Computer Systems in Engineering

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SDIR

Standardized coordinate systems / layouts of robots

Intro KUKA KR 30-L16

- AC Servomotors
- Repeatability of 0.07mm
- Cyclical absolute positioning system
- Lifetime 10 – 15 years
- Installation positions
  - Base mounted
  - Roof mounted
  - Wall mounted

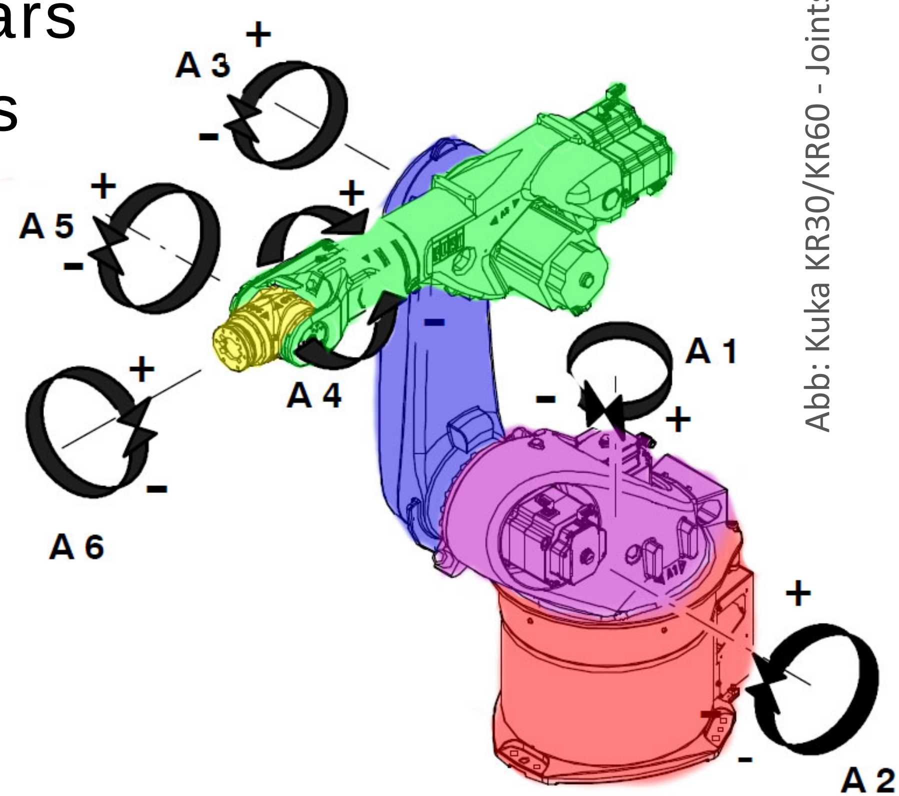


Abb: Kuka KR30/KR60 - Joints

Intro KUKA KR 30-L16

Workspace

KR 30 L16

Zentralhand, Nenn-Traglast 16 kg

Achse	Bewegungsbereich softwarebegrenzt	Geschwindigkeit
1	±185°	100 °/s
2	+35° bis -135°	80 °/s
3	+158° bis -120°	80 °/s
4	±350°	230 °/s
5	±130°	165 °/s
6	±350°	249 °/s

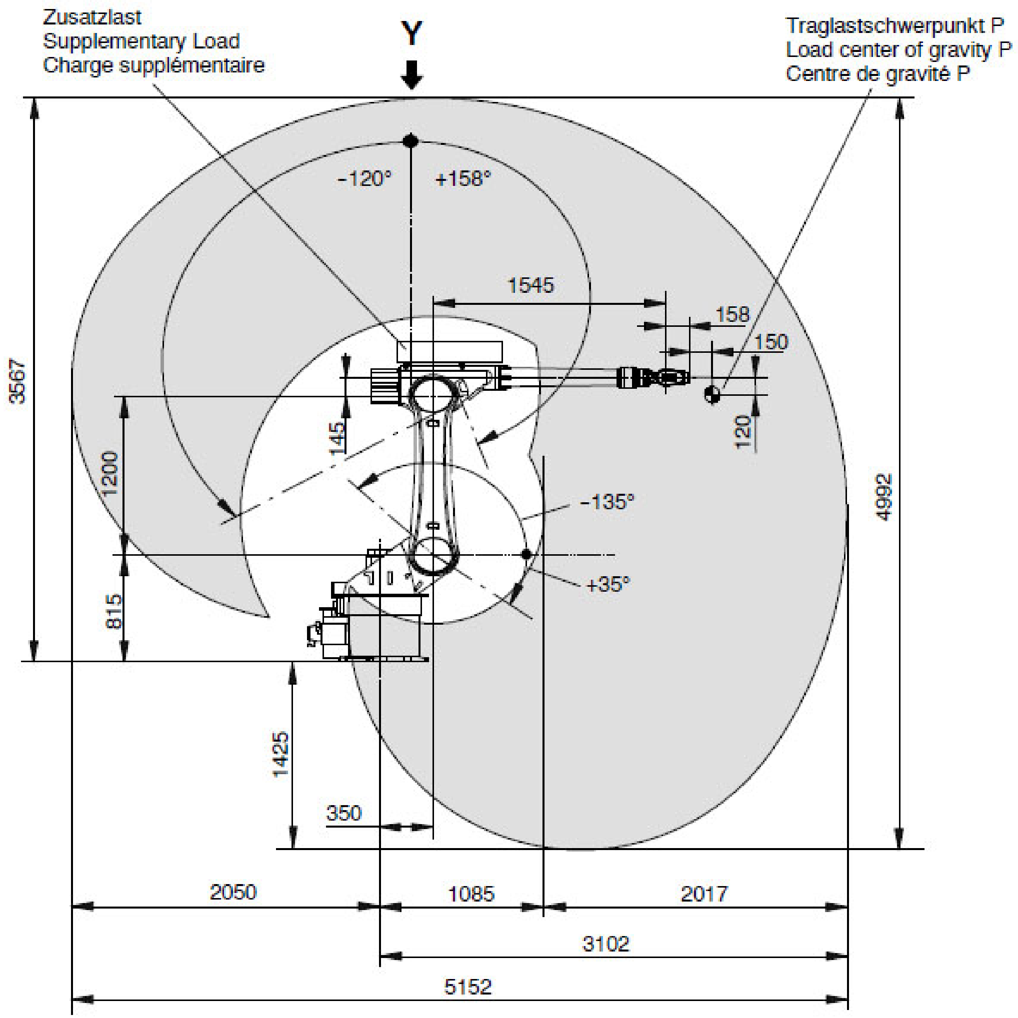
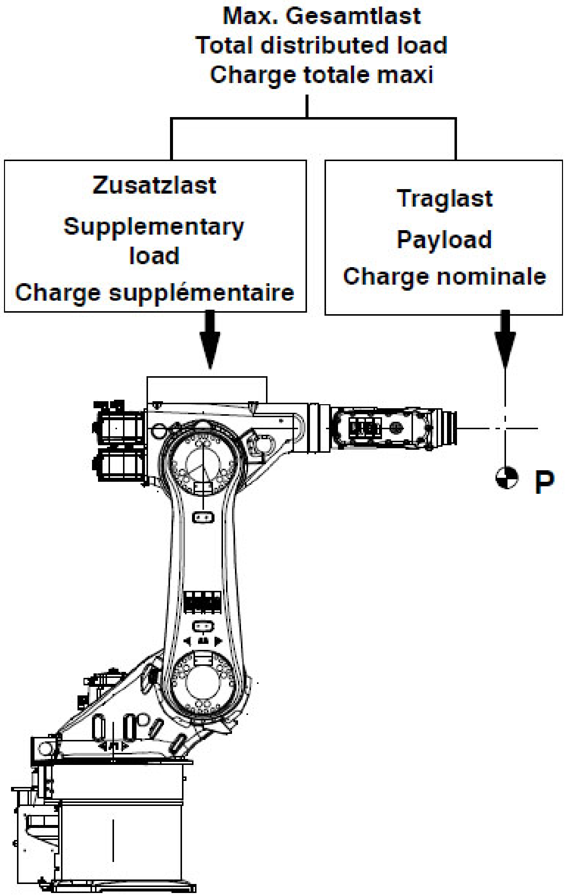


Abb: Kuka KR30 L16 - Workspace

Intro KUKA KR 30-L16

- Weight: 700kg
- Payload: 16kg
- Supplementary load: 35kg



Kinematic Chain

- Given the angles of every single rotary joint (in a kinematic chain)
- one can compute position and orientation of the ToolCenterPoint (TCP)
- by performing coordinate transformations for every joint consecutively (forward kinematics)

