Modeling Feedback Loops in Consumer Spending, Inflation, and Interest Rates

Thy Bui and Cayla Bovell
Denison University – Math 435
May 10, 2025

Abstract

This report explores the interaction between consumer spending, inflation, and interest rates using a discrete-time feedback model. The model investigates whether these relationships stabilize over time or result in diverging behavior, such as inflationary spirals. We develop one primary model, justify assumptions from economic theory, and fit the model to empirical U.S. economic data from 2000 to 2019. The report includes parameter estimation through informal fitting, equilibrium and stability analysis to see if our model is stable, and sensitivity analysis to see what factors have the biggest impact on consumer spending. Finally, we make suggestions for refining the model to incorporate psychological and income-based dynamics.

Introduction

Of course, no simple discrete-time model can perfectly mirror the full richness of real-world macro dynamics. Our framework necessarily abstracts away factors like income distribution, fiscal policy, supply shocks, and consumer sentiment. As a result, it is best viewed as a tool for qualitative insight, highlighting which feedback paths tend to stabilize the system and which can amplify shocks, rather than as a precise forecasting engine. Consumer spending, inflation, and interest rates form a central feedback loop in macroeconomics. When prices rise, consumer purchasing power falls, which may reduce spending. In turn, central banks raise interest rates to curb inflation, but higher rates may also suppress economic activity [1]. Understanding how these variables interact dynamically is key to analyzing both inflationary periods and monetary policy effects.

Methodology

Model Assumptions

- Consumer spending decreases when inflation or interest rates rise.
- inflation increases compared to consumer spending increases (demand-pull effect).
- Interest rates rise in response to rising inflation.
- All variables respond to changes, not absolute levels.
- The model is implemented in discrete time with yearly intervals. Each index i represents one year.

Model Equations

To explore how consumer spending, inflation, and interest rates interact over time, we developed one discrete-time feedback model. Using discrete steps also makes the model easier to simulate, compare to real data, and observe how small monthly changes accumulate into broader economic trends over time. The model updates values yearly and captures how changes in inflation (I_i), interest rates (R_i), and consumer spending (C_i) affect each other.

We work in annual discrete time (i indexes years) and let

$$\Delta I_{i-1} = I_{i-1} - I_{i-2}, \quad \Delta C_{i-1} = C_{i-1} - C_{i-2}, \quad \Delta R_{i-1} = R_{i-1} - R_{i-2}.$$

All variables are updated by responding to these year-to-year changes.

$$\begin{aligned} C_i &= C_{i-1} - \alpha\,\Delta I_{i-1}\,C_{i-1}, \quad &&\alpha: \text{ sensitivity of spending to inflation changes,} \\ I_i &= I_{i-1} + \beta\,\Delta C_{i-1}\,I_{i-1}, \quad &&\beta: \text{ inflation response to spending growth,} \\ R_i &= R_{i-1} + \gamma\,\Delta I_{i-1}\,R_{i-1}, \quad &&\gamma: \text{ central-bank aggressiveness to inflation.} \end{aligned}$$

If inflation increases compared to the previous year's inflation rate, the term ΔI_{i-1} becomes positive, reducing current spending.

If spending rises from the previous year, it contributes to demand-pull inflation and make the term ΔC_{i-1} become positive, then increases the inflation.

Interest rates rise only when inflation is larger than the previous year, representing reactive central bank behavior.

Data

We collect data from the U.S. Bureau of Labor Statistics for consumer spending [3] and inflation [4]. The FED interest rate is collected from FRED [2]. Significant economic disruptions, such as COVID-19, have marked the period after 2019. Therefore, we chose data from 2000 to 2019 to minimize the impact of unusual economic events and focus on a period of relative stability.

Mathematical Equilibrium

We define an equilibrium as a point where all three variables remain constant over time:

$$C_i = C_{i-1} = C_{i-2} = C^*, \quad I_i = I_{i-1} = I_{i-2} = I^*, \quad R_i = R_{i-1} = R_{i-2} = R^*$$

Substituting into the discrete-time model equations:

$$\begin{aligned} C^* = C^* - \alpha(I^* - I^*)C^* \Rightarrow C^* = C^* \\ I^* = I^* + \beta(C^* - C^*)I^* \Rightarrow I^* = I^* \\ R^* = R^* + \gamma(I^* - I^*)R^* \Rightarrow R^* = R^* \end{aligned}$$

Since all three equations reduce to identities, **any constant values** (C^*, I^*, R^*) satisfy the model. Therefore, the model has **infinitely many equilibria**, and a unique fixed point must be determined by external conditions or economic assumptions.

