Notes Week 7

Examples: Recurrences

- Recurrence: $T(n) = 2T(n-1) + 1 \rightarrow O(2^n) \rightarrow$ Fibonacci Numbers

- Recurrence: $T(n) = T(n-2) + n \rightarrow O(b^2) \rightarrow$ Fun, made up problem

Solving Recurrences by the Master Method

If the recurrence takes the form:

$$T(n) = aT(\frac{n}{b}) + \Theta(n^k)$$

where $a \geq 1$ and $b > 1$ then

$$
\begin{aligned}
O(n^k) & \quad \text{if } a < b^k \\
O(n^k log_2 n) & \quad \text{if } a == b^k \\
O(n^{log_b a}) & \quad \text{if } a > b^k
\end{aligned}
$$

Examples

- Recurrence: $T(n) = 9T(\frac{n}{3}) + n$
  $a = 9, b = 3, k = 1 \rightarrow$ Since $a > b^k$, $O(n^{log_3 9}) = O(n^2)$

- Recurrence: $2T(\frac{n}{2}) + n$

- Recurrence: $T(\frac{n}{2}) + 1$