Задорожний Антон 219/5

Задорожний Антон 219/5 Методы интегрирования

№ 56 1) $\int (7-2x)^3 dx$

$\cancel{\int (-8x^3 + 84x^2 - 284x + 343)\, dx}$

$(7-2x) = t$ $\qquad \cancel{\int (t)^3 dt}$

$\int d(7-2x) = dt$

$-2dx = dt$

$dx = -\frac{1}{2} dt$

$$\int \frac{(t)^3}{-2} dt = -\frac{1}{2}\int t^3 dt = -\frac{1}{2} \cdot \frac{t^4}{4} + C =$$

$$-\frac{1}{2} \cdot \frac{(7-2x)^4}{4} + C = -\frac{(7-2x)^4}{8} + C, \ C \in R$$

Ответ: $-\frac{(7-2x)^4}{8} + C, \ C \in R$

2) $\int (5t-1)^4 dt = \frac{1}{5}\int x^4 dx = \frac{1}{5} \cdot \frac{x^5}{5} + C =$

$(5t-1) = x$ $\qquad \frac{1}{5} \cdot \frac{(5t-1)^5}{5} + C = \frac{(5t-1)^5}{25} + C,$

$d(5t-1) = dx$ $\qquad C \in R$

$5dt = dx$

$dt = \frac{1}{5} dx$

3) $\int \frac{dx}{(4-3x)^2} = \int \frac{1}{(4-3x)^2}dx = -\frac{1}{3}\int \frac{1}{(t)^2}dt =$

$(4-3x)^2 = t$ $\quad -\frac{1}{3}\cdot\left(-\frac{1}{(2-1)\cdot t^{2-1}}\right)+c = -\frac{1}{3}\cdot\left(-\frac{1}{t}\right)+c =$

$d(4-3x) = dt$ $\quad -\frac{1}{3}\cdot\left(-\frac{1}{4-3x}\right) = \frac{1}{12-9x}+c,\ c\in R$

$-3dx = dt$

$dx = -\frac{1}{3}dt$

4) $\int \frac{dz}{(5z+1)^3} = \int \frac{1}{(5z+1)^3}dz = \frac{1}{5}\int \frac{1}{(t)^3}dt =$

$(5z+1) = t$ $\quad \frac{1}{5}\cdot\left(-\frac{1}{(3-1)\cdot t^{3-1}}\right)+c = \frac{1}{5}\cdot\left(-\frac{1}{2t^2}\right)+c =$

$d(5z+1) = dt$ $\quad \frac{1}{5}\cdot\left(-\frac{1}{2(5z+1)^2}\right) = -\frac{1}{10(5z+1)^2}+c,\ c\in R$

$5dz = dt$

$dz = \frac{1}{5}dt$

59. 1) $\int \sqrt[3]{(3x+1)^2}\,dx = \frac{1}{3}\int \sqrt[3]{(t)^2}\,dt =$ $\quad \sqrt[n]{a^m} = a^{\frac{m}{n}}$

$(3x+1) = t$ $\quad \frac{1}{3}\cdot\int t^{\frac{2}{3}}dt = \frac{1}{3}\cdot\frac{t^{\frac{2}{3}+1}}{\frac{2}{3}+1}+c =$

$d(3x+1) = dt$ $\quad \frac{1}{3}\cdot\frac{t^{\frac{5}{3}}}{\frac{5}{3}}+c = \frac{(3x+1)^{\frac{5}{3}}}{5}+c,\ c\in R$

$3dx = dt$

$dx = \frac{1}{3}dt$

Ответ: $\frac{(3x+1)^{\frac{5}{3}}}{5}+c,\ c\in R$

2) $\int \sqrt{2x-1}\,dx = \frac{1}{2}\int \sqrt{t}\,dt = \frac{1}{2}\int t^{\frac{1}{2}}dt =$

$(2x-1) = t$ $\quad \frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+c = \frac{(2x-1)^{\frac{3}{2}}}{3}+c,\ c\in R$

$2dx = dt$

$dx = \frac{1}{2}dt$

3) $\int \sqrt[3]{(4-3t)^2}\,dt = -\frac{1}{3}\int \sqrt[3]{(x)^2}\,dx = -\frac{1}{3}\int x^{\frac{2}{3}}dx =$

