ECE 329 | Fields and Waves | Fall 17

University of Illinois | He, Ilie, Lee and Waldrop

Exam 1

Wednesday, Sept 27, 2017 — 7:00-8:15 PM

Please clearly PRINT your name in CAPITAL LETTERS and circle your section in the boxes below.

Name:	
Section:	C X E

This is a closed book exam and calculators are not allowed. Please show all your work and make sure to include your reasoning for each answer. All answers should include units wherever appropriate. You may use the back of the pages as scratch paper.

Problem 1 (25 points)	
Problem 2 (25 points)	
Problem 3 (25 points)	
Problem 4 (25 points)	
TOTAL (100 points)	

1. (25 pts) Two charges in free space $Q_1 = 1\text{C}$ and $Q_2 = -2\text{C}$ are located along the $\hat{x}$ axis at $(x, y, z) = (-1, 0, 0)$ and at $(x, y, z) = (1, 0, 0)$, respectively. Point P is located at $(x, y, z) = (0, 1, 0)$.

a) (9 pts) Determine the electric field vector $\mathbf{E}$ at point P .

$$\bar{E} = \sum_{i=1}^{2} \frac{Q_i}{4\pi\varepsilon_0 |\bar{r}_p - \bar{r}_i|^2} \cdot \frac{\bar{r}_p - \bar{r}_i}{|\bar{r}_p - \bar{r}_i|} = \frac{1}{4\pi\varepsilon_0(\sqrt{2})^2} \frac{(1,1,0)}{\sqrt{2}} + \frac{-2}{4\pi\varepsilon_0(\sqrt{2})^2} \frac{(-1,1,0)}{\sqrt{2}}$$

$$= \frac{3\hat{x} - \hat{y}}{8\sqrt{2}\pi\varepsilon_0} \ \frac{V}{m}$$

Your Answer (include appropriate units):

$\mathbf{E} = \dfrac{3\hat{x} - \hat{y}}{8\sqrt{2}\pi\varepsilon_0} \ \dfrac{V}{m}$

b) (8 pts) Determine the electrostatic potential V at point P, assuming $V(\infty) = 0$.

Since $V = \dfrac{Q}{4\pi\varepsilon_0 r}$

$$V_P = V_{Q_1} + V_{Q_2} = \frac{1}{4\pi\varepsilon_0\sqrt{2}} + \frac{-2}{4\pi\varepsilon_0\sqrt{2}} = \frac{-1}{4\pi\sqrt{2}\varepsilon_0} \ V$$

Your Answer (include appropriate units):

$V = \dfrac{-1}{4\pi\sqrt{2}\varepsilon_0} \ V$

c) (8 pts) Determine the displacement field flux $\int \mathbf{D} \cdot \mathbf{dS}$ through the entire $y - z$ plane at $x = 0$ in the $\hat{x}$ direction.

$$\int_{y\text{-}z \text{ plane}} \bar{D} \cdot d\bar{s} = Q_{1\to zy} + Q_{2\to zy} = \frac{1}{2} + \frac{2}{2} = 1.5\,C$$

Your Answer (include appropriate units):

$\int_{\text{y-z plane}} \mathbf{D} \cdot \mathbf{dS} = 1.5\ C$

2. (25 pts) In this problem part a) and part b) are unrelated.

a) (10 pts) Given curl free electric field with $\mathbf{E} = y\hat{x} + (x+1)\hat{y}$ V/m, determine the electrostatic potential $V(1,1,1)$ if $V(0,0,0) = 3$V.

$$V(1,1,1) - V(0,0,0) = -\int_0^P \vec{E}\cdot d\vec{l}$$
$$= -\int_0^1 E_x(x,0,0)\,dx - \int_0^1 E_y(1,y,0)\,dy - \int_0^1 E_z(1,1,z)\,dz$$
$$= -\int_0^1 0\,dx - \int_0^1 2\,dy - \int_0^1 0\,dz$$
$$= -2[y]_0^1 = -2$$
$$\because V(1,1,1) - 3 = -2 \quad \therefore V(1,1,1) = 1\text{ V}$$

