These are "Answers". We don't have access to the solutions. These answers were prepared by a student in spring 2015 and cross-checked by a TA and by Prof. Goddard. If you find any possible errors in the answers please email Prof. Goddard at lgoddard@illinois.edu.

ECE 329 | Fields and Waves I | Fall 13

University of Illinois | Dragic, Gong, McKeown

Final Exam B

Monday, December 16, 2013 — 7:00-10:00 PM

Name:	Solution
Section:	10 AM 12 Noon 1 PM

Please clearly PRINT your name in CAPITAL LETTERS and circle your section in the above boxes. If you want your exam returned to a **different** section, write it here:

Special Return to Section:	10 AM 12 Noon 1 PM

This is a closed book exam and calculators are not allowed. Please show all your work. You are allowed the use of a compass, ruler, protractor, and notes on both sides of FOUR 4x6 index cards **OR** ONE sheet of 8.5x11 paper. All answers should include units wherever appropriate. **Points will be deducted for incorrect answers on multiple choice questions.**

Problem 1 (28 points)	
Problem 2 (25 points)	
Problem 3 (25 points)	
Problem 4 (24 points)	
Problem 5 (20 points)	
Problem 6 (26 points)	
Problem 7 (35 points)	
Problem 8 (17 points)	
TOTAL (100 points)	

1. (28 points) Answer each sub-question independently. Show all work and circle your final answer.

a) (5 points) Could $\mathbf{E} = \frac{z^2}{2}\hat{x} + (z+x)\,\hat{y} + (zx+y)\,\hat{z}$ be a valid **static** electric field? Prove your answer.

$$\nabla\times E = \begin{pmatrix} x & y & z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{z^2}{2} & z+x & zx+y \end{pmatrix}$$

$$= x(1-1) - y(2-z) + z(1-0)$$

$$= -2y + yz + z \neq 0 \quad \text{not valid.}$$

b) (5 points) Could $\mathbf{B} = x^2y\,\hat{x} - y^2x\,\hat{y} + ye^x\,\hat{z}$ be a valid magnetic field if $\mathbf{J} \neq 0$? Prove your answer.

$$\nabla\cdot B = 2xy - 2xy + 0 = 0$$

$$\nabla\times H = \nabla\times\frac{B}{\mu_0} = \begin{vmatrix} x & y & z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x^2y}{\mu_0} & -\frac{y^2x}{\mu_0} & \frac{ye^x}{\mu_0} \end{vmatrix} = \frac{1}{\mu_0}\left(\hat{x}(e^x-0) - \hat{y}(ye^x-0) + \hat{z}(-y^2-x^2)\right) \neq 0$$

valid magnetic field with $J \neq 0$

c) (5 points) In the presence of a charge density, the electric potential is given as $V = V_0x^3$. Calculate the electric field and **total** charge density in the region.

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

$$E = -\nabla V = -V_0\cdot 3x^2 \ (V/m)$$

$$\rho = -\nabla^2 V\cdot\varepsilon_0$$

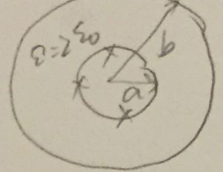
$$= -\varepsilon_0\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right)$$

$$= -\varepsilon_0(6xV_0)$$

$$= -6V_0x\varepsilon_0 \ (C/m^2)$$

d) (13 points) Consider an infinitely long co-axial capacitor with inner conductor radius a and outer conductor radius b. The region $a < r < b$ is filled with a perfect dielectric with permittivty $\varepsilon = 2\varepsilon_0$. A surface charge density of 50 $\mathrm{C/m^2}$ is present on the inner conductor.

i. (5 pts) If $\mathbf{E} = 0$ for $r > b$, calculate the surface charge density on the outer conductor in terms of a and b.



$$\nabla \cdot D = \rho$$

$$\oint D \cdot ds = Q_a$$

$$Q_a = Q_b \Rightarrow 2\pi a l \rho_a = 2\pi b l \rho_b$$

$$\rho_b = \frac{a}{b} \cdot \rho_a = 50 \frac{a}{b} \, (C/m^2)$$

ii. (8 pts) Determine $\mathbf{E}$, $\mathbf{D}$, and $\mathbf{P}$ in the region $a < r < b$ if $a = 1$ cm.

