

Задорожний Антон 219/5 Частные производные

$$1b. 1) z = x^3 - 3x^2y + 4x^3y^2 - y^3$$

$$\frac{\partial z}{\partial x} = 3x^2 - 6x^1y + 12x^2y^2$$

$$\frac{\partial z}{\partial y} = -3x^2 + 8x^3y - 3y^2$$

$$2) z = \frac{3x}{y} = 3x \cdot y^{-1}$$

$$\frac{\partial z}{\partial x} = 3 \cdot y^{-1} = \frac{3}{y}$$

$$\frac{\partial z}{\partial y} = 3x \cdot (y^{-2}) = -\frac{3x}{y^2}$$

$$3) z = e^{-x \cdot y^{-1}}$$

$$\frac{\partial z}{\partial x} = e^{-x \cdot y^{-1}} \cdot (-x \cdot y^{-1})' = e^{-x \cdot y^{-1}} \cdot (-y^{-1}) = -\frac{1}{y} \cdot e^{-\frac{x}{y}}$$

$$\frac{\partial z}{\partial y} = e^{-x \cdot y^{-1}} \cdot (-x \cdot (-y^{-2})) =$$

$$3) z = \frac{y-3x}{x+4y} = \frac{(y-3x)(x+4y)^{-1}}{(x+4y)^1} = \frac{(y-3x)(x+4y)^{-2}}{(x+4y)^2}$$

$$\frac{\partial z}{\partial x} = \frac{-13x}{x^2} = \frac{(y-3x)'(x+4y) - (x+4y)'(y-3x)}{(x+4y)^2} = \frac{15y}{(x+4y)^2}$$

$$\frac{\partial z}{\partial y} = \frac{15y}{(x+4y)^2} = \frac{(x+4y) - 14y - 12x}{(x+4y)^2} = \frac{15x}{(x+4y)^2}$$

$$5) z = \ln(2x-y)$$

$$\frac{\partial z}{\partial x} = \frac{1}{(2x-y)} \cdot (2x-y)' = \frac{1}{2x-y} \cdot 2 = \frac{2}{2x-y}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2x-y} \cdot (2x-y)' = \frac{1}{(2x-y)} \cdot (-1) = \frac{-1}{2x-y}$$

$$14. \gamma) z = \frac{x-2y}{x+y}$$

$$\textcircled{1} \frac{\partial z}{\partial x} = \frac{(x-2y)'(x+y) - (x+y)'(x-2y)}{(x+y)^2} = \frac{(x+y) - (x-2y)}{(x+y)^2} = \frac{3y}{(x+y)^2}$$

$$\textcircled{2} \frac{\partial z}{\partial y} = \frac{(-2x-2y) - (x-2y)}{(x+y)^2} = \frac{-3x}{(x+y)^2}$$

$$\textcircled{1} M(2; -1) \Rightarrow \frac{-3}{1} = -3$$

$$\textcircled{2} M(2; -1) \Rightarrow \frac{-6}{1} = -6$$

$$2) z = e^{\frac{3x}{y}}$$

$$\frac{\partial z}{\partial x} = \frac{3}{y} e^{\frac{3x}{y}} \quad M(1; 1) = 3e^3$$

$$\frac{\partial z}{\partial y} = -\frac{3x}{y^2} e^{\frac{3x}{y}} \quad M(1; 1) = -3e^3$$

$$3) z = \ln(x^2 + y^2)$$

$$\frac{\partial z}{\partial x} = 2x \frac{1}{x^2 + y^2} \quad M(2; 1) = 2 \cdot \frac{1}{5} = \frac{2}{5}$$

$$\frac{\partial z}{\partial y} = 2y \frac{1}{x^2 + y^2} \quad M(2; 1) = \frac{2}{5} = \frac{2}{5}$$

$$4) z = \frac{y}{x} + x$$

$$\frac{\partial z}{\partial x} = -\frac{y}{x^2} + 1$$

$$M(1; -2) = -2 + 1 = -1$$

$$\frac{\partial z}{\partial y} = \frac{1}{x}$$

$$M(1; -2) = \frac{1}{1} = 1$$

N15

$$1) z = \frac{y}{x+y}$$

$m(2; -1)$

$$\frac{\partial z}{\partial x} = -\frac{y}{x^2} = -\frac{1}{4}$$

$$\frac{\partial z}{\partial y} = \frac{x}{(x+y)^2} = 2$$

$$dz = -\frac{1}{4}dx + 2dy$$

$$2) z = (x^2 + y^2) \quad x=1, y=2 \quad dx=0,1 \quad dy=0,2$$

$$\frac{\partial z}{\partial x} = \cos(x^2 + y^2) \cdot 2x = 2\cos(5)$$

$$dz =$$

$$\frac{\partial z}{\partial y} = \cos(x^2 + y^2) \cdot 2 = 2\cos(5)$$

$$3) z = e^{\frac{x}{x+y}} \quad x=2, y=1 \quad dx=0,2, dy=0,1$$

$$\frac{\partial z}{\partial x} = e^{\frac{x}{x+y}} \cdot \frac{1}{2y} = \frac{1}{2}e$$

$$\frac{\partial z}{\partial y} = e^{\frac{x}{x+y}} \cdot \frac{-1}{2y^2} = -\frac{1}{2}e$$

$$dz = \frac{1}{2}e \cdot \frac{1}{5} + \left(-\frac{1}{2}e\right) \cdot \frac{1}{10} = -\frac{e}{10}$$

$$4) z = \ln(2x+y)$$

$m(1; e)$

$$\frac{\partial z}{\partial x} = \frac{2}{2x+y} = 1$$

$$\frac{\partial z}{\partial y} = \frac{1}{2x+y} = \frac{1}{2}$$

$$dz = 1dx + 0,5dy$$