Project Topic: Combinatorial Game Theory

1. Project Summary:

a. What we want to explore

- In this project we are going to explore the application of Combinatorial Game Theory (CGT) to analyse and solve classic combinatorial games, focusing on impartial games and their strategies. Specifically,

b. How are we planning to explore it

- Our exploration will involve both theoretical analysis and practical implementation. We will delve into the mathematical foundations of CGT, studying the principles of impartial games and their connection to Sprague-Grundy theory. Additionally, we will create Python programs to simulate and solve various impartial games, applying the theoretical insights gained during my study.

c. How it's related to the course

- This project applies principles from the Discrete Mathematics course, as outlined in 'Discrete Mathematics and Its Applications' by Kenneth H. Rosen. Specifically, we draw upon topics such as game trees, impartial games, and game values, which are fundamental to Combinatorial Game Theory (CGT). By utilising these concepts to solve real-world games, this project reinforces the practical relevance of discrete mathematics, bridging theory and application**.**

2. Detailed Explanation of Theoretical Part

- In the theoretical portion of our project, I will provide comprehensive explanations of the fundamental concepts and mathematical foundations of CGT, with a focus on impartial games and game values:-

    I. **Impartial Games:** We will delve into the definition of impartial games, explaining how they are characterised by identical move options for both players at each game position.

    II. **Game Values:** We will discuss the concept of game values and how they are used to determine the outcome of impartial

games. Specifically, we shall explain the process of assigning Grundy numbers (nimbers) to
game positions.

III. **Sprague-Grundy Theorem:** We will explore the Sprague-Grundy theorem and its significance in CGT. This theorem provides a crucial framework for analysing impartial games and determining winning strategies.

IV. **Nim:** As a classic example, We will provide an in-depth analysis of the game of Nim, demonstrating how to calculate nimbers, determine winning positions, and devise optimal strategies.

V. Each theoretical part will be accompanied by mathematical proofs, examples, and practical insights to ensure a comprehensive understanding of the underlying principles.

Group Members:

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