Results

Policy-Based Equilibrium Scenario

To analyze model behavior, we define a baseline equilibrium based on common macroeconomic policy targets:

- Consumer spending growth: $C^* = 2.0\%$ [6]
- Inflation rate: $I^* = 2.0\%$ [5]
- Interest rate: $R^* = 1.5\%$ [5]

These targets reflect a low-inflation, stable-growth economy and serve as reference points for stability testing.

Rationale for Policy-Defined Equilibrium Because our model governs only period-to-period changes and places no restriction on levels, any constant triple (C^*, I^*, R^*) is an equilibrium, resulting in infinitely many fixed points To give our stability and sensitivity analysis con rete meaning, we anchor the benchmark at widely recognized policy targets—namely, a 2% consumer-spending growth rate ($C^* = 2\%$), 2% inflation ($I^* = 2\%$), and a 1.5% policy rate ($R^* = 1.5\%$) This choice provides a unique, real-world reference point for assessing local stability and interpreting deviations in a policy-relevant context.

Local Stability Analysis

To assess local stability analytically, we rewrite the linearized system as a first-order recurrence on the 6-dimensional state vector.

$$x_i = \begin{pmatrix} \delta C_i \\ \delta C_{i-1} \\ \delta I_i \\ \delta I_{i-1} \\ \delta R_i \\ \delta R_{i-1} \end{pmatrix},$$

with update $x_i = J\,x_{i-1}$, where the Jacobian J at the equilibrium (C^*, I^*, R^*) is

$$J = \begin{pmatrix} 0 & 1 & \alpha\,C^* & -\alpha\,C^* & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -\beta\,I^* & \beta\,I^* & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\gamma\,R^* & \gamma\,R^* & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Substituting $C^* = 2\%$, $I^* = 2\%$, and $R^* = .5\%$ along with $\alpha = 0.98$, $\beta = 0.18$, $\gamma = 0.2$, we compute the eigenvalues of J. We chose these parameters as they reflect a realistic scenario: consumer sensitivity to inflation is high, and the central bank responds moderately to inflation changes — both of which are reasonable assumptions given recent economic conditions. These values serve as a practical starting point for stability analysis and will be refined later through data-informed parameter fitting.

Let $\{\lambda_k\}$ be these eigenvalues and define the spectral radius

$$\rho(J) = \max_k |\lambda_k|.$$

Numerical evaluation yields $\rho(J) \approx 1.306 > 1$. Since at least one eigenvalue has modulus greater than unity, the equilibrium is **locally unstable**: small deviations will grow rather than decay.

We assess local stability by introducing small deviations from equilibrium and linearizing the model:

$$\begin{aligned} \delta C_i &= \delta C_{i-1} - \alpha C^*(\delta I_{i-1} - \delta I_{i-2}) \\ \delta I_i &= \delta I_{i-1} + \beta I^*(\delta C_{i-1} - \delta C_{i-2}) \\ \delta R_i &= \delta R_{i-1} + \gamma R^*(\delta I_{i-1} - \delta I_{i-2}) \end{aligned}$$

Using parameters $\alpha = 0.98$, $\beta = 0.18$, and $\gamma = 0.2$, we simulate these equations over 30 steps with a 1% initial deviation.

