1. Short circuited at both ends. As we know, on the surface of perfect conductors, the tangential component of electrical field is zero. Considering the conditions given by the problem, the conductors located at z = 0 and z = l must be perpendicular to the $\pm z$ axis (because of the z-direction propagation). Thus, E_x is zero, as the tangential component. Furthermore, since the voltage is the path integral of E_x, the voltages at both ends are zero, which means short circuited.

2.

   a) The voltage and current phasors can be assumed as

$$\begin{cases} \tilde{V}(z) = \tilde{V}^+ e^{-j\beta z} + \tilde{V}^- e^{j\beta z} \\ \tilde{I}(z) = \frac{\tilde{V}^+}{Z_0} e^{-j\beta z} - \frac{\tilde{V}^-}{Z_0} e^{j\beta z} \end{cases}$$

where$\tilde{V}^+$ and $\tilde{V}^-$ correspond to the waves travelling along $+\hat{z}$ and$-\hat{z}$ directions, respectively. Based on the conclusion of Problem 1, at resonant frequencies, the voltage phasor should vanish at$z = -l$ (the left end) and $z = 0$ (the right end, refer to the coordinate system on Page 1 of Lecture 33):

$$\begin{cases} \tilde{V}^+ e^{j\beta z} + \tilde{V}^- e^{-j\beta z} = 0 \\ \tilde{V}^+ + \tilde{V}^- = 0 \end{cases}$$

$$\therefore \begin{cases} \tilde{V}^+ = -\tilde{V}^- \\ \tilde{V}^+ (e^{j\beta z} - e^{-j\beta z}) = 0 \end{cases}$$

$$\therefore\ \tilde{V}^+ \cdot 2j \sin \beta l = 0$$

$$\because\ \tilde{V}^+ \neq 0$$

$$\therefore\ \beta l = n\pi \quad or \quad \beta = \frac{n\pi}{l}$$

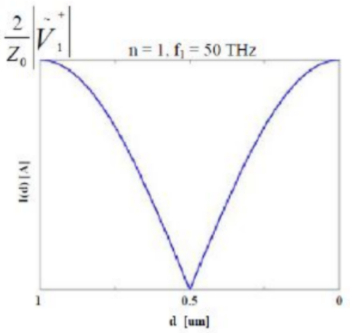
where n is an arbitrary integer. Thus, resonant frequencies are:

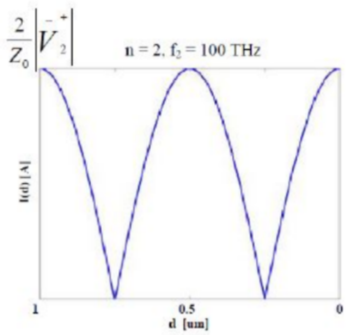
$$f_n = \frac{v}{\lambda} = \frac{\beta v}{2\pi} = \frac{n\pi}{l} \cdot \frac{v}{2\pi} = 50n \times 10^{12} [Hz] = 50n\ [THz]$$

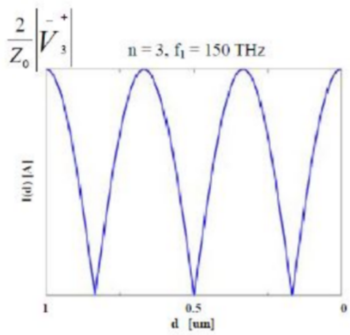
   b) At resonant frequencies,

$$\tilde{I}(d) = \tilde{I}(-z) = \frac{\tilde{V}^+}{Z_0}(e^{+j\beta d} + e^{-j\beta d}) = 2\frac{\tilde{V}_n^+}{Z_0}\cos(\frac{n\pi}{l}d)\ [A]$$

$$\therefore\ |\tilde{I}(d)| = \frac{2}{Z_0}|\tilde{V}_n^+ \cos(\frac{n\pi}{l}d)|\ [A]$$



