OTTO VON GUERICKE
UNIVERSITÄT
MAGDEBURG

INF

FAKULTÄT FÜR
INFORMATIK

Inverse Kinematics for Kuka KR30-3: Positioning of the wrist

Softwaredevelopment for Industrial Robotics

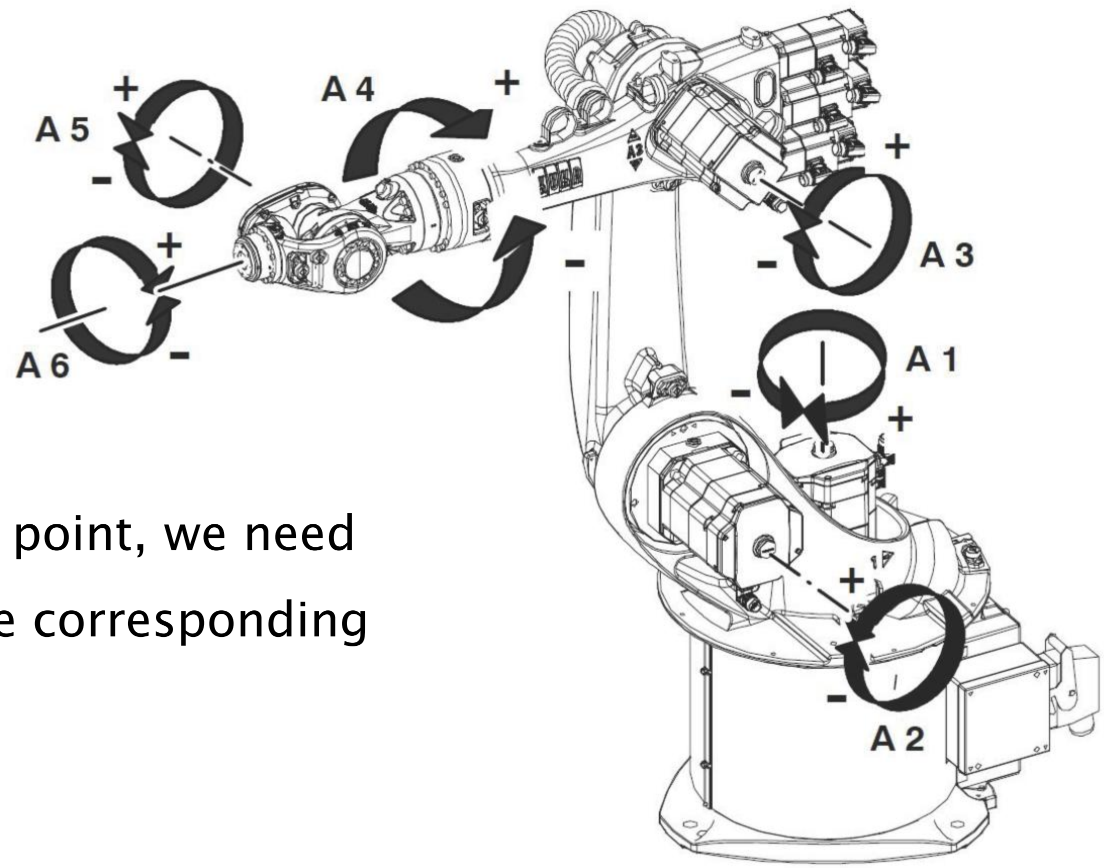
Marcel Grotzke, Gabriel Kögler, Henri Hamann 25.11.2013

OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

Table of content

OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

1.Overview: Structure of Kuka KR30-3

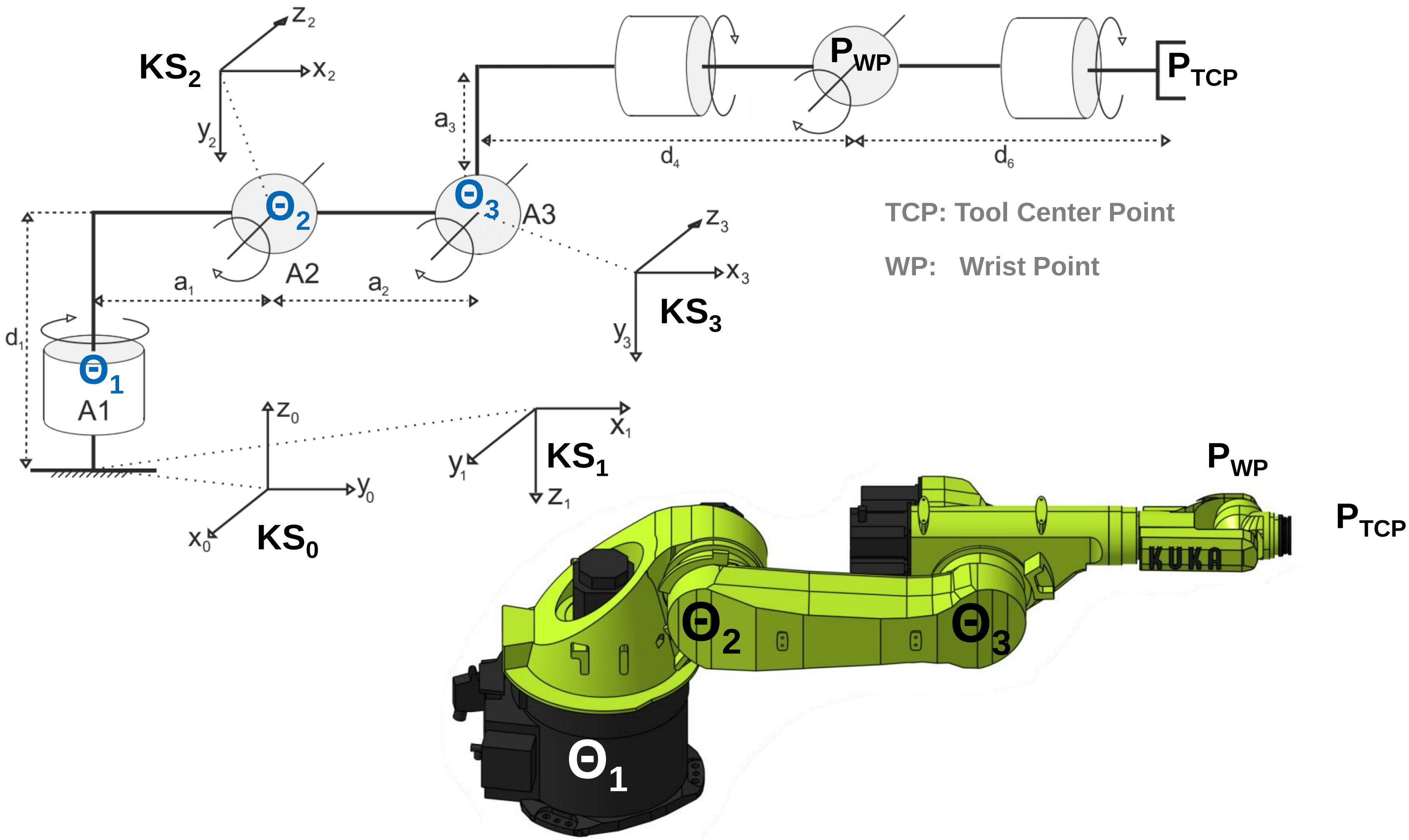


For positioning of wrist point, we need joint **A1**, **A2**, **A3** and the corresponding angles Θ_1, Θ_2, and Θ_3.

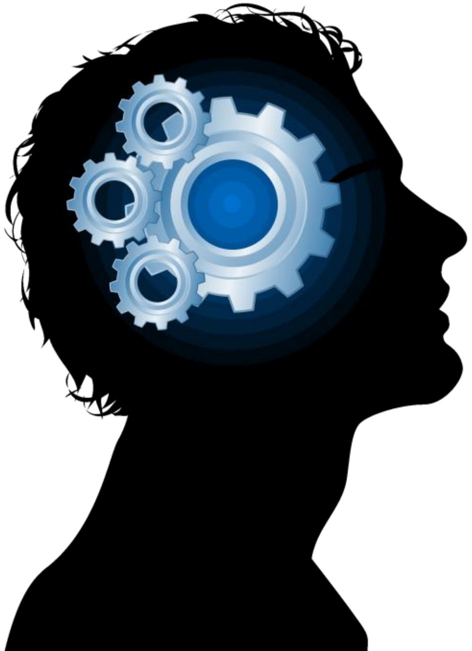
OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