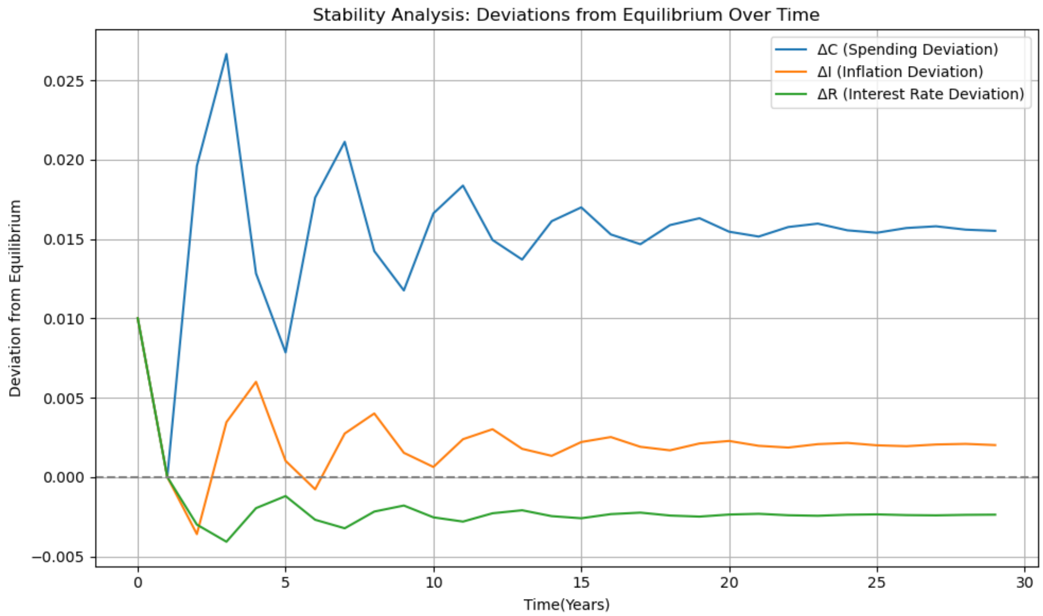

Figure 1: Deviation from equilibrium over time (in years) Persistent growth in deviations suggests instability around the policy-defined equilibrium.

Combined Interpretation At first glance, the 30-year simulation with a one-percentage-point shock may look as though deviations merely oscillate around zero, suggesting short-run stability. However, the Jacobian's spectral radius $\rho(J) \approx 1.306 > 1$ signals fundamental instability: any small disturbance will eventually grow larger rather than decay. In real-world terms—using U.S. data from 2000–2019—this implies that shocks similar in size to the 2008 oil-price spike or the mild inflation uptick in the early 2010s could have been amplified into prolonged volatility without additional damping mechanisms such as fiscal support or measures to bolster consumer confidence.

Parameter Fitting

Next, we will investigate whether our parameter choices can produce stable and data-aligned dynamics.

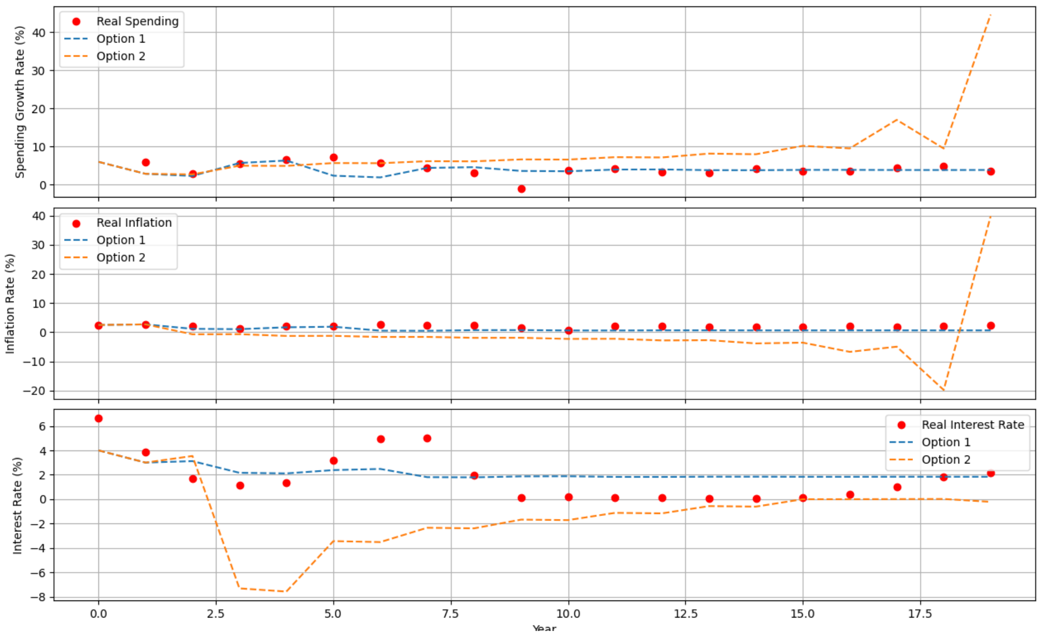

Figure 2: Option 1 is when β is kept moderate (0.1–0.2), the model remains stable and realistic even with high α (0.98). Option 2 is when β exceeds 0.4, inflation overreacts to spending, even with small $\alpha = 0.25$, causing sharp oscillations or runaway growth in consumer behavior.

