Assignment 2023-08-22

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Section 1

1. Confirm the claimed solution in example 1.1.5 is actually a solution to the differential equation.

   ODE: $y' + 3y = 2$
   General Solution: $y = \frac{2}{3} + Ce^{-3t}$

   Proof.

   $$\begin{aligned} y' &= -3Ce^{-3t} \\ y' + 3y &= -3Ce^{-3t} + 3(\frac{2}{3} + Ce^{-3t}) \\ &= -3Ce^{-3t} + frac63 + 3Ce^{-3t} \\ &= 2. \end{aligned}$$

   □

2. Show the following are solutions to the indicated differential equations.

   (a) $y = e^{-x}$ is a solution to $y' + y = 0$

   Proof.

   $$\begin{aligned} y' &= -e^{-x} \\ y' + y &= -e^{-x} + e^{-x} = 0. \end{aligned}$$

   □

(b) $y = x^2$ is a solution to $xy' = 2y$

Proof.

$$\begin{aligned} y' &= 2x \\ xy' &= x * 2x = 2x^2 = 2y. \end{aligned}$$

□

(c) $y = \sqrt{1+x^2}$ is a solution to $(1+x^2)y' = xy$

Proof.

$$\begin{aligned} y' &= \frac{1}{2}(1+x^2)^{-\frac{1}{2}} * 2x \\ &= x(1+x^2)^{-\frac{1}{2}} \\ (1+x^2)y' &= (1+x^2) * x(1+x^2)^{-\frac{1}{2}} \\ &= x(1+x^2)^{\frac{1}{2}} \\ &= xy. \end{aligned}$$

□

(d) $y = ae^x + be^{-x}$ is a solution to $y'' - y = 0$

Proof.

$$\begin{aligned} y' &= ae^x - be^{-x} \\ y'' &= ae^x + be^{-x} \\ y'' - y &= ae^x + be^{-x} - (ae^x + be^{-x}) \\ &= 0. \end{aligned}$$

□

Section 2

1. Determine the order of each differential equation and whether it is linear or non-linear. If it is linear, determine if its is homogenous or non-homogenous.

   (a) $y'' - \cos(t)y = e^t y'$

   Order: 2, Linear, Homogenous

   (b) $(y')^3 = 2y$

   Order: 1, Non-Linear

   (c) $e^{y^{(4)}} + y'' - y = y'$

   Order: 4, Linear, Homogenous

   (d) $ty'' + \cos(t) + y = 0$

   Order: 2, Linear, Non-Homogeneous