$$\oint D \cdot ds = Q$$

$$2\varepsilon_0 E \cdot 2\pi r \cdot l = 2\pi a l \rho_a$$

$$E = \frac{a \rho_a}{2 r \varepsilon_0} \, V/m = \frac{10^{-2} \times 50}{2 r \varepsilon_0} \hat{r} \, (V/m)$$

$$E = \frac{0.25}{r \varepsilon_0} \hat{r} \, (V/m)$$

$$D = \varepsilon E = \frac{2\varepsilon_0 \times 0.25}{r \varepsilon_0} = \frac{0.5}{r} \, (C/m^2)$$

$$P = D - \varepsilon_0 E = \frac{0.5}{r} - \frac{0.25}{r} = \frac{0.25}{r} \, (C/m^2)$$

iii. (4 pts) **(OPTIONAL)** If $V(a) - V(b) = \frac{1}{8\varepsilon_0} \ln 4$ and $a = 1$ cm, what is the outer radius b?

$$E = -\nabla V$$

$$V(a) - V(b) = \int_a^b \frac{0.25}{r \varepsilon_0} \hat{r} \, dr$$

$$= \left(\ln r \cdot \frac{0.25}{\varepsilon_0} \right) \Big|_a^b$$

$$= \frac{0.25}{\varepsilon_0} \cdot (\ln b - \ln a)$$

$$= \frac{0.25}{\varepsilon_0} \ln \frac{b}{a} = \frac{1}{8\varepsilon_0} \ln 4$$

$$\Rightarrow \frac{1}{4} \ln b = \frac{1}{8} \ln 4 \Rightarrow 2 \ln b = \ln 4 \Rightarrow b = 2 \, cm$$

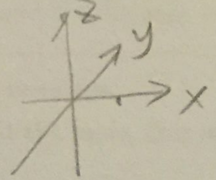
2. (25 points) Multiple choice/short answer questions. No explanation necessary.

a) (10 points: 2 pts each) Indicate the DIRECTION of the vector in each situation below. Choose ONLY ONE unit vector. Specify the unit vector AND indicate positive or negative in the selected direction. Mark "ZERO" if the requested vector is zero. No partial credit on an individual sub-question.

Example: The force on a charge Q at a point (x, y, z) exerted by a charge $-Q$ at the origin.

$\boxed{-\hat{r}}$

i. The $\mathbf{E}$ field at $(1, 0, 0)$ on the x-axis due to a spherical shell of charge with radius $r = 3$ centered at the origin.

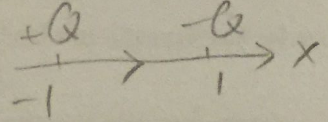


$\boxed{\text{ZERO}}$

ii. The $\mathbf{E}$ field at the point $(0, 0, 4)$ on the z-axis due to the spherical shell in Part i.

$\boxed{\hat{z}}$

iii. The $\mathbf{E}$ field at the origin if a charge of $+Q$ is located at $(-1, 0, 0)$ and a charge of $-Q$ is located at $(1, 0, 0)$ on the x-axis.



$\boxed{\hat{x}}$

iv. The force on a stationary charge, $+Q$, in a region with magnetic field $\mathbf{B} = 5\hat{z}$ Wb/m^2.

$\boxed{\text{ZERO}}$

$F = Qv \times B = 0$

v. The force on a charge, $-Q$, moving with velocity $\mathbf{v} = 2\hat{y} - 3\hat{z}$ m/s in a region with magnetic field $\mathbf{B} = 2\hat{y}$ Wb/m^2.

$\boxed{-\hat{x}}$

$$F = Qv \times B$$
$$= -Q(2\vec{y} - 3\vec{z}) \times (2\vec{y})$$
$$= Q(3\vec{z} - 2\vec{y}) \times (2\vec{y})$$
$$= Q(-6\vec{x})$$

b) (15 points, 3pts ea) Multiple choice: no explanation necessary. **Choose ALL correct answers.**

i. If we have electrostatics in a region where $\mathbf{J_f} = 0$, $\rho_f = 0$, $\mu = \mu_0$ but $\epsilon = \epsilon(x)$ depends on position:

A. $\nabla \cdot \mathbf{D} = 0$

B. $\nabla \cdot \mathbf{P} = 0$

C. $\nabla \cdot \mathbf{E} = 0$

D. $\mathbf{E} = \nabla V$

$P = D - \varepsilon_0 E$ $\quad \nabla \cdot D = \rho = 0$ $\quad \frac{\partial \varepsilon}{\partial x}$

$D = \varepsilon E \quad \nabla \cdot \varepsilon E = 0$

ii. For two conducting parallel plates with stored charge Q, potential difference V, plate area A, and distance between the plates d, that are filled with a dielectric with permittivity ε:

A. If ε is doubled, but Q is kept the same, V is reduced by 2x.

B. If both the permittivity, ε, and the distance between the plates, d, are doubled, Q and V can remain unchanged.

C. Capacitance decreases as A increases.

D. If ε is doubled, but V is kept the same, Q is reduced by 2x.

iii. For a bounded material with conductivity $\sigma > 0$, permeability $\epsilon > 0$, and an initial charge distribution $\rho(x, y, z)$:

