1.

a) For the wave described by $\mathbf{E}_1 = 4\cos(\omega t - \beta z)\hat{x}\,\frac{\mathrm{V}}{\mathrm{m}}$,

i. the corresponding phasors are given by

$$\tilde{\mathbf{E}}_1 = \hat{x}4e^{-j\beta z}\,\frac{\mathrm{V}}{\mathrm{m}}$$

$$\tilde{\mathbf{H}}_1 = \hat{y}\frac{4}{\eta_o}e^{-j\beta z}\,\frac{\mathrm{A}}{\mathrm{m}}.$$

ii. It is **linearly polarized** in $\hat{x}$-direction.

b) For the wave described by $\mathbf{E}_2 = 5\cos(\omega t - \beta y)\hat{x} + 5\sin(\omega t - \beta y)\hat{z}\,\frac{\mathrm{V}}{\mathrm{m}}$,

i. the corresponding phasors are given by

$$\tilde{\mathbf{E}}_2 = \hat{x}5e^{-j\beta y} - \hat{z}5je^{-j\beta y} = 5e^{-j\beta y}\left(\hat{x} - j\hat{z}\right)\,\frac{\mathrm{V}}{\mathrm{m}}$$

$$\tilde{\mathbf{H}}_2 = \frac{5}{\eta_o}e^{-j\beta y}\left(-\hat{z} - j\hat{x}\right)\,\frac{\mathrm{A}}{\mathrm{m}}.$$

ii. Given that the wave propagates along $\hat{y}$ direction, it is seen that the wave is **left-hand-circularly** polarized.

c) For the wave described by $\mathbf{H}_3 = 2\cos(\omega t + \beta z + \frac{\pi}{3})\hat{x} + 2\sin(\omega t + \beta z - \frac{\pi}{6})\hat{y}\,\frac{\mathrm{A}}{\mathrm{m}}$,

i. the corresponding phasors are given by

$$\tilde{\mathbf{H}}_3 = \hat{x}2e^{j\beta z}e^{j\frac{\pi}{3}} - \hat{y}2je^{j\beta z}e^{j(\frac{\pi}{3}-\frac{\pi}{2})} = 2e^{j(\beta z+\frac{\pi}{3})}\left(\hat{x} - \hat{y}\right)\,\frac{A}{\mathrm{m}}$$

$$\tilde{\mathbf{E}}_3 = 2\eta_o e^{j(\beta z+\frac{\pi}{3})}\left(\hat{y} + \hat{x}\right)\,\frac{V}{\mathrm{m}}.$$

ii. Thus the wave is **linearly polarized** in $\frac{\hat{x}+\hat{y}}{\sqrt{2}}$ direction.

d) For the wave described by $\mathbf{H}_4 = 4\cos(\omega t - \beta x)\hat{z} - 3\sin(\omega t - \beta x)\hat{y}\,\frac{\mathrm{A}}{\mathrm{m}}$,

i. the corresponding phasors are given by

$$\tilde{\mathbf{H}}_4 = \hat{z}4e^{-j\beta x} + \hat{y}3je^{-j\beta x} = e^{-j\beta x}\left(4\hat{z} + 3j\hat{y}\right)\,\frac{\mathrm{A}}{\mathrm{m}}$$

$$\tilde{\mathbf{E}}_4 = \eta_o e^{-j\beta x}\left(4\hat{y} - 3j\hat{z}\right)\,\frac{V}{\mathrm{m}}.$$

ii. Since the two components have different magnitudes, the wave is **elliptical** polarized.

e) For the wave described by $\mathbf{H}_5 = 2\sin(\omega t + \beta y)\hat{x} - 2\sin(\omega t + \beta y - \frac{\pi}{4})\hat{z}\,\frac{\mathrm{A}}{\mathrm{m}}$,

i. the corresponding phasors are given by

$$\tilde{\mathbf{H}}_5 = -\hat{x}2je^{j\beta y} + \hat{z}2je^{j(\beta y-\frac{\pi}{4})} = 2je^{j\beta y}\left(-\hat{x} + e^{-j\frac{\pi}{4}}\hat{z}\right)\,\frac{\mathrm{A}}{\mathrm{m}}$$

$$\tilde{\mathbf{E}}_5 = 2\eta_o je^{j\beta y}\left(\hat{z} + e^{-j\frac{\pi}{4}}\hat{x}\right)\,\frac{V}{\mathrm{m}}.$$

ii. Since the phase angle between $\tilde{\mathbf{E}}_5$ and $\tilde{\mathbf{H}}_5$ is $\frac{\pi}{4}$, not an integer multiple of $\frac{\pi}{2}$, it is seen that the wave is **elliptical** polarized.