1.Overview: Zero Configuration of KR30-3

Angles Θ_1, Θ_2, and Θ_3 = 0°

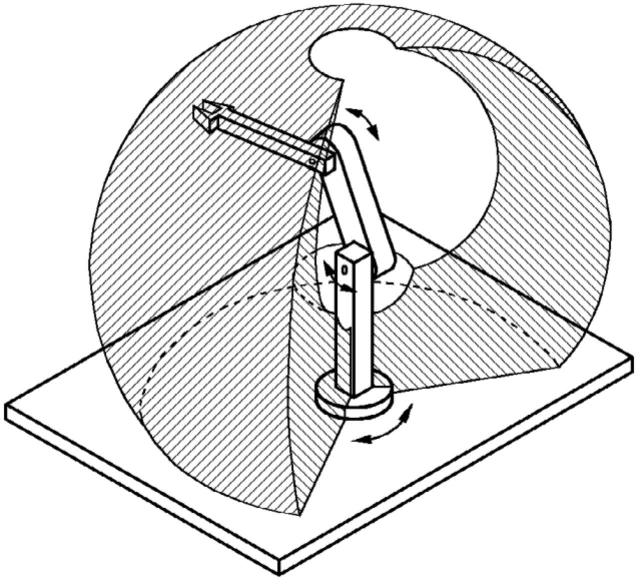


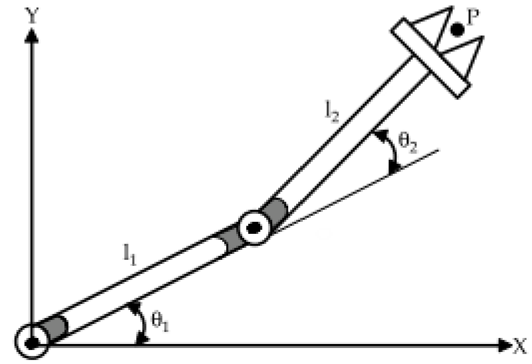
OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG
INF FAKULTÄT FÜR INFORMATIK



2. Solution statement

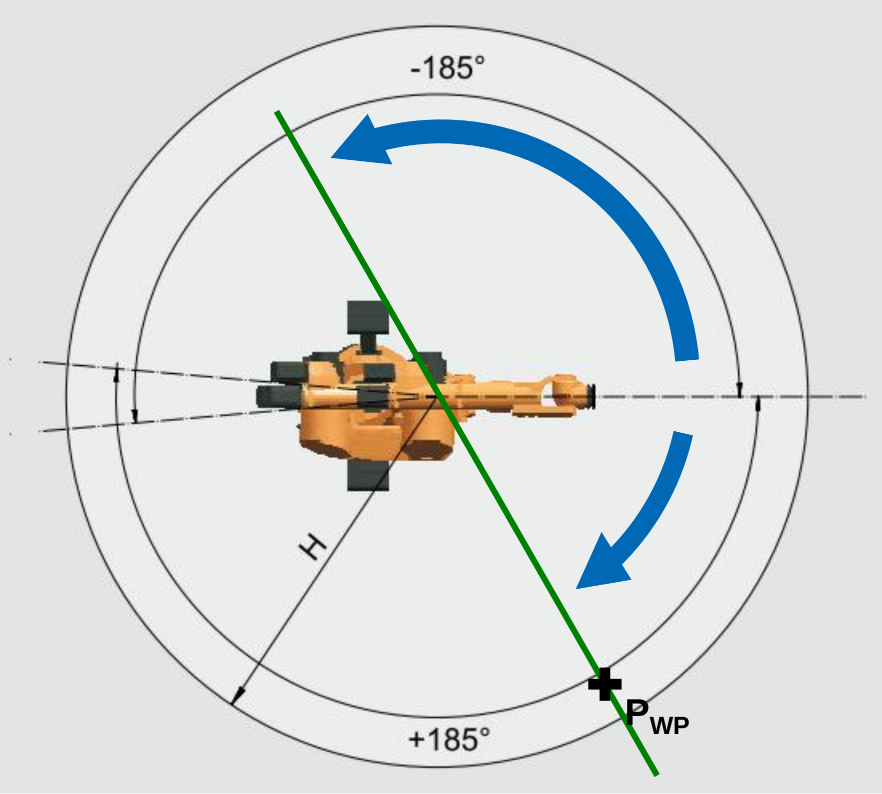
Split 3D problem of inverse kinematics in several 2D problems:





2. Solution statement [2]

Task: Reach the target point P_{WP} with the wrist of the robot



First 2D problem in topview:

Rotation of the robot arm to the plane of the wrist point (P_{WP}) [Θ_1].

→ „**WP Plane**“

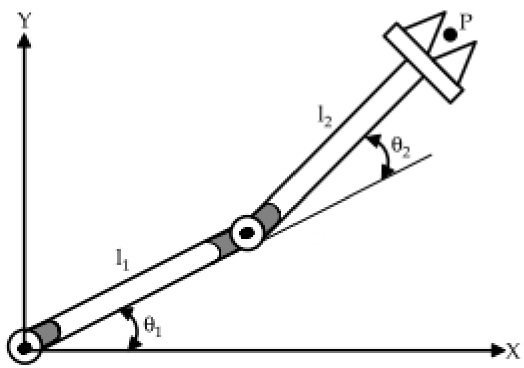
OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

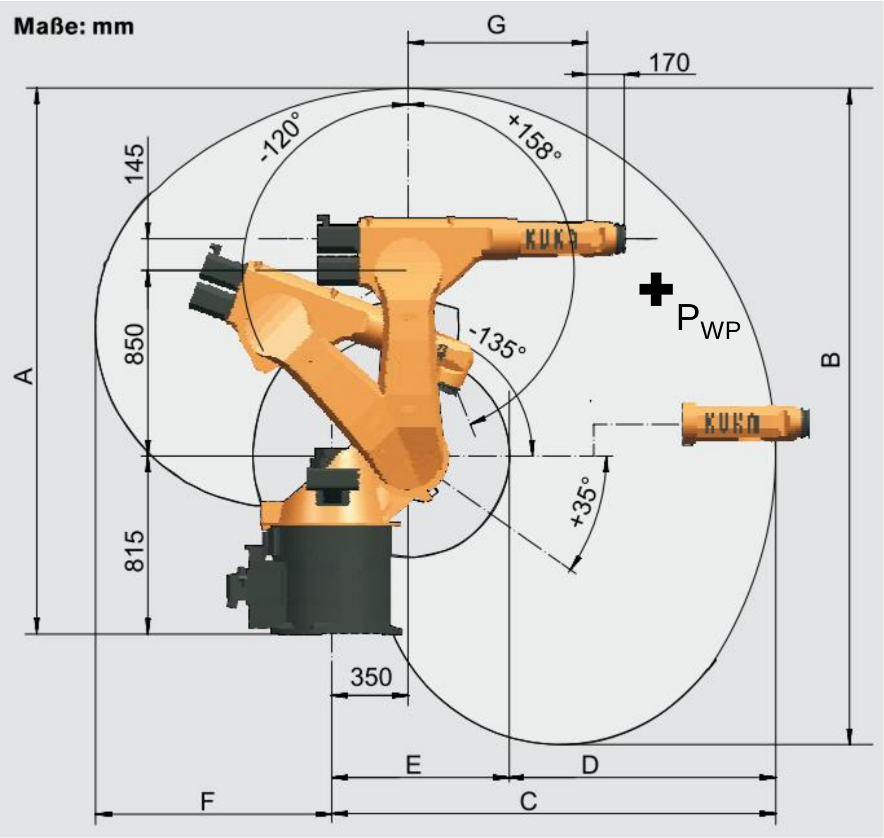
2. Solution statement [3]

