Section 1.2: Row Reduction and Echelon Forms

- A rectangular matrix is in **<u>row echelon form</u>** if it has the following three properties:

    1. All nonzero rows are above any rows of all zeros.
    2. Each leading entry of a row is in a column to the right of the leading entry of the row above it.
    3. All entries in a column below a leading entry are zeros.

- If a matrix in echelon form satisfies the following additional conditions, then it is **<u>reduced row echelon form</u>**:

    1. The leading entry in each nonzero row is 1.
    2. Each leading 1 is the only nonzero entry in its column.

- An **<u>echelon matrix</u>** (respectively, **<u>reduced echelon matrix</u>**) is one that is in echelon form (respectively, reduced echelon form).

- Any nonzero matrix may be **<u>row reduced</u>** (ie, transformed by elementary row operations) into more than one matrix in echelon form, using different sequences of row operations. However, the reduced echelon form one obtains from a matrix is unique.

Theorem 1 (Uniqueness of the Reduced Echelon Form) *Each matrix is row equivalent to one and only one reduced echelon matrix.*

- If a matrix A is row equivalent to an echelon matrix U, we can U an **echelon form of** A; if U is in reduced echelon form, we call U the **reduced echelon form of** A.

- A **<u>pivot position</u>** in a matrix A is a location in A that corresponds to a leading 1 in the reduced echelon form of A. A **<u>pivot column</u>** is a column of A that contains a pivot position.

- **Using Row Reduction to Solve a Linear System**

    1. Write the augmented matrix of the system.
    2. Use the row reduction algorithm to obtain an equivalent augmented matrix in echelon form. Decide whether the system is consistent. If there is no solution, stop; otherwise, go to the next step.
    3. Continue row reduction to obtain the reduced echelon form.
    4. Write the system of equations corresponding to the matrix obtained in step 3.
    5. Rewrite each nonzero equation from step 4 so that its one basic variable is expressed in terms of any free variables appearing in the equation.