ECE 329 REVIEW FOR EXAM 2

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EXAM 2 INFO

- ECEB 2013 (for Section C)
- ECEB 2015 (for Section X)
- ECEB 2017 (for Section E)
- Wednesday 7- 8: 15 PM

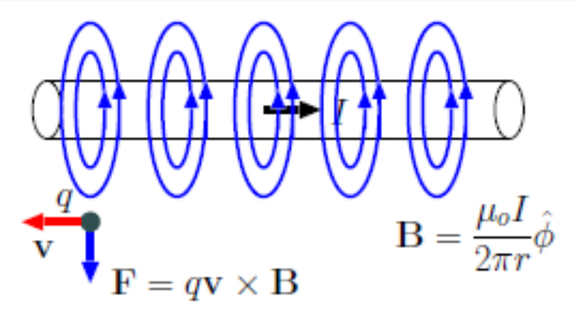
- Wednesday 5: 30 – 6:45 PM (for Conflict)

- ONE 3 X 5 inch card of notes allowed

CONCEPT LIST

- Ampere's law (lecture 12 & 13)
- Faraday's law (lecture 14)
- Inductance (lecture 15)
- Charge conservation (lecture 16 & 17)
- TEM wave solutions (lecture 18 & 19)

AMPERE'S LAW



- Ic stands for the net sum of all filament currents In crossing any surface S bounded by path C

- The direction of flow is given by the "right-hand-rule"

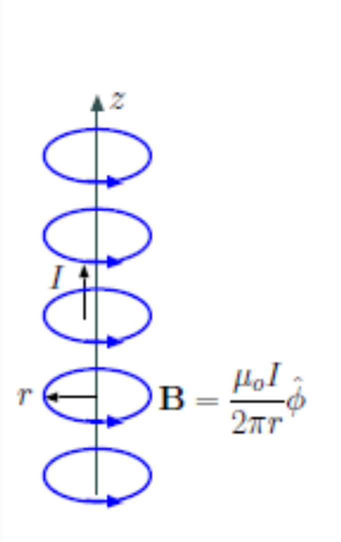
$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_o I_C$$

$$\mathbf{H} \equiv \mu_o^{-1}\mathbf{B}$$

$$I_C = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$
$$\nabla \cdot \mathbf{B} = 0$$

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S}$$



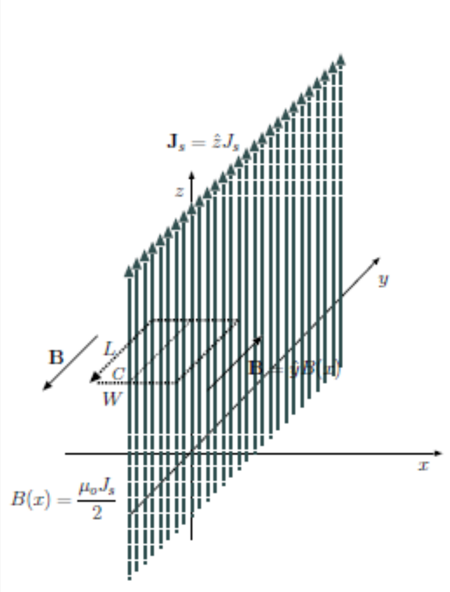
$$\mathbf{J}(x, y, z) = \mathbf{J}_s(y, z)\delta(x - x_o)$$