$(4-3t) = x$ $\quad -\frac{1}{3}\cdot\frac{x^{\frac{5}{3}}}{\frac{5}{3}}+c = -\frac{(4-3t)^{\frac{5}{3}}}{5}+c$

$-3dt = dx$

$dt = -\frac{1}{3}dx$

4) $\int \frac{dx}{\sqrt{(3x-1)^3}} = \int \frac{1}{\sqrt{(3x-1)^3}} dx = \frac{1}{3}\int \frac{1}{\sqrt{(t)^3}} dt =$

$(3x-1) = t$ $\quad \frac{1}{3} \cdot \int \frac{1}{t^{\frac{3}{2}}} dt = \frac{1}{3} \cdot \left(-\frac{1}{(\frac{3}{2}-1) \cdot t^{\frac{3}{2}-1}}\right) + C =$

$3dx = dt$ $\quad \frac{1}{3} \cdot \left(-\frac{1}{\frac{1}{2} \cdot t^{\frac{1}{2}}}\right) = -\frac{1}{\frac{3}{2}(3x-1)^{\frac{1}{2}}} + C, C \in \mathbb{R}$

$dx = \frac{1}{3}dt$

5) $\int \frac{dx}{\sqrt[3]{(3x-5)^2}} = \int \frac{1}{\sqrt[3]{(3x-5)^2}} dx = \frac{1}{3}\int \frac{1}{\sqrt[3]{(t)^2}} dt =$

$(3x-5) = t$ $\quad \frac{1}{3} \cdot \int \frac{1}{t^{\frac{2}{3}}} dt = \frac{1}{3} \cdot \left(-\frac{1}{(\frac{2}{3}-1) \cdot t^{\frac{2}{3}-1}}\right) +$

$3dx = dt$

$dx = \frac{1}{3}dt$ $\quad \frac{1}{3} \cdot \left(-\frac{1}{-\frac{1}{3} \cdot t^{-\frac{1}{3}}}\right) + C =$

$-\frac{1}{-\frac{1}{3} \cdot \left(\frac{1}{t^{\frac{1}{3}}}\right)} + C = -\frac{1}{-\frac{1}{3} \cdot \left(\frac{1}{\sqrt[3]{3x-5}}\right)} + C, C \in \mathbb{R}$

60\. 1) $\int (x^2+3)^5 x dx = \int (t)^5 \cancel{x} \cdot \frac{1}{2\cancel{x}} dt = \frac{1}{2}\int (t)^5 dt =$

$(x^2+3) = t$ $\quad \frac{1}{2} \cdot \frac{t^6}{6} + C = \frac{(x^2+3)^6}{12} + C, C \in \mathbb{R}$

$d(x^2+3) = dt$

$2x dx = dt$

$dx = \frac{1}{2x} dt$

2) $\int 4(x^4-1)^2 x^3 dx = 4\int (x^4-1)^2 x^3 dx =$

$(x^4-1) = t$ $\quad 4\int (x^4-1)^2 \cancel{x^3} \cdot \frac{1}{4\cancel{x^3}} dt =$

$d(x^4+1) = dt$ $\quad 4 \cdot \frac{1}{4} \int (t)^2 dt = \int \frac{t^3}{3} + C =$

$4x^3 dx = dt$

$dx = \frac{1}{4x^3} dt$ $\quad \frac{(x^4-1)^3}{3} + C, C \in \mathbb{R}$

3) $\int \frac{6z^2 dz}{(1-2z^3)^4} = \int \frac{6z^2}{(1-2z^3)^4} dz = -\int \frac{\cancel{6z^2}}{(t)^4} \cdot \frac{1}{\cancel{6z^2}} dt$

$(1-2z^3) = t$ $\quad -6z^2 dz = dt$

$d(1-2z^3) = dt$ $\quad dz = -\frac{1}{6z^2} dt$

$= -1\int \frac{1}{t^4}\,dt = -1\cdot\left(-\frac{1}{(4-1)t^{4-1}}\right)+c = -1\cdot\left(-\frac{1}{3t^3}\right)+c =$

$= \frac{1}{3(1-2z^3)^3} + C,\ C\in R$

4) $\int \frac{x^3\,dx}{(5x^4+3)^5} = \int \frac{x^3}{(5x^4+3)^5}\,dx = \int \frac{\cancel{x^3}}{(t)^5}\cdot\frac{1}{20\cancel{x^3}}\,dt =$