Kinematic Chain

- robotic arm abstracted as series of:
  - **single joints**
  - **rigid bodies**

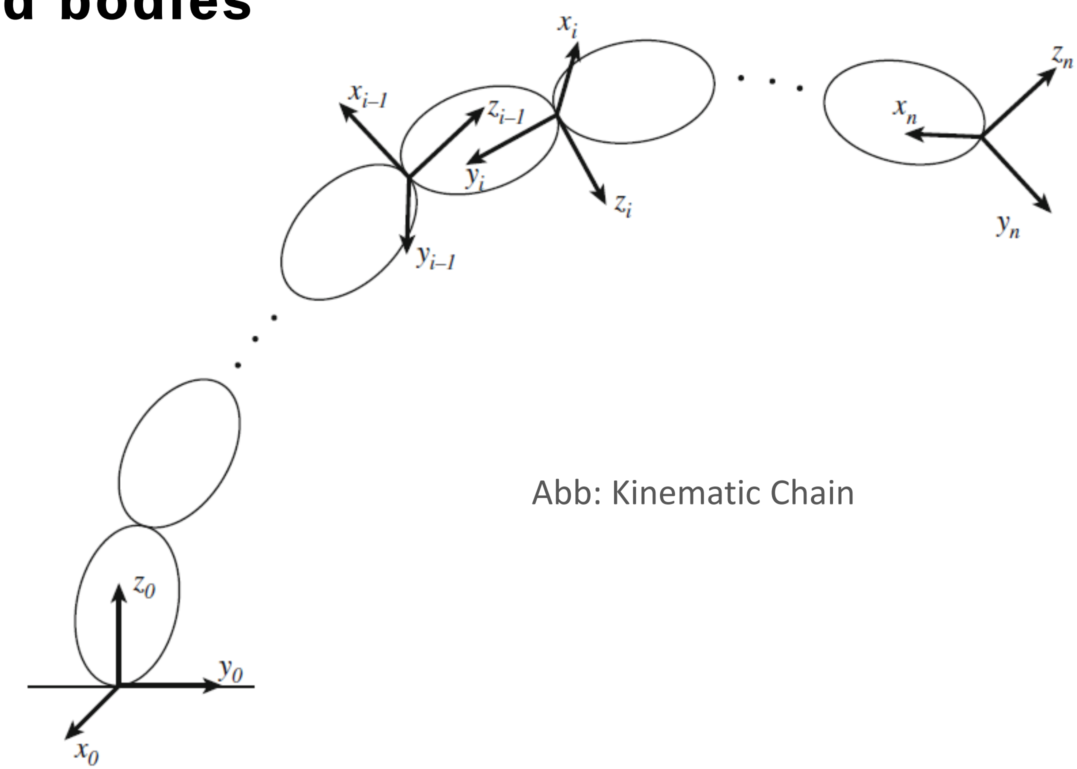


Abb: Kinematic Chain

What we want to explain!

- how to calculate the position of a robot arm with given joint angles
- a mathematical convention for **coordinate system transformation**

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Denavit Hartenberg Convention

Denavit-Hartenberg-Convention

- mathematical convention

- describes transformation of
  - a local coordinate system
  - within a kinematic chain

- with the help of 4 (system specific) parameters
  - **d**
  - **a**
  - **α**
  - **θ**

Some lexical conventions

Denavit-Hartenberg-Convention

- each transformation performed from the ***i-th*** to ***i+1-th*** coordinate system
- corresponding parameters get index ***i***
- Transformation Matrix is $\boldsymbol{T_i^{i+1}}$

The Convention

Denavit-Hartenberg-Convention

- Convention is based on how to place the specific coordinate systems in each axis

- Two basic constraints:
  - **DH1** x_1 is perpendicular to z_0
  - **DH2** x_1 intersects the z_0

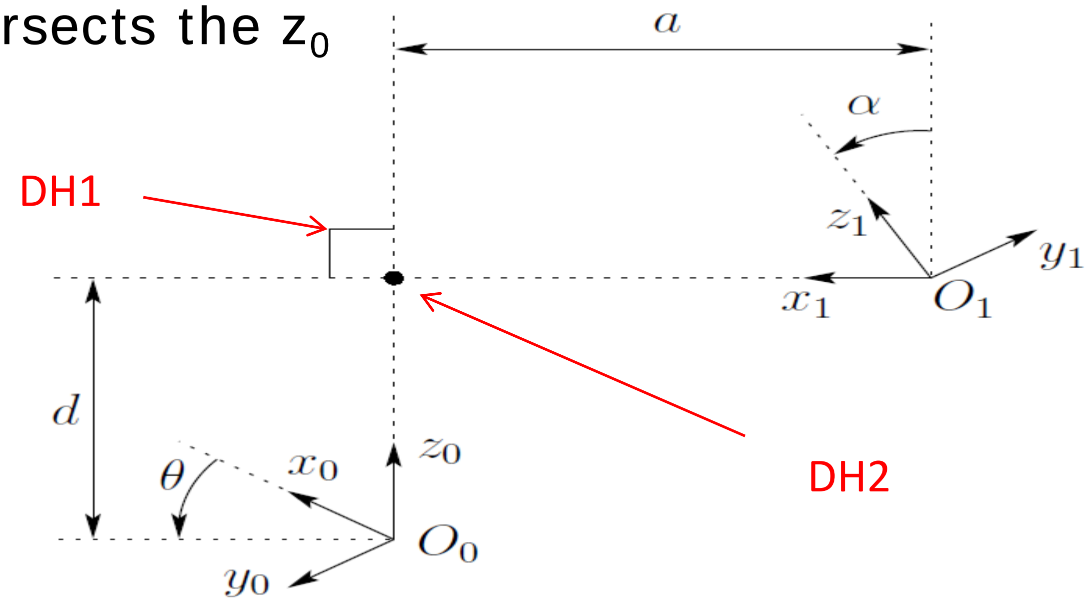

Abb: DH constraints

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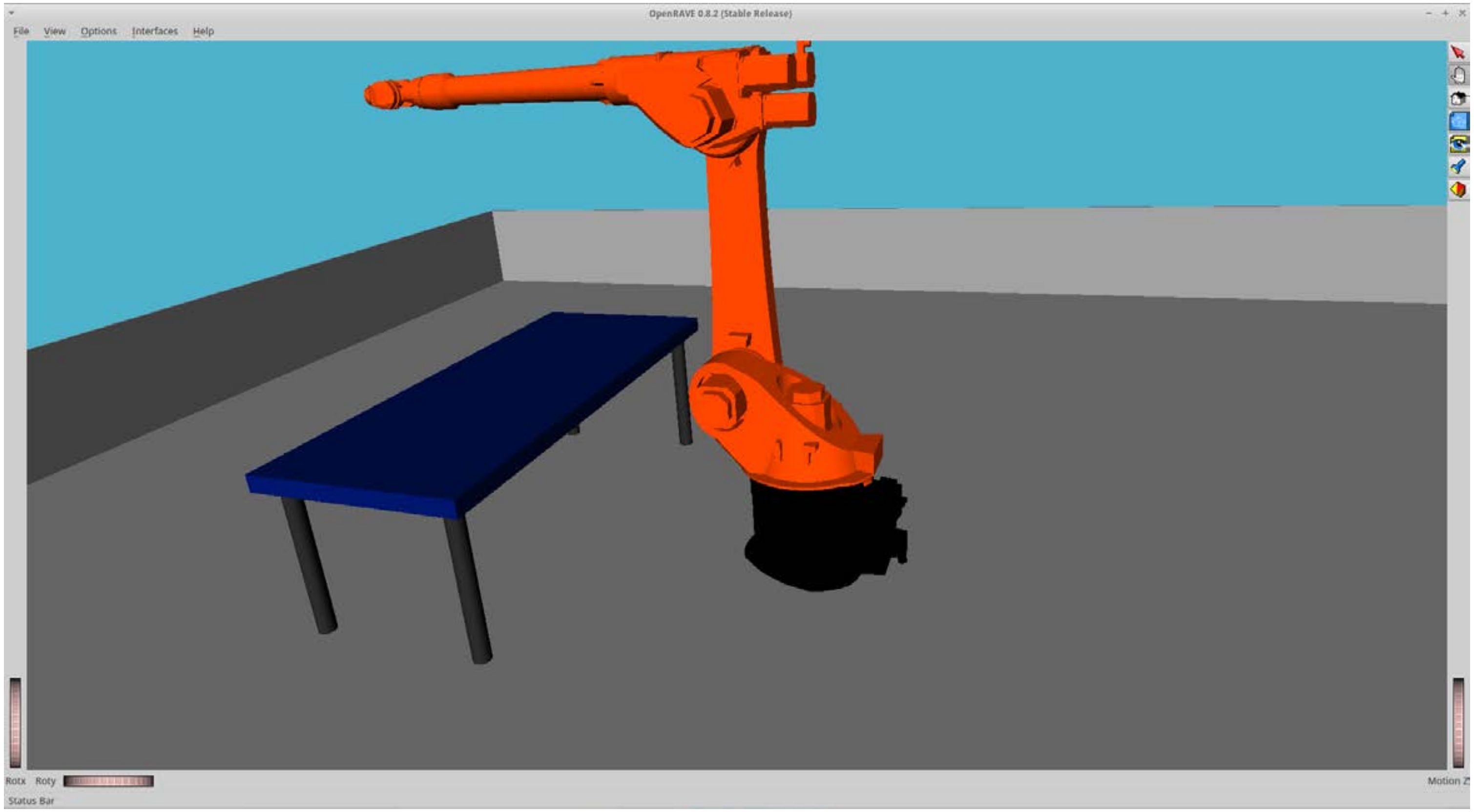
But how to fulfill these constraints?