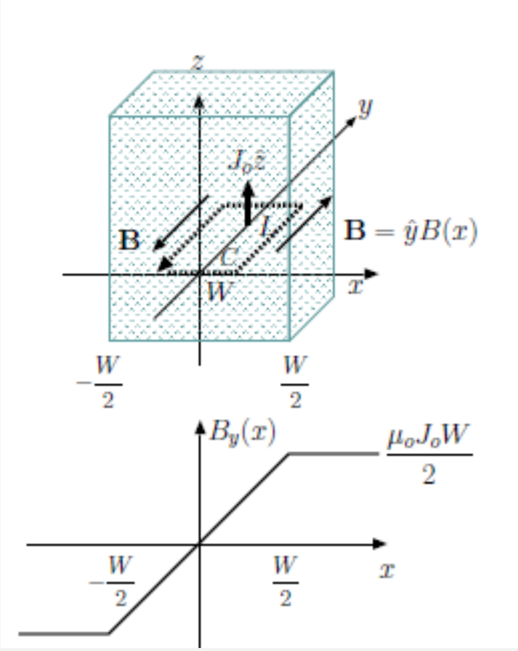
- This is the volumetric current density representation of a surface current density $J_s(x, y)$ measured in A/m units flowing on $x = x_o$ surface.

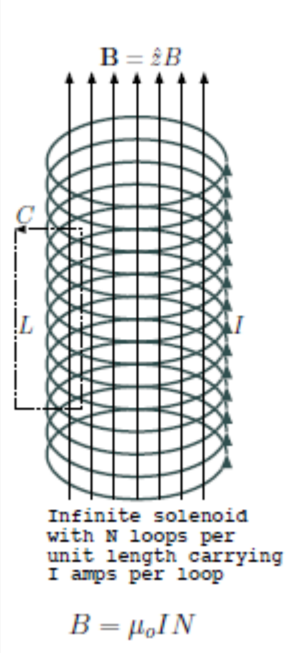
$$\mathbf{J}(x, y, z) = \hat{z}I(z)\delta(x - x_o)\delta(y - y_o)$$

- This is the line current I(z) measured in A units flowing in z-direction along a filament defined by the intersections of $x = x_o$ and $y = y_o$ surfaces.

DRAW THE LOOP







$$B(x)L + 0 - B(-x)L + 0 = \mu_o J_s L$$

$$\mathbf{B} = \hat{y}\frac{\mu_o J_s}{2}\text{sgn}(x) \text{ and } \mathbf{H} = \hat{y}\frac{J_s}{2}\text{sgn}(x)$$

$$B(x)L + 0 - B(-x)L + 0 = \mu_o J_o 2xL \Rightarrow B(x) = \mu_o J_o x$$

$$B(x)L + 0 - B(-x)L + 0 = \mu_o J_o W L \Rightarrow B(x) = \mu_o J_o \frac{W}{2}$$

$$\mathbf{H} = \begin{cases} \hat{y}J_o x, & |x| < \frac{W}{2} \\ \hat{y}J_o \frac{W}{2}\,\text{sgn}(x), & \text{otherwise} \end{cases}$$

$$\oint_C \mathbf{B}\cdot d\mathbf{l} = \mu_o I_C \Rightarrow LB = \mu_o INL$$

$$B = \mu_o IN \text{ and } \mathbf{H} = \hat{z}IN$$

$$\mathbf{B} = 0 \text{ for the exterior region}$$

FARADAY'S LAW

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\Psi \equiv \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\mathcal{E} = -\frac{d\Psi}{dt}, \qquad \text{Faraday's law}$$

$$\mathcal{E} = \oint_C \mathbf{E} \cdot d\mathbf{l} \qquad I = \frac{\mathcal{E}}{R}$$

$$\mathcal{E} = \oint_C (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

- The right hand side of the integral form equation above includes the flux of rate of change of magnetic field B over surface S.
- Ψ is the rate of change of magnetic flux.
- Induced emf (short for electro-motive force) represents the work done per unit charge moved once around path C.
- According to Faraday's law it appears that a non-zero emf can always be caused by magnetic flux variations dΨ/dt independent of how the variations are produced — the possibilities are:

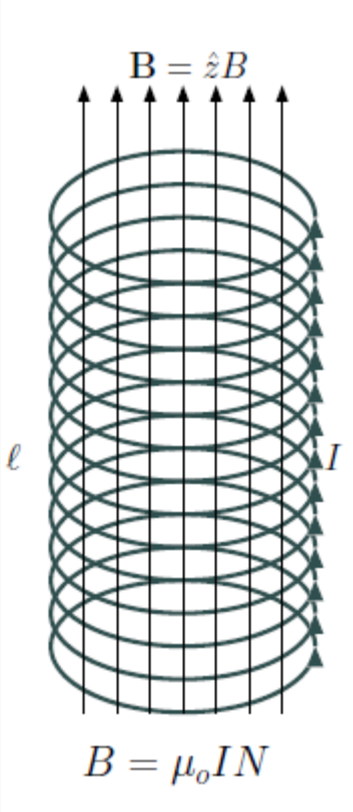
1. Fixed C, but time-varying B,
2. B =const. (in space and time), but time-varying C (rotating or changing size),
3. An inhomogeneous static B = B(r) in the measurement frame and C in motion.

INDUCTANCE

$$L \equiv \frac{n\Psi}{I}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

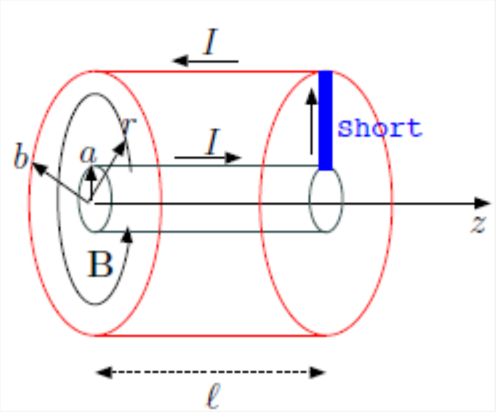
- This is the self-inductance and emf for an inductor consisting of n-loops.



- **Inductance of long solenoid:**
- Consider a long solenoid with length ℓ, cross-sectional area A, density of N loops per unit length and n = Nℓ is the number of turns of the solenoid.

$$\mathbf{B} = \mu_o I N \hat{z}$$

$$L = \frac{n\Psi}{I} = \frac{N\ell(\mu_o I N)A}{I} = N^2\mu_o A\ell$$



- **Inductance of shorted coax:**
- Consider a coaxial cable of some length ℓ which is "shorted" at one end (with a wire connecting the inner and outer conductors)

$$\oint_C \mathbf{B}\cdot d\mathbf{l} = \mu_o I_C$$

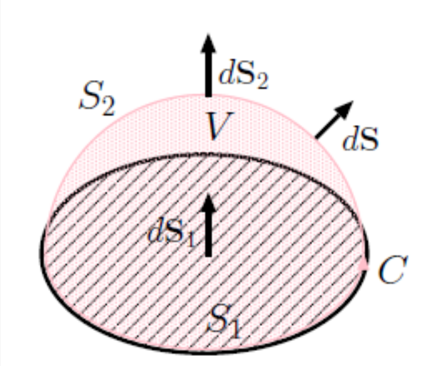
$$B_\phi 2\pi r = \mu_o I$$

$$B_\phi = \frac{\mu_o I}{2\pi r}$$

$$\Psi = \int_S \mathbf{B}\cdot d\mathbf{S} = \ell\frac{\mu_o}{2\pi}I\int_a^b \frac{dr}{r} = \ell\frac{\mu_o}{2\pi}\ln\frac{b}{a}I$$

$$L \equiv \frac{\ln\frac{b}{a}}{2\pi}\ell\mu_o$$

CHARGE CONSERVATION



$$\int_V \frac{\partial \rho}{\partial t} dV = -\oint_S \mathbf{J} \cdot d\mathbf{S}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{J}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$$

- Continuity equation is a mathematical re-statement of the principle of conservation of charge.

$$\begin{aligned} \nabla \cdot \mathbf{D} &= \rho && \text{Gauss's law} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} && \text{Faraday's law} \\ \nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, && \text{Ampere's law} \end{aligned}$$

