1. According to telegrapher's equation

$$Z_o = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}},\ v = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}}.$$

Therefore, we have

$$\mathcal{L} = \frac{Z_o}{v} = \frac{50}{2\times 10^7} = 2.5\times 10^{-6}\,\frac{\text{H}}{\text{m}}.$$
$$\mathcal{C} = \frac{1}{Z_o v} = \frac{1}{50\times 2\times 10^7} = 1\times 10^{-9}\,\frac{\text{F}}{\text{m}}.$$

2.

a) The geometrical factor of the RG-59 coax cable is given by

$$\text{GF} = \frac{2\pi}{\ln\left(\frac{0.056}{0.016}\right)} = 5.015,$$

from which we obtain

$$\mathcal{L} = \frac{\mu_o}{\text{GF}} = \frac{4\pi\times 10^{-7}}{5.015} = 250.6\,\frac{\text{nH}}{\text{m}},$$
$$\mathcal{C} = \epsilon_o\text{GF} \approx \frac{10^{-9}}{36\pi}\times 5.015 = 44.3\,\frac{\text{pF}}{\text{m}},$$
$$Z_o = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \frac{1}{\text{GF}}\sqrt{\frac{\mu_o}{\epsilon_o}} \approx 5.015\times 120\pi = 75.17\,\Omega,$$
$$v = \frac{1}{\sqrt{\epsilon_o\mu_o}} = c \approx 3\times 10^8\,\frac{\text{m}}{\text{s}}.$$

b) The geometrical factor of the RG-58 coax cable will not change since RG-58 has the same inner and outher conductor diameters. On the other hand, the inductance, capacitance, characteristic impedance, and the propagation velocity will be

$$\mathcal{L} = \frac{\mu_o}{\text{GF}} = \frac{4\pi\times 10^{-7}}{2\pi/\ln(3.5)} = 2.506\times 10^{-7}\,\frac{\text{H}}{\text{m}} = 250.6\,\frac{\text{nH}}{\text{m}},$$
$$\mathcal{C} = 2.25\epsilon_o\text{GF} \approx \frac{2.25\times 10^{-9}}{36\pi}\,\frac{2\pi}{\ln(3.5)} = 99.8\,\frac{\text{pF}}{\text{m}},$$
$$Z_o = \frac{1}{\text{GF}}\sqrt{\frac{\mu_o}{2.25\epsilon_o}} \approx \frac{\ln(3.5)}{2\pi}\,\frac{120\pi}{\sqrt{2.25}} = 50.11\,\Omega,$$
$$v = \frac{1}{\sqrt{2.25\epsilon_o\mu_o}} = \frac{2}{3}c \approx 2\times 10^8\,\frac{\text{m}}{\text{s}}.$$

due to the change in the permittivity of the dielectric filling.

3. For twin-lead transmission lines, the geometrical factor is given by

$$\mathrm{GF} = \frac{\pi}{\cosh^{-1}\left(\frac{D}{2a}\right)}.$$

Since

$$Z_o = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \frac{1}{\mathrm{GF}}\sqrt{\frac{\mu}{\epsilon}} = \frac{\cosh^{-1}\left(\frac{D}{2a}\right)}{\pi}\sqrt{\frac{\mu}{\epsilon}},$$

we can can find that

$$D = 2a\cosh\left(Z_o\pi\sqrt{\frac{\epsilon}{\mu}}\right).$$

Assuming$\epsilon = \epsilon_o$,$\mu = \mu_o$, and$a = 0.5\,\mathrm{mm} = 0.5 \times 10^{-3}\,\mathrm{m}$, let us calculate$D$ as follows:

a) For$Z_o = 75\,\Omega$,

$$D = 1 \times 10^{-3}\cosh\left(\frac{75\pi}{120\pi}\right) \approx 1.20 \times 10^{-3}\,\mathrm{m} = 1.20\,\mathrm{mm}.$$

b) For$Z_o = 300\,\Omega$,

$$D = 1 \times 10^{-3}\cosh\left(\frac{300\pi}{120\pi}\right) \approx 6.13 \times 10^{-3}\,\mathrm{m} = 6.13\,\mathrm{mm}.$$

