ECE 329 | Fields & Waves I | Fall 17

University of Illinois | He, Ilie, Lee, Waldrop

Exam 3

Wednesday, Nov 15, 2017 — 7:00-8:15 PM

Please clearly PRINT your name in CAPITAL LETTERS and circle your section in the boxes below.

Name:	Solution
Section:	11 AM 12 Noon 1 PM

If you want your exam returned to a **different** section, write it here:

Special Return to Section:	11 AM 12 Noon 1 PM

This is a closed book exam and calculators/electronic devices are not allowed. Please show all your work and make sure to include your reasoning for each answer. All answers should include **units** wherever appropriate. The exam is double sided. You may use the back of the exam as scratch paper.

Physical constants, a table, and a reference figure are included on THE LAST PAGE of this exam (you can tear it out for convenience).

Problem 1 (10 points)	
Problem 2 (15 points)	
Problem 3 (25 points)	
Problem 4 (25 points)	
Problem 5 (25 points)	
TOTAL (100 points)	

1. (10 points) TRUE or FALSE questions (2 points each). Circle either **TRUE** or **FALSE**. Note that some of these questions require some calculation to get the right answer, but you do not need to show your work to get credit. No partial credit is given on these questions.

   a) TRUE or FALSE: Sea water can be treated as an imperfect dielectric in some cases.

   $\frac{\sigma}{\omega\epsilon} \ll 1$ for imperfect dielectric. At high ω, can be imperfect dielectric

   b) TRUE or FALSE: $\nabla \cdot (\mathbf{E} \times \mathbf{H})$ accounts for energy transport in Poynting Theorem

   c) TRUE or FALSE: Linear-polarized EM waves are radiated by rotating charges

   Circular polarized waves come from rotating charges (Lecture 24)

   d) TRUE or FALSE: $\mathbf{E} = 4\sin(\omega t - \beta z + \phi)\hat{x} + 4\sin(\omega t - \beta z + \phi)\hat{y}$ describes a circular polarized wave.

   e) TRUE or FALSE: In a region of free space where a standing wave condition exists, the term $(\frac{1}{2}\epsilon\mathbf{E}\cdot\mathbf{E} + \frac{1}{2}\mu\mathbf{H}\cdot\mathbf{H}) = 0$ $\frac{\partial}{\partial t}$ of above is zero, but energy is present

2. (15 points) Short answer and multiple choice questions. Some questions require some calculation to get the right answer, but you do not need to show your work to get credit. No partial credit.

   (i) (2 points) Express $-\frac{\partial \mathbf{B}}{\partial t}$ as a phasor

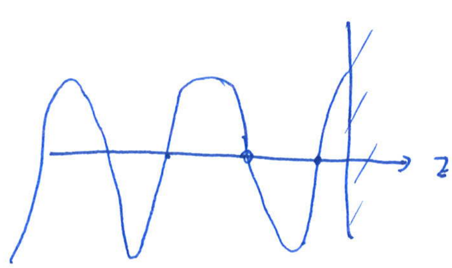
   $-j\omega\tilde{B}$

   (ii) (3 points) A plane TEM wave with $\hat{x}$ polarized $\mathbf{E}$ propagating along $+\hat{z}$ is incident on a perfectly conducting mirror at $z = 0$. Sketch $E_x$vs z near the mirror at some moment in time t_1. Hint: the position of the nodes is what matters most.

   perfect conductor, $\vec{E}$ must have a node at $z = 0$

   No penetration of wave to conductor

   (iii) (3 points) For the same TEM wave as above, sketch $H_y$vs z near the mirror at the same moment in time t_1. Hint: the position of the nodes is what matters most.



   - Node shifted left by $\frac{\lambda}{4}$
   - Current sheet induced on surface of mirror = source of reflected wave

(iv) (3 points) The magnetic field of a linearly polarized wave in a lossless magnetic material having the permitivity of free space ϵ_0 and intrinsic impedence of $\eta = 150\Omega$ is $\mathbf{H} = -3Re[e^{j(\omega t - \beta y)}]\hat{x}\ [A/m]$. The corresponding electric field in the time domain in units of $[V/m]$ is:

a) $\mathbf{E} = -450\cos(\omega t + \beta y)\hat{z}$

b) $\mathbf{E} = 450\cos(\omega t - \beta y)\hat{z}$

c) $\mathbf{E} = \frac{3}{150}cos(\omega t + \beta y)\hat{z}$

d) $\mathbf{E} = -\frac{3}{150}cos(\omega t - \beta y)\hat{z}$

e) None of these.

a) is close, but needs to be $-\beta y$ instead of $+\beta y$

(v) (2 points) A voltage source $f_i(t) = u(t)$ is connected to a transmission line with $R_g = 25\,\Omega$, $Z_o = 50\,\Omega$, and $R_L = 25\,\Omega$.
After a long time, what is the voltage across the load, V_L?