2. The phasors form of surface current densities are

$$\tilde{\mathbf{J}}_{s1} = \hat{z} J_1 e^{-j\phi} \, \frac{\mathrm{A}}{\mathrm{m}} \quad (x = 0),$$

$$\tilde{\mathbf{J}}_{s2} = \hat{y} J_2 \, \frac{\mathrm{A}}{\mathrm{m}} \quad (x = \frac{\lambda}{4}).$$

Then, recalling $\beta = \frac{2\pi}{\lambda}$, the corresponding electric fields propagating in the region $x > \frac{\lambda}{4}$ are given by

$$\begin{aligned} \tilde{\mathbf{E}}_1 &= -\hat{z} \frac{\eta_o}{2} J_1 e^{-j\phi} e^{-j\beta x} = \hat{z} \frac{\eta_o}{2} J_1 e^{j(-\beta x - \phi + \pi)} \, \frac{\mathrm{V}}{\mathrm{m}}, \\ \tilde{\mathbf{E}}_2 &= -\hat{y} \frac{\eta_o}{2} J_2 e^{-j\beta(x - \frac{\lambda}{4})} = \hat{y} \frac{\eta_o}{2} J_2 e^{j(-\beta x + \frac{3\pi}{2})} \, \frac{\mathrm{V}}{\mathrm{m}}. \end{aligned}$$

a) The total field is $\tilde{\mathbf{E}} = \tilde{\mathbf{E}}_1 + \tilde{\mathbf{E}}_2 = \frac{\eta_o}{2} J e^{j(-\beta x + \pi)} (\hat{y} e^{j\frac{\pi}{2}} + \hat{z} e^{-j\phi})$ where $J = J_1 = J_2$.

i. The direction of propagation is $+\hat{x}$ and the wave is **RHC**. Then, the $\hat{y}$ component needs to lead the $\hat{z}$ component by 90^o. Therefore

$$\phi = -\frac{\pi}{2} + \frac{\pi}{2} + 2n\pi = 2n\pi,$$

where n is an arbitrary integer. The electric field phasor for the region will be $\tilde{\mathbf{E}} = \frac{\eta_o}{2} J e^{j(-\beta x + \pi)} (\hat{y} e^{j\frac{\pi}{2}} + \hat{z} e^{j2n\pi}) \, \frac{\mathrm{V}}{\mathrm{m}}$.

ii. To have **LHC** polarization, the $\hat{z}$ component needs to lead by 90^o the $\hat{y}$ component. Therefore

$$\phi = -\frac{\pi}{2} - \frac{\pi}{2} + 2n\pi = 2n\pi - \pi,$$

where n is an arbitrary integer. The electric field phasor for the region will be $\tilde{\mathbf{E}} = \frac{\eta_o}{2} J e^{j(-\beta x + \pi)} (\hat{y} e^{j\frac{\pi}{2}} - \hat{z} e^{j2n\pi}) \, \frac{\mathrm{V}}{\mathrm{m}}$.

iii. To have linear polarization, the $\hat{z}$ component needs to be in phase with the $\hat{y}$ component or off by 180^o. Thus, we write

$$\phi = -\frac{\pi}{2} + n\pi,$$

where n is an arbitrary integer. The electric field phasor for the region will be $\tilde{\mathbf{E}} = \frac{\eta_o}{2} J e^{j(-\beta x + \pi)} (\hat{y} e^{j\frac{\pi}{2}} - \hat{z} j e^{jn\pi}) \, \frac{\mathrm{V}}{\mathrm{m}}$.

b) The corresponding magnetic field is

$$\tilde{\mathbf{H}} = \frac{1}{2} J e^{j(-\beta x + \pi)} (\hat{z} e^{j\frac{\pi}{2}} - \hat{y} e^{-j\phi}) \, \frac{\mathrm{A}}{\mathrm{m}},$$

where $J = J_1 = J_2 = 1\,\mathrm{A/m}$. Therefore, the time-averaged Poynting vector is

$$\begin{aligned} \langle \mathbf{S} \rangle &= \frac{1}{2} \mathbf{Re} \left\{ \tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^* \right\} = \frac{1}{2} \frac{\eta_o}{4} J^2 (\hat{x} + \hat{x}) \\ &= \hat{x} \frac{\eta_o}{4} \end{aligned}$$

This result does not depend on the angle ϕ, therefore the time-averaged Poynting vector

$$\langle \mathbf{S} \rangle = \hat{x}\, 30\pi \, \frac{\mathrm{W}}{\mathrm{m}^2}$$

will be the same for cases (i) and (ii).