4.

a) In this circuit, the**injection coefficient** is found as

$$\tau_g = \frac{Z_o}{R_g + Z_o} = \frac{1}{2}.$$

The load voltage reflection coefficient is given by

$$\Gamma_{L_V} = \frac{R_L - Z_o}{R_L + Z_o} = \frac{1}{2},$$

whereas the generator voltage reflection coefficient is

$$\Gamma_{g_V} = \frac{R_g - Z_o}{R_g + Z_o} = 0.$$

The corresponding current reflection coefficients are

$$\Gamma_{L_I} = -\Gamma_{L_V} = -\frac{1}{2}, \text{ and}$$

$$\Gamma_{g_I} = -\Gamma_{g_V} = 0.$$

Referring to the full expressions of$V(z,t)$ and$I(z,t)$ given in page 7 of Lecture 21, we can write

$$V(z,t) = \frac{1}{2}\delta(t - \frac{3z}{c}) + \frac{1}{4}\delta(t + \frac{3z}{c} - 2\mu\mathrm{s})$$
$$I(z,t) = \frac{1}{100}\delta(t - \frac{3z}{c}) - \frac{1}{200}\delta(t + \frac{3z}{c} - 2\mu\mathrm{s}).$$

Using these information, we can build the following "bounce diagrams" for the voltage$V(z,t)$ and current$I(z,t)$ on the transmission line.

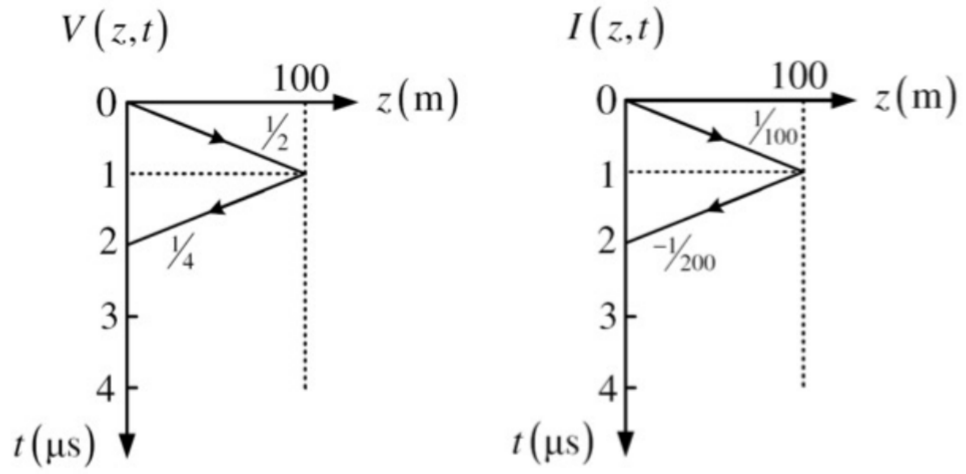


Figure 1: Voltage and current bounce diagrams for voltage source $f(t) = \delta(t)$.

b) Evaluating the $V(z,t)$ and $I(z,t)$ expressions at $z = \frac{l}{2} = 50\,\text{m}$, we have

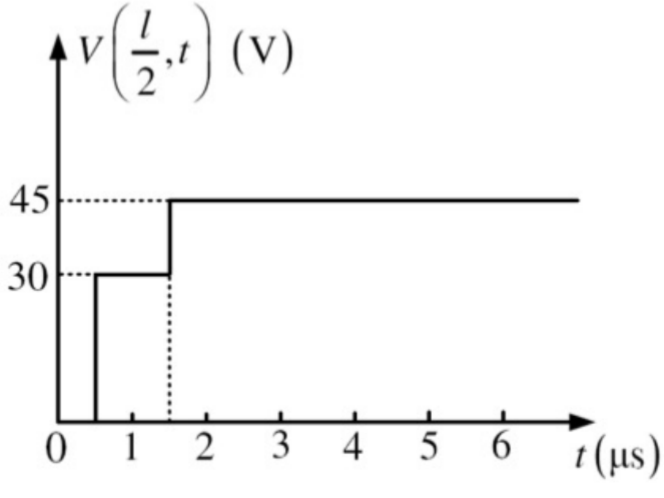
$$V(\frac{l}{2},t) = \frac{1}{2}\delta(t - 0.5\mu\text{s}) + \frac{1}{4}\delta(t - 1.5\mu\text{s})$$
$$I(\frac{l}{2},t) = \frac{1}{100}\delta(t - 0.5\mu\text{s}) - \frac{1}{200}\delta(t - 1.5\mu\text{s}).$$