$(5x^4+3) = t$

$d(5x^4+3) = dt$

$20x^3\,dx = dt$

$dx = \frac{1}{20x^3}\,dt$

$\frac{1}{20}\int \frac{1}{t^5}\,dt = \frac{1}{20}\cdot\left(-\frac{1}{(5-1)t^{5-1}}\right)+c =$

$\frac{1}{20}\cdot\left(-\frac{1}{4t^4}\right) = -\frac{1}{100(5x^4+3)^4} + c,\ c\in R$

61. 1) $\int \sqrt{4x^3+1}\,x^2\,dx = \int \sqrt{t}\,\cancel{x^2}\cdot\frac{1}{12\cancel{x^2}}\,dt = \frac{1}{12}\int t^{\frac{1}{2}}\,dt =$

$(4x^3+1) = t$

$d(4x^3+1) = dt$

$12x^2\,dx = dt$

$dx = \frac{1}{12x^2}\,dt$

$\frac{1}{12}\cdot\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}+c = \frac{(4x^3+1)^{\frac{3}{2}}}{12\cdot\frac{3}{2}}+c = \frac{(4x^3+1)^{\frac{3}{2}}}{18} + C,\ C\in R$

2) $\int \sqrt{(x^4-1)^3}\,x^3\,dx = \int \sqrt{t^3}\,\cancel{x^3}\cdot\frac{1}{4\cancel{x^3}}\,dt = \frac{1}{4}\int t^{\frac{3}{2}}\,dt =$

$(x^4-1) = t$

$4x^3\,dx = dt$

$dx = \frac{1}{4x^3}\,dt$

$\frac{1}{4}\cdot\frac{t^{\frac{3}{2}+1}}{\frac{3}{2}+1}+c = \frac{t^{\frac{5}{2}}}{4\cdot\frac{5}{2}}+c = \frac{(x^4-1)^{\frac{5}{2}}}{10} + c,\ c\in R$

3) $\int \sqrt{2\sin x-1}\,\cos x\,dx = \int \sqrt{u}\,\cancel{\cos x}\,\frac{1}{2\cos x}\,du = \frac{1}{2}\int u^{\frac{1}{2}}\,du =$

$\frac{1}{2}\cdot\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{u^{\frac{3}{2}}}{2\cdot\frac{3}{2}} = \frac{(2\sin x-1)^{\frac{3}{2}}}{3} + c,\ c\in R$

$(2\sin x-1) = u$

$d(2\sin x-1) = du$

$2\cos x\,dx = du$

$dx = \frac{1}{2\cos x}\,du$

4) $\int \sqrt{e^x+1}\,e^x\,dx = \int \sqrt{u}\,\cancel{e^x}\,\frac{1}{e^x}\,du = 1\int u^{\frac{1}{2}}\,du =$

$\frac{u^{\frac{3}{2}}}{\frac{3}{2}}+c = \frac{(e^x+1)^{\frac{3}{2}}}{\frac{3}{2}}\ \cancel{du} + c,\ c\in R$

$(e^x+1) = u$

$d(e^x+1) = du$

$dx = \frac{1}{e^x}\,du$

62. 1) $\int \sqrt{(3z^4+2)^3}\, z^3 dz = \int (3z^4+2)^{\frac{3}{2}} z^3 dz = \frac{1}{12}\int x^{\frac{3}{2}} dx =$ $x=(3z^4+2)$

$\frac{x^{\frac{5}{2}}}{30} = \frac{(3z^4+2)^{\frac{5}{2}}}{30} + C$

2) $\int \sqrt[3]{(1-3x^2)^4}\, x\,dx = \int x\,(1-3x^2)^{\frac{4}{3}} = -\frac{1}{6}\int t^{\frac{4}{3}} dt =$

$t=(1-3x^2)$ $-\frac{t^{\frac{7}{3}}}{14} = -\frac{(1-3x^2)^{\frac{7}{3}}}{14} + C$

№63 1) $\int \frac{x\,dx}{\sqrt{x^2+1}} = \int \frac{x}{\sqrt{x^2+1}} dx = \frac{1}{2}\int \frac{1}{\sqrt{t}} dt = \sqrt{t} =$