c) If $J_2 = 0$, then $\tilde{\mathbf{E}}_2 = 0$, and $\tilde{\mathbf{H}}_2 = 0$. The magnetic field due to $\tilde{\mathbf{E}}_1$ is

$$\tilde{\mathbf{H}}_1 = -\hat{y}\frac{1}{2}J_1 e^{j(-\beta x - \phi + \pi)}\ \frac{\mathrm{A}}{\mathrm{m}},$$

therefore the time-averaged Poynting vector is

$$\begin{aligned}\langle \mathbf{S}_1 \rangle &= \frac{1}{2}\mathbf{Re}\left\{\tilde{\mathbf{E}}_1 \times \tilde{\mathbf{H}}_1^*\right\} = \frac{1}{2}\frac{\eta_o}{4}J_1^2 \\ &= \hat{x}\frac{\eta_o}{8}J^2 = \hat{x}\,15\pi\ \frac{\mathrm{W}}{\mathrm{m}^2}.\end{aligned}$$

d) From the results of (b) and (c), we can see that in case of circularly polarized waves the power content is twice that of a linearly polarized wave field of an equal instantaneous peak electric field magnitudes.

3. When a wave is incident on a boundary between two differnt media, a reflected wave is produced. In addition, if the second medium is not a perfect conductor, a transmitted wave is set up. Together, these waves satisfy the boundary conditions at the interface of the two media. We shall assume that a $(+)$ wave is incident from medium 1 ($z < 0$) onto the interface, thereby setting up a reflected $(-)$ wave in that medium, and a transmitted wave in medium 2 ($z > 0$). Then we can write the solution for the complex field components in medium 1 to be

$$\tilde{\mathbf{E}}_{1x} = E_1^+ e^{-j\beta_1 z}\hat{x} + E_1^- e^{j\beta_1 z}\hat{x}$$

$$\begin{aligned}\tilde{\mathbf{H}}_{1y} &= H_1^+ e^{-j\beta_1 z}\hat{y} + H_1^- e^{j\beta_1 z}\hat{y} \\ &= \frac{1}{\eta_1}(E_1^+ e^{-j\beta_1 z} - E_1^- e^{j\beta_1 z})\hat{y}\end{aligned}$$

where $\beta_1 = \frac{\omega}{v_1} = \omega\sqrt{\mu_1\epsilon_1}$ and $\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$. The complex filed components in medium 2 are given by

$$\tilde{\mathbf{E}}_{2x} = E_2^+ e^{-j\beta_2 z}\hat{x}$$

$$\begin{aligned}\tilde{\mathbf{H}}_{2y} &= H_2^+ e^{-j\beta_2 z}\hat{y} \\ &= \frac{E_2^+ e^{-j\beta_2 z}}{\eta_2}\hat{y}\end{aligned}$$

and $\beta_2 = \frac{\omega}{v_2} = \omega\sqrt{\mu_2\epsilon_2}$ and $\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$.
To satisfy the boundary conditions at $z = 0$, we note that both electric and magnetic fields are tangential to the surface and no current exists on the surface. Hence, we have

$$\tilde{\mathbf{E}}_{1x}(z=0) = \tilde{\mathbf{E}}_{2x}(z=0)$$

$$\tilde{\mathbf{H}}_{1y}(z=0) = \tilde{\mathbf{H}}_{2y}(z=0)$$

Applying these to the solution pairs,

$$E_1^+ + E_1^- = E_2^+$$

$$\frac{1}{\eta_1}(E_1^+ - E_1^-) = \frac{1}{\eta_2}E_2^+$$

The E component of the incident wave is given by $E(z,t) = A_1 \cos(\omega t - \beta_1 z)\hat{x}$, therefore, $E_1^+ = A_1$. Solving for the equations above, the phasors of the reflected TEM wave at the interface are shown as

$$\tilde{\mathbf{E}}_1^- = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} A_1 e^{j\beta_1 z}\hat{x}$$

$$\tilde{\mathbf{H}}_1^- = -\frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}\frac{A_1}{\eta_1} e^{j\beta_1 z}\hat{y}$$

Retrieving the time-dependent forms of the reflected wave, we obtain

$$E_1^-(z,t) = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} A_1 \cos(\omega t + \beta_1 z)\hat{x} = \frac{\sqrt{\frac{\mu_2}{\epsilon_2}} - \sqrt{\frac{\mu_1}{\epsilon_1}}}{\sqrt{\frac{\mu_1}{\epsilon_1}} + \sqrt{\frac{\mu_2}{\epsilon_2}}} A_1 \cos(\omega t + \omega\sqrt{\mu_1 \epsilon_1} z)\hat{x}$$

$$H_1^-(z,t) = -\frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}\frac{A_1}{\eta_1} \cos(\omega t + \beta_1 z)\hat{y} = -\frac{\sqrt{\frac{\mu_2}{\epsilon_2}} - \sqrt{\frac{\mu_1}{\epsilon_1}}}{\sqrt{\frac{\mu_1}{\epsilon_1}} + \sqrt{\frac{\mu_2}{\epsilon_2}}} A_1 \sqrt{\frac{\epsilon_1}{\mu_1}} \cos(\omega t + \omega\sqrt{\mu_1 \epsilon_1} z)\hat{y}$$