A. If $\sigma \neq \infty$ then a steady-state electric field can exist inside the material.

B. In steady-state charge will be present on the surface of the material.

C. ρ will evolve in time to a spatially uniform value ρ_0.

D. The total free charge will go to zero as $t \to \infty$.

iv. If $\rho(t, r, \theta, \phi) = e^{-t}$ C/m^3 for the region $r < 5$:

A. The displacement field flux through the sphere bounding $r < 5$ is increasing with time.

B. $\nabla \cdot \mathbf{J} > 0$ for $r < 5$.

C. The charge at the origin is always increasing.

D. The outflux of current for the sphere bounding $r < 5$ is positive.

v. Consider **two different** media (medium 1 and 2) bounded by a flat surface. The displacement flux vector **D** and electric field **E** perpendicular to and tangential with the boundary are denoted with subscript n and t, respectively. Which statement(s) are **always** true:

A. $\mathbf{D}_{1n} = \mathbf{D}_{2n}$ X

B. If there is a surface current at the interface, then the **normal** **B** field is discontinuous. X

C. $\mathbf{E}_{1t} = \mathbf{E}_{2t}$

D. If the media are perfect dielectrics, then the electric field inside each is reduced from it's free space value due to the polarization of free charges.

Sine and cosine are periodic, the argument of these functions is not uniquely defined well enough to solve for beta*x. The arguments have wt-8 and wt-16, so it's possible that beta x = 8 or that beta x + 2 m pi = 8 for an integer m. Thus, there's not enough information for parts a.i and a.ii.

3. (25 points) A TEM wave is propagating in the $\pm\hat{x}$ directions from its current sheet source in the $x = 0$ plane through a non-magnetic ($\mu = \mu_0$), perfect dielectric material having permittivity $\epsilon \neq \epsilon_0$.The time-dependent electric fields at $x = 20$ km and $x = -40$ km are observed, respectively, to be

$$\mathbf{E}(20\text{ km}, t) = 3\cos(3\times 10^4 t - 8)\hat{y} - 3\sin((3\times 10^4 t - 8)\hat{z}\text{ V/m}$$

$$\mathbf{E}(-40\text{ km}, t) = 3\cos(3\times 10^4 t - 16)\hat{y} - 3\sin((3\times 10^4 t - 16)\hat{z}\text{ V/m}$$

a) (16 points, 4pts each) **Choose ONE correct answer** to each of the following questions:

i. What is the propagation velocity of this TEM wave?

A. $v = c$

B. There is not enough information given above to answer this question.

C. $v = 2c$

D. $v = \frac{1}{2}c$

E. $v = \frac{1}{4}c$

$\beta x = 8$

$\beta = \frac{8}{20\times}$

$= \frac{8}{20}\times$

$\beta = \frac{2\pi}{\lambda}$ $\lambda f = v$

$f = \frac{\omega}{2\pi} \Rightarrow \frac{2\pi}{\beta}\cdot\frac{\omega}{2\pi} = v$

ii. What is the intrinsic impedance η of the material medium?

A. $\eta = \frac{1}{2}\eta_0$

B. $\eta = \eta_0$

C. $\eta = \frac{1}{4}\eta_0$

D. $\eta = 2\eta_0$

E. There is not enough information given above to answer this question.

$v = \frac{1}{\sqrt{\mu\varepsilon}}$

$\mu = \mu_0$ $\varepsilon = 16\varepsilon_0$

iii. Which of the following expressions represents the phasor electric field $\tilde{\mathbf{E}}$, propagating in the $+\hat{x}$ direction as observed above at $x = 20$ km?

A. $(\hat{y} - j\hat{z})3e^{j8}$

B. $(\hat{y} - j\hat{z})3e^{-j8}$

C. $(\hat{y} + j\hat{z})3e^{-j8}$

D. $(\hat{y} + j\hat{z})3e^{j8}$

$3e^{-j8}(\hat{y} - ze^{-j\frac{\pi}{2}})$

$= 3e^{-j8}(\hat{y} + jz)$ $e^{-j\frac{\pi}{2}} = \cos(\frac{\pi}{2}) - j\sin($

$= 0 -$

iv. What is the polarization of the wave propagating in the $\hat{x}$ direction, with the phasor form found in part (c) above?