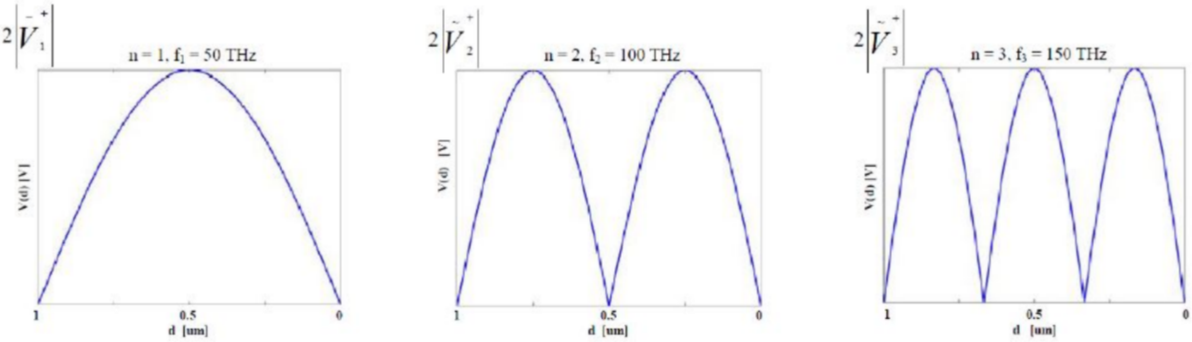




c) At resonant frequencies,

$$\tilde{V}(d) = \tilde{V}(-z) = \tilde{V^+}(e^{+j\beta d} - e^{-j\beta d}) = 2j\tilde{V_n^+}\sin(\frac{n\pi}{l}d)\ [V]$$

$$\therefore\ |\tilde{V}(d)| = 2|\tilde{V_n^+}\sin(\frac{n\pi}{l}d)|\ [V]$$



d) For all cases (three lowest resonance frequencies), although the voltage and current distribution along the transmission line are different, we always have zero voltage at the source end (short circuited). Therefore, for all three cases, we have a series resonance at the source end $z_{in} = 0$.

e) We need to find out the points where the voltage equals to zero even without a short to ground. Based on the voltage distribution along the transmission line, we can see (1) for n = 0, no such points at all; (2) $d = 0.5$ $[\mu m]$; (3) $d = \frac{1}{3}$ $[\mu m]$ and $d = \frac{2}{3}$ $[\mu m]$.

3.

a) From Lecture 32

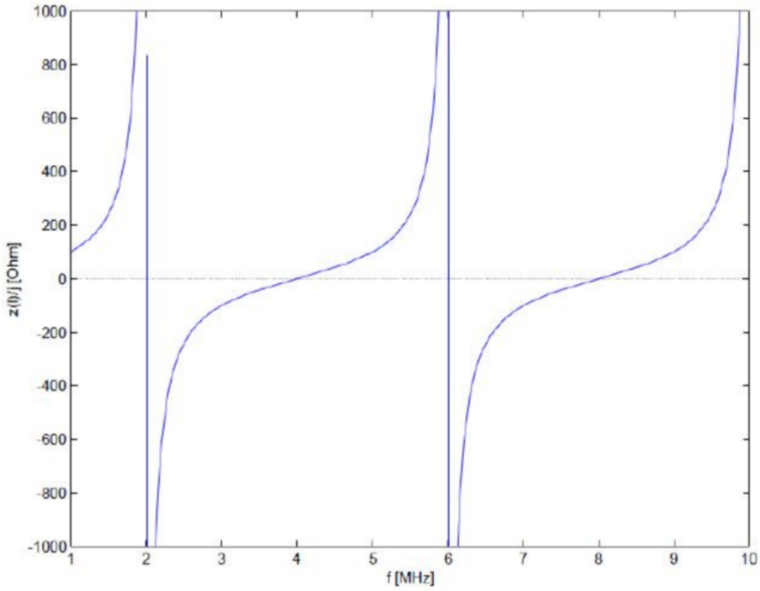
$$\begin{cases} \tilde{V}(d) = \tilde{V}^+e^{j\beta z} + \tilde{V}^-e^{-j\beta z} \\ \tilde{I}(d) = \frac{\tilde{V}^+}{Z_0}e^{j\beta z} - \frac{\tilde{V}^-}{Z_0}e^{-j\beta z} \end{cases}$$

$$V^+ + V^- = 0$$

$$\tilde{V}(d) = \tilde{V^+}(e^{j\beta d} - e^{-j\beta d}) = j2V^+\sin(\beta d)$$

$$\tilde{I}(d) = \frac{\tilde{V^+}}{Z_0}e^{j\beta d} + \frac{\tilde{V^+}}{Z_0}e^{-j\beta d} = \frac{2V^+}{Z_0}\cos(\beta d)$$

$$Z(l) = \frac{\tilde{V}(l)}{\tilde{I}(l)} = jZ_0\tan(\beta l) = j100\tan(\frac{\pi}{4\times 10^6}f)$$



b) When the length of the line equals to multiples of half wavelength, the input impedance is 0, so is the voltage at input. Solve for