First Rule: Z-axis

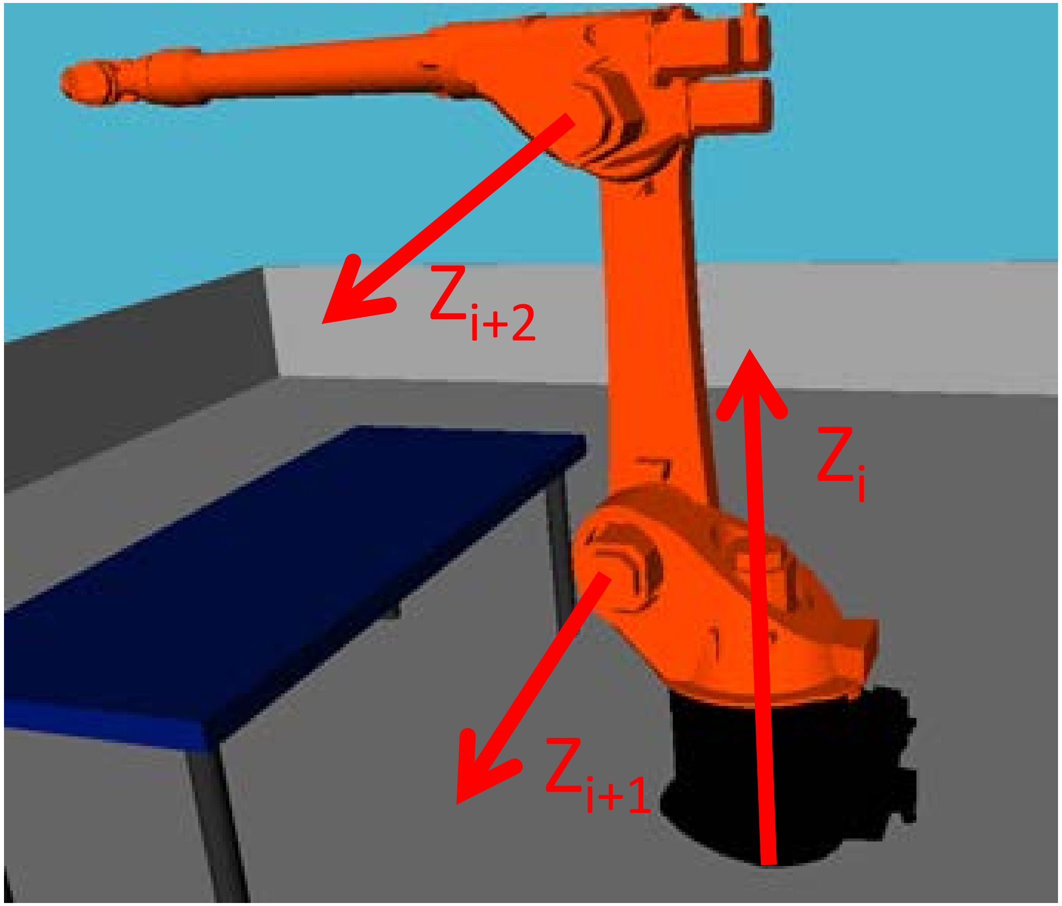
- Visualisation example Kuka KR30 L16 from the openRave VM



z-axis

First Rule: z-axis

- z-axis always placed in the rotation axis



y-axis

Third Rule: y-axis

- y-axis always creates a **right handed** coordinate system with z- and x-axis

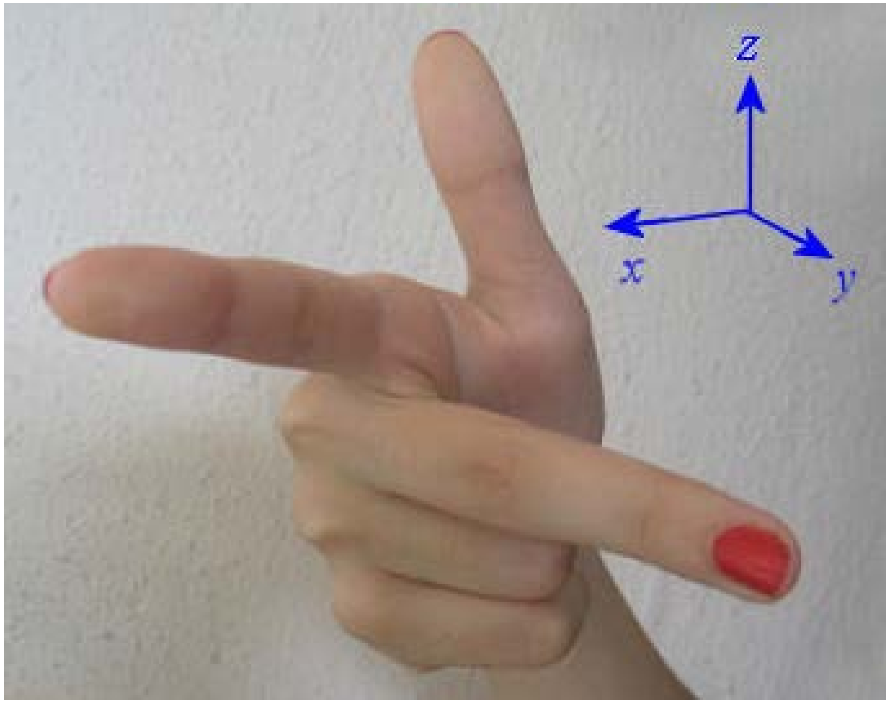


Abb: http://middletownhighschool.wikispaces.com

x-axis and
the origin of the coordinate system

X-axis

- 4 different constellations for z_i and z_{i+1}

1. base coordinate system
2. z_i and z_{i+1} are not coplanar
3. z_i and z_{i+1} parallel
4. z_i and z_{i+1} intersects

X-axis

- 4 different constellations for z_i and z_{i+1}

1. **base coordinate system**
2. z_i and z_{i+1} are not coplanar
3. z_i and z_{i+1} parallel
4. z_i and z_{i+1} intersects

x-axis: base coordinate system

z-axis:

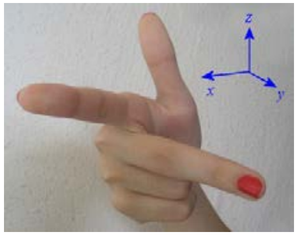
- rotation axis

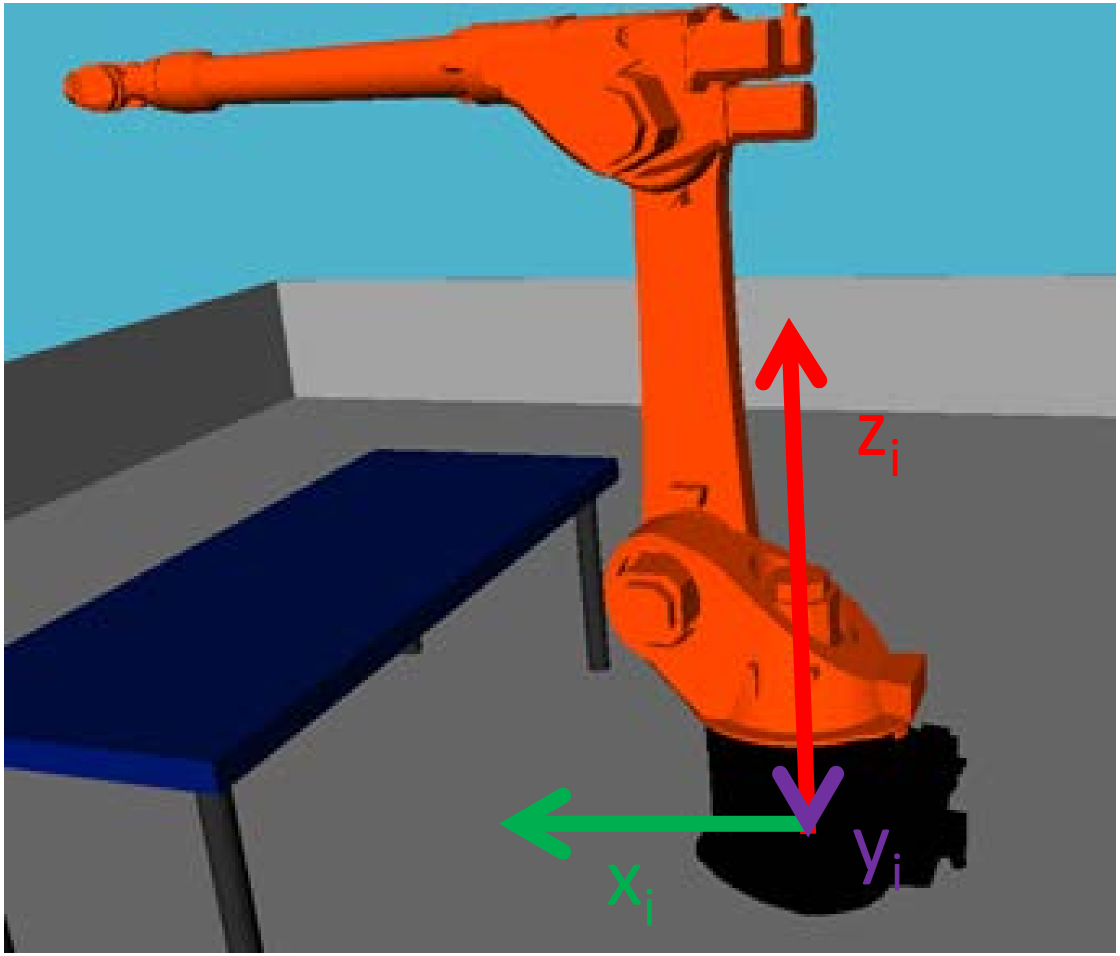
x-axis:

- free of choice

y-axis:

- right hand rule





X-axis

- 4 different constellations for z_i and z_{i+1}

1. base coordinate system
2. **z_i and z_{i+1} are not coplanar**
3. z_i and z_{i+1} parallel
4. z_i and z_{i+1} intersects

not coplanar z-axes

- 2 vectors are coplanar, if they are within the same plane

- not coplanar =>
  - not parallel
  - no intersection

Not coplanar z-axes

z-axis:

- rotation axis

x-axis:

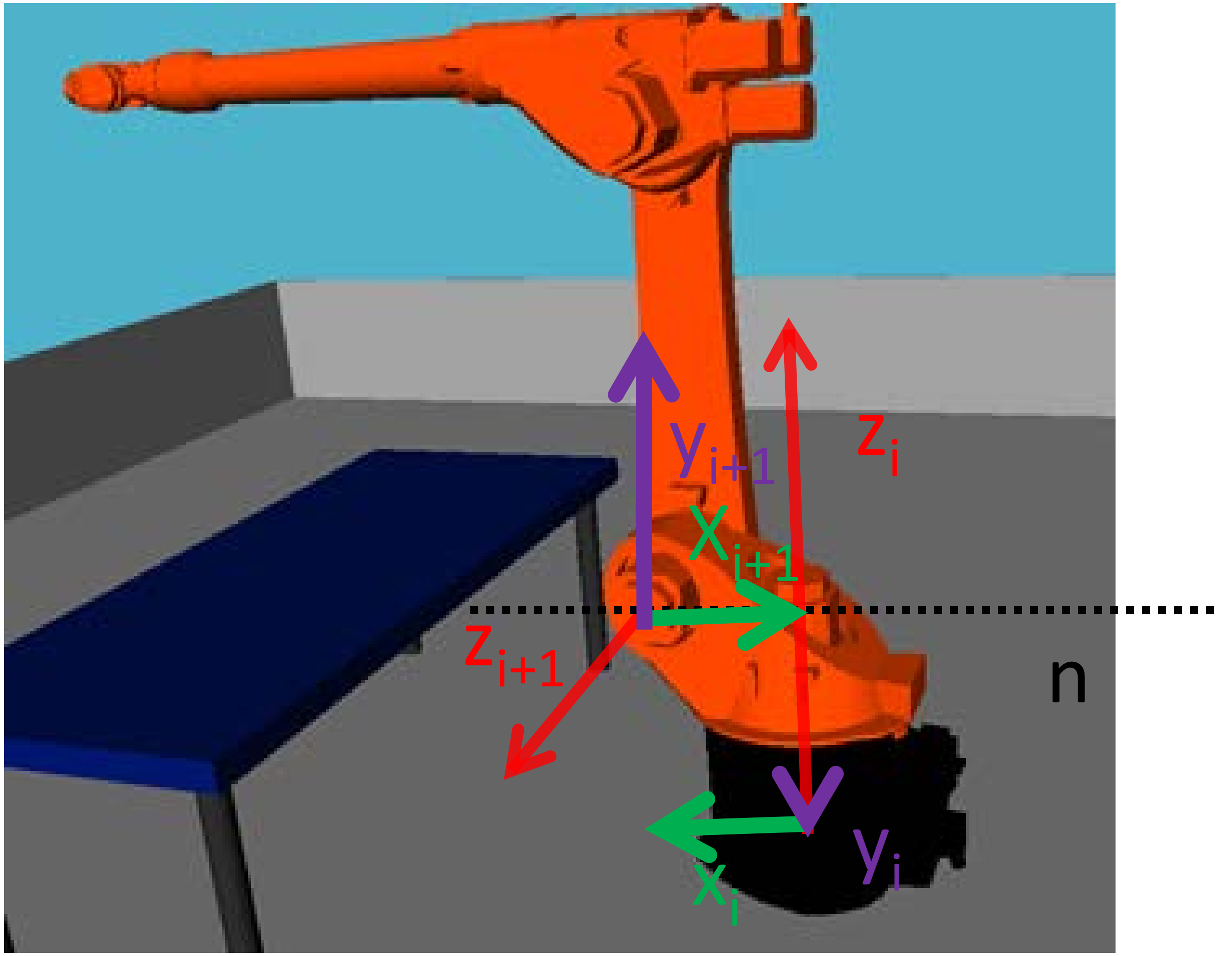
- normal to z-axes

y-axis:

- right hand rule

Origin:

- intersection xi+1 – zi+1



X-axis

- 4 different constellations for z_i and z_{i+1}

1. base coordinate system
2. z_i and z_{i+1} are not coplanar
3. **z_i and z_{i+1} parallel**
4. z_i and z_{i+1} intersects

parallel z-axes

z-axis:

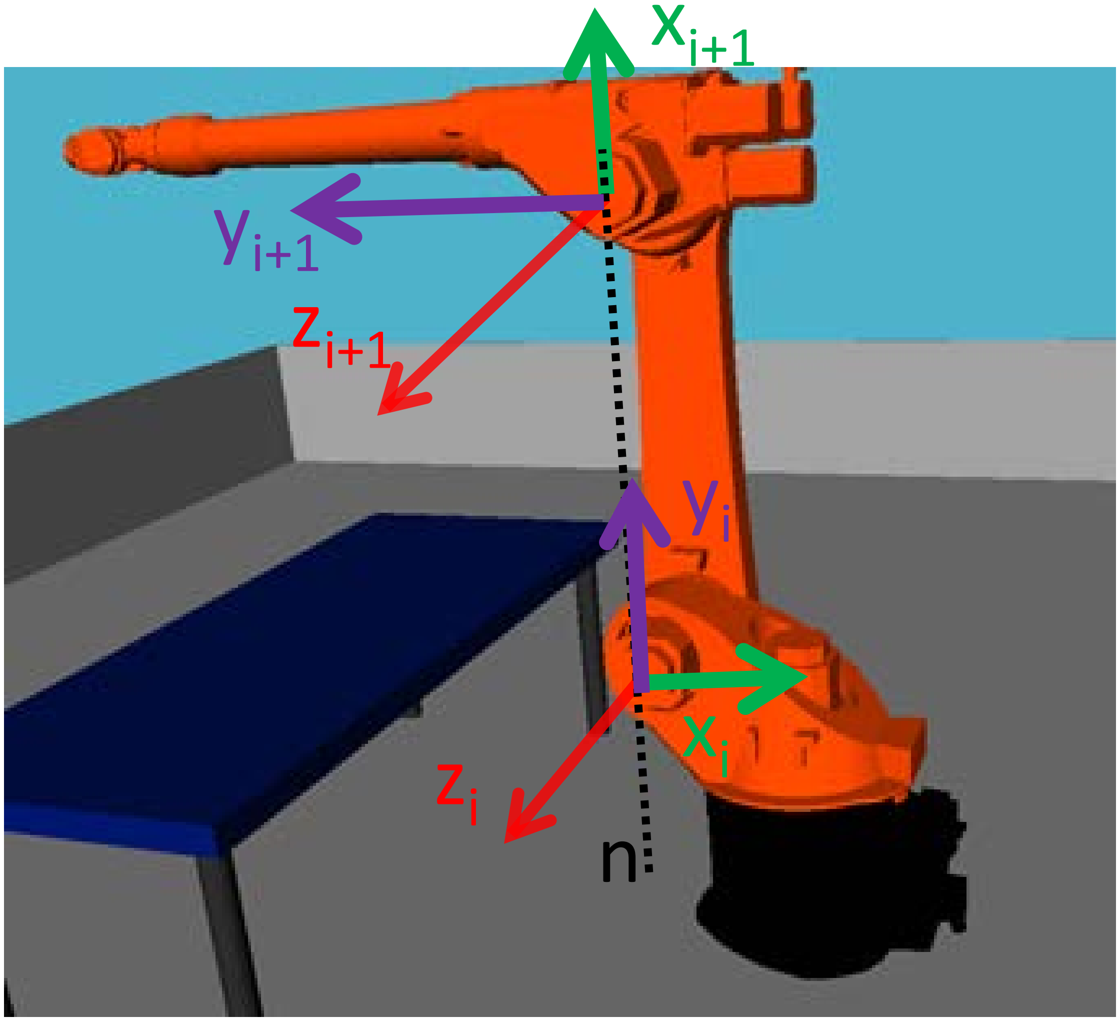
- rotation axis

x-axis:

- infinite normal
- Choose normal through origin of last cs
- points towards new joint

y-axis:

- right hand rule



X-axis

- 4 different constellations for z_i and z_{i+1}

1. base coordinate system
2. zi and zi+1 are not coplanar
3. zi and zi+1 parallel
4. **z_i and z_{i+1} intersects**

intersecting z-axes

- z-axis:
  rotation axis

- x-axis:
  - normal to the plane of z_i and z_{i+1}
  - direction of x-axis is arbitrary

- place origin in the intersection point of z_i and z_{i+1}

- y-axis:
  right hand rule

Deriving the parameters

DH-parameters

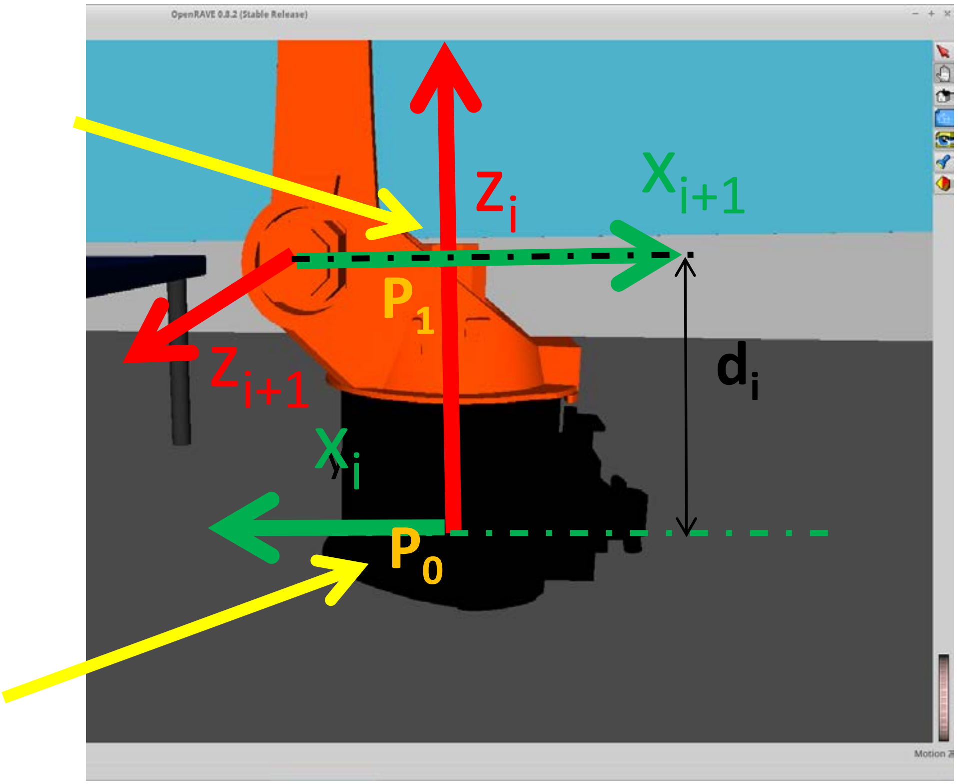
- We need the 4 DH-parameters
  - **d**
  - **a**
  - **α**
  - **θ**

- describe the relation between two successive coordinate systems

- used for transformation from coordinate system into successor

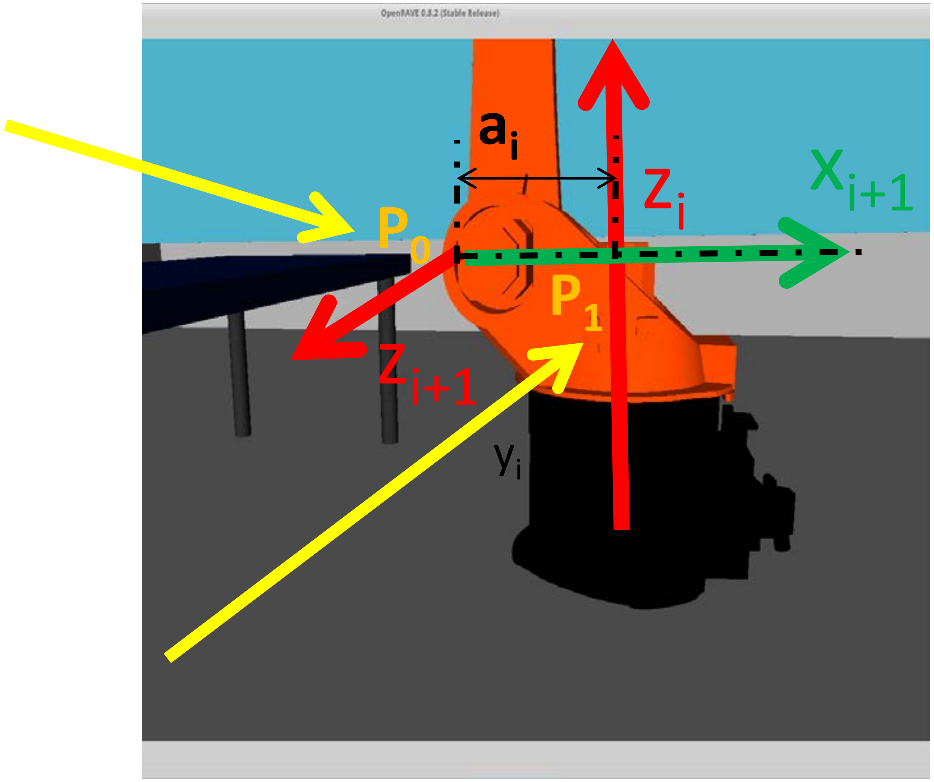
d-parameter

- $\mathbf{d_i}$ is the depth between the origin i (P_0) and the intersection of the z_i- and the x_{i+1}-axis (P_1)