c) By setting $t = 2.5\,\mu s$ in part (a), we obtain

$$V(z, 2.5\mu s) = \frac{1}{2}\delta(2.5\mu s - \frac{3z}{c}) + \frac{1}{4}\delta(0.5\mu s + \frac{3z}{c})$$
$$I(z, 2.5\mu s) = \frac{1}{100}\delta(2.5\mu s - \frac{3z}{c}) - \frac{1}{200}\delta(0.5\mu s + \frac{3z}{c}).$$

d) If $f(t) = 60u(t)$, then

$$V(\frac{l}{2},t) = 30u(t - 0.5\mu\text{s}) + 15u(t - 1.5\mu\text{s})$$



e) The source is a DC source, so as $t \to \infty$, every point on the transmission line takes the same voltage, and the transmission line can be viewed as a lumped circuit. So, we have $V(\frac{l}{2}, \infty) = 45V$.

5.

a) Looking at the voltage waveform plot given in the problem, it can be seen that the amplitude of the incident pulse at $t = 1\,\mu\text{s}$ is $20\,\text{V}$. Applying the voltage division formula as we did in page 3 of Lecture 28, i.e. $f(t) = \tau_g f_i(t)$ where $f_i(t)$ is the generator voltage. Thus, we can write

$$20 = 50 \times \frac{Z_o}{R_g + Z_o} \implies Z_o = \frac{50}{3}\,\Omega.$$

b) Again looking at the plot in the problem, one can see that the amplitude of the first reflected pulse is $10\,\text{V}$. Therefore, the reflection coefficient is

$$\Gamma_L = \frac{10}{20} = \frac{R_L - Z_o}{R_L + Z_o}$$

from which we obtain the load resistance as $R_L = 3Z_o = 50\,\Omega$.

c) Since at $z = 300\,\text{m}$, the incident pulse is delayed by $1\,\mu\text{s}$, the propagation speed $v_p = \frac{300}{1\times10^{-6}} = 3 \times 10^8 m/s$. The time interval between the incident pulse and the second reflected pulse is $4\,\mu\text{s}$ which is equal to the two-way travel time. Then, the time a pulse travels in one-way along the distance L is $\tau_L = 3\,\mu\text{s}$. Thus, we can find that the length of the line is $L = v_p \times \tau_L = 300 \times 3 = 900\,\text{m}$.

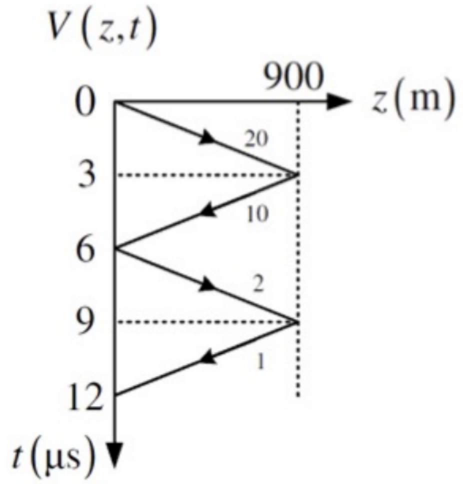
d) $\Gamma_g = \frac{R_g - Z_o}{R_g + Z_o} = \frac{1}{5}$, which should be equal to the magnitude ratio of the second and the third pulses, therefore $A = 10 \times \Gamma_g = 2$.