Voltage divider at long time,
$V_L = 0.5\,V$

(vi) (2 points) For the T.L. circuit of the previous problem, the function is swapped to $f_i(t) = 10rect(t)$. After a long time, what is the voltage across the load, V_L?

0 V

3. (25 points) For a plane TEM wave in a "good conductor," parameters γ and η satisfy

$$\gamma\eta = j\omega\mu \text{ and } \frac{\gamma}{\eta} = \sigma + j\omega\epsilon$$

The electric field of such a plane wave propagating in a non-magnetic material ($\mu = \mu_0$) is given by:

$$\mathbf{E} = e^{-3x}\sin(2\pi \times 10^4 t - 4x)\hat{y} \text{ V/m}$$

Noting physical constants given on the last page:

a) (3 pts) The wave's attenuation constant α, wave number β, and propagation constant γ

Your Answer (include appropriate units):

$\alpha =$ 3 /m $\qquad \beta =$ 4 rad/m $\qquad \gamma =$ 3 + 4j /m

b) (3 pts) The wavelength λ, linear frequency f, and phase velocity v_p of the wave

$\lambda = \frac{2\pi}{\beta}$ $\qquad \omega = 2\pi f$, $2\pi \times 10^4 = 2\pi f$ $\qquad v = \frac{\omega}{\beta} = \frac{2\pi \cdot 10^4 \text{ rad/s}}{4 \text{ rad/m}}$

Your Answer (include appropriate units):

$\lambda =$ $\frac{\pi}{2}$ m $\qquad f =$ 10 kHz $\qquad v_p =$ $\frac{\pi}{2} \times 10^4$ $\frac{m}{s}$

c) (6 pts) The amplitude and phase of the intrisic impedance η of the medium. The angle *does not* have to be solved explicitly, but can be left in terms of appropriate constants and trigonometric functions.

$$\eta = \frac{j\omega\mu}{\gamma}, \quad |\eta| = \left|\frac{\omega\mu}{\gamma}\right| = \frac{2\pi \times 10^4 \cdot 4\pi \times 10^{-7}}{5} = \frac{8}{5}\pi^2 \times 10^{-3}$$

$$\hookrightarrow \frac{j\omega\mu}{3+4j} \cdot \frac{3-4j}{3-4j} = \frac{j3\omega\mu + 4\omega\mu}{25} \Rightarrow \angle\eta = \tan^{-1}\left(\frac{3}{4}\right)$$

Your Answer (include appropriate units):

$|\eta| =$ $\frac{8}{5}\pi^2 \times 10^{-3}$ or $\frac{1}{625}\pi^2$ Ω $\qquad \tau = angle(\eta) =$ $\tan^{-1}\left(\frac{3}{4}\right)$ radians

d) (8 pts) The associated magnetic field in time domain ($\mathbf{H}$) and in phasor domain $\tilde{\mathbf{H}}$.

$\vec{E} \Rightarrow \hat{y}$ $\qquad \vec{H} \Rightarrow \hat{y} \times \hat{z} = \hat{x}$ $\qquad$ x, z y

$\vec{S} \Rightarrow \hat{x}$

Your Answer (include vector direction and appropriate units):

$\mathbf{H} =$ $\frac{e^{-3x}}{|\eta|}\sin(2\pi \times 10^4 t - 4x - \tau)\,\hat{z}$ $\frac{A}{m}$

$\hat{\mathbf{H}} =$ $-j\, e^{-3x} \frac{1}{|\eta|} e^{-j4x} e^{-j\tau}\,\hat{z}$ A/m

e) (5 points) Consider a TEM wave propagating in a good conductor. Is the electric field phasor $\hat{\mathbf{E}} = \hat{x}e^{0.5z - j0.5z}$V/m a valid wave field solution to Maxwell's equations? Explain.