Through our informal fitting, we observed that the parameters α and β have a significant impact on model behavior, while changes in γ have little to no effect. Changing the value of γ even from 0.1 to 0.98 does not significantly affect the behavior of the model. This is because γ only influences the update of the interest rate R_i, through the term $\gamma(I_{i-1} - I_{i-2})R_{i-1}$ and the change in inflation is typically small, making the entire product negligible regardless of how large γ is.

We found that when β exceeds 0.04, the model becomes highly unstable. Even small increases in α cause sharp oscillations or runaway growth in consumer spending. This reflects a volatile feedback loop where even modest increases in spending rapidly drive up inflation, which in turn suppresses spending through a strong α, creating economic instability similar to that seen in overheating economies.

However, when β is kept within a moderate range (0.1 to 0.2), the system behaves much more robustly. Even with a high α (up to 0.98), the model remains stable, and consumer spending adjusts smoothly in response to inflation. This produces behavior that aligns more closely with real-world data, leading to a better visual and qualitative fit.

Sensitivity Analysis

To understand how different parameters influence the system's behavior, we conduct a sensitivity analysis.

We selected $\alpha = 0.98$, $\beta = 0.18$, and $\gamma = 0.2$ through informal fitting as it captures better the real-world scenario and behaves better. A formal least-squares fitting approach using minimizing error via R^2 proved ineffective because our model is relatively simple and structurally nonlinear, leading to poor overall fit metrics even when dynamics match qualitatively. Moreover, our dataset spans only 20 years, limiting statistical power and making parameter estimation unstable.

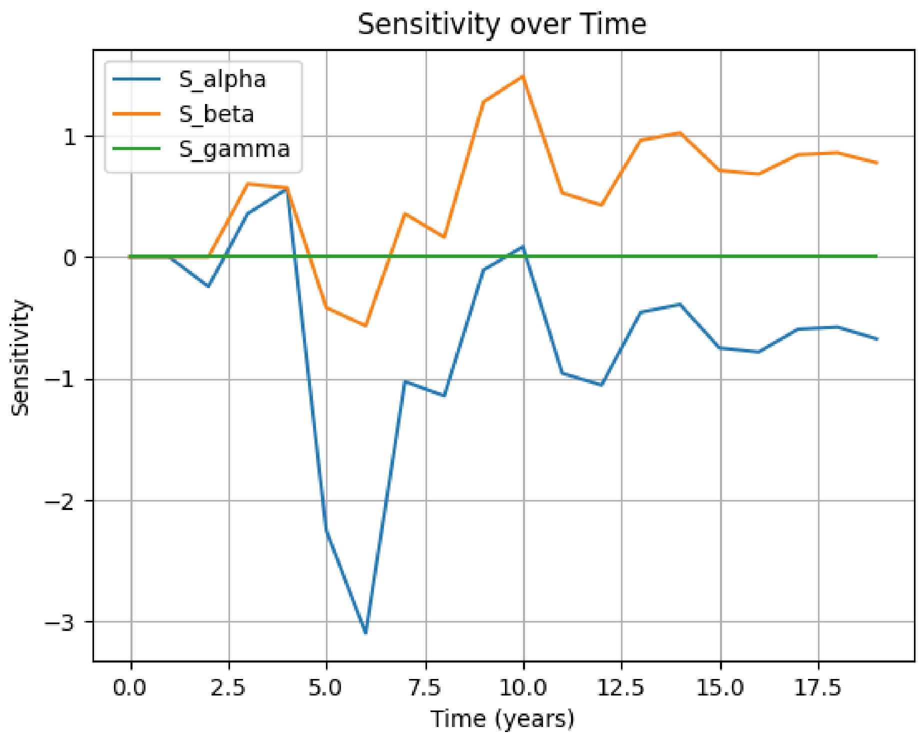

Figure 3: α and β shows the most strongest sensitivity on consumer spending

The graph shows that consumer spending is most sensitive to two factors: how inflation responds to spending (β) and how strongly spending reacts to inflation (α). A 1% increase in α leads to a 3% decrease in consumer spending, while a 1% increase in β results in a 1% increase in spending. In contrast, changes in γ have no meaningful impact, as indicated by the flat sensitivity line. The effect of α is strongest around year 6, while β has its greatest influence around year 10. Although both parameters influence spending in opposite directions, α has a larger overall impact, confirming our earlier parameter testing that consumer spending is more sensitive to changes in inflation responsiveness.

