Q.1)

a) Comparison of Sampling Methods:

- **Direct Sampling:**
  - **Strengths:** Simple to implement, efficient when the probability distribution is known.
  - **Weaknesses:** Relies on the accuracy of the probability distribution. If the distribution is incorrect, the samples may be biased.
- **Rejection Sampling:**
  - **Strengths:** Can be used for complex distributions where direct sampling is difficult.
  - **Weaknesses:** Can be inefficient, especially when the target distribution is much smaller than the proposal distribution. Many samples may be rejected.
- **Gibbs Sampling:**
  - **Strengths:** Can be used for high-dimensional distributions, and it can handle complex dependencies between variables.
  - **Weaknesses:** Can be computationally expensive, especially for large datasets. Convergence to the target distribution can be slow.

In the context of the given travel dataset, direct sampling might be suitable for simple probabilities like the probability of air travel or train travel. However, for more complex probabilities, like the probability of stress given a mode of travel, Gibbs sampling could be a better choice. Rejection sampling might be useful for specific scenarios, but its efficiency would depend on the choice of the proposal distribution.

b) Estimating Travelers for Leisure:

Given:

- Probability of leisure travel given train preference = 0.400
- Number of people who prefer train travel = 30

Expected number of leisure travelers by train = **Probability x Number of train travelers** = 0.400 x 30 = 12

c) Probability of Air Travel and Business:

Given:

- Probability of air travel = 0.80
- Probability of business travel given air travel = 0.20

Probability of air travel and business = **P(Air Travel) x P(Business | Air Travel)** =

0.80 x 0.20 = 0.160

d) Effect of Increasing Sample Size on Accuracy and Precision:

Increasing the sample size generally leads to:

- **Increased accuracy:** The sample mean becomes a more reliable estimate of the population mean.
- **Increased precision:** The standard error of the estimate decreases, leading to narrower confidence intervals.

In the context of the given dataset, a larger sample size would provide more accurate estimates of probabilities like the proportion of people who prefer air travel or the proportion of stressed travelers. This would lead to more reliable conclusions and better decision-making. However, it's important to balance the benefits of a larger sample size with the costs of data collection and analysis.

Q.2)

a) Identifying Random Variables and Expressing Statements

Let's define the following random variables:

- **A:** Person reads books

- **J:** Person accesses academic journals
- **B:** Person participates in book clubs

Now, we can express the given statements in probability notation:

$P(A \cup J) = 0.910$

$P(J \mid A) = 0.400$, $P(\neg J \mid A) = 0.600$

$P(B \mid A) = 0.320$

$P(J \cap \neg A) = 0.227$

$P(\neg A \cap \neg J) = 0.090$

$P(J \mid \neg A) = 0.716$

$P(B \cap J) = 0.088$

$P(B \cup J) = 0.631$

$P(J \mid B) = 0.400$

$P(J) = 0.500$

$P(B \mid \neg A) = 0.0044$

b) Verifying the Probability Distribution

To verify the validity of the probability distribution, we need to ensure that the following axioms hold:

1. **Non-negativity:** All probabilities must be non-negative.
2. **Normalization:** The sum of all probabilities in the sample space must equal 1.
3. **Additivity:** For mutually exclusive events, the probability of their union is the sum of their individual probabilities.

Based on the given probabilities, we can verify that these axioms hold. For instance, $P(A \cup J) + P(\neg A \cap \neg J) = 0.910 + 0.090 = 1.0$, satisfying the normalization axiom.

c) Constructing the Joint Probability Table

To construct the joint probability table, we need to calculate the probabilities of all possible combinations of the three events A, J, and B. We can use the given conditional probabilities and the law of total probability to do this.

A	J	B	P(A, J, B)
0	0	0	P(¬A ∩ ¬J ∩ ¬B)
0	0	1	P(¬A ∩ ¬J ∩ B)
0	1	0	P(¬A ∩ J ∩ ¬B)
0	1	1	P(¬A ∩ J ∩ B)
1	0	0	P(A ∩ ¬J ∩ ¬B)
1	0	1	P(A ∩ ¬J ∩ B)
1	1	0	P(A ∩ J ∩ ¬B)
1	1	1	P(A ∩ J ∩ B)

We can calculate these probabilities using the given information and the laws of probability. For example, P(A ∩ J ∩ B) = P(B | A ∩ J) * P(A ∩ J) = P(B | J) * P(J | A) * P(A).

d) Checking for Conditional Independence

To check for conditional independence, we need to verify if the following conditions hold:

- P(A | J, B) = P(A | J)
- P(B | A, J) = P(B | A)
- P(A | B) = P(A)

We can calculate these conditional probabilities using the joint probability table and the definition of conditional probability. If any of these conditions hold, then the corresponding variables are conditionally independent.

By calculating the joint probabilities and checking for conditional independence, we can gain a deeper understanding of the relationships between the random variables A, J, and B.