Your Answer (include appropriate units):

$V(1,1,1) =$ 1 V

b) (15 pts) Given an electrostatic potential $V(x,y,z) = 3x^2 - 2x$ V in a certain region of free space, determine the corresponding

i. electrostatic field $\mathbf{E}$.

$$\vec{E} = -\nabla V$$
$$= -\frac{\partial}{\partial x}(3x^2-2x)\hat{x} - \frac{\partial}{\partial y}(3x^2-2x)\hat{y} - \frac{\partial}{\partial z}(3x^2-2x)\hat{z}$$
$$= -(6x-2)\hat{x} + 0 + 0$$

Your Answer (include appropriate units):

$\mathbf{E} =$ $(2-6x)\hat{x}$ V/m

ii. charge density ρ in the region.

(Using Gauss law).

$$\rho = \nabla\cdot\vec{D}$$
$$= \nabla\cdot(\varepsilon_0\vec{E})$$
$$= \varepsilon_0\left(\frac{\partial}{\partial x}E_x + \frac{\partial}{\partial y}E_y + \frac{\partial}{\partial z}E_z\right)$$
$$= -6\varepsilon_0 + 0 + 0$$

Your Answer (include appropriate units):

$\rho =$ $-6\varepsilon_0$ C/m^3

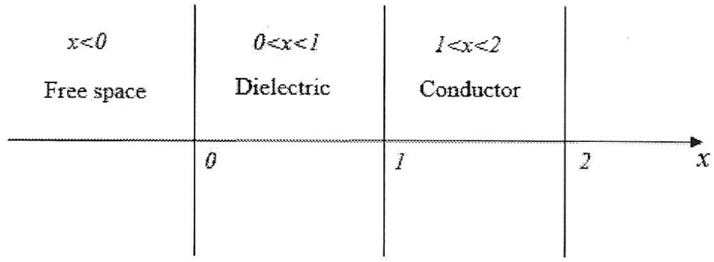
iii. curl $\nabla\times\mathbf{E}$.

$$\nabla\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2-6x & 0 & 0 \end{vmatrix} = \hat{x}(0) - \hat{y}\left[0 - \frac{\partial}{\partial z}(2-6x)\right] + \hat{z}\left[0 - \frac{\partial}{\partial y}(2-6x)\right]$$
$$= 0 - 0 + 0$$
$$= 0$$

Your Answer (include appropriate units):

$\nabla\times\mathbf{E} =$ 0

3. (25 pts) Consider in free space a slab of perfect dielectric with $\epsilon = 2\epsilon_0$ at $0 \le x < 1$m and a slab of perfect electric conductor at $1 \le x < 2$m. Both slabs are perpendicular to $\hat{x}$ axis. The electric field is known to be $\mathbf{E} = 2\hat{x} - 2\hat{y}$ V/m for in region $x < 0$. Furthermore, field $\mathbf{E}$ is uniform in each of regions $x < 0$, $0 < x < 1$m, and $1 < x < 2$m.



a) (5 pts) Determine the displacement field vector $\mathbf{D}$ in the region $x < 0$ m.

$$\vec{D} = \varepsilon_0 \vec{E} = 2\epsilon_0 \hat{x} - 2\epsilon_0 \hat{y}$$

Your Answer (include appropriate units):

$\mathbf{D} =$ $2\epsilon_0 \hat{x} - 2\epsilon_0 \hat{y}$ C/m^2

b) (10 pts) Determine the electric field vector $\mathbf{E}$ and displacement field vector $\mathbf{D}$ in the region $0 < x < 1$ m.

$$\vec{E} = \hat{x} + E_y \hat{y}$$

$$\vec{D} = 2\epsilon_0 \hat{x} + D_y \hat{y} \quad , \text{where } D_y = 2\epsilon_0 E_y.$$

Your Answer (include appropriate units):

$\mathbf{E} =$ $\hat{x} + E_y \hat{y}$ V/m

$\mathbf{D} =$ $2\epsilon_0 \hat{x} + D_y \hat{y}$ C/m^2

c) (5 pts) Determine the electric field vector $\mathbf{E}$ in region $1 < x < 2$ m.

