Section 1.2: Row Reduction and Echelon Forms

- A rectangular matrix is in **row echelon form** if it has the following three properties:
  1. All nonzero rows are above any rows of all zeros.
  2. Each leading entry of a row is in a column to the right of the leading entry of the row above it.
  3. All entries in a column below a leading entry are zeros.
- If a matrix in echelon form satisfies the following additional conditions, then it is **reduced row echelon form**:
  1. The leading entry in each nonzero row is 1.
  2. Each leading 1 is the only nonzero entry in its column.
- An **echelon matrix** (respectively, **reduced echelon matrix**) is one that is in echelon form (respectively, reduced echelon form).
- Any nonzero matrix may be **row reduced** (ie, transformed by elementary row operations) into more than one matrix in echelon form, using different sequences of row operations. However, the reduced echelon form one obtains from a matrix is unique.

Theorem 1 (Uniqueness of the Reduced Echelon Form) *Each matrix is row equivalent to one and only one reduced echelon matrix.*

- If a matrix A is row equivalent to an echelon matrix U, we can U an **echelon form of** A; if U is in reduced echelon form, we call U the **reduced echelon form of** A.
- A **pivot position** in a matrix A is a location in A that corresponds to a leading 1 in the reduced echelon form of A. A **pivot column** is a column of A that contains a pivot position.