bill is auctioned off to the highest bidder, but the second-highest bidder has to pay the amount of his last bid
- Dramatization:
<https://www.youtube.com/watch?v=pA-SNscNADk>

Regulatory mechanism design: Tragedy of the commons

- States want to set their policies for controlling emissions
 - Each state can reduce their emissions at a cost of -10 or continue to pollute at a cost of -5
 - If a state decides to pollute, -1 is added to the utility of every other state
- What is the dominant strategy for each state?
 - Continue to pollute
 - Each state incurs cost of $-5 - 49 = -54$
 - If they all decided to deal with emissions, they would incur a cost of only -10 each
- Mechanism for fixing the problem:
 - Tax each state by the total amount by which they reduce the global utility (**externality cost**)
 - This way, continuing to pollute would now cost -54

Review: Game theory

- Normal form representation of a game
- Dominant strategies
- Nash equilibria
- Pareto optimal outcomes
- Pure strategies and mixed strategies
- Examples of games
- Mechanism design
 - Auctions: ascending bid, sealed bid, sealed bid second-price, “dollar auction”
 - Regulatory mechanism design