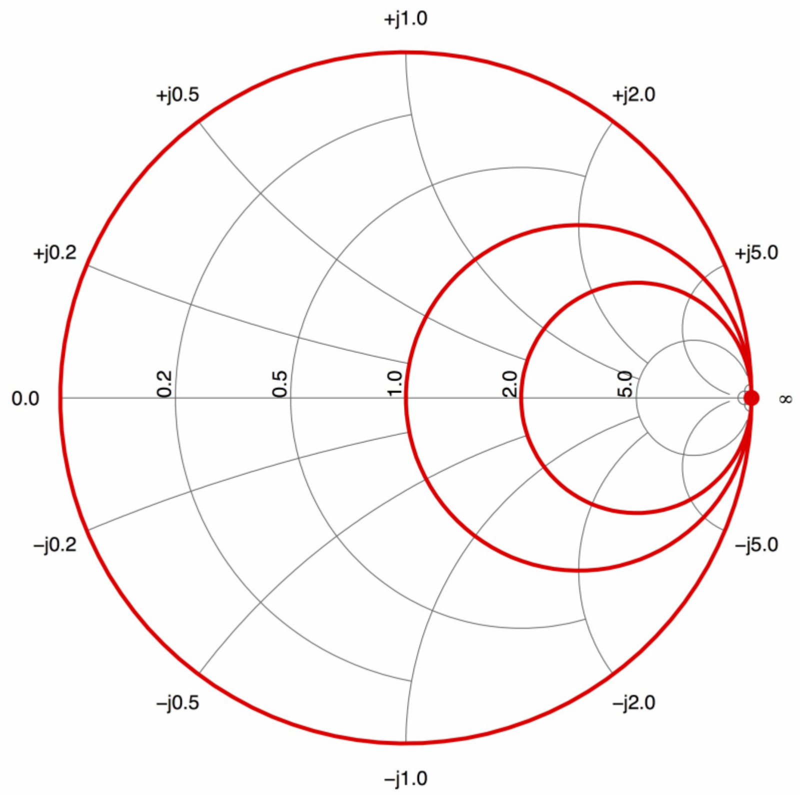
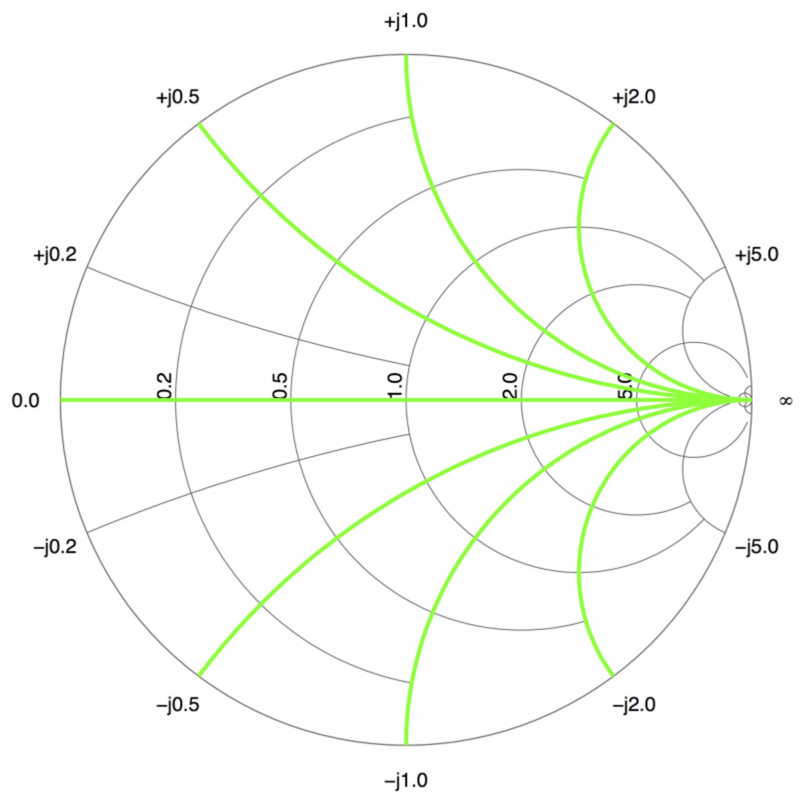
ECE 329 **Homework 13 - Solution** **Due: Tuesday Dec 5, 2017, 6 PM**