$\sqrt{x^2+1} + C$

2) $\int \frac{x^3 dx}{\sqrt[3]{(5x^4+2)^2}} = \frac{1}{20}\int \frac{1}{t^{\frac{2}{3}}} dt = \frac{3\sqrt[3]{t}}{20} = \frac{3\sqrt[3]{5x^4+2}}{20} + C$

$t=(5x^4+2)$

3) $\int \frac{x^2 dx}{\sqrt{(x^3-1)^3}} = \int \frac{x^2}{\sqrt{(x^3-1)^3}} dx = \frac{1}{3}\int \frac{1}{t^{\frac{3}{2}}} dt = -\frac{2}{3\sqrt{t}} =$

$t=(x^3-1)$

$-\frac{2}{3\sqrt{x^3-1}} + C$

4) $\int \frac{\cos x\,dx}{\sqrt{1-\sin x}} = -\int \frac{1}{\sqrt{t}} dt = -2\sqrt{t} = -2\sqrt{1-\sin x} + C$

$t=(1-\sin x)$

5) $\int \frac{e^x dx}{(e^x+1)^3} = \int \frac{1}{t^3} dt = -\frac{1}{2t^2} = -\frac{1}{2(e^x+1)^2} + C$

$t=(e^x+1)$

№64 1) $\int \frac{z^3 dz}{1+z^3} = \frac{1}{3}\int \frac{z^3}{t} dt = \frac{\ln(t)}{3} = \frac{\ln(1+z^3)}{3} + C$

$t=(1+z^3)$

2) $\int \frac{e^{3x} dx}{e^{3x}+1} = \frac{1}{3}\int \frac{1}{t} dt = \frac{\ln(t)}{3} = \frac{\ln(e^{3x}+1)}{3} + C$

$t=(e^{3x}+1)$

3) $\int \frac{\cos x\,dx}{2\sin x+1} = \frac{1}{2}\int \frac{1}{t} dt = \frac{\ln(t)}{2} = \frac{\ln(|2\sin x+1|)}{2} + C$

$t=(2\sin x+1)$

№65 1) $\int tg\,3x\,dx = \frac{1}{3}\int tg\,t\,dt = -\frac{\ln(\cos t)}{3} =$ $t=3x$

$-\frac{\ln(|\cos 3x|)}{3} + C$

2) $\int ctg\,kx\,dx = \frac{1}{k}\int ctg(t)\,dt = \frac{\ln(\sin(|kx|))}{k}$ $t=kx$

3) $\int ctg\left(\frac{x}{2}\right)dx = 2\int ctg(t)\,dt = 2\ln(\sin t) =$ $t=\frac{x}{2}$

$2\ln\left(|\sin\frac{x}{2}|\right) + C$

№66 1) $\int\frac{dx}{\sin(2x)}\cdot\frac{\sin 2x}{\sin 2x} = \int\frac{\sin 2x}{\sin^2(2x)}\,dx = \int\frac{\sin 2x}{1-\cos(2x)^2}\,dx =$ $t=\cos(2x)$

$\int\frac{\cancel{\sin 2x}}{1-\cos(2x)^2}\cdot\frac{1}{-\cancel{\sin(2x)}\cdot 2}\,dt = \int-\frac{1}{2(1-\cos(2x)^2)}\,dt = \int-\frac{1}{2(1-t^2)}\,dt =$

$-\frac{1}{2}\int\frac{1}{1-t^2}\,dt = -\frac{1}{2}\cdot\frac{1}{-(2\cdot 1)}\cdot\ln\left(\left|\frac{t-1}{t+1}\right|\right) =$

$\frac{1}{4}\cdot\ln\left(\left|\frac{\cos(2x)-1}{\cos(2x)+1}\right|\right) = \frac{1}{4}\cdot\ln(|\cos(2x)-1|) - \frac{1}{\cancel{4}}\cdot\cancel{\ln 2} - \frac{1}{2}\cdot$

$\ln(\cos(x)) + C = \frac{1}{4}\cdot\ln(|\cos(2x)-1|) - \frac{1}{2}\cdot\ln(\cos(x)) + C$

2) $\int\frac{dx}{\sin\frac{x}{3}} = \int\frac{1}{\sin(t)}\cdot\frac{1}{\frac{1}{3}}\,dt = \int\frac{3}{\sin(t)}\,dt = 3\int\frac{1}{\sin t}\,dt =$ $t=\frac{x}{3}$ $z=\cos t$