Your Answer (include appropriate units):

$\mathbf{E} =$ 0

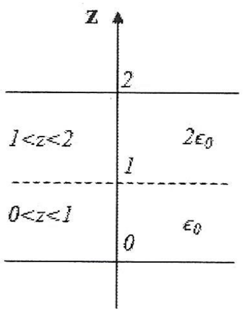
d) (5 pts) Determine the **free** surface charge density ρ_s at $x = 1$m.

$0 - D_n = \rho_s$

$\rho_s = -D_n = -2\epsilon_0 \ C/m^2$

Your Answer (include appropriate units):
At$x = 1$m, $\rho_s = -2\epsilon_0 \ C/m^2$

4. (25 pts) A pair of infinitely large perfect conductors are located in regions $z < 0$ and $z > 2$ m. Regions $0 < z < 1$ m and $1 < z < 2$ m are occupied by perfect dielectrics with permittivities of ϵ_0 and $2\epsilon_0$, respectively. At $z = 0$, electrostatic potential is $V(0) = 0$ and surface charge density is $\rho_s = 4\epsilon_o C/m^2$.



a) (5 pts) Determine the displacement field vector **D** in regions $0 < z < 1$ m and $1 < z < 2$ m.

$D = \rho_s = 4\epsilon_0 \; c/m^2$

Your Answer (include appropriate units):

In region $0 < z < 1$ m, $\mathbf{D} = 4\epsilon_0 \hat{z} \; c/m^2$

In region $1 < z < 2$ m, $\mathbf{D} = 4\epsilon_0 \hat{z} \; c/m^2$

b) (5 pts) Determine the electric field vector **E** in regions $0 < z < 1$ m and $1 < z < 2$ m.

$\vec{D} = \epsilon \vec{E}$

Your Answer (include appropriate units):

In region $0 < z < 1$ m, $\mathbf{E} = 4\hat{z} \; V/m$

In region $1 < z < 2$ m, $\mathbf{E} = 2\hat{z} \; V/m$

c) (5 pts) Determine the electrostatic potential V at $z = 2$m.

$V(0) - V(2) = \int_0^2 \vec{E} \cdot d\vec{l} = \int_0^1 4 \, dz + \int_1^2 2 \, dz$

$V(2) = -6 \; V$

Your Answer (include appropriate units):

$V(2) = -6 \; V$

d) (5 pts) Determine the capacitance per unit area c of the two conducting plates.

$$C = \frac{Q}{V} = \frac{\rho_s \cdot A}{V}$$

$$c = \frac{\rho_s}{V} = \frac{4\epsilon_0}{6} = \frac{2\epsilon_0}{3}$$

Your Answer (include appropriate units):
$c = \frac{2\epsilon_0}{3}$ F/m^2

e) (5 pts) If instead region $0 < z < 2$m is occupied by dielectric with permittivity varying along $\hat{z}$ direnction as $\epsilon(z) = \frac{\epsilon_0}{3-z}$, determine which of the following equations are correct in this region. (Circle all the correct answers.)

i. $\nabla \cdot \mathbf{E} = 0$

ii. $\nabla V = 0$

iii. $\epsilon \nabla \cdot \mathbf{E} = 0$

iv. $\nabla \cdot (\epsilon \mathbf{E}) = 0$

v. $\nabla^2 V = 0$

$\nabla \cdot D = \rho = 0$

$\nabla \cdot (\epsilon \vec{E}) = 0$