Q.3)

a) Formulating the Problem Using Bayesian Inference

We need to update our belief about the likelihood of adversarial perturbations (P) causing a misclassification given new information about the increased prevalence of backdoor attacks (B). Bayesian inference provides the framework for this.

Definitions:

1. Event M: A misclassification alarm is raised.
2. Event P: The misclassification is caused by adversarial perturbations.
3. Event B: The misclassification is caused by backdoor attacks.

Initially, P and B are considered independent. The misclassification alarm M acts as a common effect of both P and B. Using Bayesian inference, we update our belief about P given B and M. We need to compute

$P(P \mid M, B)$ (the probability of P given the alarm M and evidence B).

b) Defining Probabilities

Prior Probabilities:

1. $P(P)$: Prior probability of adversarial perturbations causing a misclassification.
2. $P(B)$: Prior probability of backdoor attacks causing a misclassification.

Likelihood:

1. $P(M \mid P)$: Probability of a misclassification alarm given adversarial perturbations.
2. $P(M \mid B)$: Probability of a misclassification alarm given backdoor attacks.
3. $P(M \mid P, B)$: Probability of a misclassification alarm when both P and B occur.

Posterior:

Using Bayes' theorem, the posterior probability of P given M and B is:

$$P(P \mid M, B) = \{ P(M \mid P, B) \cdot P(P \mid B) \} / P(M \mid B)$$

To compute $P(P \mid B)$, consider that P and B are not truly independent after conditioning on M (common-effect relationship).

Key Relationship:

When conditioned on the common effect M, the probability of one cause (e.g., P) depends on the likelihood of the other cause (e.g., B) through:

$$\cdot P(P \mid M) \propto P(P) \cdot P(M \mid P)$$

However, $P(P \mid M,B)$ incorporates the influence of B on M.

c) How Conditioning on B Changes Beliefs About P (2.5 Marks)

Before Conditioning on B:

- $P(P \mid M)$ is influenced only by P(P), P(M∣P), and the total probability of M from all causes.

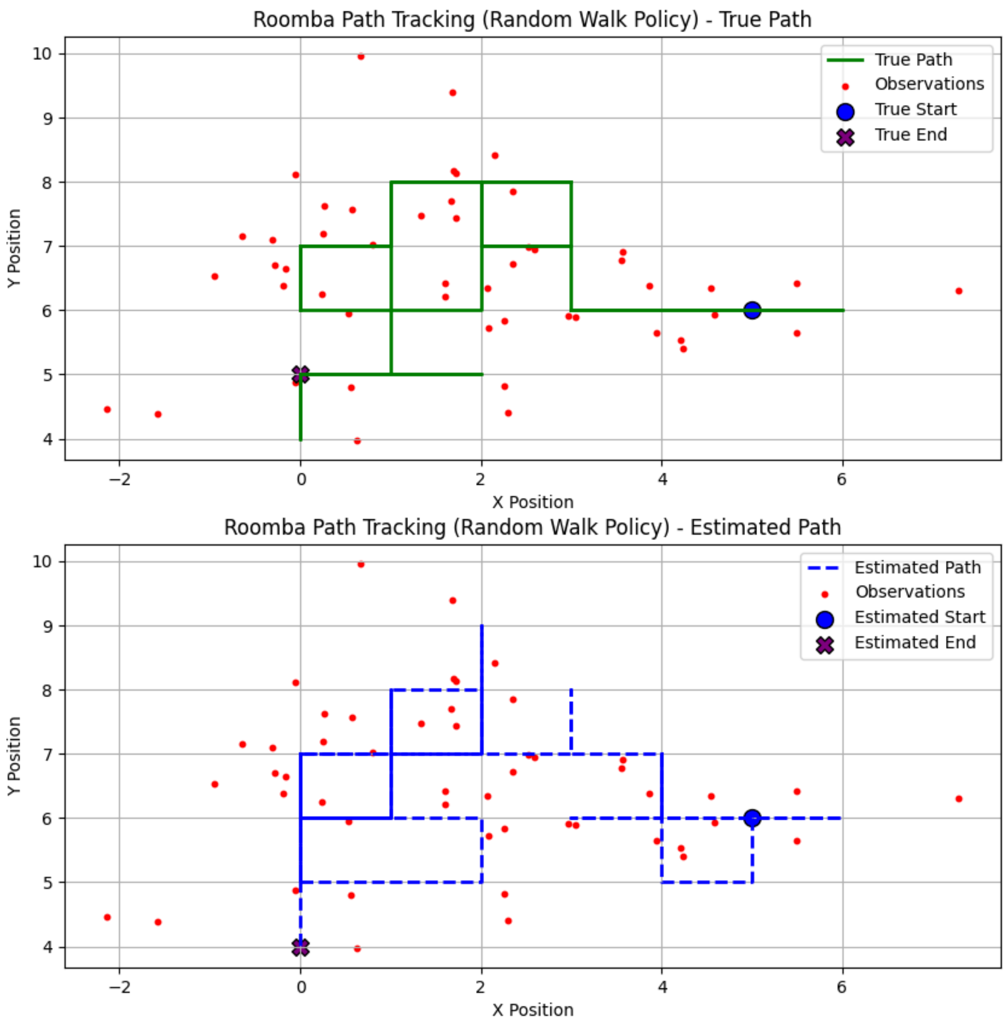
After Conditioning on B:

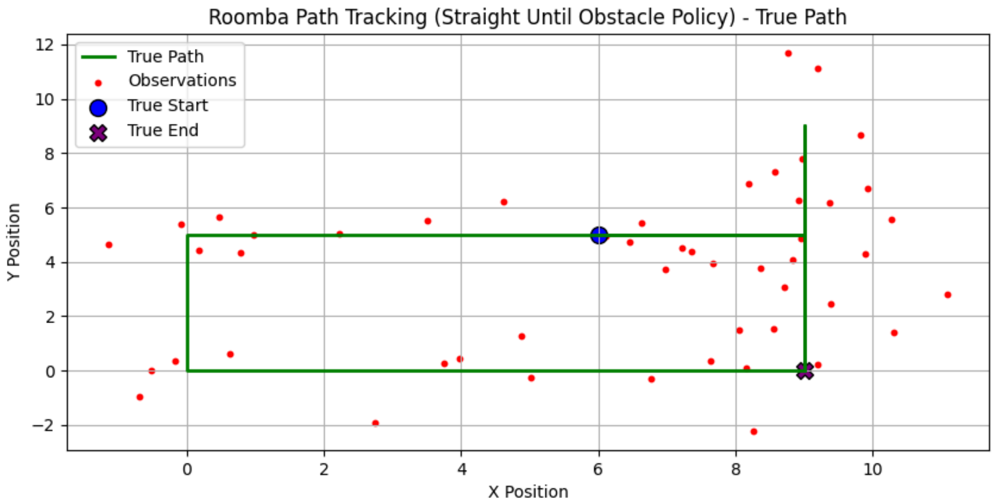
- New evidence shows that B (backdoor attacks) are increasingly prevalent in recent datasets.
- Since $P(M \mid B)$ is likely high (backdoor attacks cause misclassification reliably), observing M increases the posterior probability of B.
- This means $P(B \mid M)$ increases, which reduces the likelihood that P (adversarial perturbations) caused M.

Impact of the New Information:

The increased prevalence of B makes it more likely that M was caused by backdoor attacks, thus decreasing the belief in adversarial perturbations *($P(P \mid M,B)$)* as the cause of M. This phenomenon is due to explaining away: when one potential cause becomes more probable, it diminishes the necessity of the other cause to explain the observed effect.

Q.5)


Roomba Path Tracking (Random Walk Policy) - True Path
True Path
Observations
True Start
True End
X Position
Y Position
Roomba Path Tracking (Random Walk Policy) - Estimated Path
Estimated Path
Observations
Estimated Start
Estimated End
X Position
Y Position


Roomba Path Tracking (Straight Until Obstacle Policy) - True Path
True Path
Observations
True Start
True End
Y Position
X Position
12
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4
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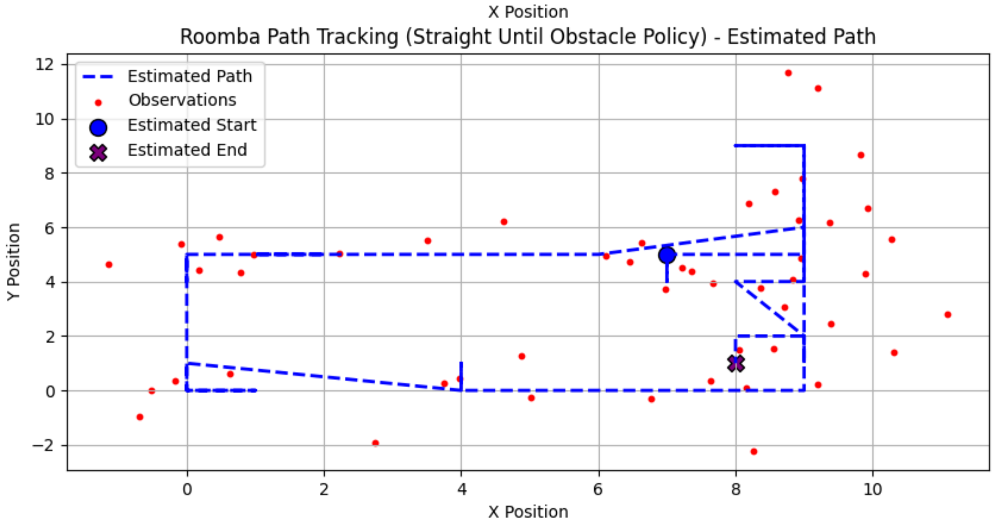
Roomba Path Tracking (Straight Until Obstacle Policy) - Estimated Path
Estimated Path
Observations
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