A. LH circular

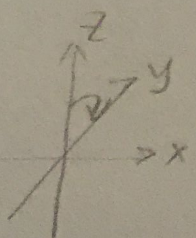
B. linear

C. RH circular

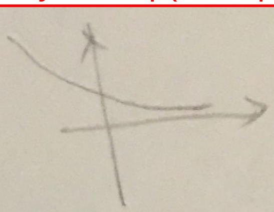
D. none of the above

$(\hat{y} + e^{-j\frac{\pi}{2}} z)\, 3e^{-j8}$

z leading $\hat{y}$



Part b.i should be false. Since the field is generated at x=0, the forward wave decays to exp(-20 alpha) while the backwards wave decays to exp(-40 alpha)



$e^{40\alpha}$ $e^{-20\alpha}$

b) (4 points) If the wave above were propagating instead through a highly conducting material, which of the following statements would be true, assuming that the observation of $\mathbf{E}(20\text{ km}, t)$ above remains the same? $e^{-\alpha x}$

i. ~~**TRUE**~~ or **FALSE**: The amplitude of the wave at $x = -40$ km would be larger than that observed at 20 km.

ii. **TRUE** or **FALSE**: $\langle \mathbf{J} \cdot \mathbf{E} \rangle$ would increase as the wave propagates in the $+\hat{x}$-direction

iii. **TRUE** or **FALSE**: The associated magnetic field would propagate in phase with the electric field.

iv. **TRUE** or **FALSE**: $\langle \mathbf{S} \rangle$ would decrease as the wave propagates in the $+\hat{x}$-direction

c) (5 points) If the conductivity of the material were increased, which of the following parameters would **not change**? (*choose all that apply*)

i. The speed of propagation X

ii. The amount of power transported across the $z = \frac{1}{10}$ m plane at $t = 0$ X

$\oint |\langle E \times H \rangle| \, ds$

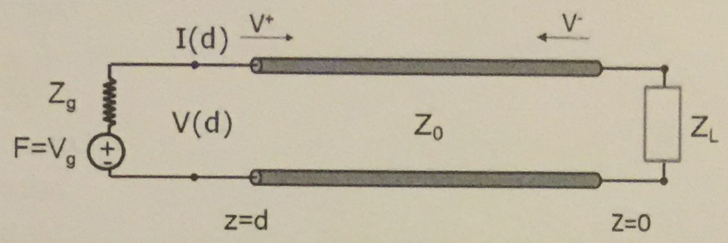
iii. The amplitude of $\mathbf{H}(0, 0)$ X

iv. The penetration depth X

v. The intrinsic impedance of the material X

vi. The direction of polarization

4. (24 points, 3 pts each) In the sytem shown in the figure below, the tramssion line is in steady state. **Choose ALL correct answers** to each of the following questions:



a) β is the wave number for the waves propagating on the transmission line. $V^+$and V^-represent the magnitude of forward and backward traveling wave on the transmission line respectively. What is the phasor form expression of the voltage and current waveform traveling towards the load impedance?

i. Voltage: $V^+e^{j\beta d}$ Current:$V^+e^{-j\beta d}/Z_0$

ii. Voltage: $V^+e^{j\beta d}$ Current:$V^+e^{j\beta d}/Z_0$

iii. Voltage: $V^+e^{j\beta d}$ Current:$-V^+e^{j\beta d}/Z_0$

iv. Voltage: $V^+e^{-j\beta d}$ Current:$V^+e^{-j\beta d}/Z_0$

v. Voltage: $V^+e^{-j\beta d}$ Current:$-V^+e^{j\beta d}/Z_0$

vi. Voltage: $V^+e^{-j\beta d}$ Current:$V^+e^{j\beta d}/Z_0$

b) What is the phasor form expression of the voltage and current waveform traveling towards the source?

i. Voltage: $V^-e^{j\beta d}$ Current:$V^-e^{-j\beta d}/Z_0$

ii. Voltage: $V^-e^{j\beta d}$ Current:$V^-e^{j\beta d}/Z_0$

iii. Voltage: $V^-e^{j\beta d}$ Current:$-V^-e^{-j\beta d}/Z_0$

iv. Voltage: $V^-e^{-j\beta d}$ Current:$V^-e^{-j\beta d}/Z_0$

v. Voltage: $V^-e^{-j\beta d}$ Current:$-V^-e^{j\beta d}/Z_0$

vi. Voltage: $V^-e^{-j\beta d}$ Current:$V^-e^{j\beta d}/Z_0$

c) If the reflection coefficient at the load is given as: $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$, which of the following expressions represent the votlage ($V(d)$) and current ($I(d)$) phasor at the input of the transmission line ($z = d$) ?