No. The $e^{0.5z}$ term is a growing exponential in the positive z-direction. Simultaneously, the wave is propagating along $+\hat{z}$. It should decay in the good conductor, <u>not</u> grow.

4. (25 points) A plane wave with $\mathbf{E} = 4\cos(\omega t - \beta z)\hat{x} + 4\sin(\omega t - \beta z + \phi)\hat{y}$ travels through free space and encounters a perfect dielectric at $z = 0$ with permeability μ_o and permittivity $4\epsilon_o$.

a) (2 pts) What value of ϕ makes the incident wave RHCP?

$$\phi = 0$$

b) (2 pts) What value of ϕ makes the reflected wave RHCP?

$$\phi = 180°$$

c) (4 pts) Determine Γ, τ.

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \eta_2 = \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{1}{2}\eta_0 \qquad \tau = 1 + \Gamma$$

$$\Gamma = \frac{\frac{1}{2} - 1}{\frac{1}{2} + 1} = \underline{-\frac{1}{3}} \qquad = \underline{\frac{2}{3}}$$

d) (12 pts) What are the phasors $\tilde{\mathbf{E}}_{\mathbf{i,r,t}}$ and $\tilde{\mathbf{H}}_{\mathbf{i,r,t}}$ where **i, r,** and **t** stand for incident, reflected, and transmitted, respectively? Express the answer explcitly, and ***do not*** leave them in terms of Γ, τ.

One acceptable form of answer

$$\tilde{E}_i = 4e^{-j\beta z}(\hat{x} - je^{j\phi}\hat{y}), \quad \tilde{H}_i = \frac{4}{\eta_0}e^{-j\beta z}(\hat{y} + je^{j\phi}\hat{x})$$

$$\tilde{E}_r = -\frac{4}{3}e^{+j\beta z}(\hat{x} - je^{j\phi}\hat{y}), \quad \tilde{H}_r = -\frac{4}{3\eta_0}e^{+j\beta z}(-\hat{y} - je^{j\phi}\hat{x})$$

$$\tilde{E}_t = \frac{8}{3}e^{-j\beta_2 z}(\hat{x} - je^{j\phi}\hat{y}), \quad \tilde{H}_t = \frac{16}{3\eta_0}e^{-j\beta_2 z}(\hat{y} + je^{j\phi}\hat{x})$$

$$\underline{\beta_2 = 2\beta}$$

Recall:

$$e^{-j\frac{\pi}{2}} = \cos\left(-\frac{\pi}{2}\right) + j\sin\left(-\frac{\pi}{2}\right) = -j$$

So answer can also be nicely expressed like this:

Your Answer (include vector directions and appropriate units):

$\tilde{\mathbf{E}}_\mathbf{i} = 4e^{-j\beta z}(\hat{x} + e^{j(\phi - \frac{\pi}{2})}\hat{y})$

$\tilde{\mathbf{H}}_\mathbf{i} = \frac{4}{\eta_0}e^{-j\beta z}(\hat{y} - e^{j(\phi - \frac{\pi}{2})}\hat{x})$

$\tilde{\mathbf{E}}_\mathbf{r} = -\frac{4}{3}e^{j\beta z}(\hat{x} + e^{j(\phi - \frac{\pi}{2})}\hat{y})$

$\tilde{\mathbf{H}}_\mathbf{r} = -\frac{4}{3\eta_0}e^{j\beta z}(\hat{y} - e^{j(\phi - \frac{\pi}{2})}\hat{x})$

$\tilde{\mathbf{E}}_\mathbf{t} = \frac{8}{3}e^{-j\beta_2 z}(\hat{x} + e^{j(\phi - \frac{\pi}{2})}\hat{y})$

$\tilde{\mathbf{H}}_\mathbf{t} = \frac{16}{3\eta_0}e^{-j\beta_2 z}(\hat{y} - e^{j(\phi - \frac{\pi}{2})}\hat{x})$

e) (5 pts) What is the time-averaged Poynting vector of the transmitted wave?