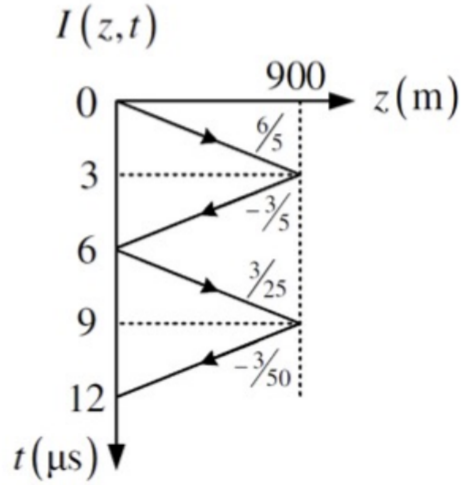
e) The expression for the voltage and current waveforms are

$$V(z,t) = 20\delta(t - \frac{z}{v_p}) + 10\delta(t + \frac{z}{v_p} - 6\,\mu\text{s}) + 2\delta(t - \frac{z}{v_p} - 6\,\mu\text{s}) + \delta(t + \frac{z}{v_p} - 12\,\mu\text{s})\,\text{V}.$$

$$I(z,t) = \frac{6}{5}\delta(t - \frac{z}{v_p}) - \frac{3}{5}\delta(t + \frac{z}{v_p} - 6\,\mu\text{s}) + \frac{3}{25}(t - \frac{z}{v_p} - 6\,\mu\text{s}) - \frac{3}{50}\delta(t + \frac{z}{v_p} - 12\,\mu\text{s})\,\text{A}.$$

Bounce diagrams for $0 < t < 10\,\mu\text{s}$ are given below:





6. This problem is similar to that given in Example 3 in Lecture 30.

   a) The voltage in the first line will be a sum of incident V^+ and reflected voltages V^-, i.e. $V_1 = V^+ + V^-$ whereas in the second line it will be $V_2 = V_R + V^{++}$ where V_R is the voltage across the resistor. Therefore, the KVL equation at the junction is

   $$V_1 = V_2 \implies V^+ + V^- = V_R + V^{++}.$$

   The current flowing in the first line is the sum of the incident and reflected currents, i.e. $I_1 = \frac{V^+}{Z_1} - \frac{V^-}{Z_1}$. However, this current must also be equal to the current flowing through the resistor and also to the current transmitted to the second line. Thus, the KCL equation at the junction is given by

   $$\frac{V^+ - V^-}{Z_1} = \frac{V_R}{R} = \frac{V^{++}}{Z_2}.$$

   b) Combining the previous equations by eliminating V^{++} and V_R we have

   $$\frac{V^+}{Z_1} - \frac{V^-}{Z_1} = \frac{V^+ + V^-}{R + Z_2} \quad \rightarrow \quad \left(\frac{1}{Z_1} - \frac{1}{Z_{eq}}\right) V^+ = \left(\frac{1}{Z_1} + \frac{1}{Z_{eq}}\right) V^-,$$

   where $Z_{eq} = R + Z_2$. Thus, the reflection coefficient is

   $$\Gamma_{12} = \frac{V^-}{V^+} = \frac{Z_{eq} - Z_1}{Z_{eq} + Z_1},$$

   Similarly, transmission coefficient is simply found by referring to $\tau_{12} = \frac{V^{++}}{V^+}$. Hence, we have

   $$\begin{aligned} \tau_{12} &= (1 + \Gamma_{12}) \frac{Z_2}{R + Z_2} \\ &= \left(1 + \frac{Z_{eq} - Z_1}{Z_{eq} + Z_1}\right) \frac{Z_2}{Z_{eq}} \\ &= \frac{2Z_2}{Z_{eq} + Z_1}, \end{aligned}$$

   where $\frac{Z_2}{R+Z_2}$ is a voltage division factor and $Z_{eq} = R + Z_2$.

c) Considering $Z_1 = 100\,\Omega$, $Z_2 = 200\,\Omega$, and $R = 50\,\Omega$, we can find that $Z_{eq} = 250\,\Omega$. Then, we calculate

$$\Gamma_{12} = \frac{Z_{eq} - Z_1}{Z_{eq} + Z_1} = \frac{250 - 100}{250 + 100} = \frac{3}{7},$$

and

$$\tau_{12} = \frac{2Z_2}{R + Z_1 + Z_2} = \frac{2 \times 200}{50 + 200 + 100} = \frac{8}{7}.$$

Note that in this case $1 + \Gamma_{12} \neq \tau_{12}$.