Section 1.3: Vector Equations

- A matrix with only one column is called a **column vector**, or simply a **vector**.
- An example of a vector with two entries, where w_1 and w_2 are any real numbers, is:

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

- The set of all vectors with 2 entries is denoted by $\mathbb{R}^2$.
- The $\mathbb{R}$ stands for the real numbers that appear as entries in the vector, and the exponent 2 indicates that each vector contains 2 entries.
- Two vectors in $\mathbb{R}^2$ are **equal** if and only if their corresponding entries are equal.
- Given two vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^2$, their **sum** is the vector $\mathbf{u} + \mathbf{v}$ obtained by adding the corresponding entries of $\mathbf{u}$ and $\mathbf{v}$.
- Given a vector $\mathbf{u}$ and a real number $\mathbf{c}$, the **scalar multiplication** of $\mathbf{u}$ by $\mathbf{c}$ is the vector $\mathbf{cu}$ obtained by multiplying each entry in $\mathbf{u}$ by $\mathbf{c}$.
- Consider a rectangular coordinate system in the plane. Because each point in the plane is determined by an ordered pair of numbers, we can identify a geometric point (a, b) with the column vector $\begin{bmatrix} a \\ b \end{bmatrix}$.

Theorem 1 (Existence and Uniqueness Theorem) *A linear system is consistent if and only if the rightmost column of the augmented matrix is not a pivot column; ie, if and only if an echelon form of the augmented matrix has no row of the form* $[0 \cdots 0b]$ *with* b *nonzero.*