MAT341 - Project 1 - Matrix Theory

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October 26, 2021

1 Introduction

This project demonstrates Matrix Theory when applied to a network throughput graph. The scenario to be solved is as follows.

A sender is transmitting data with a total rate of 150 megabits per second along the network graph in Figure 1. The data is transmitted from the sender to the receiver over a network of five different routers. These routers are labeled A, B, C, D, and E. The connections and data rates between the routers are labeled as x_1, x_2, x_3, x_4, x_5. This project will find a solution to the data transfer through the specific routers in this network. Two methods will be used to find the solution: *LU* Factorization and Cramer's Rule. At the end will be a recommendation on network changes to suit the problem.

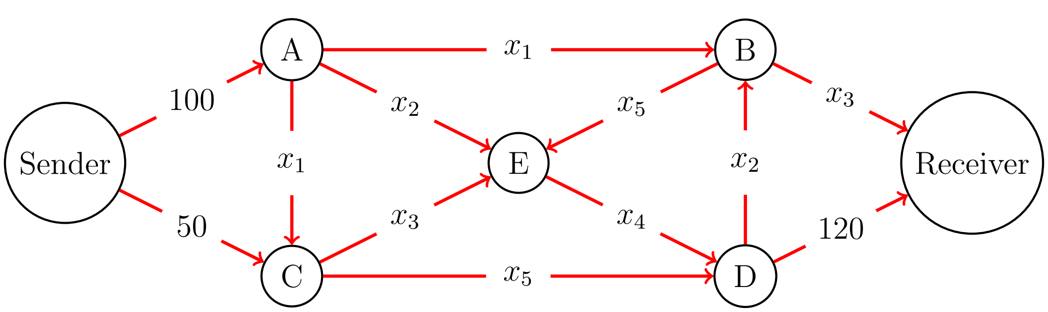

Figure 1: The Network Graph

2 A System of Linear Equations

This problem can be solved by utilizing a system of linear equations. Each equation represents the input and output of a specific router node. First we must construct a table of the input and output flow of each node. The input and output must be equivalent for each node. Solving for variables for each node gives us a system of linear equations.

Node	**Input**	**Output**	**Equation**
A	100	$2x_1 + x_2$	$2x_1 + x_2 = 100$
B	$x_1 + x_2$	$x_3 + x_5$	$x_1 + x_2 - x_3 - x_5 = 0$
C	$50 + x_1$	$x_3 + x_5$	$x_1 - x_3 - x_5 = -50$
D	$x_4 + x_5$	$x_2 + 120$	$-x_2 + x_4 + x_5 = 120$
E	$x_2 + x_3 + x_5$	x_4	$x_2 + x_3 - x_4 + x_5 = 0$

Utilizing this system of equations, we can construct a matrix equation of the form $Ax = b$ representing it. This becomes equation (1) below:

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & -1 \\ 1 & 0 & -1 & 0 & -1 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ -50 \\ 120 \\ 0 \end{bmatrix} \tag{1}$$

3 LU Factorization

The first method to solve this system of equations will be LU Factorization. This involves decomposing the coefficient matrix A into two factors, L and U, resulting in an equation of the form $A = LU$. Doing so gives:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 & 0 \\ \frac{1}{2} & -1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 1 & 0 \\ 0 & 2 & -\frac{3}{2} & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -1 & 0 & -1 \\ 0 & 0 & -2 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Using the LU decomposition it's possible to decompose the original matrix equation into two separate equations that become easily solved:

$$Ux = y \tag{2}$$

$$Ly = b \tag{3}$$