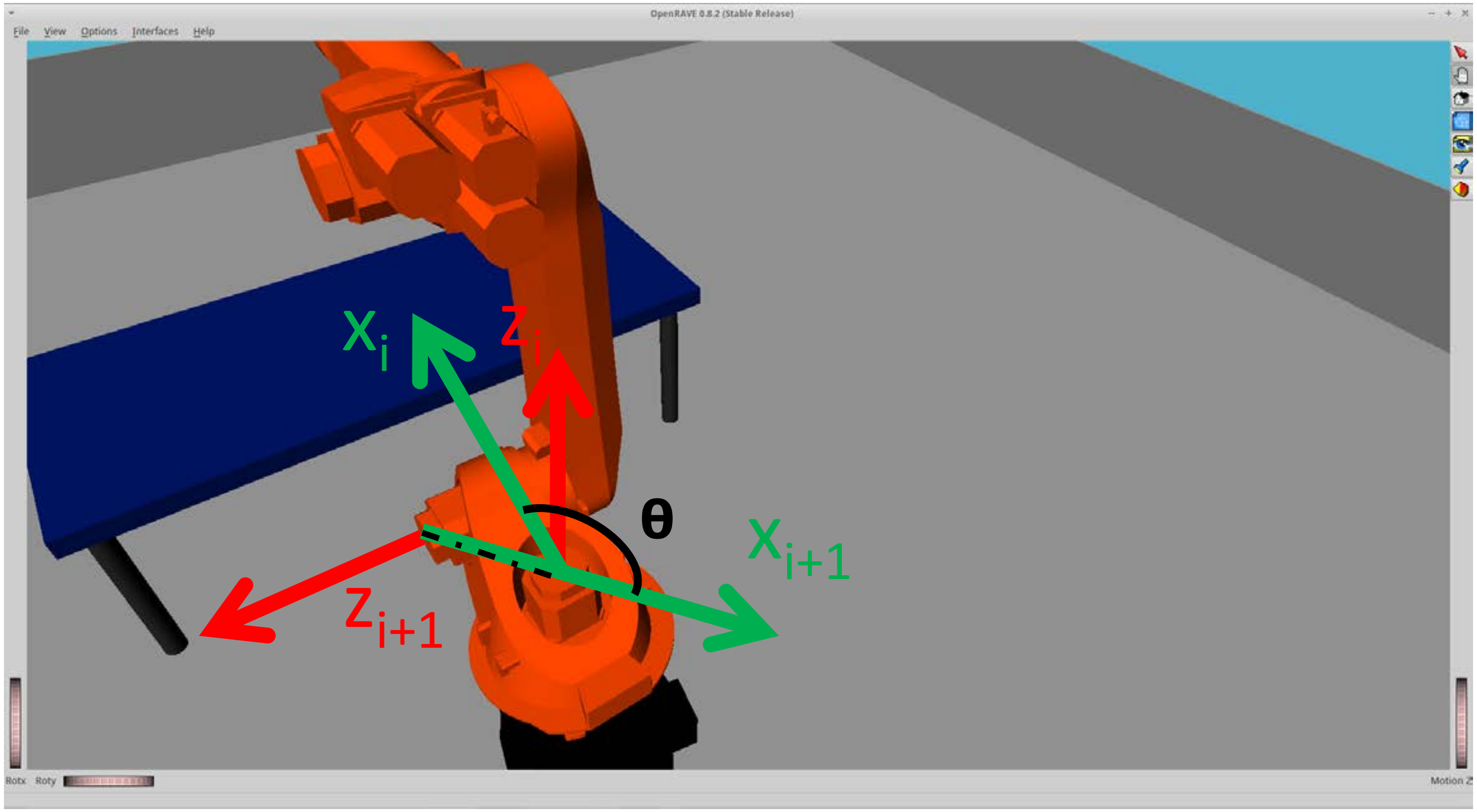
a-parameter

- **a_i** is the distance between the z-axes along the new x_{i+1} –axis
- distance origin *i+1* and intersection x_{i+1} and z_i



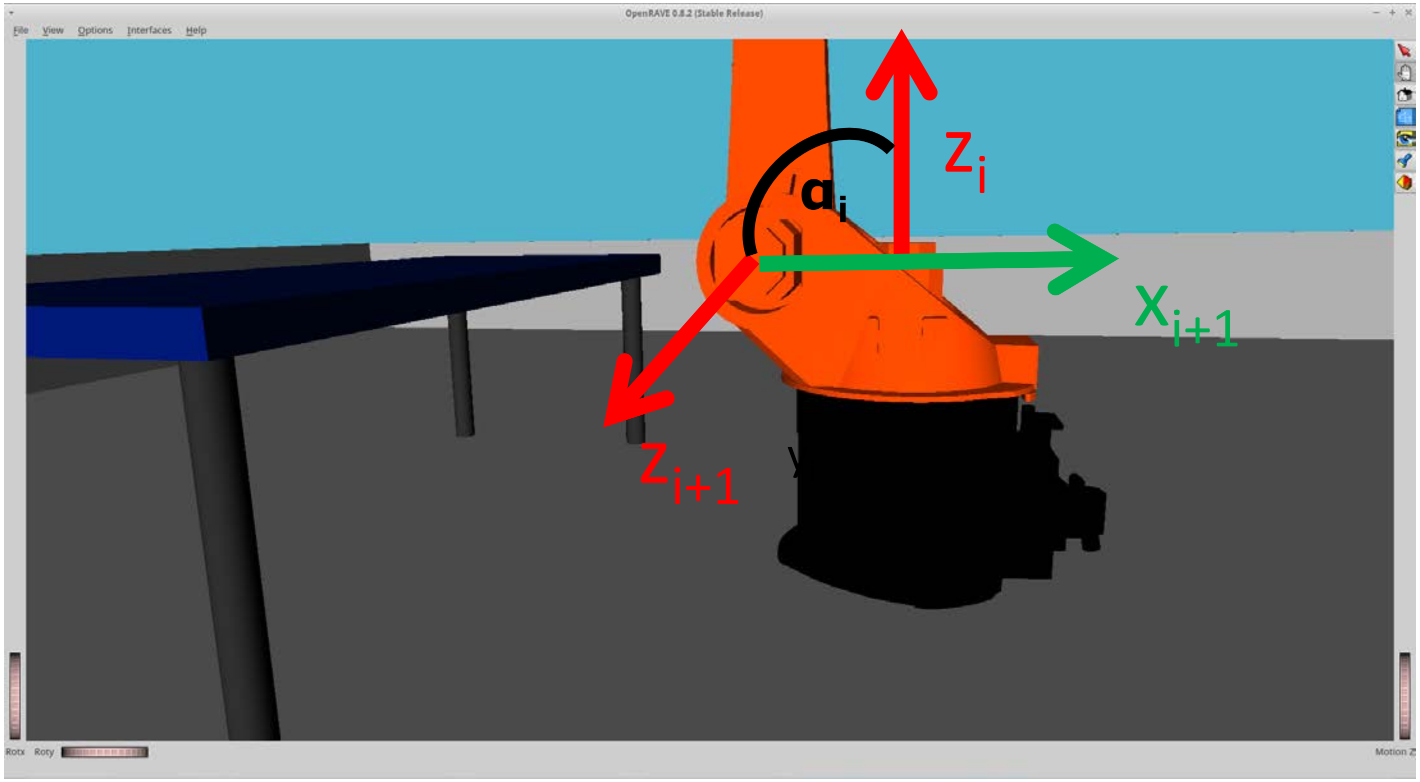
θ-parameter

- **θ_i** is the angle between the old and the new x-axis rotating around z_i-axis



α-parameter

- α_i is the angle between the z-axes rotating around x_{i+1}-axis



Using the parameters

Using the parameters

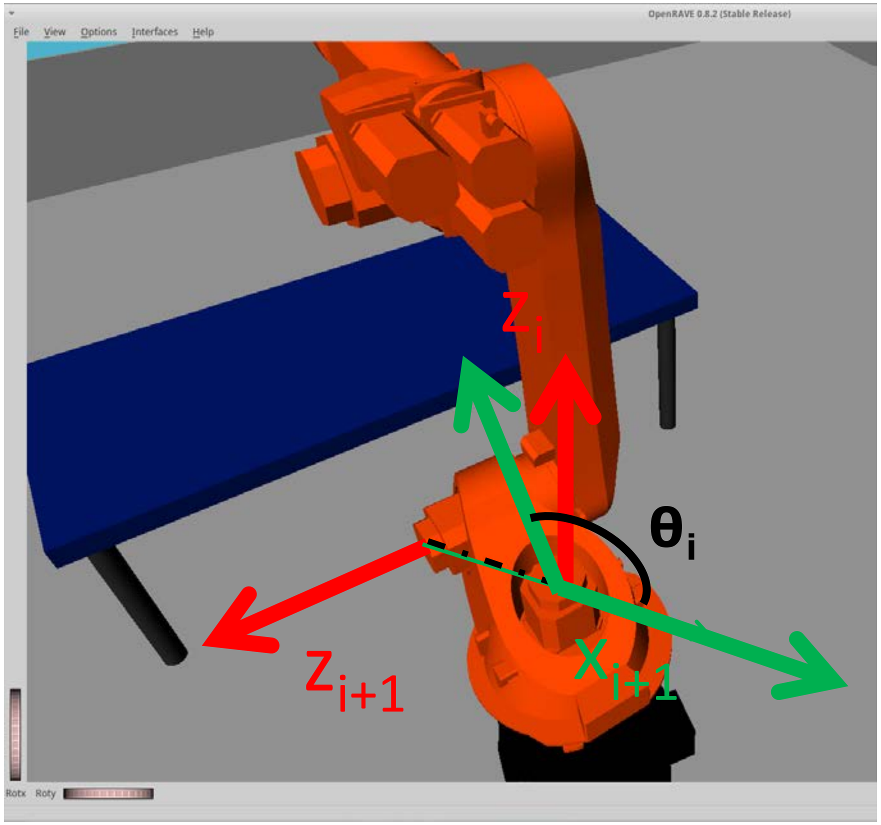
- performing the transformation by using the parameters

