Theory

Q.1)

Formalized Logic:

1. **G ∨ Y ∨ R** (only one state is true at any time)
2. **(G → Y) ∧ (Y → R) ∧ (R → G)** (strict order)
3. **¬(G ∧ Cycle(3))** (limiting consecutive cycles)

Explanation:

- **∨:** OR
- **∧:** AND
- **→:** IMPLIES
- **¬:** NOT
- **Cycle(3):** A predicate representing a 3-cycle.

Q.2)

FOL Representation and Axioms for a Graph Coloring Problem

Step 1: Defining Predicates

We define predicates to represent the properties of nodes, colors, edges, and relationships between them:

- Node(n): n is a node in the graph.
- Color(n, c): Node n has color c.
- Edge(n1, n2): There is a directed edge from node n1 to node n2.
- Yellow(n): Node n has the color yellow.

- Red(n): Node n has the color red.
- Green(n): Node n has the color green.
- Clique(n1, n2): Nodes n1 and n2 belong to the same clique.
- Distance(n1, n2, d): The shortest distance between nodes n1 and n2 is d.

Step 2: Formalizing Rules as FOL Axioms

Here are the FOL axioms representing the given rules:

Rule 1: Connected nodes don't have the same color.

$\forall n1, n2\ (Edge(n1, n2) \rightarrow \neg(Color(n1, c) \wedge Color(n2, c)))$

This axiom states that for any two connected nodes n1 and n2, if there is an edge between them, they cannot have the same color.

Rule 2: Exactly two nodes are allowed to wear yellow.

$\exists n1, n2\ (Yellow(n1) \wedge Yellow(n2)) \wedge \forall n3\ (Yellow(n3) \rightarrow (n3 = n1 \vee n3 = n2))$

This axiom ensures that there are exactly two nodes that are colored yellow, and no other node is yellow.

Rule 3: Starting from any red node, you can reach a green node in no more than 4 steps.

$\forall n1\ (Red(n1) \rightarrow \exists n2\ (Green(n2) \wedge Distance(n1, n2, d) \wedge d \leq 4))$

This axiom states that for every red node n1, there exists a green node n2 such that the distance between them is at most 4.

Rule 4: Every color in the palette is assigned to at least one node.

$$\forall c\ \exists n\ (Color(n, c))$$

This axiom ensures that every color in the palette is assigned to at least one node.

Rule 5: The nodes are divided into |C| disjoint non-empty cliques, one for each color. This rule can be represented by the following axioms:

Each color has a clique:

$$\forall c\ \exists n1, n2\ (Color(n1, c) \land Color(n2, c) \land Clique(n1, n2))$$

Disjoint cliques:

$$\forall n1, n2\ (Clique(n1, n2) \rightarrow Color(n1, c) \land Color(n2, c))$$

Non-empty cliques:

$$\forall c\ \exists n\ (Color(n, c))$$

These axioms together ensure that the nodes are divided into disjoint cliques based on their colors, and each clique is non-empty.

Q.3)

Let's break this problem down into two parts: **representation using Propositional Logic (PL) and First-Order Logic (FOL)**, and **satisfiability checks using resolution refutation**. The goal is to capture the logical relationships between literacy, reading, and intelligence in dolphins.

Section 1: Problem Statement

- **Goal:** To represent and analyze the logical relationships between literacy, reading, and intelligence in dolphins.
- **Methodology:** Use Propositional Logic (PL) and First-Order Logic (FOL) for representation, and resolution refutation for satisfiability checks.

Section 2: Propositional Logic (PL) Representation

- **Propositional Variables:**
  - R(x): (x) can read.
  - L(x): (x) is literate.
  - I(x): (x) is intelligent.
  - D(x): (x) is a dolphin.
- **Statements:**
  - List the five statements as given in the prompt, using PL notation.

Section 3: First-Order Logic (FOL) Representation

- **Predicates:** Same as in PL.
- **Statements:**
  - List the five statements as given in the prompt, using FOL notation.

Section 4: Satisfiability Check using Resolution Refutation

- **Fourth Statement:**
  - Negation:
  - Resolution steps:
  - Conclusion:
- **Fifth Statement:**

    - Negation:
    - Resolution steps:
    - Conclusion:

Section 5: Conclusion

- Summarize the findings from the satisfiability checks.

Additional Notes:

- **Formatting:** Use headings, subheadings, and bullet points to improve readability.
- **Clarity:** Ensure that the statements and reasoning are clear and easy to follow.
- **Completeness:** Include all necessary details for understanding the problem and solution.
- **Consistency:** Maintain consistency in notation and terminology.

By following this format, you can create a well-structured and informative Google Doc that effectively presents the problem and its solution.

Computational

Q.1)

(a) Top 5 busiest routes based on the number of trips:

- ('5722', 318)
- ('5721', 318)
- ('674', 313)
- ('593', 311)
- ('5254', 300)

(b) Top 5 stops with the most frequent trips:

- ('10225', 4115)
- ('10221', 4049)
- ('149', 3998)
- ('488', 3996)
- ('233', 3787)

(c) The top 5 busiest stops based on the number of routes passing through them:

- ('488', 107)
- ('10225', 101)
- ('149', 99)
- ('231', 95)
- ('18721', 94)

(d) The top 5 pairs of stops (start and end) that are connected by exactly one direct route, sorted by the combined frequency of trips passing through both stops:

- (('233', '148'), 6448)
- (('11476', '10060'), 6438)
- (('10225', '11946'), 6238)
- (('11844', '180120'), 5732)
- (('11845', '10120'), 5608)