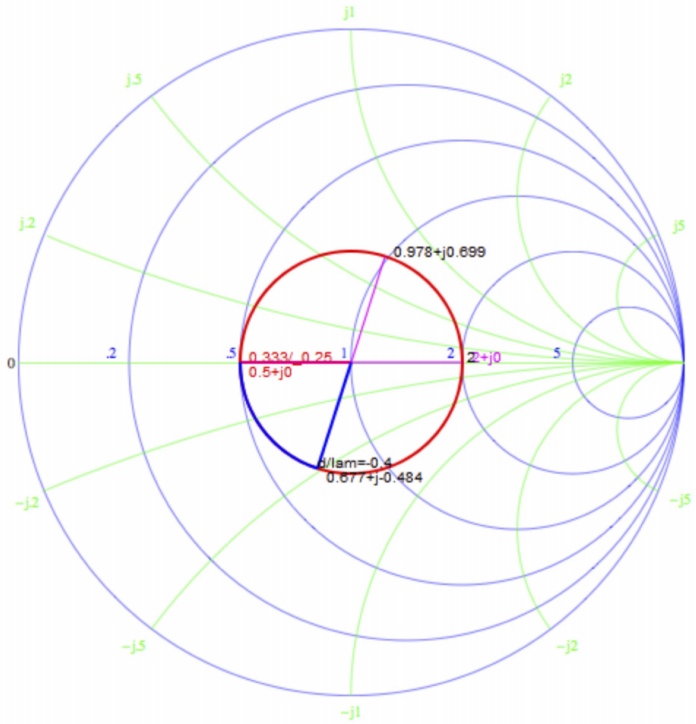
1. Smith chart:





2.

a) (i) $z_L = Z_L/Z_o = 0.5$. Locating the point z_L on the Smith Chart, we can find the corresponding $\Gamma_L = 0.333\angle 180^o$. (ii) (iii) Rotating along the constant $|\Gamma|$ circle towards the generator (clockwise) by a distance of $l = 0.4\lambda$, we find $\Gamma(l) = 0.333\angle\theta^o$, where $\theta = 180^o - \frac{0.4\lambda}{0.5\lambda} \times 360^o = -108^o$, and $z(l) = 0.675 - j0.481$. Then, $Z(l) = Zo \times z(l) = 67.5 - j48.1\ \Omega$.



b)

$$V(l) = \frac{Z(l)}{Z(l) + Z_g} V_g = \frac{67.5 - j48.1}{67.5 - j48.1 + 100} \times 10 = 4.48 - j1.58 = 4.76\angle - 19.4^o\ V.$$

c)

$$V(l) = V^+(e^{j\beta l} + \Gamma_L e^{-j\beta l}) = V^+(e^{0.8\pi} - 0.333e^{-0.8\pi}),$$
$$\therefore \qquad V^+ = -4.05 - j2.94 = 5\angle - 144^o\ V.$$

d)

$$V(0) = V^+(1 + \Gamma_L) = V^+(1 - 0.333) = -2.70 - j1.97 = 3.34\angle - 144^o\ V.$$

e)

$$I(0) = \frac{V(0)}{Z_L} = -0.054 - j0.0394 = 0.0668\angle - 144^o\ A.$$

3. In this case, $Z_L = Z_o$ (matched), so we do not need a Smith Chart.

a)

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = 0,$$
$$\Gamma(l) = \Gamma_L e^{-2j\beta d} = 0,$$
$$Z(l) = Z_0 = 100\ \Omega.$$

b)

$$V(l) = \frac{Z(l)}{Z(l) + Z_g} V_g = \frac{100}{100 + 100} \times 10 = 5\ V.$$

c)

$$V(l) = V^+ e^{j\beta l} = V^+ e^{0.8\pi},$$

$$\therefore \qquad V^+ = 5e^{-0.8\pi} = 5\angle - 144^o\ V.$$

d)

$$V(0) = V^+ = 5\angle - 144^o\ V.$$

e)

$$I(0) = \frac{V(0)}{Z_L} = -0.05\angle - 144^o\ A.$$

4.

a) (i) $Z_L = R + \frac{1}{j\omega C} = 50 - j200\ \Omega$, $z_L = Z_L/Z_o = 0.5 - 2j$. Locating the point z_L on the Smith Chart, we can find the corresponding $\Gamma_L = 0.82\angle - 50.9^o$. (ii) (iii) Rotating along the constant $|\Gamma|$ circle towards the generator (clockwise) by a distance of $l = 0.4\lambda$, we find $\Gamma(l) = 0.82\angle 21.75^o$, and $z(l) = 2.155 + j4.13$. Then, $Z(l) = Z_o \times z(l) = 215.5 + j413\,\Omega$.

b)

$$V(l) = \frac{Z(l)}{Z(l) + Z_g} V_g = \frac{215.5 + j413}{215.5 + j413 + 100} \times 10 = 8.83 + j1.53 = 8.96\angle 9.82^o\ V.$$

c)

$$V(l) = V^+(e^{j\beta l} + \Gamma_L e^{-j\beta l}) = V^+(e^{j0.8\pi} + 0.82\angle - 50.9^o \cdot e^{-j0.8\pi}),$$

$$\therefore \qquad V^+ = -4.034 - j2.97 = 5.01\angle - 143.68^o\ V.$$

d)

$$V(0) = V^+(1 + \Gamma_L) = V^+(1 + 0.82\angle - 50.9^o) = -8.01 - j1.93 = 8.24\angle - 166.43^o\ V.$$

e)

$$I(0) = \frac{V(0)}{Z_L} = 3.29 \times 10^{-4} - j0.04 = 0.04\angle - 90.47^o\ A.$$