$3\int\frac{1}{\sin t}\,dt\cdot\frac{\sin t}{\sin t} = 3\int\frac{\sin t}{\sin^2 t}\,dt = 3\int\frac{\sin t}{1-\cos^2(t)}\,dt =$

$-3\cdot\int\frac{1}{1-z^2}\,dz = -3\cdot\frac{1}{-(2\cdot 1)}\cdot\ln\left(\left|\frac{z-1}{z+1}\right|\right) = \frac{3}{2}\cdot\ln\left(\left|-tg^2\left(\frac{x}{6}\right)\right|\right)$

$\frac{3}{2}\cdot 2\ln\left(\left|tg\left(\frac{x}{6}\right)\right|\right) = 3\ln\left(tg\frac{x}{6}\right) + C$

$t = \sin(3x)$ 3) $\int \frac{dx \cdot 1}{\cos 3x} \cdot \frac{\cos 3x}{\cos 3x} dx = \int \frac{\cos 3x}{\cos^2 3x} dx = \int \frac{\cos 3x}{1-\sin^2(3x)} dx =$

$\int \frac{\cos 3x}{1-\sin^2(3x)} \cdot \frac{1}{\cos(3x) \cdot 3} dt = \frac{1}{3} \int \frac{1}{1-t^2} dt =$

$-\frac{1}{3} \int \frac{1}{t^2-1} dt = -\frac{1}{3} \cdot \frac{1}{2} \cdot \ln\left(\left|\frac{t-1}{t+1}\right|\right) = -\frac{1}{6} \cdot \ln\left(\left|\frac{\sin 3x-1}{\sin 3x+1}\right|\right.$

$-\frac{1}{6} \cdot \ln\left(\left|\frac{\sin(3x)-1}{\sin(3x)+1}\right|\right) + C$

$t = \frac{x}{2}$ 4) $\int \frac{dx}{\cos(\frac{x}{2})} = \int \frac{1}{\cos(\frac{t}{2})} \cdot \frac{1}{\frac{1}{2}} dt = \int \frac{1}{\cos(t)} \cdot 2 \, dt =$

$z = \sin t$ $2 \int \frac{1}{\cos(t)} dt \cdot \frac{\cos t}{\cos t} 1 = 2 \int \frac{\cos(t)}{\cos(t)^2} dt = 2 \int \frac{\cos(t)}{1-\sin^2(t)} dt$

$2 \int \frac{\cancel{\cos t}}{1-\sin(t)^2} \cdot \frac{1}{\cancel{\cos t}} dz = 2 \int \frac{1}{1-z^2} dz = -2 \cdot \frac{1}{2} \cdot \ln\left(\left|\frac{z-1}{z+1}\right|\right.$

$-1 \cdot \ln\left(\left|\frac{\sin t-1}{\sin t+1}\right|\right) = -\ln\left(\left|\frac{\sin\frac{x}{2}-1}{\sin\frac{x}{2}+1}\right|\right) + C$

$t = (\ln x + 1)$ № 67 1) $\int \frac{1}{x(\ln(x)+1)} dx \quad \frac{dt}{dx} = \int \frac{1}{t} dt =$

$\ln(t) = \ln(|\ln(t)+1|) + C$

$x = \ln t$ 2) $\int \frac{(2-\ln t)\, dt}{t} = \int \frac{2}{t} - \frac{\ln t}{t} dt = 2\ln(|t|) -$

$\int x \, dx = 2\ln(|t|) - \frac{x^2}{2} = 2\ln(|t|) - \frac{\ln^2 t}{2}$

$t = x^4$ № 68 1) $\int a^{x^4} \cdot x^3 dx = \frac{1}{4} \int a^t dt =$

$\frac{1}{4} \cdot \frac{a^t}{\ln(a)} = \frac{a^{x^4}}{4\ln a} + C$

$dx =$

2) $\int a^{2x} b^{2x} dx = \int (ab)^t dt = \frac{ab^{t+1}}{t+1} = \frac{ab^{2x+1}}{2x+1} + C$ $t=2x$

№6 1) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int 2 dt = 2e^{\sqrt{x}} + C$ $t=e^{\sqrt{x}}$