<u>Second 2D problem in lateral view:</u>

Calculation of the other

angles [Θ_2,Θ_3] in the „**WP Plane**“



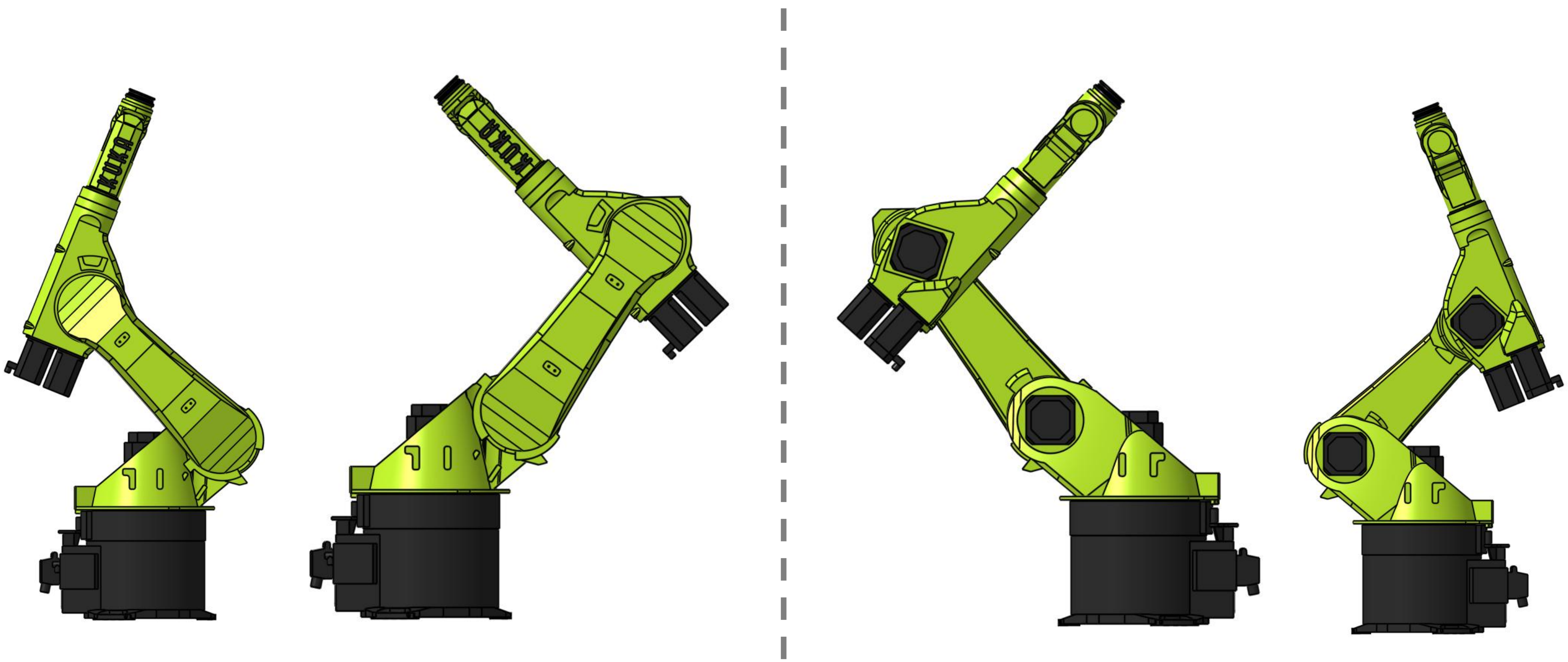


2. Solution statement: Possible Solutions

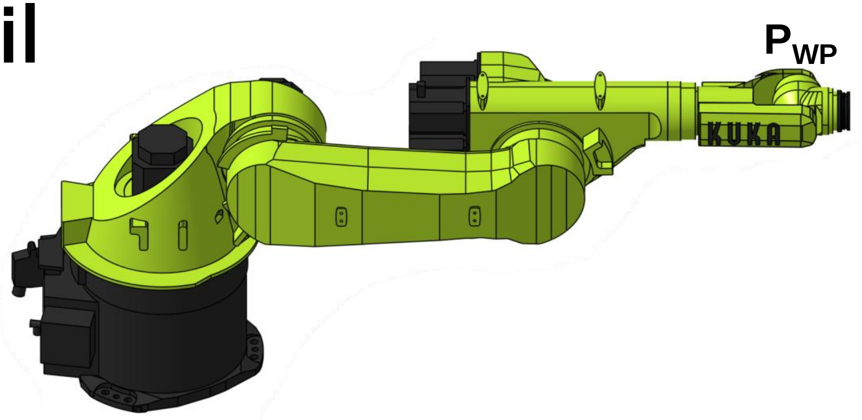
4 possible solutions:

Shoulder: right — left

Elbow: up, down — down, up



OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG
INF FAKULTÄT FÜR INFORMATIK



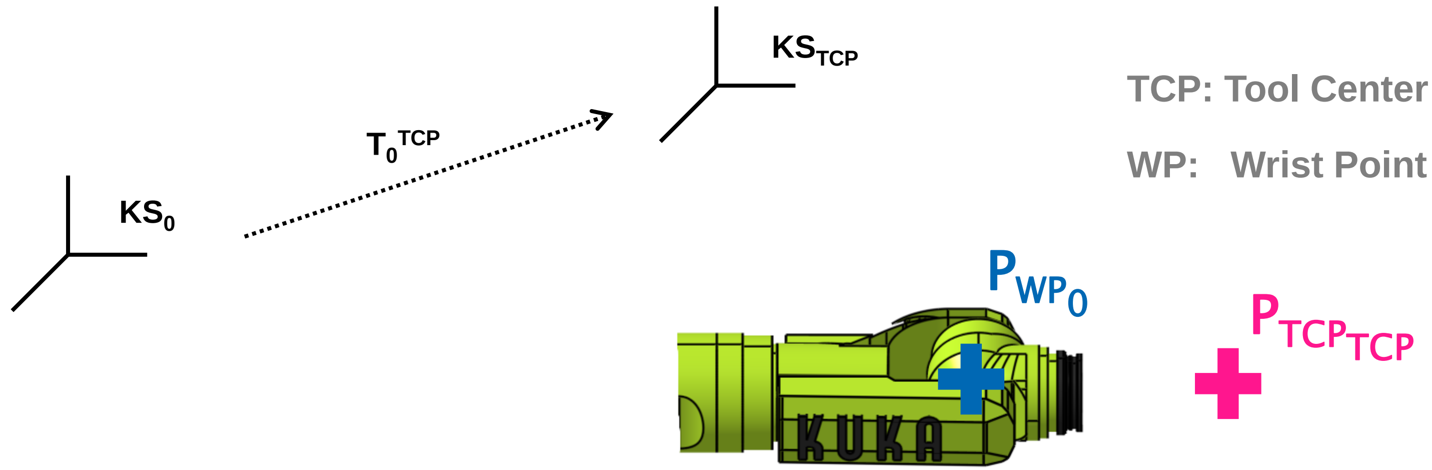
OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

3.1 P_{WP}: Calculation of Wrist Point

Input: T_0^{TCP} (transformation of KS_0 to KS_{TCP})

TCP: Tool Center Point

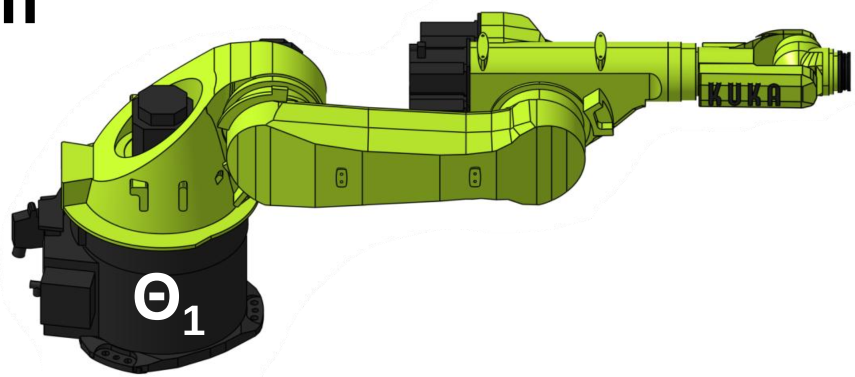
WP: Wrist Point



$P_{WP_{TCP}}$ = (0, 0, $-d_6$) … (depends on how KS_{TCP} looks like)

P_{WP_0} = $T_0^{TCP} \cdot (0, 0, -d_6)$

OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK



OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

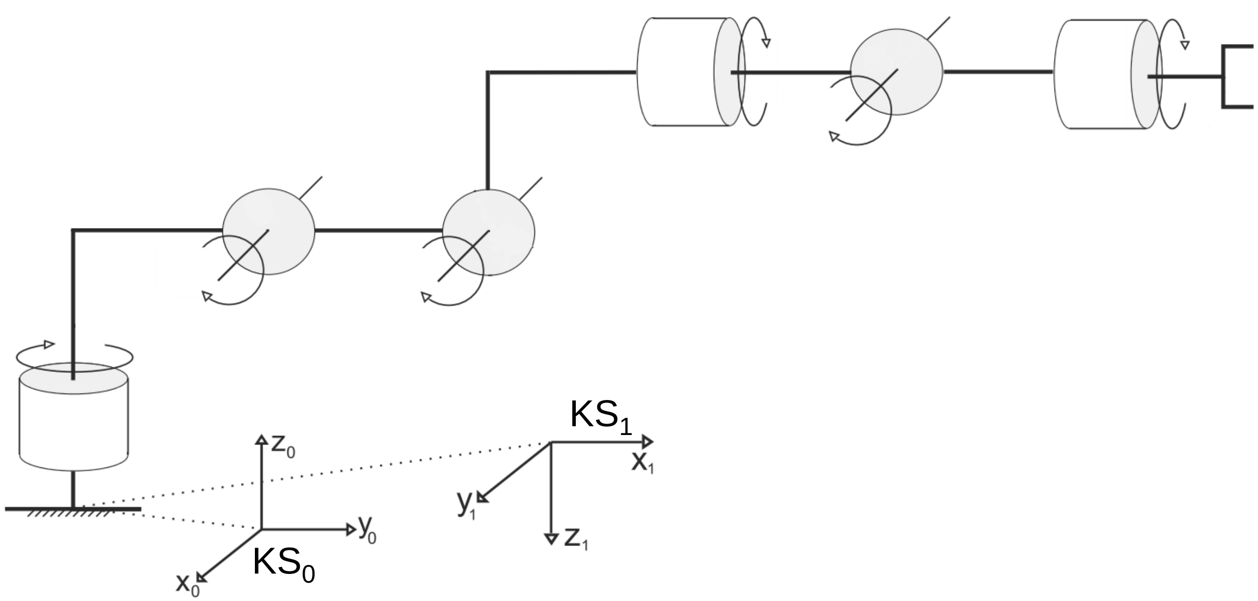
3.2 Θ_1: Transformation of P_{WP_0} to KS_1