i. $V(d) = V^+e^{j\beta d}(1 - \Gamma_L e^{-2j\beta d})$ and $I(d) = \frac{V^+e^{j\beta d}(1+\Gamma_L e^{-2j\beta d})}{Z_0}$

ii. $V(d) = V^-e^{-j\beta d}(\Gamma_L e^{2j\beta d} + 1)$ and $I(d) = \frac{V^-e^{-j\beta d}(\Gamma_L e^{2j\beta d}+1)}{Z_0}$

iii. $V(d) = V^-e^{-j\beta d}(\frac{1}{\Gamma_L}e^{2j\beta d} + 1)$ and $I(d) = \frac{V^-e^{-j\beta d}(\frac{1}{\Gamma_L}e^{2j\beta d}-1)}{Z_0}$

iv. $V(d) = V^+e^{j\beta d}(1 + \Gamma_L e^{-2j\beta d})$ and $I(d) = \frac{V^+e^{j\beta d}(1+\Gamma_L e^{-2j\beta d})}{Z_0}$

v. $V(d) = V^+e^{j\beta d}(1 - \Gamma_L e^{-2j\beta d})$ and $I(d) = \frac{V^+e^{j\beta d}(1-\Gamma_L e^{-2j\beta d})}{Z_0}$

$V(d) = V^+ + V^-$

$= V^+e^{j\beta d} + V^-e^{-j\beta d}$

$V^- = V^+ \cdot T_L$

$= V^+e^{j\beta d} + V^+T_L e^{-j\beta d}$

$= V^+e^{j\beta d}(1 + T_L e^{-2j\beta d})$

d) What is the smallest distance d from the load, **IN GENERAL,** for which the input voltage = the load voltage?

i. $d_{min} = 0.75\lambda$

ii. $d_{min} = \lambda$

iii. $d_{min} = 2\lambda$

iv. $d_{min} = 0.125\lambda$

v. $d_{min} = 0.25\lambda$

vi. $d_{min} = 0.5\lambda$

e) What is the smallest distance d from the load for which the input impedance (Z_{in}) = the load impedance (Z_L)?

i. $d_{min} = 0.75\lambda$

ii. $d_{min} = \lambda$

iii. $d_{min} = 2\lambda$

iv. $d_{min} = 0.125\lambda$

v. $d_{min} = 0.25\lambda$

vi. $d_{min} = 0.5\lambda$

f) Which of the following is/are true?

i. If the load is a pure resistor, the input impedance is also resistive.

ii. If the load is an inductor, the input impedance is always capacitive.

iii. VSWR can be smaller than 1 for the above system.

iv. if $Z_L = Z_0$, the input impedance is Z_0 for any d values.

v. If the load is an inductor and a capacitor connected in paralell, it is possible that the input impedance is inductive.

vi. The VSWR is infinity only if the load is short or open. X

vii. The larger the VSWR is, the larger the magtitude of $\Gamma_L$is.

$VSWR = \frac{1+\Gamma}{1-\Gamma}$

g) If the transmission line is 2.4 λ long and reflection coefficient at the load is neither zero nor ± 1, how many voltage max points are on the transmission line for the partial standing wave ?

i. 3

ii. 4

iii. 5

iv. 2

v. 1

vi. 6

It's not clear what this question means by impedance matching. We will assume what is meant is that Z_in = Zg. In this interpretation, the possible lengths are 2.5, 3, 3.5, 4, 4.5 lambda but 2.5 lambda is not strictly between f1 and f2 so instead of 5, there are only 4

h) If the source is tunable now and can output a single frequency signal from f_1 to $f_2(f_1<f_2)$, the length of the tranmission line is 2.5λ and $4.8\ \lambda$, respectively. Also, $Z_g = Z_L \neq Z_0$. How many frequencies are between f_1 and $f_2$at which the impedance is matched?

i. 1.

ii. 3.

iii. 6.

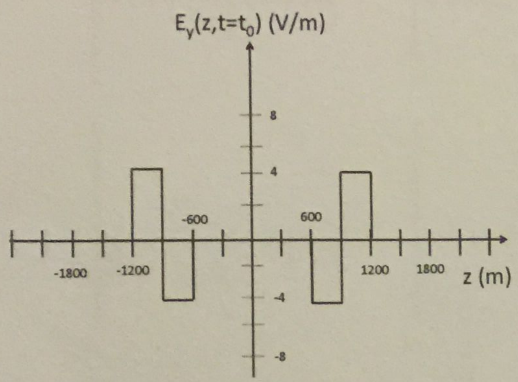
iv. 2.

v. 4.

vi. 5.

5. (20 points) A TEM waveform is propagating in free space away from its surface current sheet source in the $z = 0$ plane, where $\mathbf{J_s}(t < 0) = 0$ and $\mathbf{J_s}(t = 0) \neq 0$. A "snapshot" of the electric field as a function of distance from the sheet is shown in the figure below.