2) $\int x e^{-x^2} dx = \int -\frac{1}{2} dt = -\frac{1}{2} \cdot e^{-x^2} = -\frac{1}{2e^{x^2}} + C$ $t=e^{-x^2}$

3) $\int e^{\sin x} \cos x dx = \int e^t dt = e^t = e^{\sin x} + C$ $t=\sin x$

4) $\int \frac{e^{\frac{1}{x}} dx}{x^2} = \int \frac{e^t}{x^2} \cdot \frac{1}{-\frac{1}{x^2}} dt = -\int \frac{e^t}{\cancel{x^2}} \cdot \cancel{x^2} dt =$ $t=\frac{1}{x}$

$-e^t = -e^{\frac{1}{x}} + C$

№10 1) $\int t \sin(t^2-1) dt = \int t \sin(x) \cdot \frac{1}{2t} dx =$ $x=(t^2-1)$

$\frac{1}{2}\int \sin(x) = \frac{1}{2} \cdot (-\cos(t)) = \frac{1}{2}(-\cos(x^2-1)) =$

$-\frac{\cos(x^2-1)}{2} + C$

2) $\int \sin(\frac{z}{2}) dz = \int \sin(\frac{x}{2}) \cdot \frac{1}{\frac{1}{2}} dt = 2\int \sin t\, dt =$ $t=\frac{z}{2}$

$2 \cdot (-\cos(t)) = 2 \cdot (-\cos(\frac{x}{2})) = -2\cos(\frac{x}{2}) + C$

3) $\int x^3 \cos x^4 dx = \int \cancel{x^3} \cos(t) \cdot \frac{1}{4\cancel{x^3}} dt =$ $t=x^4$

$\frac{1}{4}\int \cos(t) dt = \frac{1}{4} \sin(t) = \frac{\sin(x^4)}{4} + C$

4) $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx = \int \frac{\cos t}{\cancel{\sqrt{x}}} \cdot 2\cancel{\sqrt{x}} = 2\int \cos t\, dt =$ $t=\sqrt{x}$

$2 \cdot \sin t = 2\sin\sqrt{x} + C$

$t=x^2+1$ 5) $\int x\cos(x^2+1)dx = \int \cancel{x}\cdot\cos(x^2+1)\cdot\frac{1}{2\cancel{x}}dt =$

$\frac{1}{2}\int\cos(t)dt = \frac{1}{2}\sin(t) = \frac{1}{2}\cdot\sin(x^2+1) =$

$\frac{\sin(x^2+1)}{2}+C$

$t=\sqrt{x}$ №71 1) $\int\frac{dx}{\sqrt{x}\cos^2\sqrt{x}} = \int\frac{2t\,dt}{t\cos^2 t} = 2tg\,t = 2tg\sqrt{x}+C$

$dx = 2t\,dt$

$t=x^3$ 2) $\int\frac{x^2dx}{\cos^2x^3} = \cancel{\frac{1}{3}tg\,x} = \int\frac{x^2}{\cos^2(t)}\cdot\frac{1}{3x^2}dt =$

$\frac{1}{3}\int\frac{1}{\cos(t)^2}dt = \frac{1}{3}\cdot tg(t) = \frac{tg\,x^3}{3}+C$

3) $\int\frac{dy}{\sin^2(\frac{\pi}{3}-y)} = -\int\frac{d(\frac{\pi}{3}-y)}{\sin^2(\frac{\pi}{3}-y)} = \cancel{+}ctg(\frac{\pi}{3}-y)+C$

$t=\frac{1}{x}$ 4) $\int\frac{dx}{x^2\sin^2\frac{1}{x}} = \int-\frac{1}{\sin^2(t)}dt = +ctg(t) =$

$-ctg(\frac{1}{x})+C$

$t=\ln(x)$ 5) $\int\frac{dx}{x\sin^2\ln x} = \int\frac{1}{x\sin^2(t)}dx = -ctg(\ln(x))+C$

№72 1) $\int\frac{e^y dy}{\sqrt{1-e^{2y}}} = \frac{1}{\sqrt{2}}\int\frac{d\sqrt{2}e^y}{\sqrt{1-2e^{2+y}}} = \frac{\arcsin\sqrt{2}e^y}{\sqrt{2}}+C$

2) $\int\frac{dz}{z\sqrt{1-\ln^2z}} = \int\frac{d\ln z}{\sqrt{1-\ln^2z}} = \arcsin\ln z+C$