For Calculation of Θ_1 in KS_1 $\rightarrow$ Transformation of P_{WP_0} to P_{WP_1}

$$T_0^{\,1} = Rot_{z0}(\Theta_0) \cdot Trans_{z0}(d_0) \cdot Trans_{x1}(a_0) \cdot Rot_{x1}(\alpha_0)$$

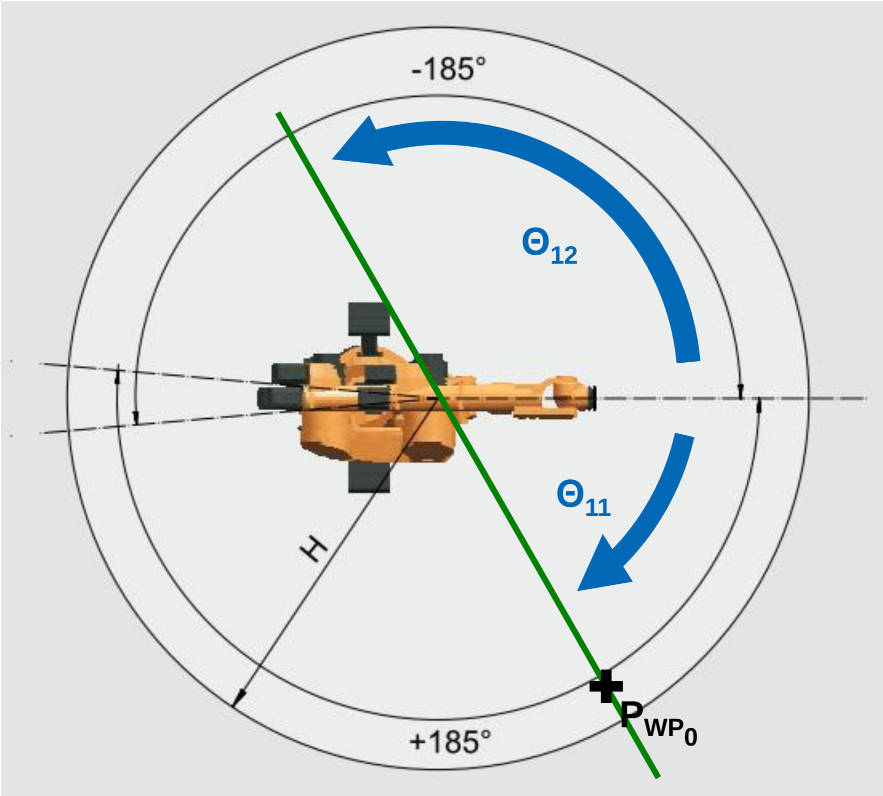
$$P_{WP_0} = T_0^{\,1} \cdot P_{WP_1} \quad \rightarrow \quad P_{WP_1} = (T_0^{\,1})^{-1} \cdot P_{WP_0}$$

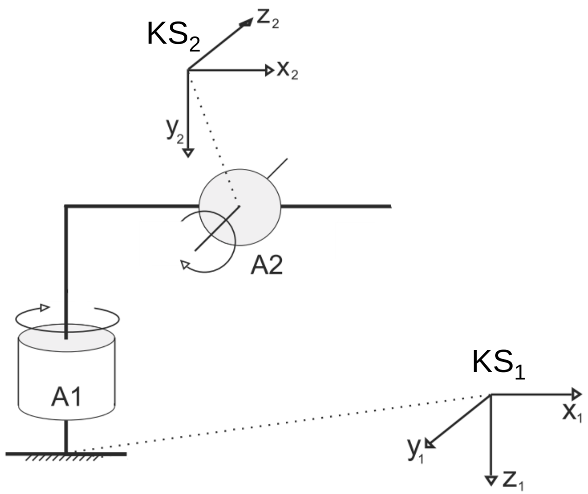
$\Theta_0 = \pi/2$
$d_0 = 0$
$a_0 = 0$
$\alpha_0 = \pi$



3.2 Θ$_1$: Calculation of Θ$_1$

Calculation of Θ$_1$ (top view) in KS$_1$

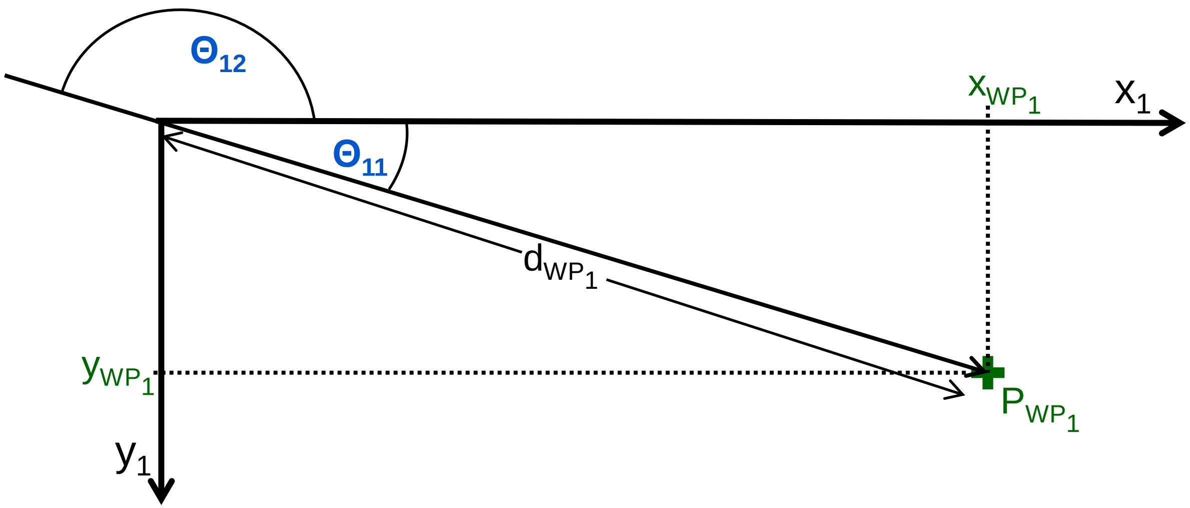




OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG | INF FAKULTÄT FÜR INFORMATIK

3.2 Θ_1: Calculation of Θ_1

Calculation of Θ_1 (topview) in KS_1



$$\tan(\Theta_{11}) = \frac{\sin(\Theta_{11})}{\cos(\Theta_{11})}$$

$$\sin(\Theta_{11}) = \frac{y_{WP_1}}{d_{WP_1}}$$

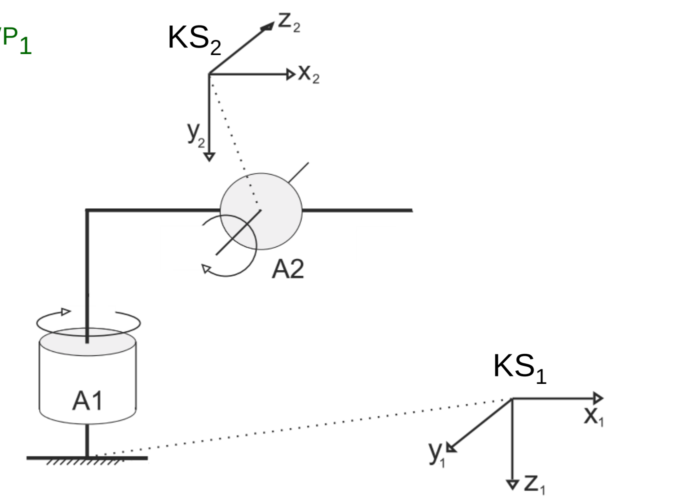
$$\cos(\Theta_{11}) = \frac{x_{WP_1}}{d_{WP_1}}$$

$$d_{WP_1} = \sqrt{x_{WP_1}{}^2 + y_{WP_1}{}^2}$$

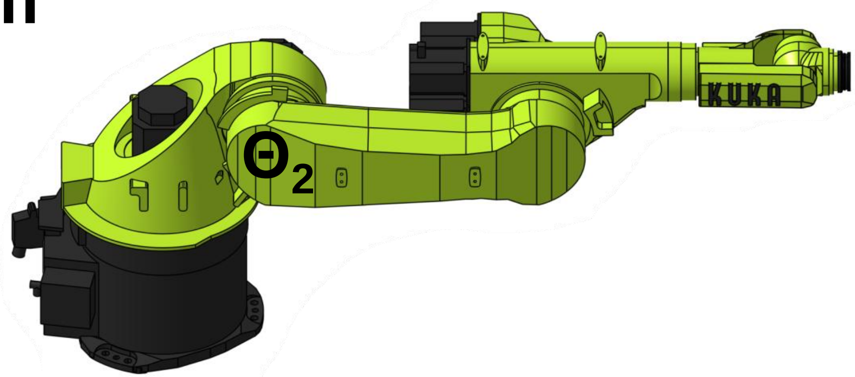
$$\Theta_{11} = \operatorname{atan2}(x_{WP_1}, y_{WP_1})$$

if $(\Theta_{11} < 0)$ then $\Theta_{12} = \Theta_{11} + \pi$

else $\Theta_{12} = \Theta_{11} - \pi$



OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG
INF FAKULTÄT FÜR INFORMATIK



