Section 1.3: Vector Equations

- A matrix with only one column is called a **column vector**, or simply a **vector**.
- An example of a vector with two entries, where w_1 and w_2 are any real numbers, is:

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

- The set of all vectors with 2 entries is denoted by $\mathbb{R}^2$.
- The $\mathbb{R}$ stands for the real numbers that appear as entries in the vector, and the exponent 2 indicates that each vector contains 2 entries.
- Two vectors in $\mathbb{R}^2$ are **equal** if and only if their corresponding entries are equal.
- Given two vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^2$, their **sum** is the vector $\mathbf{u} + \mathbf{v}$ obtained by adding the corresponding entries of $\mathbf{u}$ and $\mathbf{v}$.
- Given a vector $\mathbf{u}$ and a real number $\mathbf{c}$, the **scalar multiplication** of $\mathbf{u}$ by $\mathbf{c}$ is the vector $\mathbf{cu}$ obtained by multiplying each entry in $\mathbf{u}$ by $\mathbf{c}$.
- Consider a rectangular coordinate system in the plane. Because each point in the plane is determined by an ordered pair of numbers, we can identify a geometric point (a, b) with the column vector $\begin{bmatrix} a \\ b \end{bmatrix}$.
- We may regard $\mathbb{R}^2$ as the set of all points in the plane
- The vector whose entries are all zero is called the **zero vector** and is denoted by $\mathbf{0}$.
- For all $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, in $\mathbb{R}^2$ and scalars c and d:
  1. $u + v = v + u$
  2. $(u + v) + w = u + (v + w)$
  3. $u + 0 = 0 + u = u$
  4. $u + (-u) = 0$
  5. $c(u + v) = cu + cv$
  6. $(c + d)u = cu + du$
  7. $c(du) = (cd)u$
  8. $1u = u$
- The vector $y = c_1v_1 + \cdots + c_pv_p$ is called a **linear combination**.
- A vector Equation

$$x_1a_1 + \cdots + x_na_n = b$$

  has the same solution set as the linear system whose augmented matrix is

$$\begin{bmatrix} a_1 & a_2 & \cdots & a_n & b \end{bmatrix}$$

- If $v_1, \ldots, v_p$ are in $\mathbb{R}^2$, then the set of all linear combinations of $v_1, \ldots, v_p$ is denoted by $Span\{v_1, \ldots, v_p\}$ and is called the **subset of $\mathbb{R}^2$ spanned by $v_1, \ldots, v_p$**.