Theoretical basis - Sound field and enclosure relation

1. An isotropic source placed in free space produces a sound field that can be regarded as a set of spherical wavefronts concentric on the source position **[1]**. This means that each point on such field has a sound pressure with **[2]**:
   - an amplitude inversely proportional to its distance to the source and
   - a phase angle that tends to be aligned with the velocity as the distance from the source increases (which means that tends to a plane wave in the infinity).

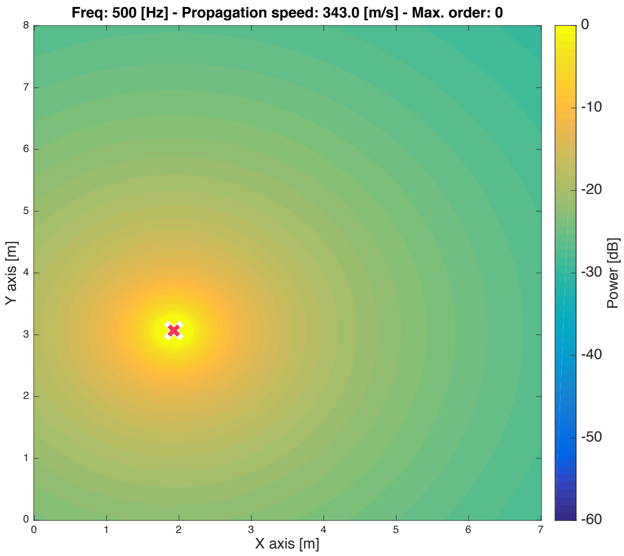

Fig. 1: monochromatic point source radiating in free space. The sound field formed is made of spherical wavefronts with decaying intensity as the distance to the source grows.

2. When that same source is placed on an enclosure, the sound field changes due to the presence of boundaries that act reflecting part of the propagated energy **[3]**. This new field can no longer be regarded as a pure spherical wave propagating to the infinite, but as the combined contributions of the point source radiation plus all new waves originated on the boundaries. The result is, in general, an irregular distribution of pressures across the enclosed space.

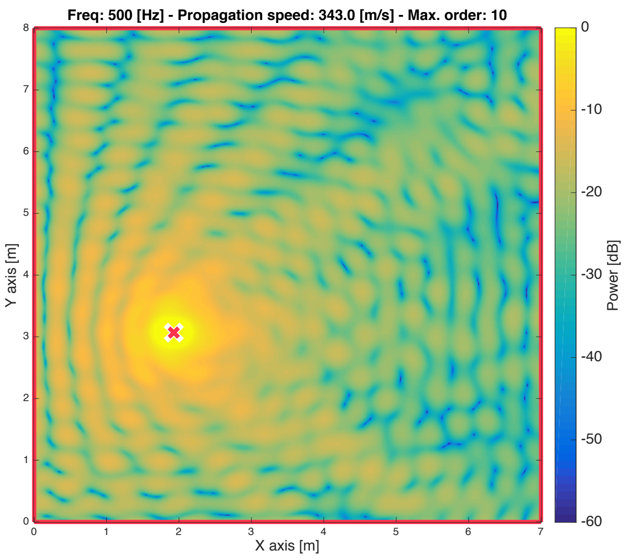

Fig. 2: monochromatic point source radiating in an enclosure. The boundaries are partially reflective so some energy is reflected and produces interference patterns on the sound field. The exterior red line marks the enclosure.

3. Consequently, for any enclosure that holds (*) the sound field is defined, univocally, by the only two elements that are involved:
   - the source characteristics in terms of directivity
   - the enclosure characteristics:
     - shape of boundaries
     - absorption of boundaries (frequency dependent)

(*) The medium is considered to be homogenous and isotropic, thus the sound propagation speed does not change in all the enclosure (homogeneous) and does not depend on the displacement direction (isotropic). If this does not hold through the enclosed space, the medium uniformity will be a third parameter.

4. From a pure physical mode, the enclosure can be understood as a series of infinitesimal particles forming a closed surface. Those, when excited by the source, vibrate and produce new waves that are in turn propagated inside the enclosure **[4]**. These new waves act in the same manner as the one produced by the original source, and the whole process continues over time until the acoustic energy is dissipated due to the viscosity of the media (which means that the propagation occurs with a thermal consumption of energy).

Note:

It can be easily thought of as a linear system with feedback. The initial input is due to the original source wave that excites the system, which responds with new waves that in turn excite the system again, but with less energy since there exists attenuation in the propagation.

5. The relation that exists between the vibration of an enclosure boundaries and the sound field produced inside, when the enclosure is free of sources, is given by the Kirchhoff-Helmholtz integral. It states that the sound pressure is fully determined within the space contained when it is known across all points in its surface.

$$P(\overline{r}) = \frac{1}{4\pi}\oint_S \left[P\frac{\partial}{\partial \hat{n}}\left(\frac{e^{jkd}}{d}\right) - \frac{e^{jkd}}{d}\left(\frac{\partial P}{\partial \hat{n}}\right)\right]\partial S$$

*Fig. 3: Kirchhoff-Helmholtz integral for a monochromatic wave. It defines the sound pressure on a point r within a closed surface **S**, with **d** being the distance from the point to the surface element. The differentiation is done respect to the surface normal, that is considered to point inside the enclosed volume.*

In the case of study, the enclosure is not free of sources, but it is equivalent to say that, the excitation introduced by the original source is instead produced by a combination of all boundaries vibrating in such manner that the same sound field exists. This principle is used in the technique of wave-field-synthesis **[5]**, and it leads to a new model where the original source is removed and the boundaries are now active elements able to produce waves on its own. Since the sound fields in both situations are the same, they are said to be acoustically equivalent scenarios.

6. It is therefore shown that there exists a unique and well defined relationship between the enclosure characteristics and the sound field present inside. It can be concluded that, if the sound field is known, information about the boundaries can be obtained.

Accordingly, the problem is formulated as
   - to define the proper tools that can provide the needed information about the existing sound field and
   - to use that information to find out about the boundaries.

References

[1] Room acoustics, page 12, Heinrich Kuttruff, 3rd edition, 1991.
[2] Fundamentals of acoustics, page 158, Lawrence E. Kinsler, 2nd ed, 1962.
[3] Fundamentals of acoustics, page 128, Lawrence E. Kinsler, 2nd ed, 1962.
[4] Room acoustics, page 29, Heinrich Kuttruff, 3rd edition, 1991.
[5] The theory of wave field synthesis. AES 124th convention, Amsterdam, 17-20 May 2008. Spors, S. M., R. Rabenstein, and J. Ahrens.

Before this theory:

We're trying to improve the methods to find about the room geometry and materials characteristics.
- what are the current "manual" way of finding the information about a room?
- how do we improve them with our method?
- what would the typical consultant do to find the behaviour of the room? (measure all surfaces and know all materials and then simulate it with a software like Odeon)

After this theory:

What do we do to solve the parts of:
- find about the sound field
- from the known / estimated sound field, work the way to the boundaries

Each part should also specify the limitations / precision of the system, as well as assumptions in the methods and choices made.