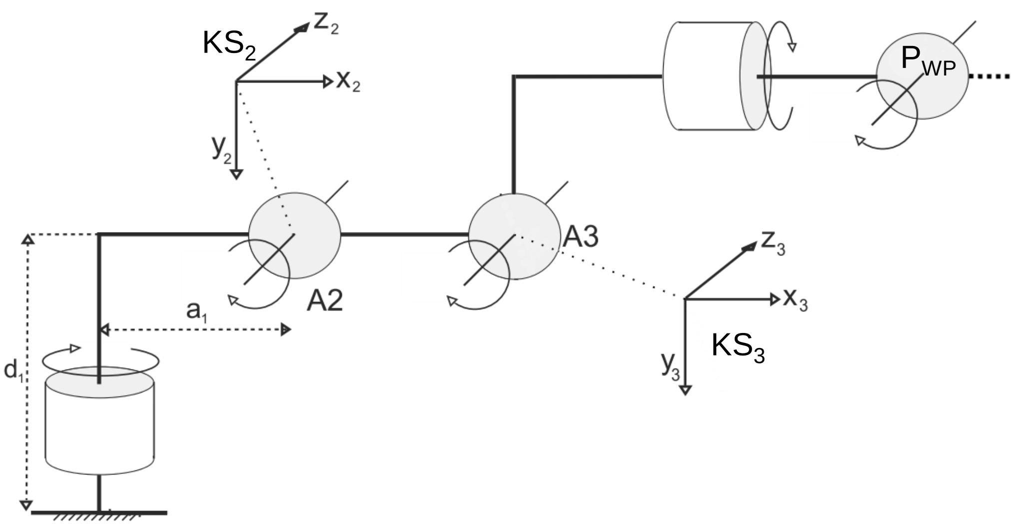
3.3 Θ_2: Distinction of cases for Θ_1

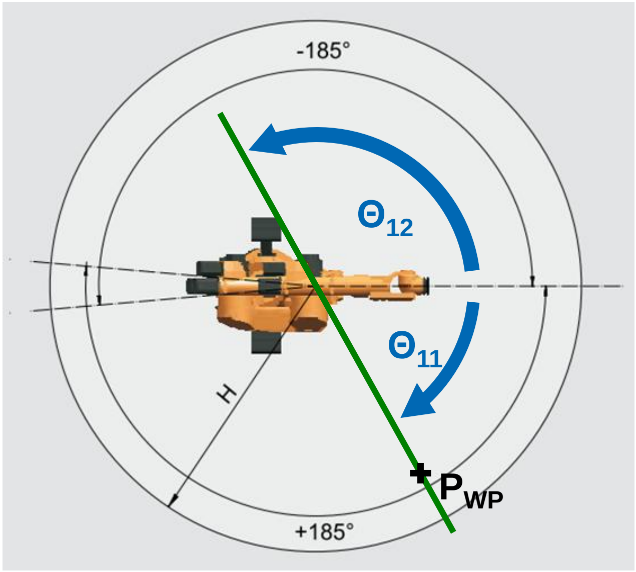
For Calculation of Θ_2 in KS_2 → Transformation of P_{WP_1} to P_{WP_2}

$$\mathbf{T_1}^2 = Rot_{z_1}(\Theta_1) \cdot Trans_{z_1}(d_1) \cdot Trans_{x_2}(a_1) \cdot Rot_{x_2}(\alpha_1)$$

$$\mathbf{P_{WP_1}} = T_1^{\,2} \cdot P_{WP_2} \quad \rightarrow \quad \mathbf{P_{WP_2}} = (T_1^{\,2})^{-1} \cdot P_{WP_1}$$

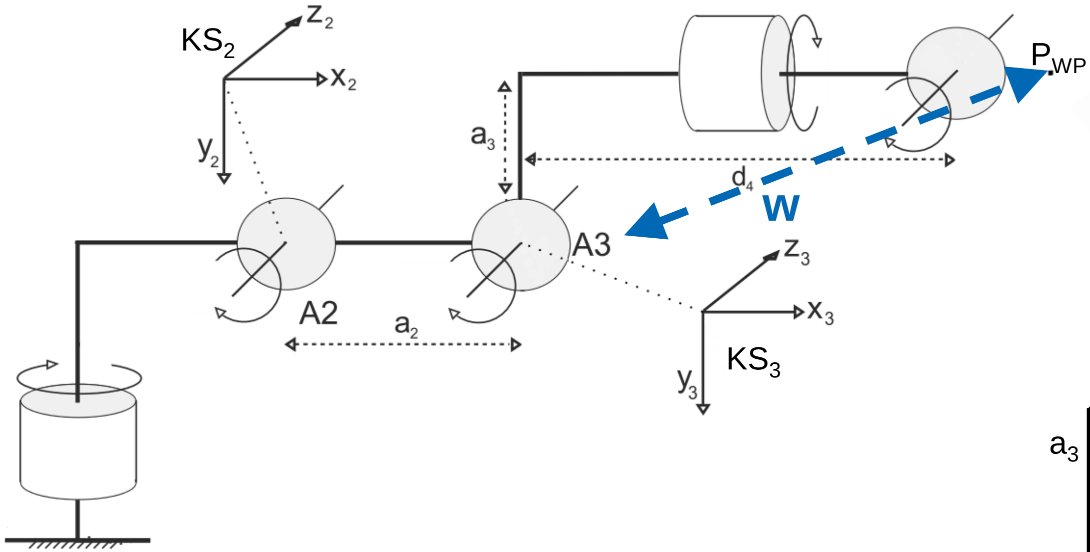
Distinction of cases for Θ_{11} and Θ_{12} → $\mathbf{P_{WP_{21}}}$ and $\mathbf{P_{WP_{22}}}$

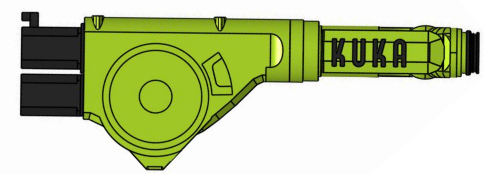


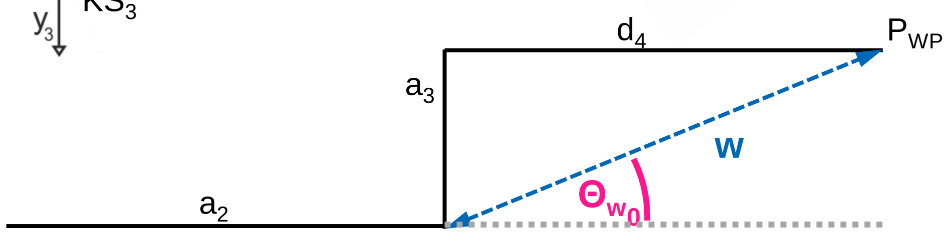


3.3 Θ_2: Simplification of the problem

For simplification: uniquely calculation of *w* and Θ_{w_0}







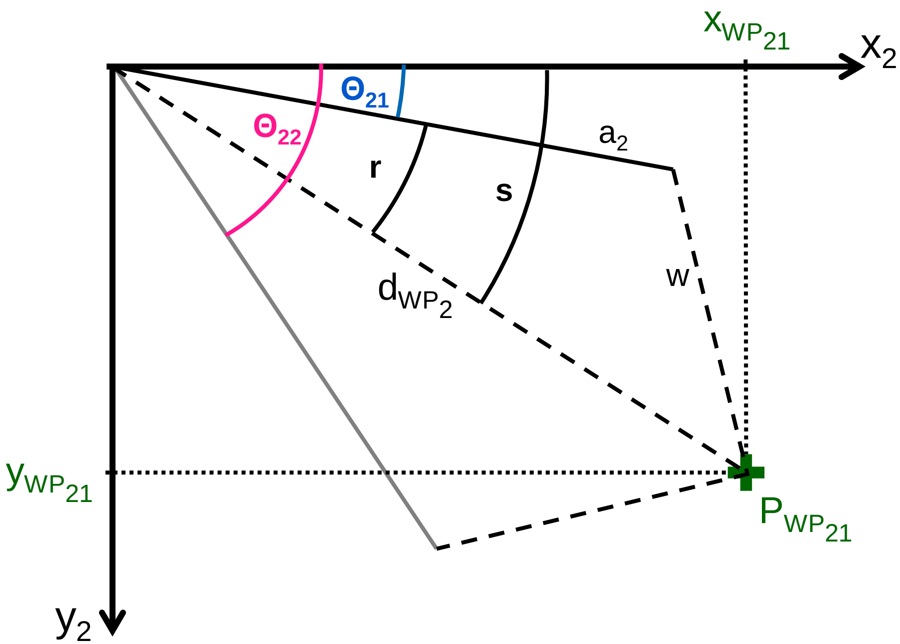
$\mathbf{w} = \sqrt{a_3^2 + d_4^2}$ → Calculation of Θ_2