- 4 individual transformations
  - Rotation of **θ around z_i** : *Rot*(θ, z_i)
  - Translation of **d along z_i** : *Trans*(d, z_i)
  - Translation of **a along x_{i+1}** : *Trans*(a, x_{i+1})
  - Rotation of α **around x_{i+1}** : *Rot*(α, x_{i+1})

θ-parameter

- Rotation of **θ around z_i** : *Rot(θ, zi)*
- x_i same alignment as x_{i+1}

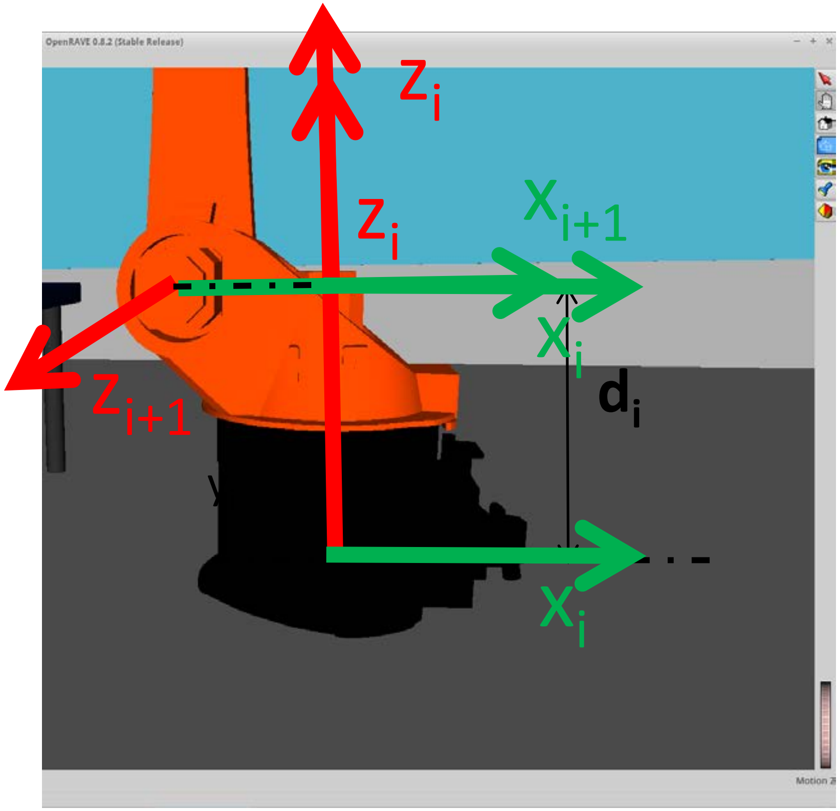
$$\begin{bmatrix} cos\theta i & -sin\theta i & 0 & 0 \\ sin\theta i & cos\theta i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



d-parameter

- Translation of **d on z_i** : *Trans(d, z_i)*

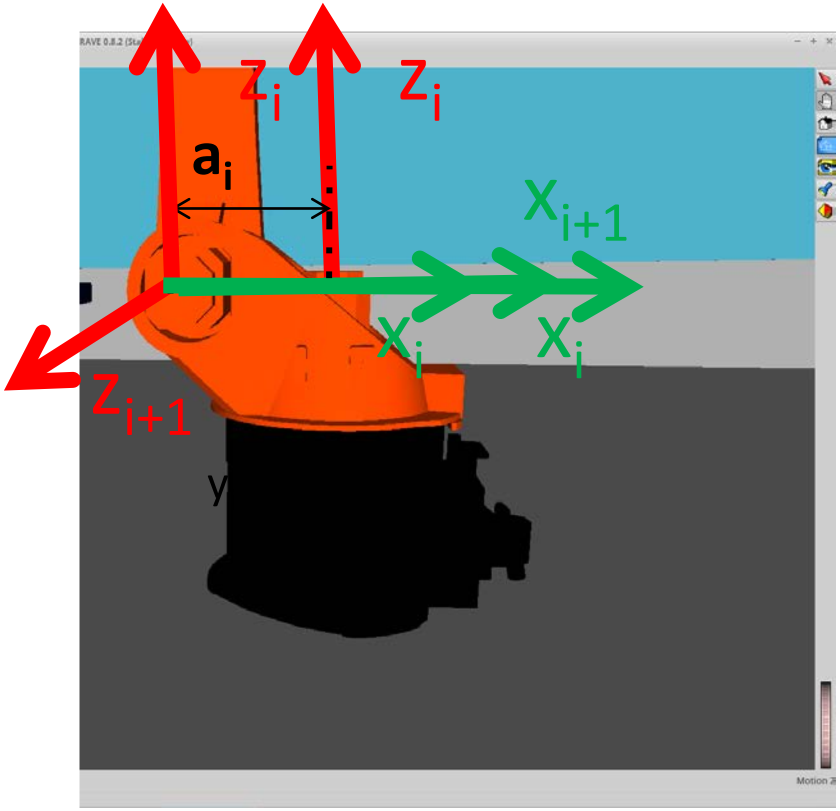
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \boldsymbol{di} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



a-parameter

- Translation of **a on $\mathbf{x_{i+1}}$** : *Trans*(a, x_{i+1})
- Origin i and i+1 at same position

$$\begin{bmatrix} 1 & 0 & 0 & \boldsymbol{ai} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



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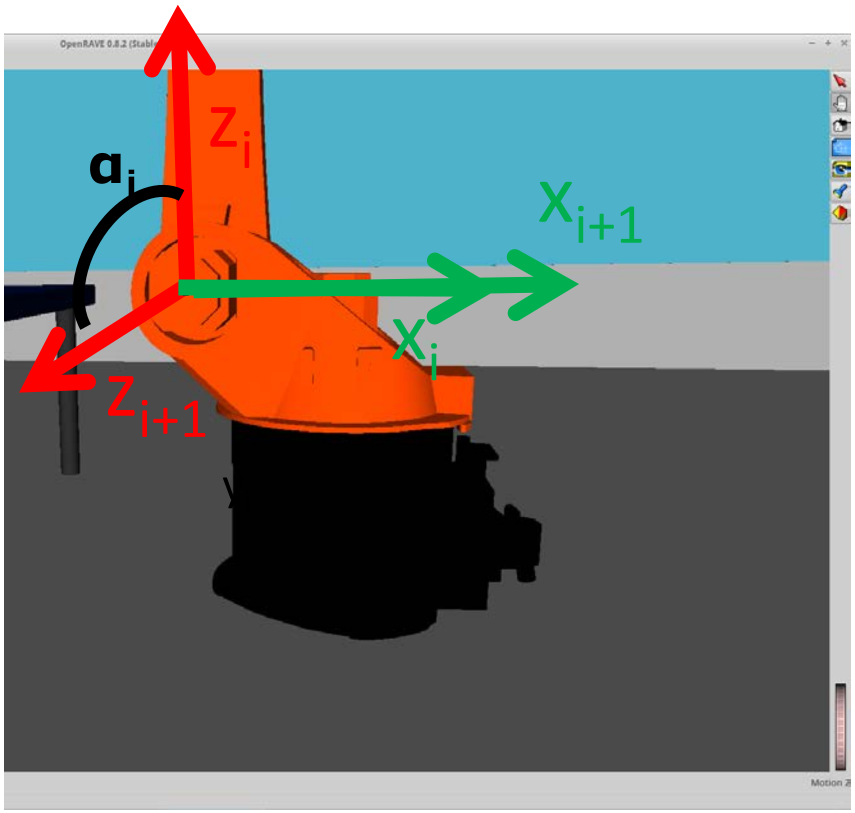
α-parameter