- The modified Ampere's law postulated by Maxwell under the assumption that Gauss's law is also valid under time-varying conditions, leads to some specific predictions about how time-varying fields should behave.

$$\begin{aligned} \hat{n} \cdot (\mathbf{D}^+ - \mathbf{D}^-) &= \rho_s \\ \hat{n} \cdot (\mathbf{B}^+ - \mathbf{B}^-) &= 0 \\ \hat{n} \times (\mathbf{E}^+ - \mathbf{E}^-) &= 0 \\ \hat{n} \times (\mathbf{H}^+ - \mathbf{H}^-) &= \mathbf{J_s} \end{aligned}$$

$$\begin{aligned} \mathbf{D} &= \epsilon \mathbf{E} \\ \mathbf{B} &= \mu \mathbf{H} \end{aligned}$$

- M is referred to as magnetization field; χm is a dimensionless parameter called magnetic susceptibility.

$$\mathbf{H} = \mu_o^{-1} \mathbf{B} - \mathbf{M}$$

$$\mathbf{B} = \mu_o (1 + \chi_m) \mathbf{H} = \mu \mathbf{H}$$

TEM WAVE SOLUTIONS

$$-\nabla^2\mathbf{E} = -\mu\frac{\partial}{\partial t}\nabla\times\mathbf{H}$$

$$\nabla^2\mathbf{E} = \mu\epsilon\frac{\partial^2\mathbf{E}}{\partial t^2}$$

$$\frac{\partial^2\mathbf{E}}{\partial x^2} + \frac{\partial^2\mathbf{E}}{\partial y^2} + \frac{\partial^2\mathbf{E}}{\partial z^2} = \mu\epsilon\frac{\partial^2\mathbf{E}}{\partial t^2}$$

$$\mathbf{E}(\mathbf{r},t) = \hat{x}E_x(z,t)$$

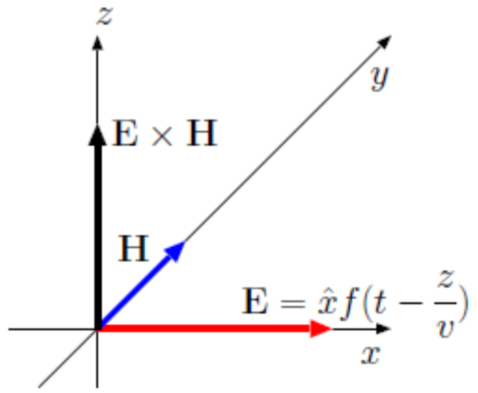
$$\frac{\partial^2 E_x}{\partial z^2} = \mu\epsilon\frac{\partial^2 E_x}{\partial t^2}$$

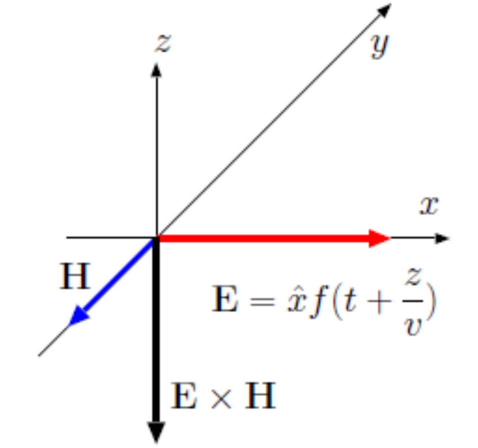
$$E_x = \cos(\omega(t \mp \frac{z}{v})) \qquad v \equiv \frac{1}{\sqrt{\mu\epsilon}}$$

$$\mathbf{H} = \pm\hat{y}\sqrt{\frac{\epsilon}{\mu}}\cos(\omega(t \mp \frac{z}{v})) \qquad \eta \equiv \sqrt{\frac{\mu}{\epsilon}}$$