Hint: Rect(t/T) is the rectangle function centered at t=0 with width T and amplitude 1.



a) (3 points) At what time t_0 (in μs) the above "snapshot" taken?

$$E(z,t) = -\hat{y}\,\eta_0 \frac{J_s(t \mp z/c)}{2}$$

$$t_0 = \frac{1200}{v=c} = \frac{1200\ m}{300\ m/\mu s} = 4\ \mu s$$

b) (5 points) Write the expression for the time-dependent surface current source $\mathbf{J}_s(t)$.

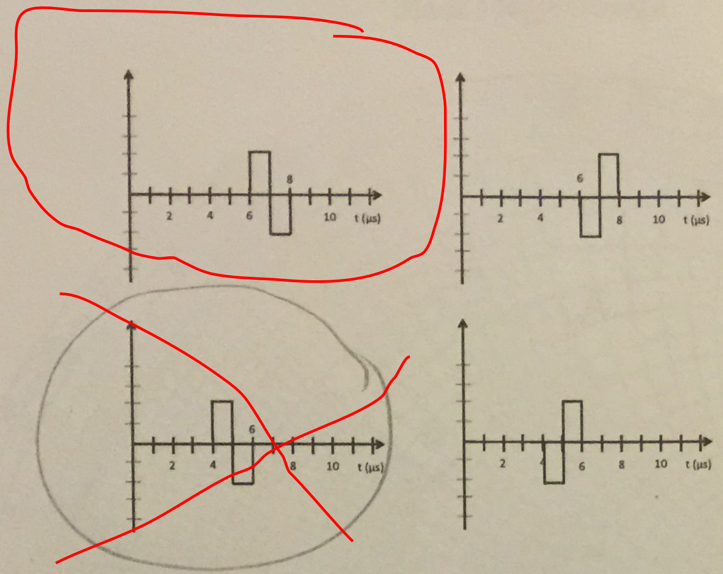
$$E_y(z,t) = -4\,\mathrm{rect}\left(\frac{z-750}{150}\right) + 4\,\mathrm{rect}\left(\frac{z-1050}{150}\right)$$

$$\cancel{= -4\,\mathrm{sgn}(z \mp 1000)\,\mathrm{rect}\left(\frac{z \mp 1000}{600}\right)\hat{y} \quad z \gtrless 0}$$

$$\cancel{J_s(t) = \frac{8}{\eta_0}\,\mathrm{sgn}(1-t)\,\mathrm{rect}\left(\frac{t}{2}\right)}$$

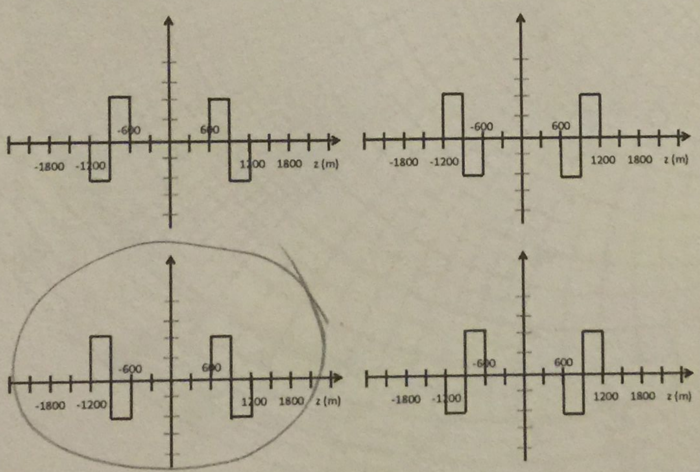
The first step gives the correct Ey for the wave propagating in the +z direction. But we need to instead use Ey(z,t)=-eta0/2 Js(4-z/c) = -4 rect(z/150-5) + 4 rect(z/150-7) and then define your time variable as t=4-z/c. When you plug this in as z=(4-t)/c, you get:
-eta0/2 Js(t) = -4 rect((4-t)*2-5) + 4 rect((4-t)*2-7) = -4 rect(3-2t) + 4 rect(1-2t) and thus
Js(t) = 8/eta0 rect(3-2t) - 8/eta0 rect(1-2t) where time is measured in micro seconds.
You can check that this function is zero for t<0, the second term: rect(1-2t) turns on at t=0 and turns off at t=1 -- this gives the part of the wave for z=1200m (farthest edge) to z=900m while the first term: rect(3-2t) turns on at t=1 and turns off at t=2 -- this gives the part of the wave for z=900m to z=600m.

c) (4 points) Choose which of the following diagrams represents $E_y(z = -1.8 \text{ km}, t)$, where t has units of μs.



$E_y(-z, t)$

d) (4 points) Choose which of the following diagrams represents the magnitude of the associated magnetic field at time $t = t_0$.



