1. a) According to Gauss's law

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q_V$$

Since the caps of the cylinder will have no electric field lines going through them, the flux of the caps will be 0. We only need to calculate the side of the cylinder

$$\epsilon_0 E_r 2\pi r L = \rho_0 L \pi R^2$$

$$\rho_0 = \frac{2\epsilon_0 E_r r}{R^2} = -100\, C/m^3$$

b) Using Gauss's law, and consider a cylinder with length L

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q_V$$

$$\epsilon_0 E_{in} 2\pi r L = \pi r^2 L \rho_0$$

$$E_{in} = \frac{r\rho_0}{2\epsilon_0} = -\frac{1}{6\epsilon_0}\, V/m$$

c) From (a), for $r > R$

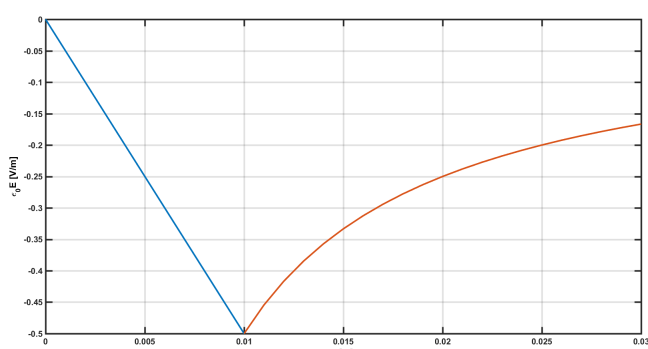
$$E_r(r) = \frac{\rho_0 R^2}{2\epsilon_0 r}$$

from (b), for r < R

$$E_{in}(r) = \frac{\rho_0 r}{2\epsilon_0}$$

at $r = R$

$$E_{in}(R) = \frac{\rho_0 R}{2\epsilon_0} = \frac{\rho_0 R^2}{2\epsilon_0 R} = E_r(R)$$



2. a) The displacement flux $\int \mathbf{D} \cdot \mathbf{dS}$ through the entire $\hat{x}\hat{z}$ plane at $y = 0$ in the $-\hat{y}$ direction is 4 C. Thus,

$$\frac{Q_1}{2} - \frac{Q_2}{2} = 4$$

Also, the displacement flux through the plane $x = 1$in the $+\hat{x}$direction is 1 C which implies that

$$\frac{Q_1}{2} + \frac{Q_2}{2} = 1$$

Using the two equations, we get

$$Q_1 = 5\,C \; and \; Q_2 = -3\,C$$

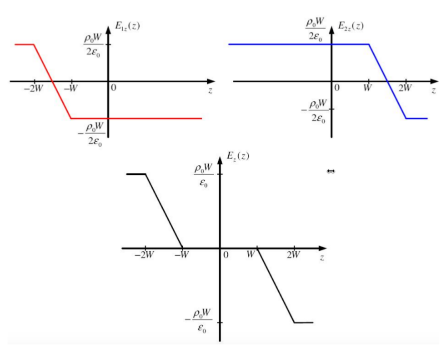
3. a) A two-slab geometry of two identical slabs of equal widths W in z$-$direction and expanding infinitely in x and y directions is considered here. The electric field generated by a single charged slab has been derived in the class notes. Making use of that result, we find that the electric fields generated by each of the two slabs are:

$$\mathbf{E_1} = \begin{cases} \frac{\rho_0 W}{2\epsilon_0}\,\hat{z} & z < -2W \\ -\frac{\rho_0}{\epsilon_0}(z + \frac{3W}{2})\,\hat{z} & -2W < z < -W \\ -\frac{\rho_0 W}{2\epsilon_0}\,\hat{z} & z > -W \end{cases}$$

$$\mathbf{E}_2 = \begin{cases} \frac{\rho_0 W}{2\epsilon_0}\,\hat{z} & z < W \\ -\frac{\rho_0}{\epsilon_0}(z - \frac{3W}{2})\,\hat{z} & W < z < 2W \\ -\frac{\rho_0 W}{2\epsilon_0}\,\hat{z} & z > 2W \end{cases}$$