№73 1) $\int\frac{\sin x\,dx}{a^2+\cos^2x} = -\int\frac{d\cos x}{a^2+\cos^2x} = \operatorname{arctg}\cos x+C$

2) $\int\frac{e^x dx}{1+e^{2x}} = \int\frac{de^x}{1+e^{2x}} = \frac{1}{3}\operatorname{arctg}e^x+C$

3) $\int \frac{x^2 dx}{1+x^6} = \frac{1}{3}\int \frac{dx^3}{1+(x^3)^2} = \frac{1}{3} arctg\, x^3 + C$

4) $\int \frac{dx}{x(1+\ln^2 x)} = \int \frac{d \ln x}{1+\ln^2 x} = arctg \ln x + C$

75. 1) $u = x$ $\quad dv = \cos x dx$

$du = dx \quad v = \sin x$

$\int x \cos x\, dx = x \sin x - \int \sin x dx = x \sin x + \cos x + C$

2) $u = 1 - x \quad dv = \sin x\, dx$

$du = -dx \quad v = -\cos x$

$\int = (1-x)\sin x\, dx = (1-x)(-(\cos x)) - \int \cos x\, dx =$

$-\cos x + x \cancel{\cos} - \sin x + C = -\cos x + x\cos x - \sin x + C$

78. 1) $u = \arcsin x \quad dv = dx \quad t = \sqrt{1-x^2}$

$du = \frac{1}{\sqrt{1-x^2}} dx \quad v = x \quad t' = \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) = \frac{1}{2t}(-2x)$

$\int \arcsin x\, dx = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx =$

$x \arcsin x \int \frac{1}{t} \cdot \frac{1}{\frac{1}{2t}} dt = x \arcsin x - \int -\frac{1}{t} \cdot \frac{1}{\frac{1}{t}} dt =$

$x \arcsin x \int -\frac{1}{t} \cdot x\, dt = x \arcsin x - \int - dt = x \arcsin x + t =$

$x \arcsin + \sqrt{1-x^2} + C$

2) $u = arctg\, x \quad dv = dx \quad t = 4x^2$

$du = \frac{1}{1+x^2} dx \quad v = x \quad t' = 2x$

$\int arctg\, x\, dx = x\, arctg\, x - \int \frac{x}{1+x^2} dx = x\, arctg\, x$

$-\int \frac{x}{t}\cdot\frac{1}{2}dt = x\,arcctg\,x - \int\frac{1}{2x}dt =$

$x\,arcctg\,t - \int\frac{1}{2t}dt = x\,arcctg\,t - \frac{1}{2}\cdot\int\frac{1}{t}dt = x\,arcctg$

$x - \frac{1}{2}\ln(t) = x\,arcctg\,x - \frac{1}{2}\ln|1+x^2| + C$

78. 1) $u = e^x$ $\quad dv = \cos x$

$du = e^x dx$ $\quad v = \sin x$

$\int e^x\cdot\cos x\,dx = e^x\sin x - \int \sin x\, e^x dx =$

$u = e^x$ $\quad dv = \sin x$

$du = e^x dx$ $\quad v = -\cos x$

$e^x\sin x - (-e^x\cos x - \int \cos x\, e^x) = e^x\sin x + e^x\cos x$

$-\int \cos x\, e^x dx = 2\int e^x\cos dx = e^x\sin x + e^x\cos x =$

$\int e^x\cos x\,dx = \frac{e^x\sin x + e^x\cos x + C}{2}$

2) $u = e^x$ $\quad dv = \sin x\,dx$

$du = e^x dx$ $\quad v = -\cos x$

$\int e^x\sin x\,dx = e^x(-\cos x) - \int -\cos x\, e^x dx = -e^x\cos x +$

$\int e^x\cos x\,dx$

$u = e^x$ $\quad dv = \cos x\,dx$

$du = e^x dx$ $\quad v = \sin x$

$= -e^x\cos x + e^x\sin x - \int e^x\sin.$

$\int e^x\sin x\,dx + \int e^x\sin x\,dx = e^x(\sin x - \cos x$

$$2\int e^x \sin x \, dx = e^x (\sin x - \cos x)$$

$$\int e^x \sin x \, dx = \frac{e^x \sin x - e^x \cos x}{2} + C$$

xavety

$e^x \cos x$

osx+