(4 points) What is the magnitude and direction of the magnetic field at $t = t_0$ and $z = 600$ m?

$$|H| = \frac{4}{\eta_0} = \frac{4}{120\pi} = \frac{1}{30\pi} \; +\hat{x}$$

6. (26 points, 2pts ea) Multiple choice: no explanation necessary and no partial credit on an individual sub-question.

The ten labeled locations on the Smith Chart (SC) below represent various **LOAD impedances** on a lossless TL having characteristic impedance $Z_0 = 50\ \Omega$. In the box, list **ALL** of the numbered points on the SC that correspond to each of the following TL properties. **If none of points meets the description below, please write down NONE to indicate.** Leaving it BLANK will NOT be considered for points.

a) load impedance Z_L is purely reactive — J, C, H

$z = j$

b) load reflection coefficient is $\Gamma_L = j$ $\quad \frac{z-1}{z+1} = j \quad z-1 = j(z+1)$ — None

c) the load is shorted $\quad z=0$ — C

d) the load is a resistor (neither short nor open is considered as a resistor) — D, E, G

e) reflected voltage phasor is $\tilde{V}^- = 0$ for all $\tilde{V}^+$ $\quad z=1$ — E

f) sequence the loads from the largest to the smallest in terms of the magnitudes of the corresponding reflection coefficients (please use brackets to group the ones with the same magtitude)

(C, J, H), D, A, I, B, G, F, E

g) load conductance is $G_L = 10$ mS — B

h) load admittance is $Y_L = \infty$ $\quad z=0$ — C

i) load impedance Z_L is purely inductive — None

$P_L = \frac{V^2}{Z}$
$\Rightarrow Z = \infty$

j) time-averaged power delivered to the load is $P_L = 0$ for all $\tilde{V}_{in}$ — C, J, H

k) load current and voltage are in phase $\quad z$ real — C, D, E, G, H

l) distance from load to first voltage minimum is $d_{min} = 0.08\lambda$ — I

m) voltage standing wave ratio $VSWR = \infty$ — H, C, J

$VSWR = \frac{1-\Gamma}{1+\Gamma} \quad \Gamma = -1$

$\Rightarrow \Gamma = \frac{z-1}{z+1} \quad z=0$

There's an error in this question. It says input admittance but gives z_in which is impedance. We will assume zin=2+j and rotate counterclockwise for 0.16lamda=115degree to find z(l)=0.42+0.3j, then Zl=z(l)*50= 21+15j, and Yl=0.03-0.02j. so the magnitude of reflection coefficient should be 0.45 and VSWR=2.64.

7. (35 pts) A transmission line with characteristic impedance $Z_0 = 50\ \Omega$ and length $l = 0.16\lambda$ is measured to have a normalized input admittance of $z_{in} = 2 + 1j$.

a) (5 points) What is the impedance Z_L in Ohms, admittance Y_L in Siemens of the load (at $d = 0$), and VSWR on the transmission line? You may calculate these directly or by using the Simth Chart on the next page.

$Z_L = 21 + 15j\ \Omega$	$Y_L = 0.03 - 0.02j$ S	$VSWR = 2.64$

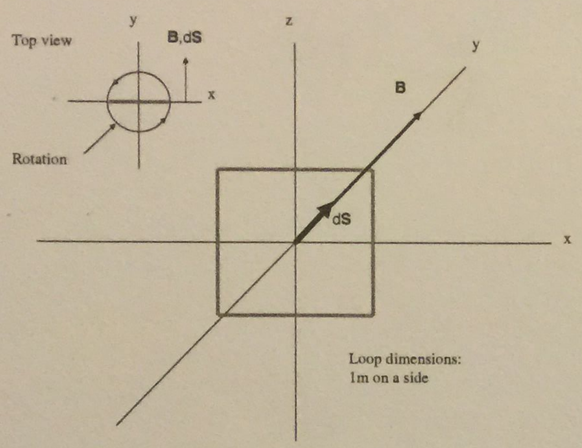
$0.16\lambda = 115^\circ$

$z(l) = 0.42 + 0.3j$

$VSWR = \frac{1+\Gamma}{1-\Gamma}$

$\Gamma = \frac{z-1}{z+1}$

8. (17 points) Consider a simple AC generator, consisting of a square conducting loop of wire that is placed within a uniform magnetic field $\mathbf{B} = 40\hat{y}$ Wb/m^2 and mechanically rotated around the $\hat{z}$-axis as shown in the figure below. If we choose to define the differential surface vector $\mathbf{dS}$ as pointing in the $\hat{y}$ direction at time $t = 0$ (assuming the center of the loop is at the origin), the rotation is such that $\mathbf{dS}$ points along the $-\hat{x}$ direction when $\omega t = \frac{\pi}{2}$ (see the inset in the top left corner).



a) (5 points) What is the magnetic flux (in Wb) through the surface of the loop as a function of time?