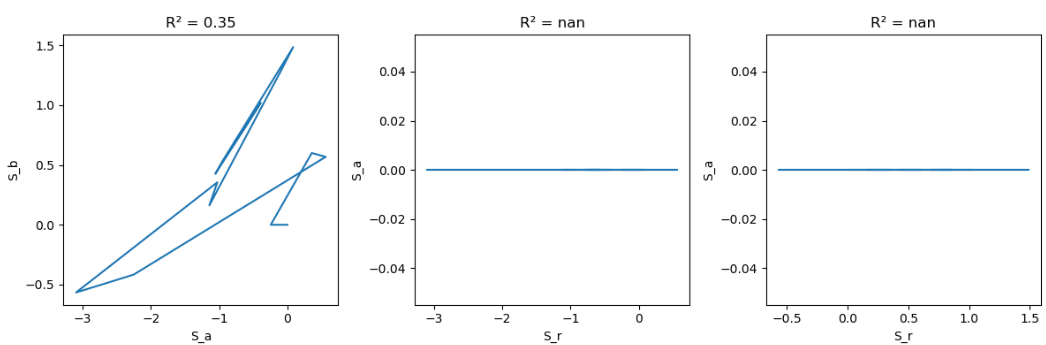

Figure 4: While the pair (α and β) is lower than 85%, both parameter pairs (α and γ) and (β and γ) values are NaN. It means that only α and β can be estimated independently

The low value of $R^2 = 0.35$ (< 0.85) for the interaction between α and β indicates that these parameters are structurally identifiable - they influence the output in sufficiently distinct ways, allowing them to be estimated independently In contrast, the plots for the pairs of parametes (α, γ) and (β, γ) return NaN, meaning that γ shows no measurable effect in the output. As a result, including γ in the parameter set may lead to nonidentifiability when estimated jointly with α or β.

However, the jagged shape of the plot between α vs β might be due to our short time span (20 steps), and our model is nonlinear and uses feedback loops, meaning that small changes in one parameter can cause sharp or unpredictable changes in the output These factors make the sensitivity relationship between α and β look unstable or erratic, even though the correlation is still low enough to consider the parameters identifiable.

In contrast, the plots for the parameter pairs (α, γ) and (β, γ) return NaN values This occurs because γ has no measurable influence on consumer spending under our current setup, resulting in a flat sensitivity line and an undefined correlation This suggests that including γ in parameter estimation may introduce non-idenfiability issues when estimating it alongside α or β.

Discussion

Our model yields valuable insights into the qualitative interaction between consumer spending, inflation, and interest rate. For instance, identifying that α and β are the most sensitive drivers of consumer spending suggests that real-world policy should focus on managing consumer inflation expectations and moderating demand-driven price pressure. However, it also has several limitations and opportunities for refinement:

Data limitations and trend clarity Because our empirical sample spans only twenty years, it isn't easy to discern long-run trends or robustly estimate sensitivity parameters. Short datasets may exaggerate the importance of one shock over another, and structural breaks (e.g. the 2008 crisis) may distort parameter estimates.

Focus on rates vs. levels By modeling only year-to-year changes, we capture immediate feedback effects but potentially miss delayed or cumulative dynamics. For example, a sustained period of moderate inflation could gradually erode purchasing power—an effect our pure-difference specification may understate.

Oversimplification of real-world dynamics Our framework abstracts from income distributions, wealth effects, and fiscal policy. In reality, consumer sentiment, wage growth, credit conditions, and balance-sheet health all influence spending and monetary transmission in complex ways.

Moreover, in the current framework, interest rates affect neither consumer spending nor inflation, meaning the interest rate pathway operates in isolation. This contradicts macroeconomic reality, where monetary policy (via interest rates) significantly influences both inflation expectations and household consumption [1]. We acknowledge that omitting these feedback channels limits the model's realism and undercuts the intended role of γ. Future versions of the model should introduce direct or lagged effects of interest rates on spending and inflation to restore this economic linkage.

Potential refinements

Sentiment and wage dynamics: Introduce variables for consumer confidence or real wage growth, allowing spending to respond to both psychological and income-driven forces.

Smoothed or lagged feedback: Replace simple difference terms with moving averages or explicit lag structures so shocks accumulate or decay over several years.

Bounded constraints: Impose saturation or floor effects (e.g. a minimum subsistence spending rate or an upper bound on inflation sensitivity) to prevent unrealistic oscillations.

These extensions would improve the model's realism and its ability to match observed economic behavior over different horizons.

Author Contribution

Both authors contributed equally to model development, data processing, analysis and visualization, and writing. Cayla focused on model calibration and equilibrium and stability analysis, while Thy focused on mathematical structure and sensitivity analysis.

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