Minimum Dominating Set of a Graph

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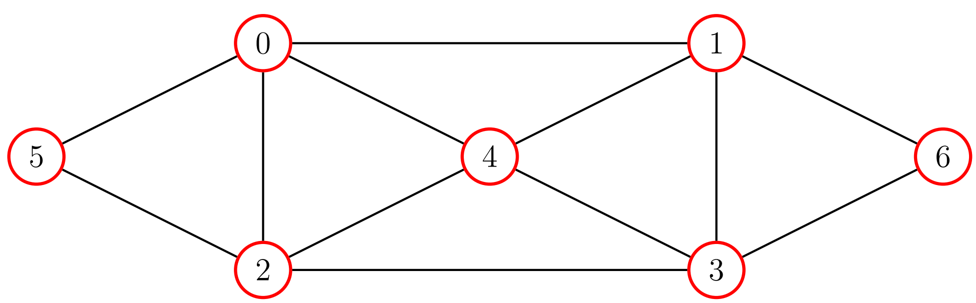


1 Introduction

In this paper, we explore the Minimum Dominating Set of a Graph and how to find it. The Minimum Domianting Set of a Graph is the Set of all nodes such that every node or one of its neighbors is in the set.

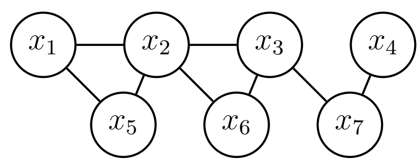
This is in demonstration of the time complexity of algorithms, often denoted using Big-O notation ($O(n)$). To date, the best known algorithms used to guarantee the minimum dominating set run in exponential scales, of the form $O(2^n)$. Solutions however are able to be verified in polynomial time, of the form $O(n^2)$. There are several algorithms used to get an approximate answer that run in much smaller scales.

In this paper, we explore the algorithm used to find the Minimum Domating Set. We also explore an approximating algorithm in order to compare the efficacy of the two, with accuracy and runtime being the major criteria.

This paper is divided into the following sections. Section 2 contains background information on the Minimum Dominating Set. Section 3 contains the algorithms used to solve for the Minimum Dominating Set. Section 4 contains experimental data when running the algorithm. Section 5 concludes the paper.

2 Background

The Minimum Dominating Set of a Graph is defined as the set of vertices for which each vertex or one of its neighbors is in the set. The size of the minimum set will be unique, however there may be multiple sets of that size that are a minimal covering. Consider the graph below.



This graph has minimum dominating sets, $\{x_2, x_4\}$ and $\{x_2, x_7\}$. The vertex x_2 covers vertices $\{x_1, x_3, x_5, x_6\}$, leaving nodes x_4 and x_7. Either vertex can dominate the other and complete a covering of the graph. In this example, both the approximate algorithm and brute force approach would find an equally sized set.

The next section explores the two algorithmic approaches used in this project.