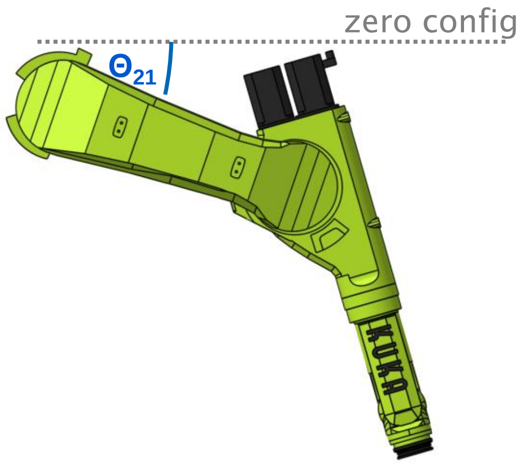
$\Theta_{w_0} = \operatorname{atan2}(d_4, a_3)$ → Calculation of Θ_3

OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG
INF FAKULTÄT FÜR INFORMATIK

3.3 Θ_2: Calculation of Θ_2

Calculation of Θ_{21} and Θ_{22} with $P_{WP_{21}}$ in x_2–y_2–plane (KS_2)

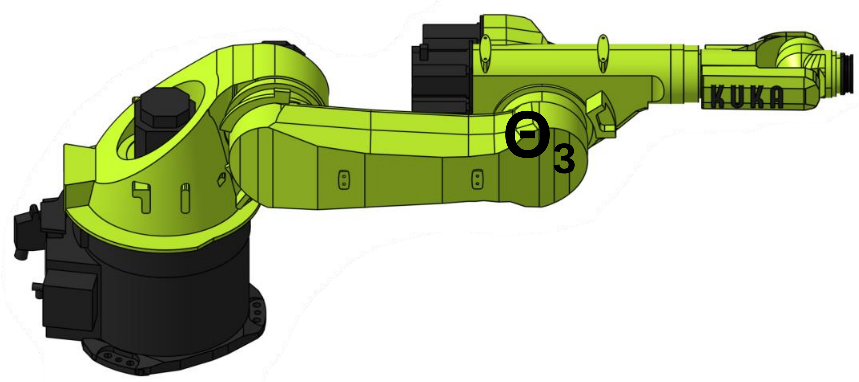




$\Theta_{21} = s - r$ $\Theta_{22} = s + r$ $s = \text{atan2}(x_{WP_{21}}, y_{WP_{21}})$

cosine rule → $w^2 = {d_{WP_{21}}}^2 + {a_2}^2 - 2 \cdot d_{WP_{21}} \cdot a_2 \cdot \cos(r)$

conversion → $r = \text{acos}\left(\frac{w^2 - {d_{WP_{21}}}^2 - {a_2}^2}{-2 \cdot d_{WP_{21}} \cdot a_2}\right)$


KUKA
Θ3

OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

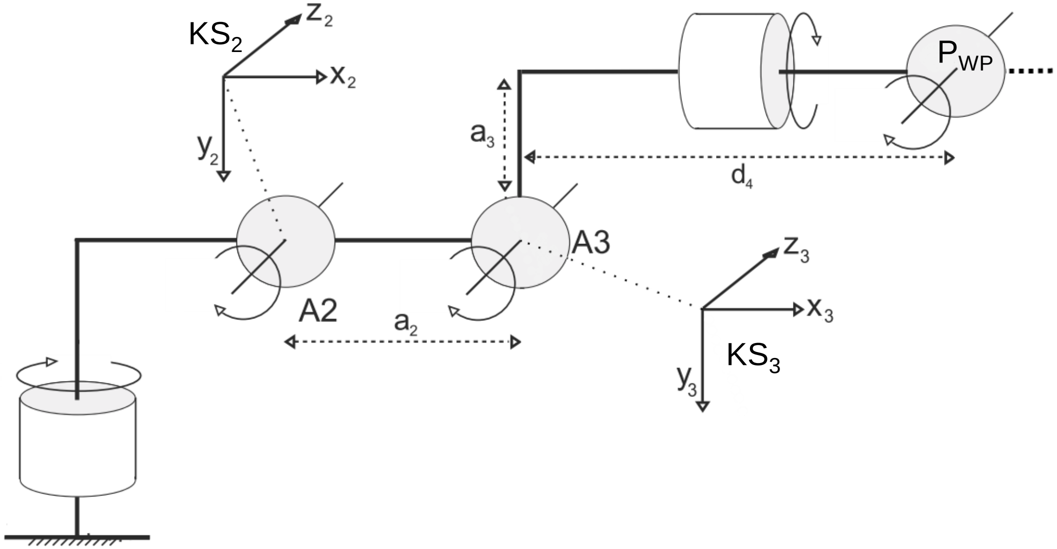
3.4 Θ_3: Distinction of cases for Θ_2

For Calculation of Θ_3 in KS_3 → Transformation of P_{WP_2} to P_{WP_3}

$$\mathbf{T_2}^{\mathbf{3}} = Rot_{z_2}(\Theta_2) \cdot Trans_{z_2}(d_2) \cdot Trans_{x_3}(a_2) \cdot Rot_{x_3}(\alpha_2)$$

$$\mathbf{P_{WP_2}} = T_2{}^3 \cdot P_{WP_3} \quad \rightarrow \quad \mathbf{P_{WP_3}} = (T_2{}^3)^{-1} \cdot P_{WP_2}$$

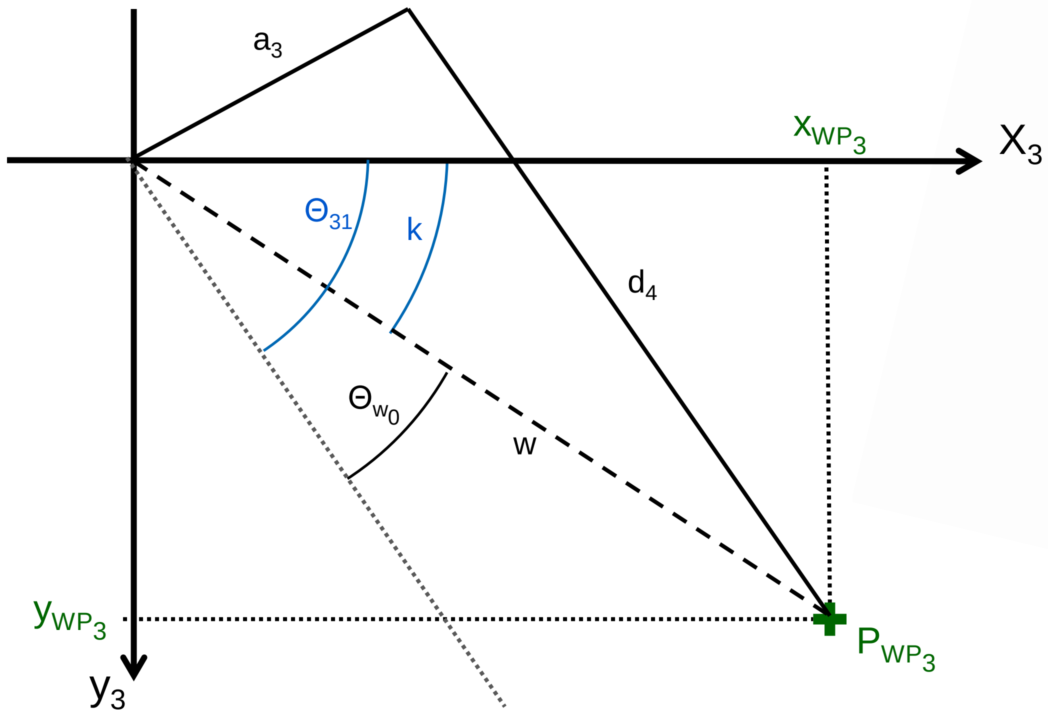
Distinction of cases for Θ_{21} and Θ_{22} → $\mathbf{P_{WP_{31}}}$ and $\mathbf{P_{WP_{32}}}$

