1.

a) The vector wave field $\mathbf{E}(x,t)$ is given by

$$\mathbf{E}(x,t) = 3\Delta(\frac{t + x/c}{\tau})\,\hat{z}\,\frac{\mathrm{V}}{\mathrm{m}}.$$

b) The associated wave field $\mathbf{H}(x,t)$ is

$$\mathbf{H}(x,t) = \frac{3}{\eta_o}\Delta(\frac{t + x/c}{\tau})\,\hat{y}\,\frac{\mathrm{A}}{\mathrm{m}}.$$

c) The Poynting vector is

$$\mathbf{S} = \mathbf{E}\times\mathbf{H} = -\frac{9}{\eta_o}\Delta^2(\frac{t + x/c}{\tau})\,\hat{x}\,\frac{\mathrm{W}}{\mathrm{m}^2},$$

and its maximum value is

$$\max\left(|\mathbf{E}\times\mathbf{H}|\right) = \frac{9}{\eta_o} \approx \frac{3}{40\pi}\,\frac{\mathrm{W}}{\mathrm{m}^2}.$$

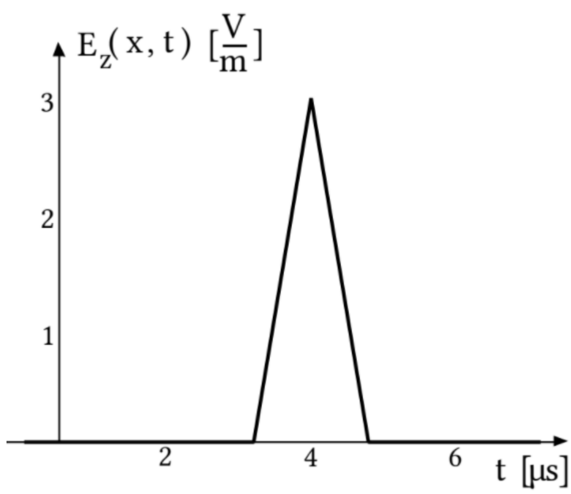
d) The location of the peak of $\mathbf{E}\times\mathbf{H}$ evolves according to

$$\frac{t + x/c}{\tau} = 0 \quad \rightarrow \quad x = -ct.$$

e) The field E_z given by

$$E_z(x,t)|_{x=-1200} = 3\triangle\left(\frac{t - \frac{1200}{3\times10^8}}{2\times10^{-6}}\right) = 3\triangle\left(\frac{t - 4\,\mu\mathrm{s}}{2\,\mu\mathrm{s}}\right)\,\frac{\mathrm{V}}{\mathrm{m}},$$

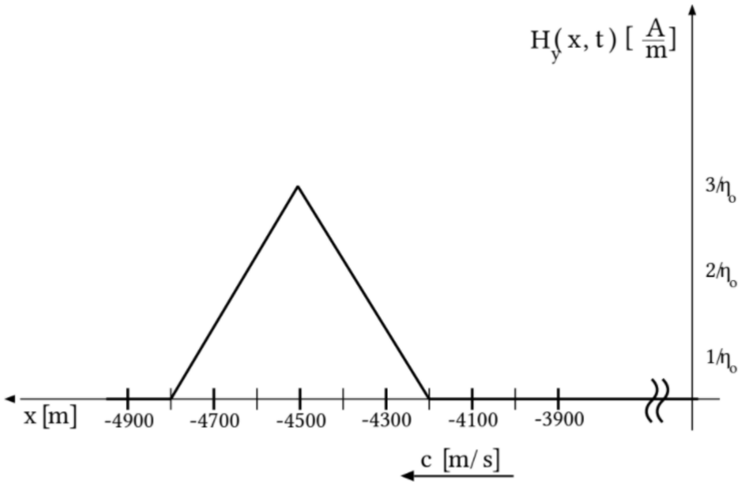
vs t at $x = -1200\,\mathrm{m}$ is plotted in the following figure:



f) The field H_y given by

$$H_y(x,t)|_{t=15\,\mu\text{s}} = \frac{3}{\eta_o}\Delta\left(\frac{15\times10^{-6}+\frac{x}{3\times10^{-8}}}{2\times10^{-6}}\right) = \frac{3}{\eta_o}\Delta(\frac{4500+x}{600})\,\frac{\text{A}}{\text{m}},$$

vs x at $t = 15\,\mu$s is plotted in the following figure:



2.

a) The intrinsic impedance of the medium is

$$\eta = \left|\frac{E_x}{H_y}\right| = 60\pi\,\Omega.$$

We have used the orthogonal pair E_x and H_y. The same relation should be valid for the orthogonal pair E_y and H_x.

b) The propagation velocity is given by

$$v = \frac{c}{2} = 1.5\times10^8\,\frac{\text{m}}{\text{s}}.$$

c) If $\epsilon = \epsilon_r\epsilon_o$ and $\mu = \mu_r\mu_o$, then the intrinsic impedance η can take the following form

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_r}{\epsilon_r}}\eta_o$$

while the wave propagation velocity v can be written as

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{\sqrt{\epsilon_r\mu_r}}.$$

Using the results of parts (a) and (b) we find that

$$\epsilon_r = 4, \mu_r = 1.$$

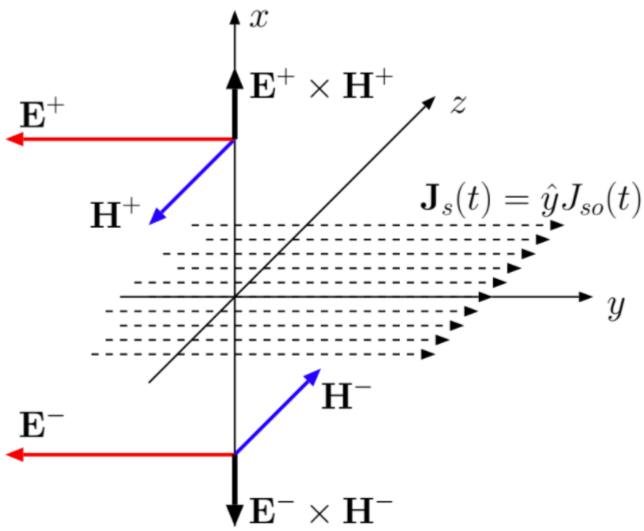
d) Finally, since $E_y = -\eta H_x$ (recall that the propagation direction is $\hat{z} = -\hat{y} \times \hat{x}$, hence the minus sign), we have that

$$g(t - \frac{z}{c/2}) = -60\pi \times (\frac{40z}{c} - 20t) = 1200\pi \times \left(t - \frac{z}{c/2}\right),$$

then

$$g(t) = 1200\pi t.$$

3. The pulse of sheet current $\mathbf{J}_s(t) = \hat{y}8t\text{rect}(\frac{t}{\tau})\,\frac{\text{A}}{\text{m}}$ will produce magnetic and electric fields as shown in the following figure.



The magnetic field is: (it's direction can be verify using the right-hand-rule for Ampere's law $\oint_C \mathbf{H} \cdot d\mathbf{l} = I_C$)

$$\begin{aligned} \mathbf{H}^{\pm}(x,t) &= \mp\frac{1}{2}J_{so}\left(t \mp \frac{x}{c}\right)\hat{z}\,\frac{\text{A}}{\text{m}} \\ &= \mp 4\left(t \mp \frac{x}{c}\right)\text{rect}\left(\frac{t \mp \frac{x}{c}}{\tau}\right)\hat{z}\,\frac{\text{A}}{\text{m}} \quad \text{for } x \gtrless 0, \end{aligned}$$

whereas the electric field is given by

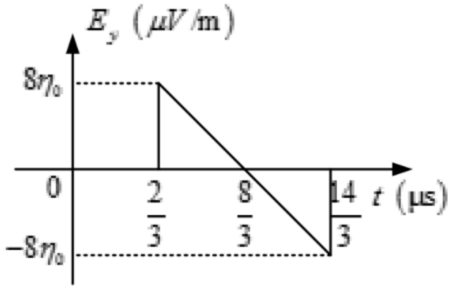
$$\mathbf{E}^{\pm}(x,t) = -\frac{\eta_o}{2}J_{so}\left(t \mp \frac{x}{c}\right)\hat{y} = -4\eta_o\left(t \mp \frac{x}{c}\right)\text{rect}\left(\frac{t \mp \frac{x}{c}}{\tau}\right)\hat{y}\,\frac{\text{V}}{\text{m}} \quad \text{for } x \gtrless 0,$$

