Notes Week 4

- Exam in Week 5 - Last Names A-M 6:00pm, N-Y 6:45pm
- Read Chapter 4 for Week 6 - after exam

Quiz 1 Review - Proof by Induction

Prove $n < 2n$ for all $n \geq 1$.

1. Base Case: $n = 1$

   $1 < 2 \times (1)$

   $1 < 2$

2. Assume this is true for $1, \ldots, k$.
3. If true for k, then it is true for $k + 1$

$k < 2k$	Independent Hypothesis
$k + 1 < 2k + 1$	Add 1 to both sides
$k + 1 < 2k + 1 + 1$	As this is an equality, can add 1 to the larger side and hold true
$k + 1 < 2k + 2$	Simplify
$k + 1 < 2(k + 1)$	Factor out the two

Example - Proof by Induction

Prove that path of length n has $n-1$ edges.

1. Base Case: $n=2$

2. Assume this is true for $1, \ldots, k$.

3. If true for k, then it is true for $k+1$

Proof By Contradiction

- Prove something by contradicting the negated hypothesis
- "Not everyone that is exactly six feet tall has red hair."
- The Steps:
  1. Assume that the hypothesis is false
  2. Show the assumption leads to a contradiction
  3. Recognize that assuming step 1 is false leads to a contradiction, implying the hypothesis is true

Example - Proof by Contradiction

Prove if n is an integer and n^2 is even, then n is even.

Suppose by way of contradiction that:

- n is an integer
- n^2 is even
- n is odd

  $\rightarrow$ $n = 2k + 1$ for some integer k (the definition of an odd number)
- $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1$
- $2(2k^2 + 2k) + 1$ is odd
- n^2 is odd

Contradiction: n^2 cannot be both even and odd.

Example - Proof by Contradiction

Prove: for every n, if $n > 2$ and n is prime, then n is odd.

Suppose by way of contradiction that:

- $n > 2$
- n is prime
- n is even
- $n = 2k$ for some integer k (definition of an even integer)
- $k! = 1$
- $n = 2(k)$, $k > 1$
- $n \& (wholenumber > 1)$ is composite

Contradiction: n cannot be both composite and prime

Example - Proof by Contradiction

Prove: No integers a & b exist for which $24a + 12b = 1$.

Suppose by way of contradiction:

- a is an integer
- b is an integer
- $24a + 12b = 1$
- Divide by 12, $2a + b = \frac{1}{12}$

Contradiction: A whole number multiplied by 2 summed with another whole number cannnot be a fraction.

Notation notes

- $B = O(A) \rightarrow$ B is upper bounded by A
- $T(n) = O(f(n))$ if there are positive constants c & n_0 such that $T(n) \leq cf(n)$ when $n \geq n_0$
- $A = \Omega(B) \rightarrow$ A is lower bounded by B
- $T(n) = \Omega(f(n))$ if there are positive constants c & n_0 such that $T(n) \geq cf(n)$ when $n \geq n_0$
- $\omega(n)$ instead of $\Omega(n)$ if not possible to be equal
- $T(n) = \mu(f(n))$ if there are positive constants c & n_0 such that $T(n) > cf(n)$ when $n \geq n_0$
- Use the lower case versions when bounded but not able to equal. Use upper case when bounded and can be equal, based on definitions above.
- $T(n) = \Theta(f(n))$ if and only if $T(n) = O(f(n))$ and $T(n) = \Omega(f(n))$

Given $f(n) = 3n^2$ and $g(n) = 5n^2$, give all true statements.

- $f(n) = O(g(n))$
- $g(g) = \Omega(f(n))$
- etc.