$\int B \cdot ds = \int 40\hat{y}\, ds$

$= 40 \cos(wt)$ Wb

b) (6 points) If the maximum electromotive force generated is $\mathcal{E} = 80\pi$ V, at what frequency f (in Hz) is the loop rotating?

$\varepsilon = -\frac{d\psi}{dt}$

$\varepsilon = -40 w \sin(wt)$

$w = 2\pi \quad f = \frac{w}{2\pi} = 1\ Hz$

c) (6 points) According to the figure above, what is the direction of current flow for $\omega t = -\pi/8$ and $\omega t = \pi/8$?

$wt = \frac{\pi}{8}$ Clockwise

$wt = -\frac{\pi}{8}$ Counter Clockwise

d) **OPTIONAL** (4 points) Describe how this generator produces an AC current.

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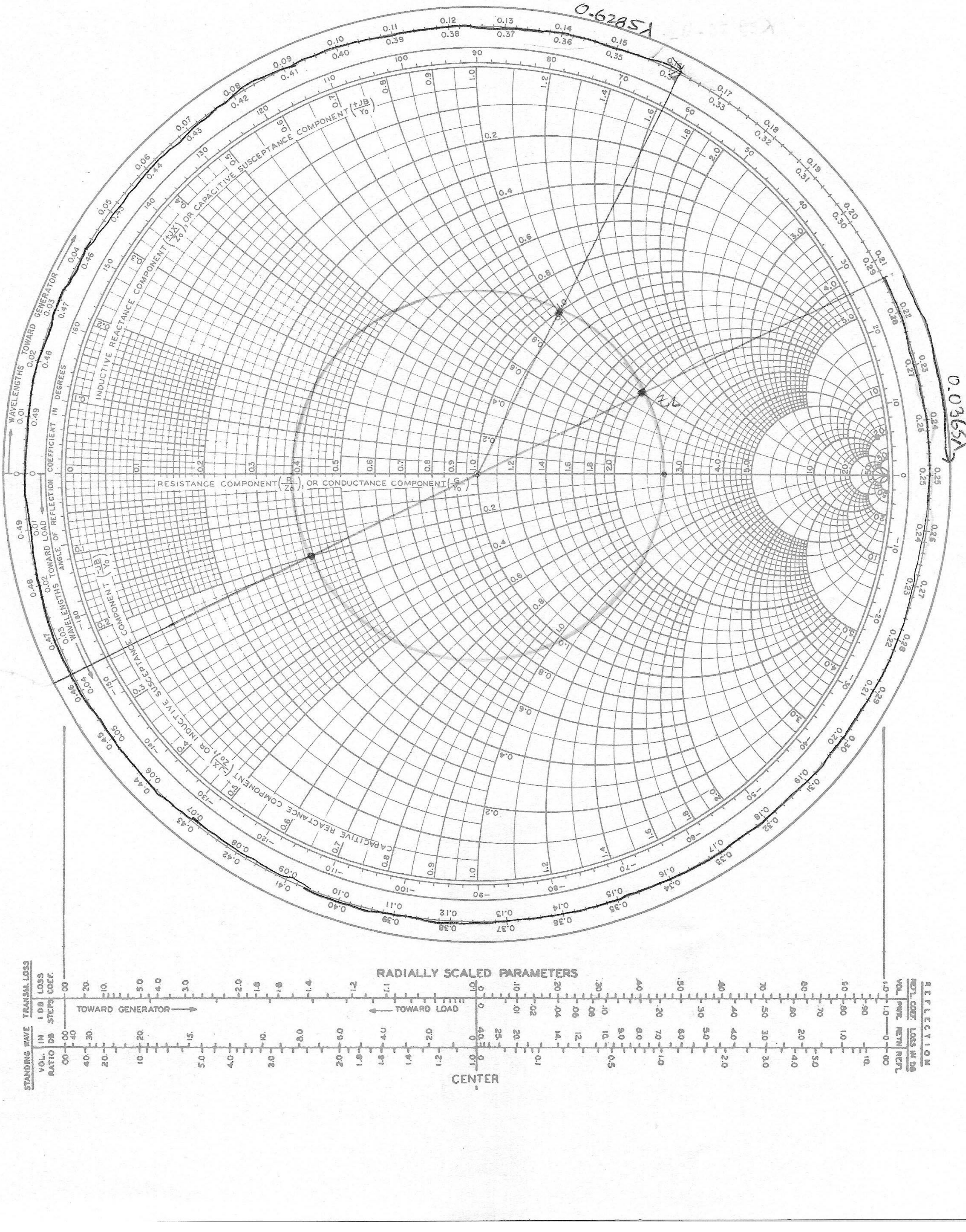

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