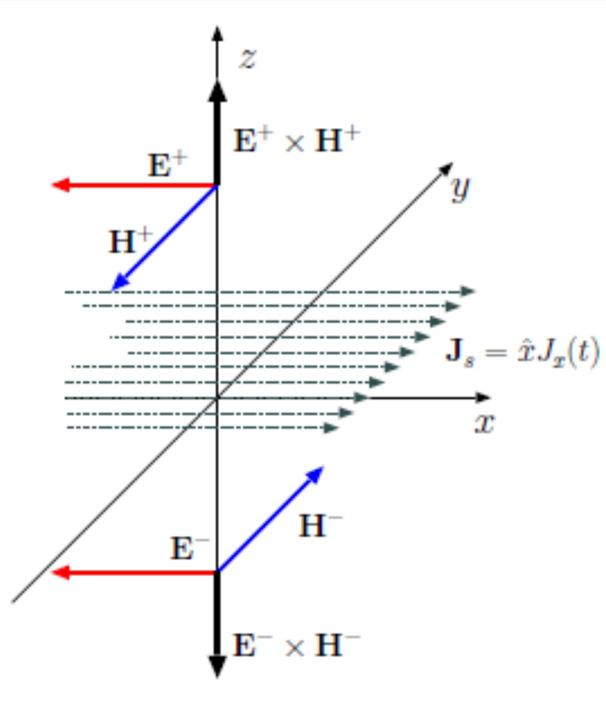
- The curl of Faraday's law combines with the Ampere's law to produce the 3D vector wave equation.

- 1D scalar wave equation is a field solution that only depends on z and t and "polarized" in x-direction.





TEM WAVE SOLUTIONS



$$\mathbf{J}_s = \hat{x}f(t) \;\; \text{on } z = 0 \;\; \text{plane}$$

$$\mathbf{E}^{\pm} = -\hat{x}\frac{\eta f(t \mp \frac{z}{v})}{2} \;\text{ and }\; \mathbf{H}^{\pm} = \mp\hat{y}\frac{f(t \mp \frac{z}{v})}{2} \;\text{ in regions } z \gtrless 0$$

1. Ex and Hy waveforms are proportional to delayed versions of surface current Jx(t) at each location z above and below the current sheet, with the reference directions of E and Js opposing one another.

2. Fields E± are continuous on z = 0 surface in compliance with tangential boundary condition equations.

3. Fields H± exhibit a discontinuity on z = 0 surface that matches the current density of the same surface, once again in compliance with tangential boundary condition equations.

4. Opposing E and Js vectors on z = 0 plane indicate that the surface is acting as a source of radiated energy

PRACTICE PROBLEMS

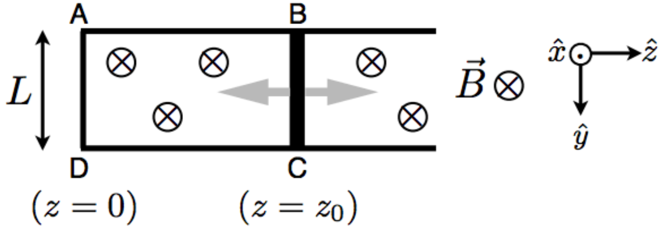
3. Consider a hollowed out cylinder centered on the z-axis with inner radius a and outer radius b such that it is described by $a < r < b$. The hollow cylinder conducts a uniform current density of $\mathbf{J} = J_0\hat{z}$ A/m^2 in the region $a < r < b$. Outside this region, that is for $r > b$ and $r < a$, the charge and current densities are zero.

   a) Using the integral form of Ampere's law find $\mathbf{B}$ everywhere. (You can simplify your answer using $I = J_0 \cdot A$)

   b) For $r > b$ what does the magnetic field $\mathbf{B}$ produced by a hollow cylinder look like? (Hint: See Lecture 12 online, this is the magnetic analog to a hollow sphere of charge).