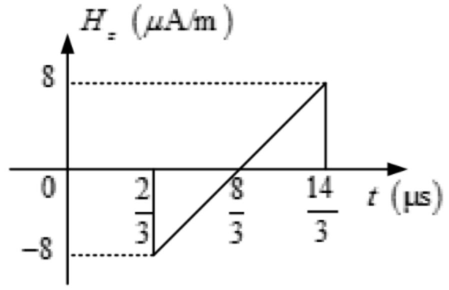
where $\eta_o = 120\pi\,\Omega$ is the intrinsic impedance of free-space.

a) The fields are given by

$$\begin{aligned} H_z(x,t)|_{x=-800\,m} &= 4\left(t - \frac{800}{3\times 10^8}\right)\text{rect}\left(\frac{t - \frac{800}{3\times 10^8}}{4\times 10^{-6}}\right)\hat{z} \\ &= 4\left(t - \frac{8}{3}\,\mu\text{s}\right)\text{rect}\left(\frac{t - \frac{8}{3}\,\mu\text{s}}{4\,\mu\text{s}}\right)\hat{z}\,\frac{A}{m} \end{aligned}$$

$$E_y(x,t)|_{x=-800\,m} = -4\eta_o\left(t-\frac{8}{3}\,\mu\text{s}\right)\,\text{rect}\left(\frac{t-\frac{8}{3}\,\mu\text{s}}{4\,\mu\text{s}}\right)\,\hat{y}\,\frac{V}{m}$$

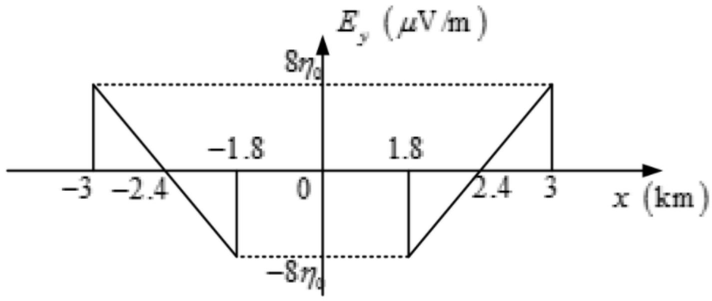


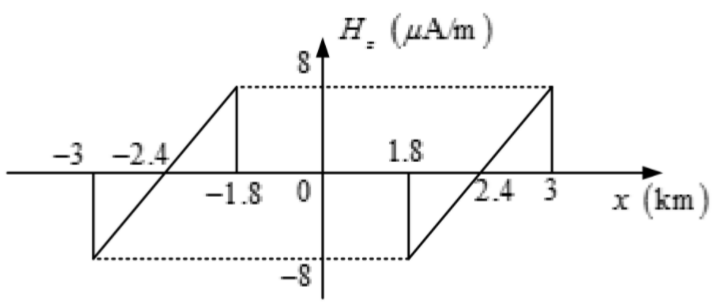


b) The fields are given by

$$H_z(x,t)|_{t=8\,\mu\text{s}} = \mp 4\left(8\,\mu\text{s}\mp\frac{x}{c}\right)\,\text{rect}\left(\frac{8\mu\text{s}\mp\frac{x}{c}}{4\,\mu\text{s}}\right)\,\hat{z}\,\frac{A}{m} \quad \text{for } x \gtrless 0$$

$$E_y(x,t)|_{t=8\,\mu\text{s}} = -4\eta_o\left(8\,\mu\text{s}\mp\frac{x}{c}\right)\,\text{rect}\left(\frac{8\mu\text{s}\mp\frac{x}{c}}{4\,\mu\text{s}}\right)\,\hat{y}\,\frac{\text{V}}{\text{m}} \quad \text{for } x \gtrless 0$$





c) Following the hint given in the problem, we can write

$$
\begin{aligned}
-\mathbf{J}_s \cdot \mathbf{E} &= -\left(\hat{y}\, 8t \,\mathrm{rect}\left(\frac{t}{\tau}\right)\right) \cdot \left(-4\eta t \,\mathrm{rect}\left(\frac{t}{\tau}\right)\, \hat{y}\right) \\
&= 32\eta_o t^2 \,\mathrm{rect}^2\left(\frac{t}{\tau}\right) \frac{\mathrm{W}}{\mathrm{m}^2}.
\end{aligned}
$$