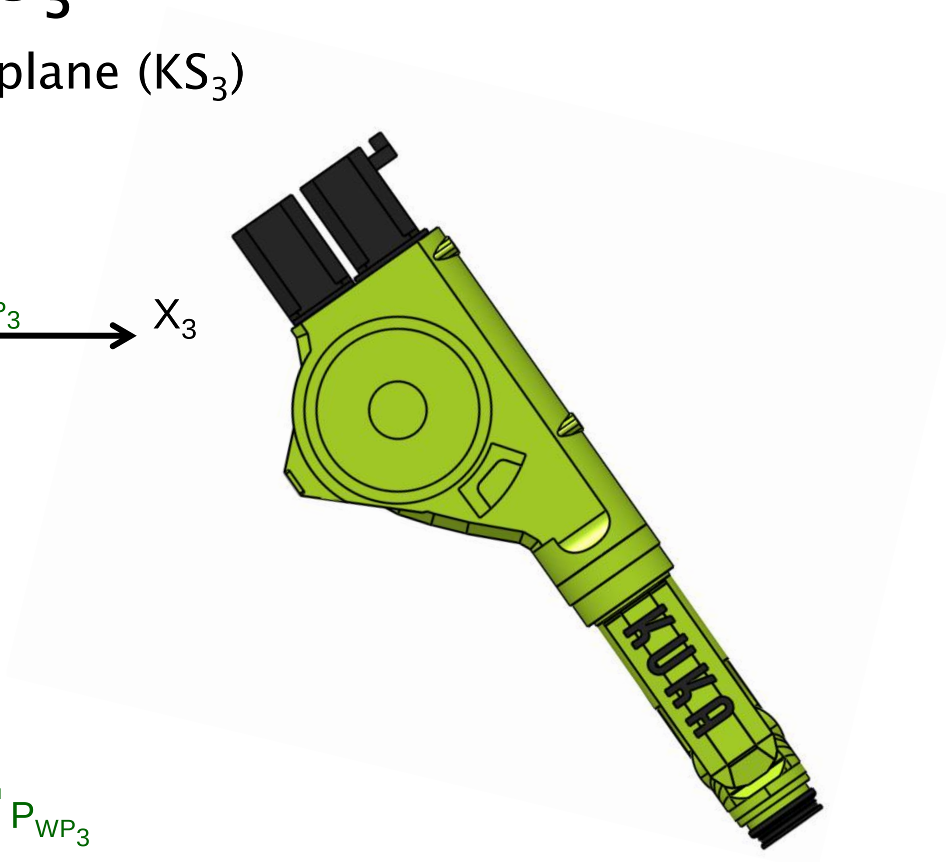


OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

3.4 Θ_3: Calculation of Θ_3

Calculation of Θ_3 with P_{WP_3} in x_3–y_3–plane (KS_3)





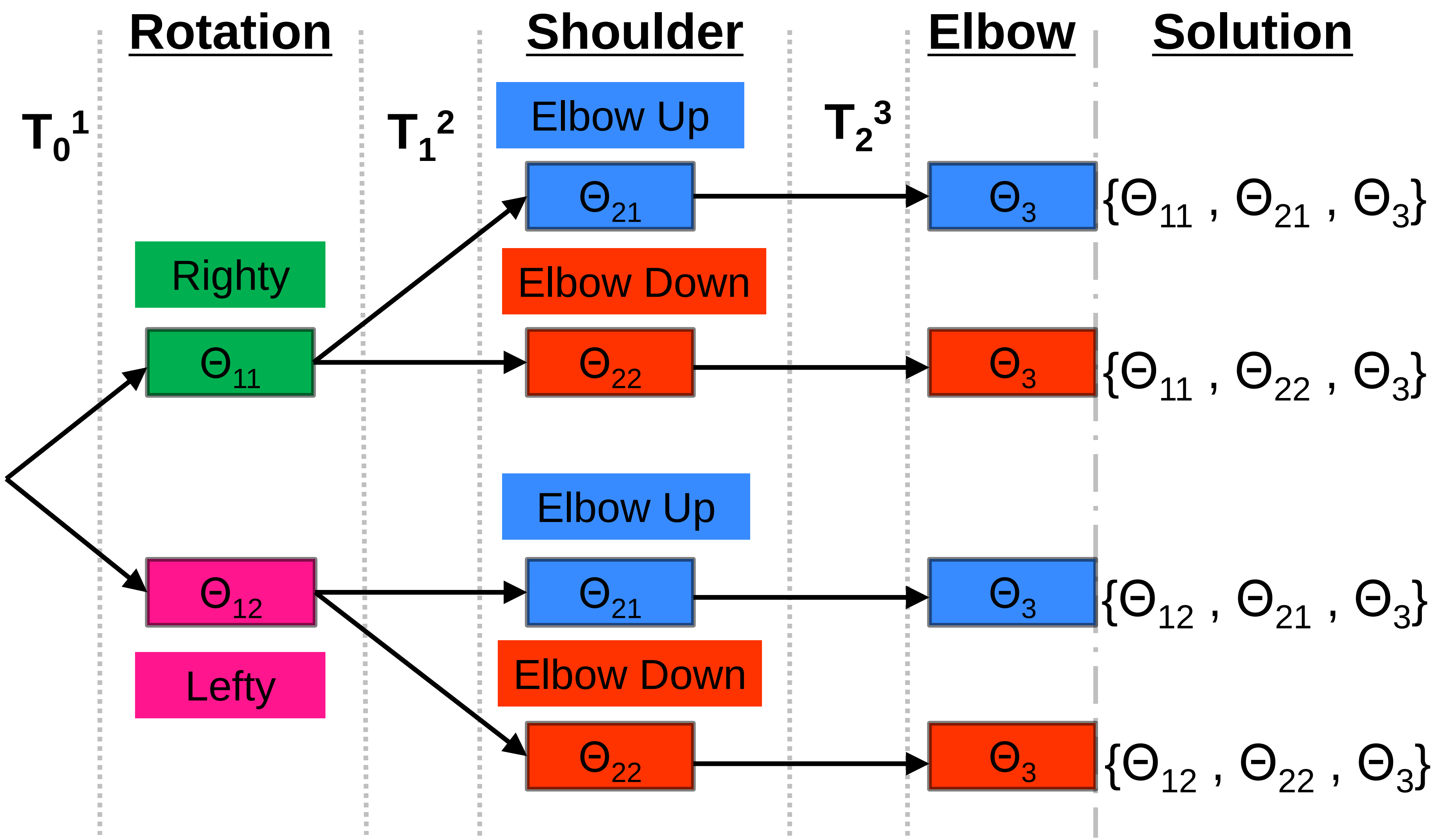
$$\Theta_3 = \Theta_{w_0} + k$$

$$k = \operatorname{atan2}(x_{WP_3}, y_{WP_3})$$

OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

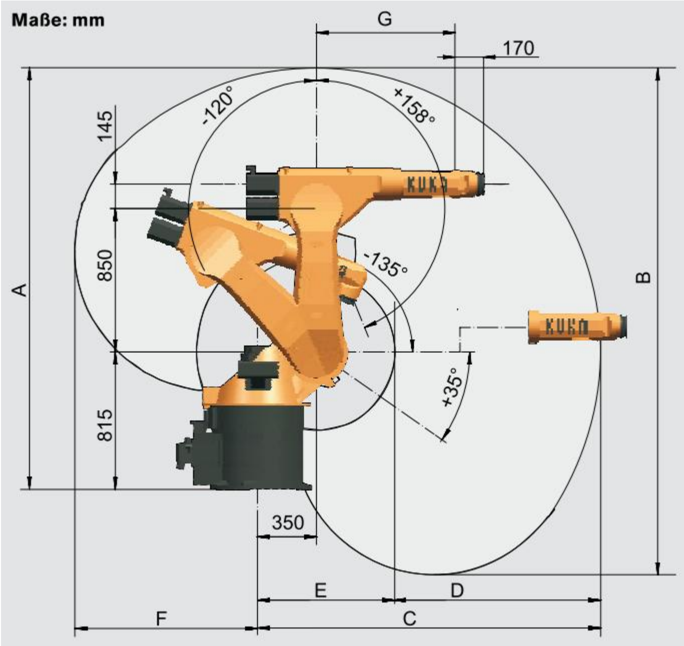
4. Conclusion: Overview of Solutions

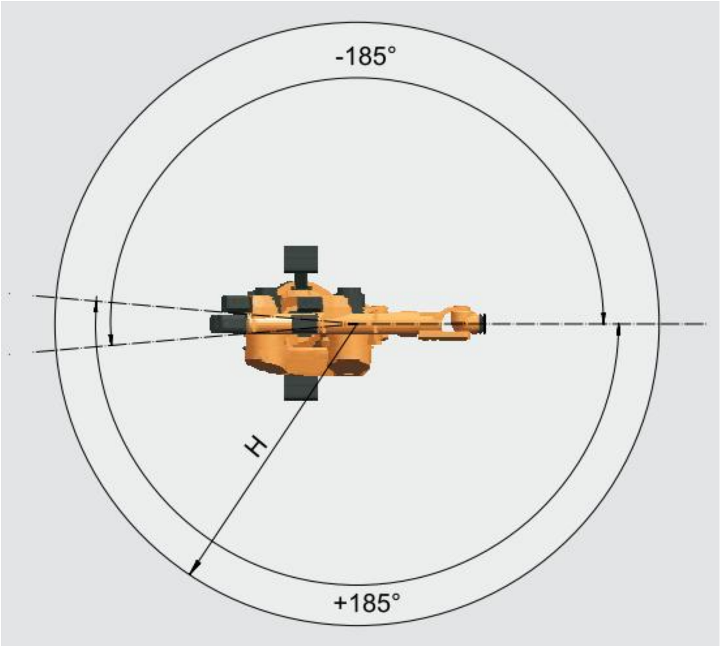


OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

4. Conclusion: Control of joint constraints

angle	min value	max value
Θ_1	-185°	+185°
Θ_2	-135°	+35°
Θ_3	-120°	+158°





OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG | INF FAKULTÄT FÜR INFORMATIK