$$n \times 0.5\lambda = 10$$

and plug in

$$\lambda = \frac{v}{f}$$

We can find that, the input impedance is 0 when

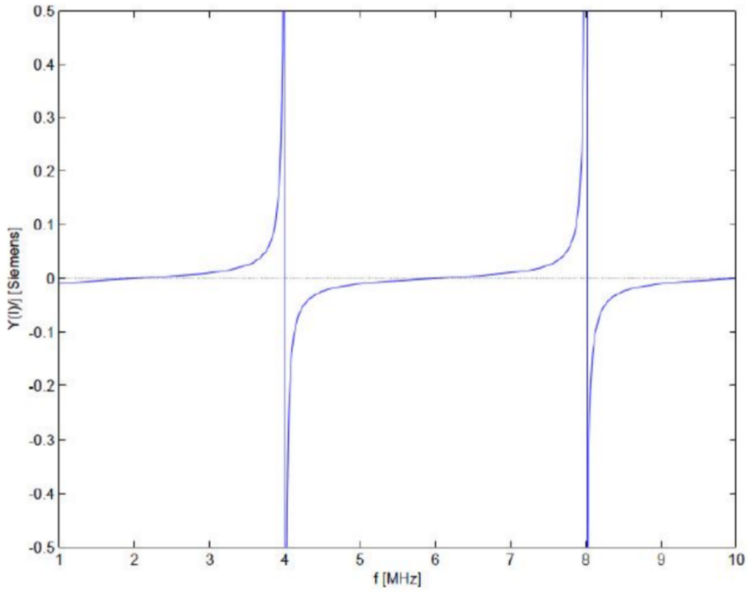
$$f = 4n\ [MHz]$$

In the given range of frequency, we know the voltage will be zero at the input end when

$$f = 4\ and\ 8\ [MHz]$$

c) Input admittance is

$$Y(l) = \frac{1}{Z(l)} = -j\frac{1}{100\tan(\frac{\pi}{4\times10^6}f)}$$



d) When the length of the line equals to odd multiples of quarter wavelength, the input admittance is 0, so is the current at input. Solve for

$$(2n+1) \times 0.25\lambda = 10$$

and plug in

$$\lambda = \frac{v}{f}$$

We can find that, the input admittance is 0 when

$$f = 2(2n+1)\ [MHz]$$

In the given range of frequency, we know the current will be zero at the input end when

$$f = 2, 6, 10\ [MHz]$$

4.

a)

$$f = \frac{\omega}{2\pi} = 5 \times 10^7\ [Hz]$$

$$\lambda = \frac{v}{f} = \frac{\frac{2}{3}c}{f} = \frac{\frac{2}{3} \times 3 \times 10^8}{5 \times 10^7} = 4\ [m]$$

b) The "open" boundary indicates that $\tilde{I}(0) = 0$ for any length.

c) Yes. Since total reflection occurs at the open end, the reflected voltage has the same magnitude as the incident voltage, which means that a standing wave will be formed. We can also verify this mathematically:

$$\tilde{V}(d) = \tilde{V^+}e^{+j\beta d} + \tilde{V^-}e^{-j\beta d} = \tilde{V^+}e^{+j\beta d} + \tilde{V^+}e^{-j\beta d} = 2V^+\cos(\beta d)$$

which is obviously a standing-wave pattern.

d) $I_R = 2\angle 0$ [A] means that the current on the transmission line is zero at d = l. Since the current is also zero at d = 0, we know that $l = n\frac{\lambda}{2}$ (n = 1, 2, 3...), which means the smallest non-zero l is $\frac{\lambda}{2} = 2$ [m].

e) If $l = \frac{\lambda}{2}$, then $\tilde{V}(\frac{l}{2}) = 0$ since there is always a voltage null exactly in the middle of two adjacent current nulls.

f) If $l = \frac{\lambda}{2}$, then $\tilde{I}(\frac{l}{2})$ is the maximum point for the current standing wave, which can't be zero.

g) $l_R = 0$means $\tilde{I}(l)$ is maximum and thus $l = (n + \frac{1}{2})\frac{\lambda}{2}$ $(n = 0, 1, 2, 3...)$. Therefore, the smallest non-zero l is $\frac{\lambda}{4} = 1$ [m].

h) Recall the logic d): $\tilde{I}(l) = 0$, $\tilde{I}(0) = 0 \Longrightarrow l = n\frac{\lambda}{2}$ (n = 1, 2, 3...). Now, $\tilde{I}(\frac{l}{2}) = 0$, $\tilde{I}(0) = 0$ $\Longrightarrow \frac{l}{2} = n\frac{\lambda}{2}$ (n = 1, 2, 3...)$\Longrightarrow l = n\lambda$ (n = 1, 2, 3...). Thus, the minimum l is = 4 [m].

i) If $l = \lambda$, then $\tilde{I}(l) = 0$ and thus $\tilde{V}(l) = I_R \cdot 50\Omega = 100\angle 0$ $[V]$.