$$\langle \tilde{S}_t \rangle = \frac{1}{2} Re \{ \tilde{E}_t \times \tilde{H}_t^* \}$$

$$= \left(\frac{8}{3}\right)^2 \frac{2}{\eta_0} \cdot \frac{1}{2} Re \{ \underbrace{(\hat{x} - j e^{j\phi} \hat{y}) \times (\hat{y} - j e^{-j\phi} \hat{x})}_{= 2\hat{z}} \}$$

$$= \frac{128}{9\eta_0} \hat{z} \frac{W}{m^2}$$

Your Answer (include vector directions and appropriate units):

$\langle \mathbf{S} \rangle = \frac{128}{9\eta_0} \hat{z} \frac{W}{m^2}$

5. (25 points) Consider a transmission line with characteristic impedance $Z_o = 50\,\Omega$, length $l = 540$ m, $v = 180\ \frac{m}{\mu s}$. A voltage source with internal resistance $R_g = 25\,\Omega$ is connected at the $z = 0$ end, while a load resistance $R_L = 100\,\Omega$ terminates the line at $z = l$.

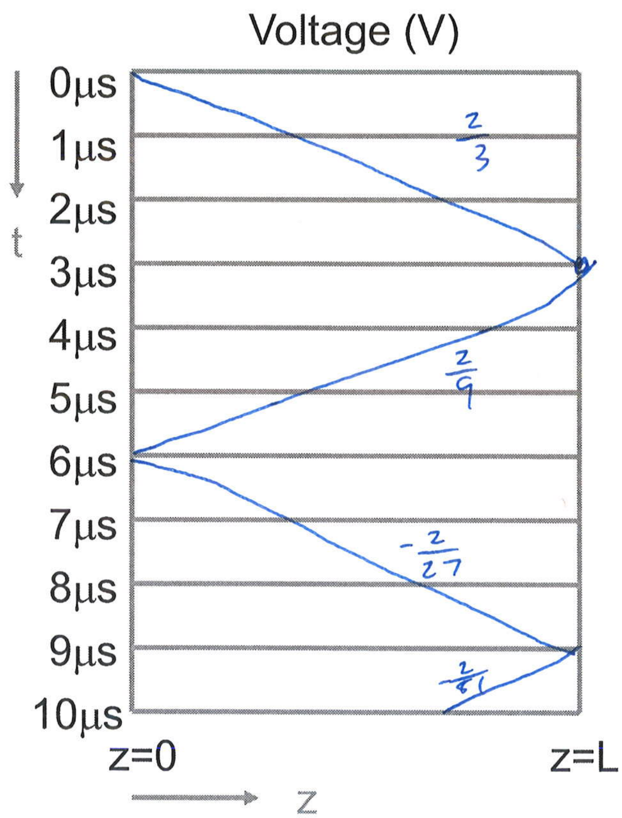
a) (6 pts) Compute the transmission line parameters τ_g, Γ_g, and Γ_L :

$\tau_g = \frac{50}{50+25}$ $\qquad$ $\Gamma_g = \frac{25-50}{25+50}$ $\qquad$ $\Gamma_L = \frac{100-50}{100+50}$

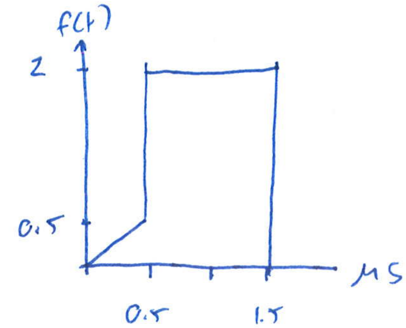
Your Answer:

τ_g = $\frac{2}{3}$ $\qquad$ Γ_g = $-\frac{1}{3}$ $\qquad$ Γ_L = $\frac{1}{3}$

b) (4 pts) Construct the voltage bounce diagram for $f(t) = u(t)$ V, where $u(t)$ is the unit-step function. Label the voltage amplitude on each bounce (you do not need to write out the functional forms for the voltage waves).



c) (5 pts) Consider the voltage source given by $f(t) = t[u(t) - u(t - 0.5\,\mu s)] + 2rect\left(\frac{t-1\,\mu s}{T}\right)$ where $T = 1\,\mu s$. Plot $f(t)$.



d) (10 points) Plot the voltage on the transmission line versus position (V vs z) at $t = 2\,\mu s$ and at $t = 4.5\,\mu s$. Be sure to label the vertical and horizontal scale in each graph.