Then, the TEM wave density energy is

$$
\begin{aligned}
\int -\mathbf{J}_s \cdot \mathbf{E}\, dt &= \int 32\eta_o t^2 \,\mathrm{rect}^2\left(\frac{t}{\tau}\right) dt \\
&= \int_{-\tau/2}^{\tau/2} 32\eta_o t^2\, dt \;=\; \frac{32}{3}\eta_o \left[t^3\right]\Big|_{-\tau/2}^{\tau/2} \\
&= \frac{8}{3}\eta_o \tau^3 = 2048\pi \times 10^{-17}\, \frac{\mathrm{J}}{\mathrm{m}^2}.
\end{aligned}
$$

4.

a) For the plane wave described by $\mathbf{E}_1 = 4\cos(\omega t - \beta z)\hat{x}\, \frac{\mathrm{V}}{\mathrm{m}}$;

i. The magnetic field $\mathbf{H}$ should satisfy $\mathbf{H} = -\frac{\mathbf{E}\times\hat{\beta}}{\eta}$, where $\hat{\beta}$ is the unit vector parallel to the propagation direction. Then, we can find the expressions for $\mathbf{H}$ field of the given plane wave as

$$
\hat{\beta}_1 = \hat{z} \rightarrow \mathbf{H}_1 = \frac{4}{\eta_o}\cos(\omega t - \beta z)\hat{y}\, \frac{\mathrm{A}}{\mathrm{m}}.
$$

ii. The instantaneous power flow density is given by the Poynting vector $\mathbf{S} = \mathbf{E}\times\mathbf{H}$. Therefore, the instantaneous power that crosses some surface A is given by $P = \int_A \mathbf{S}\cdot d\mathbf{A}$, where $d\mathbf{A} \equiv \hat{n}\cdot A$. Therefore, the Poynting vector is found as

$$
\mathbf{S}_1 = \mathbf{E}_1 \times \mathbf{H}_1 = \frac{16}{\eta_o}\cos^2(\omega t - \beta z)\hat{z}\, \frac{\mathrm{W}}{\mathrm{m}^2},
$$

and expression for instantaneous power that crosses a $1\,\mathrm{m}^2$ area (i.e. $A = 1\,\mathrm{m}^2$) in the xy-plane from $-z$ to $+z$ may be written as

$$
P_1 = \frac{16}{\eta_o}\cos^2(\omega t - \beta z)\,\mathrm{W}.
$$

iii. We can calculate the time-average of the Poynting vector using the trigonometric identity: $\cos^2\theta = \frac{1}{2}\left(1 + \cos(2\theta)\right)$. Based on the fact that the time average of the cosine wave is zero ($\frac{1}{T}\int_T \cos(\omega t)dt = 0$), we can write

$$
\langle \mathbf{S}_1 \rangle = \left\langle \frac{16}{\eta_o}\cos^2(\omega t - \beta z)\hat{z}\, \frac{\mathrm{W}}{\mathrm{m}^2} \right\rangle = \frac{8}{\eta_o}\hat{z}\, \frac{\mathrm{W}}{\mathrm{m}^2}.
$$

Therefore, the average power that crosses some surface A is given by

$$
\langle P_1 \rangle = \langle \mathbf{S}_1 \rangle \cdot \hat{n}\, A = \frac{8}{\eta_o}\,\mathrm{W}.
$$

b) For the plane wave described by $\mathbf{E}_2 = E_o \left(\cos(\omega t - \beta z)\hat{x} + \sin(\omega t - \beta z)\hat{y}\right) \frac{\mathrm{V}}{\mathrm{m}}$;

i. The propagation direction is $\hat{\beta}_2 = \hat{z}$. Thus, the magnetic field $\mathbf{H}_2$ is given by

$$\mathbf{H}_2 = \frac{E_o}{\eta_o} \left(\cos(\omega t - \beta z)\hat{y} - \sin(\omega t - \beta z)\hat{x}\right) \frac{\mathrm{A}}{\mathrm{m}}.$$