PRACTICE PROBLEMS

4. As shown in the diagram below, a pair of conducting rails separated by a distance L is connected at $z = 0$ to a fixed conducting rod (AD) and at $z = z_0$ to a conducting armature (BC) that can slide along the rail in the $\pm\hat{z}$ direction. A constant magnetic field $\mathbf{B} = -B_0\,\hat{x}$ exists in the region, which is shown pointing down into the page in the diagram. The armature is mechanically pulled in the $+\hat{z}$ direction at a constant velocity $\mathbf{v} = v_0\hat{z}$ m/s from its starting position at $z = z_0$, and the changing magnetic flux through the loop ABCD induces an electromotive force $\mathcal{E}$ and thus a current I_0 around the conducting loop.



a) What is the emf $\mathcal{E}$ induced in the loop ABCD?

b) What is the magnitude and direction of the induced current I_0 in terms of the resistance R of the conducting loop ABCD?

c) What is the magnitude and direction of the Lorentz force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ that is exerted on the armature by the magnetic field $\mathbf{B}$? **Hint**: express your answer in terms of the induced current I_0.

d) Now consider that the armature is no longer moved mechanically (though it is still free to move in response to Lorentz forces) and that a projectile of mass M is attached to it. A constant current of magnitude I' is introduced into the loop at point A such that it flows along the contour ABCD. The magnetic field generated by this current loop is negligibly small compared to the background magnetic field $\mathbf{B}$, and the mass of the armature is negligibly small compared to M. What is the magnitude and direction of the acceleration $\mathbf{a}$ of the projectile? **Hint**: $\mathbf{F} = M\mathbf{a}$.

PRACTICE PROBLEMS

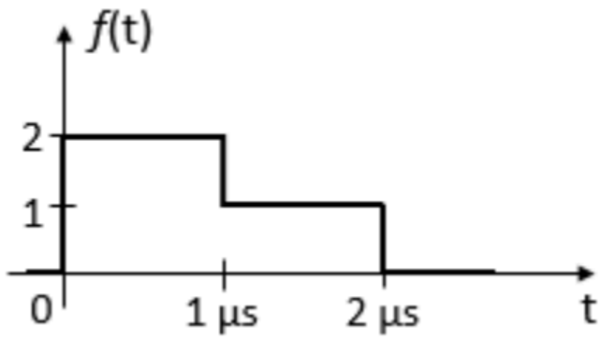
3.

a) For current density $\mathbf{J} = (5z^2\,\hat{x} + 4x^3y\,\hat{y} + 3z(y - y_o)^2\,\hat{z})$ A/m^2, which is time independent, find the charge density $\rho(0,t)$ at the origin (0,0,0) as a function of time t, if $\rho = 0$ at that location and time $t = 0$, $y_o = 2$ m, and coordinates x, y, and z are specified in meter units. **Hint:** use the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0.$$

PRACTICE PROBLEMS

3. (25 points) Consider a time varying sheet current $\mathbf{J}_s$ in $yz-$ plane at $x = 0$. In $x > 0$ region, the magnetic field of the plane wave caused by the sheet current is $\mathbf{H}(x,t) = f(t - \frac{x}{c})\hat{y}\,\frac{\mathrm{A}}{\mathrm{m}}$. Assume all region is free space. The plot for $f(t)$ is shown below.



a) (8 points) Plot $H_y(t = 4\mu\mathrm{s}, x)$ vs. x for $x \in [-1500\,\mathrm{m}, +1500\,\mathrm{m}]$. Be sure to clearly label all axis.

PRACTICE PROBLEMS

b) (8 points) Plot $H_y(t, x = 1200\,\mathrm{m})$ vs. t for $t \in [0, 10\mu\mathrm{s}]$. Be sure to clearly label all axis.

c) (9 points) Plot $\mathbf{E}(t = 4\mu\mathrm{s}, x)$ vs. x for $x \in [-1500\,\mathrm{m}, +1500\,\mathrm{m}]$. You only need to plot the non-zero components of $\mathbf{E}$. Be sure to indicate the component and clearly label all axis.


Thanks
a
Bunch!