ECE 329 review for Exam 1

by

Head TAs: Pratik Joshi and Shuo Liu

Vector calculus theorems and Coulomb's Law

Stokes' Theorem: What happens if E is a conservative field?

$$\oint \vec{E} \cdot d\vec{l} = \iint \left(\nabla \times \vec{E}\right) \cdot d\vec{S}$$

Divergence Theorem:

$$\iiint (\nabla \cdot \vec{x}) dV = \iint \vec{x} \cdot d\vec{S}$$

Force due to a point charge on another point charge (Coulomb's law)

$$\vec{F}_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{21} \qquad \vec{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{12}$$

Maxwell’s equations

Faraday’s Law	$\oint_C \vec{E} \bullet d\vec{l} = -\frac{d}{dt}\iint_S \vec{B} \bullet d\vec{S}$	$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$
Ampere’s Law	$\oint_C \vec{H} \bullet d\vec{l} = \iint_S \vec{J} \bullet d\vec{S} + \frac{d}{dt}\iint_S \vec{D} \bullet d\vec{S}$	$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$
Gauss’ Law	$\oiint_S \vec{B} \bullet d\vec{S} = 0$	$\nabla \bullet \vec{B} = 0$
Gauss’ Law	$\oiint_S \vec{D} \bullet d\vec{S} = \iiint_V \rho dV$	$\nabla \bullet \vec{D} = \rho$
Continuity Eq.	$\oiint_S \vec{J} \bullet d\vec{S} = -\frac{d}{dt}\iiint_V \rho dV$	$\nabla \bullet \vec{J} = -\frac{d\rho}{dt}$

Gauss Law (very important)

Gauss' Law for Electric Fields tells us the electric flux due to an enclosed charge

$$\oiint \vec{D} \cdot d\vec{S} = Q_{enclosed}$$

$$\nabla \cdot \vec{D} = \rho$$

To calculate E-field due to surface-charge density use symmetrical Gaussian surface which would give a uniform field on the surface.

Cylinder for a line charge

Sphere for a point charge

Boundary conditions (important)

Conductors

E-field is always zero inside!

Image charges will form in the presence of an external E-field in order to guarantee the E-field is zero inside

$\vec{J} = \sigma\vec{E}$

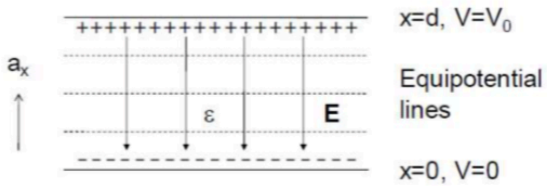
Dielectrics

Atoms can form dipoles in the presence of an electric field where the nucleus and electrons become polarized

Polarization Vector: $\vec{P} = \epsilon_0\chi_e\vec{E}_{tot}$

Connecting $\vec{D}, \vec{E}, and\ \vec{P}$: $\vec{D} = \vec{P} + \epsilon_0\vec{E}_{total} = \vec{E}_{total}\epsilon_0(1 + \chi_e), 1 + \chi_e = dielectric\ constant$

Steps for solving capacitance problems (important)



- Laplace Equation $\vec{\nabla}^2 V = 0$
- Find V using boundary conditions

$$\boxed{V(x) = V_0 \frac{x}{d}}$$

- Find **E** using $\vec{E} = -\vec{\nabla} V$
- Find **D** using $\vec{D} = \varepsilon \vec{E}$
- Get surface charge density on <u>one</u> conductor using BC $\rho_s = \vec{a}_n \bullet (D_{n1} - D_{n2})$ $\boxed{\rho = \varepsilon V_0 / d}$
- Charge $Q = (Area)(\rho_s)$
- Capacitance $C = Q / V_0$ $\boxed{C = \frac{\varepsilon A}{d}}$

Summary: few important concepts to note

- Divergence represents change **ALONG** the field
- Curl represents change **PERPENDICULAR** to the field

Important vector identities

- $\nabla.(\nabla \times \mathbf{A}) = 0$ (Here A is any vector)
- $\nabla \times (\nabla f) = 0$ (Here f is any scalar) → This property is important when you have curl-free fields (electrostatics case)

- When potential (V) is given, you have $\mathbf{E} = -\nabla V$, $\mathbf{D} = \epsilon\mathbf{E}$, $\rho_s = \nabla.\mathbf{D}$
- When **E** is given, you have potential between two points b and a is given as: $V(b) - V(a) = -\int_a^b \mathbf{E}.\mathbf{dl}$

- Inside a conductor $\mathbf{E} = \mathbf{D} = \mathbf{P} = 0$
- Inside a dielectric $\mathbf{D} = \epsilon\mathbf{E} = \epsilon_o\mathbf{E} + \mathbf{P}$, where $\mathbf{P} = \epsilon_o\mathbf{E}\, X_e$ (here X_e is susceptibility)

- When ϵ is not constant only $\nabla.\mathbf{D} = \nabla.(\epsilon\mathbf{E}) = \rho_s$ is true, however note that $\nabla.\mathbf{E} \neq \rho_s/\epsilon$.

- Capacitance (C) = Q/V, For parallel plates $C = \epsilon A/d$, For co-axial cable $C = (\epsilon l\, 2\pi) / \ln(b/a)$

Note: Review all home works; in review session we solved problems from Fall 2016, Spring 2017 exams and Q2 from Summer 2012 exam. Try to review and understand the approach and concepts for each type of problem