ii. The Poynting vector is given by

$$\begin{aligned}
\mathbf{S}_2 &= \mathbf{E}_2 \times \mathbf{H}_2 \\
&= E_o \left(\cos(\omega t - \beta z)\hat{x} + \sin(\omega t - \beta z)\hat{y}\right) \times \frac{E_o}{\eta_o} \left(\cos(\omega t - \beta z)\hat{y} - \sin(\omega t - \beta z)\hat{x}\right) \\
&= \frac{E_o^2}{\eta_o} \left(\cos^2(\omega t - \beta z)\hat{z} + \sin^2(\omega t - \beta z)\hat{z}\right) = \frac{E_o^2}{\eta_o} \hat{z} \, \frac{\mathrm{W}}{\mathrm{m}^2}.
\end{aligned}$$

Therefore, the instantaneous power crossing the area $A = 1\,\mathrm{m}^2$ is

$$P_2 = \frac{E_o^2}{\eta_o} \,\mathrm{W}.$$

iii. The Poynting vector is constant in time, thus the time-average power is

$$\langle P_2 \rangle = \frac{E_o^2}{\eta_o} \,\mathrm{W}.$$

c) For the plane wave described by $\mathbf{H}_3 = \cos(\omega t + \beta z + \frac{\pi}{3})\hat{x} - \sin(\omega t + \beta z - \frac{\pi}{6})\hat{y} \frac{\mathrm{A}}{\mathrm{m}}$;

i. The electric field $\mathbf{E}$ should satisfy $\mathbf{E} = \eta(\mathbf{H} \times \hat{\beta})$ where $\hat{\beta} = -\hat{z}$ in this case. Then, we can find the expressions for the $\mathbf{E}$ field of the given plane wave as

$$\mathbf{E}_3 = \eta_o \left(\cos(\omega t + \beta z + \frac{\pi}{3})\hat{y} + \sin(\omega t + \beta z - \frac{\pi}{6})\hat{x}\right) \frac{\mathrm{V}}{\mathrm{m}}.$$

ii. The Poynting vector is given by

$$\begin{aligned}
\mathbf{S}_3 &= \mathbf{E}_3 \times \mathbf{H}_3 \\
&= -\eta_o \left(\cos^2(\omega t + \beta z + \frac{\pi}{3})\hat{z} + \sin^2(\omega t + \beta z - \frac{\pi}{6})\hat{z}\right) \\
&= -\eta_o \left(\cos^2(\omega t + \beta z + \frac{\pi}{3})\hat{z} + \cos^2(\omega t + \beta z + \frac{\pi}{3})\hat{z}\right) \\
&= -2\eta_o \cos^2(\omega t + \beta z + \frac{\pi}{3})\hat{z} \, \frac{\mathrm{W}}{\mathrm{m}^2}.
\end{aligned}$$

Therefore, the instantaneous power crossing the area $A = 1\,\mathrm{m}^2$ is

$$P_3 = -2\eta_o \cos^2(\omega t + \beta z + \frac{\pi}{3}) \,\mathrm{W}.$$

iii. The time-average power crossing a $1\,\mathrm{m}^2$ area is

$$\langle P_3 \rangle = -\eta_o \,\mathrm{W}.$$

d) For the plane wave described by $\mathbf{H}_4 = \cos(\omega t - \beta x)\hat{z} + \sin(\omega t - \beta x)\hat{y} \frac{\mathrm{A}}{\mathrm{m}}$;

i. The propagation direction is $\hat{\beta}_4 = \hat{x}$. Thus, the electric field $\mathbf{E}_4$ is given by

$$\mathbf{E}_4 = \eta_o \left(\cos(\omega t - \beta x)\hat{y} - \sin(\omega t - \beta x)\hat{z}\right) \frac{\mathrm{V}}{\mathrm{m}}.$$

ii. The wave is propagating in the $+x$ direction, therefore there is no flux of energy flowing into the z direction. Therefore, the instantaneous power crossing a $1\,\mathrm{m}^2$ area in the xy-plane from $-z$ to z is

$$P_4 = 0\,\mathrm{W}.$$

iii. The time-average power crossing a $1\,\mathrm{m}^2$ area in the xy-plane from $-z$ to z is also

$$\langle P_4 \rangle = 0\,\mathrm{W}.$$