- Read Chapters 7 and 8 for Week 3. Quiz in Week 3.
- What governs how fast an algorithm can run?
  - The complexity of the problem; a problem has a theoretical best solution
  - The efficiency of the algorithm; an algorithm can give a close enough solution in a fraction of the time
- Sometimes writing a program that always guesses the correct solution is not always good enough.
  - Time complexity; a program taking too long for its answer; keep in mind timescales

Properites of Exponents

- $x^a * x^b = x^{a+b}$
- $(x^a)^b = x^{ab}$
- $x^a/x^b = x^{a-b}$

Properties of Logarithms

- $x^a = b$ iff $log_x b = a$
- $log_a b = log_c b/log_c a$ if $a, b, c > 0, a \neq 0$
  - Let $x = log_c b$, $y = log_c a$, $z = log_a b$
  - $b = c^x, b = a^z \rightarrow b = c^x = a^z$
  - $a = c^y$
  - $b = c^x = c^{yz}$
  - $x = yz \rightarrow z = x/y$
- $logab = loga + logb$
  - Let $x = log_2 a, y = log_2 b, z = log_2 ab$ if $a, b > 0$
  - $2^x = a, 2^y = b, 2^z = ab$
  - $2^x * 2^y = 2^z$
  - $2^{x+y} = 2^z$
  - $x + y = z$
- $log_2(a/b) = log_2 a - log_2 b$ if $a, b > 0$
  - Let $x = log_2 a, y = log_2 b, z = log_2(a/b)$
  - $2^x = a, 2^y = b, 2^z = a/b$
  - $2^x/2^y = 2^z$
  - $x - y = z$

Disproof

- Only need to find one counter example
- Example: $n! < n^2 + n^5$

  Pick $n = 9$

  $9! = 362880$ $\quad 9^2 = 81 \quad 9^5 = 59130$

  $362880 < 81 + 59130$ FALSE!

Proof by Induction

- Use proof by Induction for discrete types only.
- Happens in 3 steps:
  1. Start with a (very small) base case.
  2. Inductive Hypothesis: "Since we showed a base case in Step 1, let's assume it holds true for values through k"
  3. Show that if the Hypothesis works for value k, the Hypothesis holds for $k + 1$

Example of Induction

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

1. Let $n = 1 \rightarrow 1 = 1$
2. Assume the property is true for $1, 2, \ldots, k$
3. Show if true for k, then true for $k + 1$

$$\begin{aligned}
\textstyle\sum_{i=1}^{k+1} i &= \textstyle\sum_{i=1}^{k} + k + 1 \\
&= \tfrac{k(k+1)}{2} + k + 1 \\
&= \tfrac{k(k+1)}{2} + \tfrac{2(k+1)}{2} \\
&= \tfrac{k(k+1)+2(k+1)}{2} \\
&= \tfrac{(k+1)(k+2)}{2} \\
&= \tfrac{(k+1)(k+1+1)}{2}
\end{aligned}$$

Example 2 of Induction

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$$

1. Let $n = 1 \rightarrow 2 = 2$
2. Assume the property is true for $1, 2, \ldots, k$
3. Show if true for k, then true for $k+1$

$$\begin{aligned}
\textstyle\sum_{i=1}^{k+1} i(i+1) &= \textstyle\sum_{i=1}^{k} i(i+1) + (k+1)(k+2) \\
&= \tfrac{k(k+1)(k+2)}{3} + (k+1)(k+2) \\
&= \tfrac{k(k+1)(k+2)}{3} + \tfrac{3(k+1)(k+2)}{3} \\
&= \tfrac{k(k+1)(k+2)+3(k+1)(k+2)}{3} \\
&= \tfrac{(k+1)(k+2)(k+3)}{3} \\
&= \tfrac{(k+1)(k+1+1)(k+1+2)}{3}
\end{aligned}$$

Example 3 of Induction

Fibonnaci Numbers $\rightarrow F_0 = 1, F_1 = 1, \ldots, (F_i = F_{i-1} + F_{i-2})$
Prove that $F_i < (\frac{5}{3})^i$, for $i \geq 1$

1. $F_1 = 1 < (\frac{5}{3})^1$
2. Assume this is true for $1, \ldots, k$
3. Show if true for k, then true for $k+1$

$$\begin{aligned}
F_{k+1} &= F_k + F_{k-1} \\
F_{k+1} &< (\tfrac{5}{3})^k + (\tfrac{5}{3})^{k-1} \\
F_{k+1} &< (\tfrac{3}{5})(\tfrac{5}{3})^{k+1} + (\tfrac{3}{5})^2(\tfrac{5}{3})^{k+1} \\
F_{k+1} &< (\tfrac{5}{3})^{k+1} * ((\tfrac{3}{5})^2 + (\tfrac{3}{5})) \\
F_{k+1} &< (\tfrac{5}{3})^{k+1} * (\tfrac{15}{25} + \tfrac{9}{25}) \\
F_{k+1} &< (\tfrac{5}{3})^{k+1} * \tfrac{24}{25} \qquad < \quad (\tfrac{5}{3})^{k+1}
\end{aligned}$$