Design and Analysis of Algorithms
CS375 Fall 2020

Theory Assignment 1

Due 9/24/2020 (Thursday) at 11:59pm

Remember to include the following statement at the start of your answers with a signature by the side. "I have done this assignment completely on my own. I have not copied it, nor have I given my solution to anyone else. I understand that if I am involved in plagiarism or cheating I will have to sign an official form that I have cheated and that this form will be stored in my official university record. I also understand that I will receive a grade of 0 for the involved assignment for my first offense and that I will receive a grade of "F" for the course for any additional offense."

1. (20 points) Fill in all the missing values. For column A you need to compute the sums. For column B (the last two rows) you need to guess a function that does not contradict any of the yes/no answers already in the next three columns. Fill in each empty entry in the last three columns with either a yes/no answer.

Function A	Function B	O $A = O(B)$	Ω $A = \Omega(B)$	Θ $A = \Theta(B)$
n^4	$n^3 \lg n$			
$n\sqrt{n}$	n^2			
$(n+1)!$	$n!$			
$lg\ n$	n^k where $k > 0$			
$\sum_{i=1}^{n}(i+1) = ?$				yes
$\sum_{i=0}^{n-1} 3^i = ?$		no		

2. (20 points) Order the functions below by increasing growth rates (no justification required):

3. n^n, n, $n\ln n$, n^{ε} $(0 < \varepsilon < 1)$, $2^{\lg n}$, $\ln n$, 1000000, $n^{1/n}$, $n!$, 2^n

   Let $g_i(n)$ be the ith function from the left after the ordering (the leftmost function has the slowest growth rate). In the order, $g_i(n)$ should satisfy $g_i(n) \in O(g_{i+1}(n))$. If two or more functions are equivalent (in terms of Θ), put them in [ ] separated by comma (e.g., [n^2, $5n^2$]).

4. (15 points) Let f(n) and g(n) be asymptotically positive functions. Prove or show a counter example for each of the following conjectures.
   a) $(f(n)+g(n)) \in \Theta(\max(f(n), g(n)))$.
   b) $f(n) \in O(g(n))$ implies $2^{f(n)} \in O(2^{g(n)})$.
   c) $f(n) \in O(g(n))$ implies $g(n) \in \Omega(f(n))$.

5. (15 points) Prove the following using the original definitions of O, Ω, θ, o, $and\ \omega$.

(a) $3n^3 + 50n^2 + 4n - 9 \in O(n^3)$

(b) $1000n^3 \in \Omega(n^2)$

(c) $10n^3 + 7n^2 \in \omega(n^2)$

(d) $78n^3 \in o(n^4)$

(e) $n^2 + 3n - 10 \in \Theta(n^2)$

6. (15 points) Prove the following using limits.

   (a) $n^{1/n} \in \Theta(1)$ [Hint: you can use $x=e^{\ln x}$]

   (b) $4^n \in \omega(n^k)$

7. Solve the following recurrence equations.

   a) (8 points) Use the substitution method to show that $T(n) = T(n-1) + 2n \in O(n^2)$. You can assume $T(1) = 2$.

   b) (8 points) Use the iteration method to solve the following recurrence equation (hint: you can assume $n^{1/2^k} = 2$ for some integers n and k):

$$T(n) = \begin{cases} 0 & \text{if } n = 2 \\ T(\sqrt{n}) + 1 & \text{if } n > 2 \end{cases}$$

   c) (6 points) Use Master method to solve $T(n) = 16T(n/4) + n^2$.

   d) (8 points) Use Master method to solve $T(n) = T(9n/10) + n$.