respectively. Adding these fields, we find that the total field in space

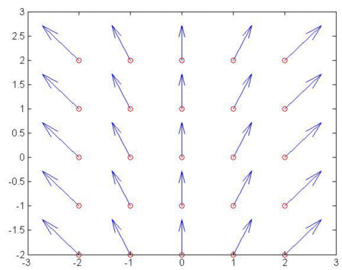
$$\mathbf{E} = \mathbf{E_1} + \mathbf{E}_2 = \begin{cases} \frac{\rho_0 W}{\epsilon_0}\,\hat{z} & z < -2W \\ -\frac{\rho_0}{\epsilon_0}(z + W)\,\hat{z} & -2W < z < -W \\ 0 & -W < z < W \\ -\frac{\rho_0}{\epsilon_0}(z - W)\,\hat{z} & W < z < 2W \\ -\frac{\rho_0 W}{\epsilon_0}\,\hat{z} & z > 2W \end{cases}$$



b) From $\mathbf{D} = \epsilon_0 \mathbf{E}$,

$$\nabla \cdot \mathbf{D} = \begin{cases} 0 & z < -2W \\ -\rho_0 & -2W < z < -W \\ 0 & -W < z < W \\ -\rho_0 & W < z < 2W \\ 0 & z > 2W \end{cases}$$

4. a) $F = x\hat{x} + 2\hat{y}$



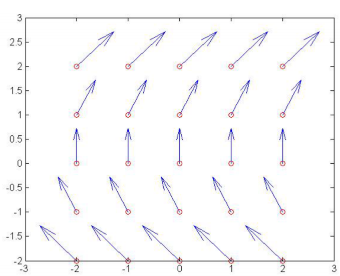
i. Curl

$$\nabla \times F = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & 2 & 0 \end{vmatrix} = 0$$

ii. Divergence

$$\nabla \cdot F = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(2) = 1$$

b) $F = y\hat{x} + 2\hat{y}$



i. Curl

$$\nabla \times F = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & 2 & 0 \end{vmatrix} = -\hat{z}$$

ii. Divergence

$$\nabla \cdot F = \frac{\partial}{\partial x}(y) + \frac{\partial}{\partial y}(2) = 0$$

c) Curl of F not equal to 0 implies that the field strength varies across the direction of the field F

d) Divergence of F not equal to 0 implies that the field strength varies along the direction of the field F

5. a) Curl of E,

$$\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ cos(y) & sin(y) & 0 \end{vmatrix} = -\frac{\partial}{\partial y}(cos(y))\hat{z} = sin(y)\text{♩}\hat{z}$$

Curl of curl of E,

$$\nabla \times (\nabla \times E) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & sin(y) \end{vmatrix} = -\frac{\partial}{\partial y}(sin(y))\hat{x} = cos(y)\text{♩}\hat{x}$$

b) Bying Gauss law(in differential form)

$$\nabla \cdot D = \epsilon_0 \nabla \cdot E = \rho$$

we find that

$$\rho = \epsilon_0 \nabla \cdot E = \epsilon_0 \frac{\partial}{\partial y}(sin(y)) = \epsilon_0 cos(y)\, C/m^3$$

6. Given $\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} = E_r\hat{r}$

a) Using the curl in spherical coordinate,

$$\nabla \times E = \frac{1}{r^2 sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & rsin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ E_r & rE_\theta & rsin\theta E_\phi \end{vmatrix} = 0$$

b) Using the divergence in spherical coordinate,

$$\nabla \cdot E = \frac{1}{r^2}\frac{\partial(r^2 E_r)}{\partial r} + \frac{1}{rsin\theta}\frac{\partial(E_\theta sin\theta)}{\partial\theta} + \frac{1}{rsin\theta}\frac{\partial(E_\phi)}{\partial\phi} = 0$$

when $r \neq 0$

7. Given an electrostatic potential $V(x, y, z) = 3z^2 - 2\,V$ in certain region of space, let us calculate the following,

a) Electrostatic field E,

$$E = -\nabla V = -\frac{\partial}{\partial x}(3z^2 - 2)\hat{x} - \frac{\partial}{\partial y}(3z^2 - 2)\hat{y} - \frac{\partial}{\partial z}(3z^2 - 2)\hat{z} = -6z\hat{z}\, V/m$$

b) Curl of E,

$$\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & -6z \end{vmatrix} = 0$$

c) Divergence of E,

$$\nabla \cdot E = \frac{\partial}{\partial z}(-6z) = -6$$

d) Charge density ρ,

$$\rho = \epsilon_0 \nabla \cdot E = -6\epsilon_0\, C/m^3$$