4. Formula summery

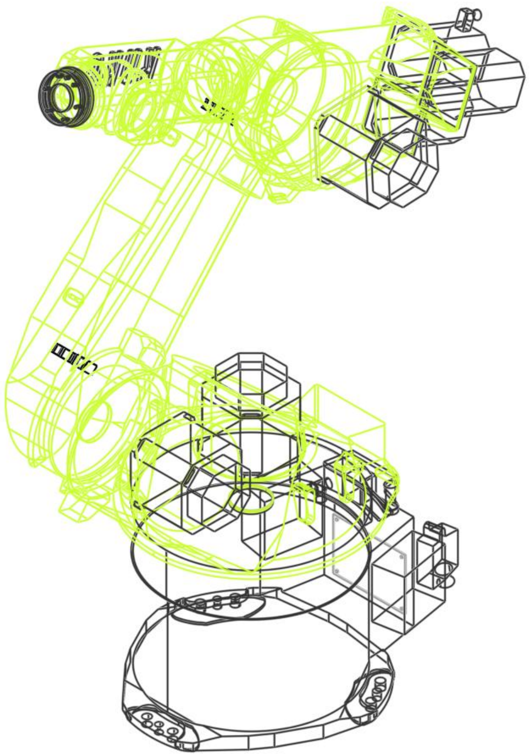
$P_{WP_0} = T_0^{TCP} \cdot (0, 0, -d_6)$	Calculation of wrist point (KS_0)
$P_{WP_1} = (T_0^{1})^{-1} \cdot P_{WP_0}$	Transformation of wrist point to KS_1
$\Theta_{11} = \operatorname{atan2}(x_{WP_1}, y_{WP_1})$ $\rightarrow \Theta_{12} = \Theta_{11} +/- \pi$	Calculation of Θ_1 in KS_1
$P_{WP_2} = (T_1^{2})^{-1} \cdot P_{WP_1}$	Transformation of P_{WP_1} with Θ_1 to KS_2
$\Theta_2 = \operatorname{atan2}(x_{WP_{21}}, y_{WP_{21}})$ $+/- \operatorname{acos}\left(\frac{w^2 - d_{WP_{21}}{}^2 - a_2{}^2}{-2 \cdot d_{WP_{21}} \cdot a_2}\right)$	Calculation of Θ_2 in KS_2
$P_{WP_3} = (T_2^{3})^{-1} \cdot P_{WP_2}$	Transformation of P_{WP_2} with Θ_2 to KS_3
$\Theta_3 = \operatorname{atan2}(d_4, a_3)$ $+ \operatorname{atan2}(x_{WP_3}, y_{WP_3})$	Calculation of Θ_3 in KS_3

Many thanks for your attention!

www.ovgu.de

OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG

INF FAKULTÄT FÜR INFORMATIK



OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

5. Appendix: DH parameters

$$T_i^{i+1} = Rot_{z_i}(\Theta_i) \cdot Trans_{z_i}(d_i) \cdot Trans_{x_{i+1}}(a_i) \cdot Rot_{x_{i+1}}(\alpha_i)$$

i	Θ	d	a	α
0	$\pi/2$	0	0	π
1	Θ_1	$-d_1$	a_1	$\pi/2$
2	Θ_2	0	a_2	0
3	$\pi/2 + \Theta_3$	0	a_3	$-\pi/2$
4	Θ_4	$-d_4$	0	$\pi/2$

Compute $\Theta_{1,}$ $\Theta_{2,}$ and Θ_3

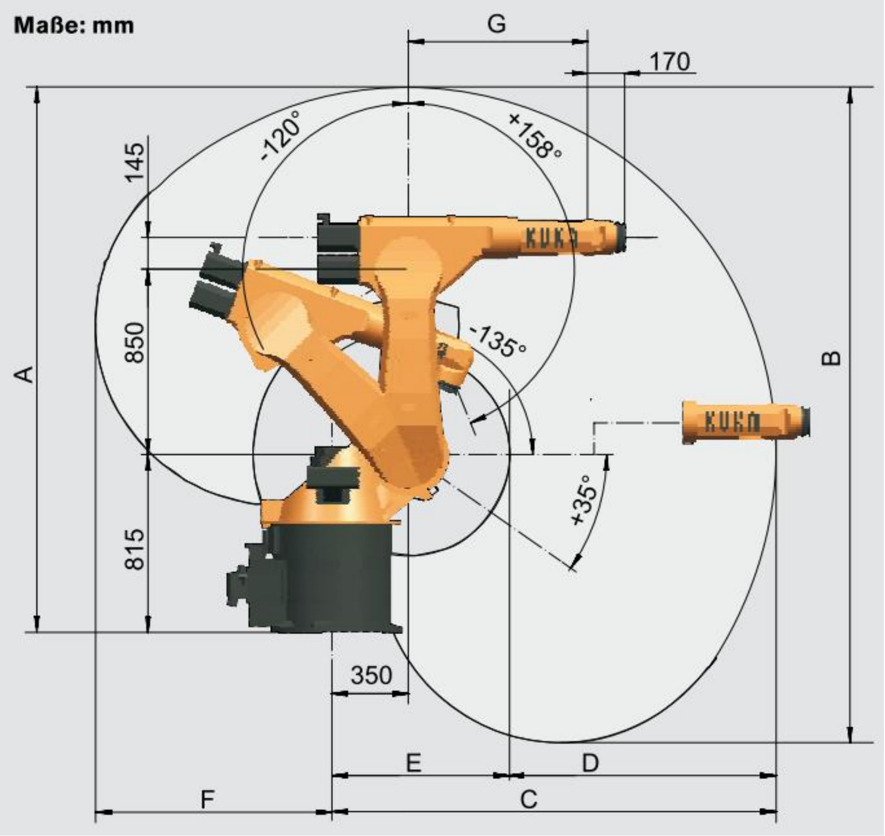
Θ_4 is not relevant for positioning problem

OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

5. Appendix: Dimensions of KR30-3

Constants	Value in mm
d_1	815
a_1	350
a_2	850
a_3	145
d_4	820
d_6	170

Dimension	Value in mm
A	2.498
B	3.003
C	2.033
D	1.218
E	815
F	1.084
G	820



OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

5. Appendix: Why we use *atan2(x, y)*?

- Two arguments: Information on the signs of inputs
  → Distinction between opposite directions in KS
- Programming: No need to take care of KS quadrants
  → less programm logic
- atan2(y, x) is the angle in radians between the positive x-axis of a plane and the point given by the coordinates (x, y)

$$\operatorname{atan2}(x,y)=\begin{cases}\arctan\left(\frac{y}{x}\right) & x>0\\ \arctan\left(\frac{y}{x}\right)+\pi & y\geq 0, x<0\\ \arctan\left(\frac{y}{x}\right)-\pi & y<0, x<0\\ +\frac{\pi}{2} & y>0, x=0\\ -\frac{\pi}{2} & y<0, x=0\\ \text{undefined} & y=0, x=0\end{cases}$$

y > 0: upper half-plane
y < 0: lower half-plane

Quelle: Wikipedia

OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

bibliography

1) Lee, Ziegler: *A geometric approach in solving the inverse kinematics of puma robots*. The University of Michigan, 1983.
2) Linnemann: *Robotortechnik* (Vorlesung). Beuth Hochschule für Technik Berlin, 2013.
3) Kuka Roboter GmbH: *KUKA KR30-3 Datenblatt*. Gefunden auf http://www.kuka-robotics.com/res/sps/f776ebab-f613-4818-9feb-527612db8dc4_PF0042_KR_30-3_KR_60-3_de.pdf. Abgerufen am 09.11.13.
4) Bjerkeng: *Coordinated Control with Obstacle Avoidance for Robot Manipulators - Chapter 3*. Norwegian University of Science and Technology, 2010.
5) Suchý: *Grundlagen der Robotik* (Vorlesung). Technische Universität Chemniz, 2010.