- Rotation of α **around $\mathbf{x_{i+1}}$** : $Rot(\alpha, x_{i+1})$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha i & -\sin\alpha i & 0 \\ 0 & \sin\alpha i & \cos\alpha i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Complete transformation

- Transformation is concatenation of each single transformation

- *Rot(θ, z_i) * Trans(d, z_i) * Trans(*a*, x_{i+1}) * Rot(α, x_{i+1})*

$$\boldsymbol{T}_{\boldsymbol{i}}^{\boldsymbol{i+1}}(\theta\boldsymbol{i}, \boldsymbol{di}, \boldsymbol{ai}, \alpha i) = \begin{bmatrix} cos\theta & -sin\theta cos\alpha & sin\theta sin\alpha & \boldsymbol{a}cos\theta \\ sinQ\theta & cos\theta cos\alpha & -cos\theta sin\alpha & \boldsymbol{a}sin\theta \\ 0 & sin\alpha i & cos\alpha i & \boldsymbol{d} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Summary

- 4 parameters used (a, d, θ, ɑ)
- Important how to allign the coordinate system
  - z points along the rotation axis
  - y completes the right hand system
  - x is the normal between z-axes
- d distance between x-axes
- a distance between z-axes
- θ rotation between x-axes around z_i
- ɑ rotation between z-axes around x_{i+1}

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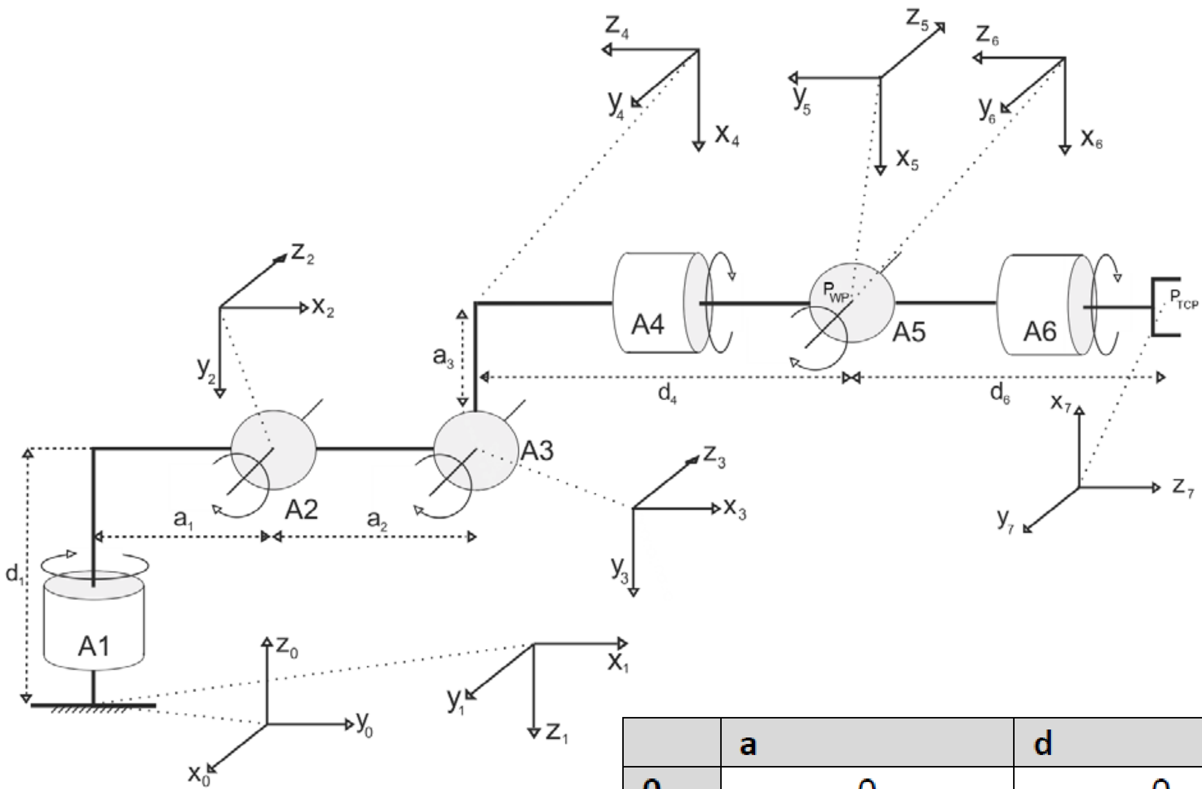
Abstraction of the Kuka KR30 L16

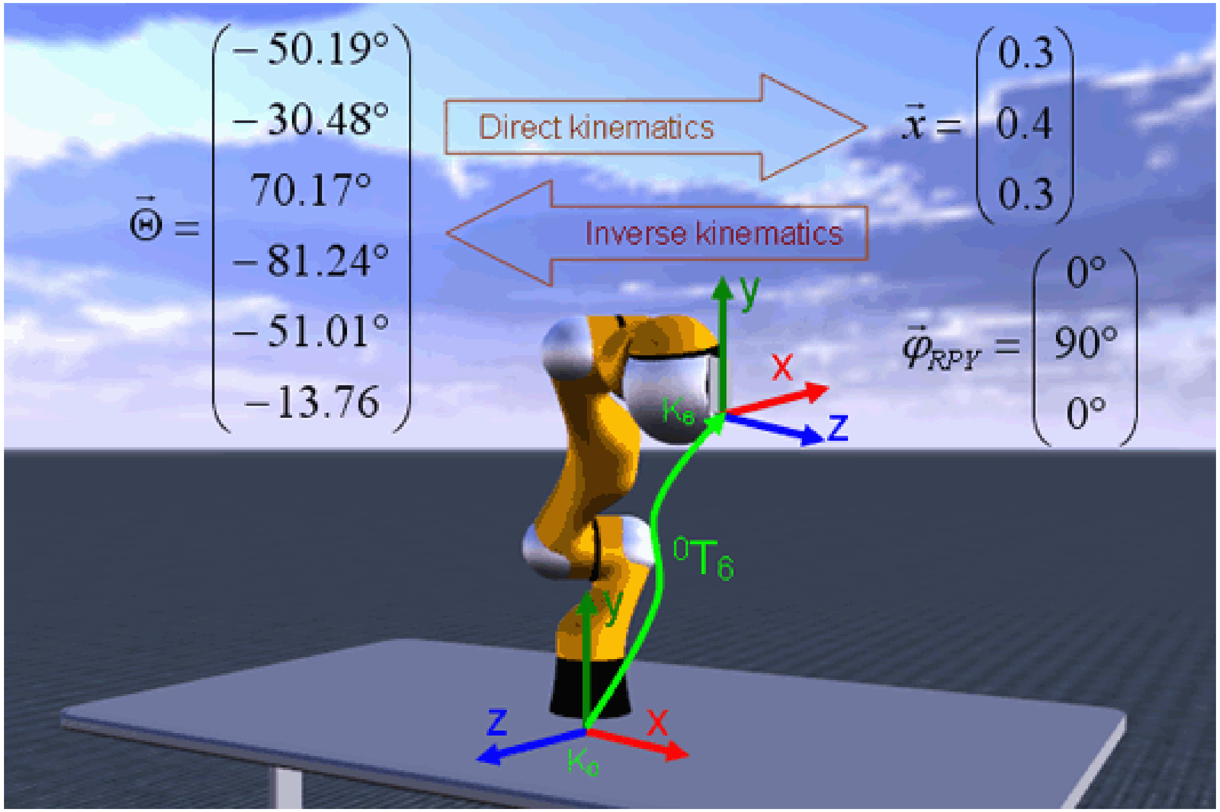


	a	d	θ	α
0	0	0	$\pi/2$	π
1	0.350	- 0.815	$\Theta 1$	$\pi/2$
2	1.200	0	$\Theta 2$	0
3	- 0.145	0	$\Theta 3 + \pi/2$	$-\pi/2$
4	0	- 1.545	$\Theta 4$	$\pi/2$
5	0	0	$\Theta 5$	$-\pi/2$
6	0	- 0.158	$\Theta 6 + \pi$	π

Abb: Abstraction of the Kuka KR30 L16

Direct Kinematic

- Problem: Given the angles of each rotary joint of a robot, what is the position and the orientation of the TCP



TCP Orientation and Position

$$M = \left(\begin{array}{ccc|c} & R & & T \\ & & & \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

- The rotation matrix **R** describes the relative
- **orientation** to the base coordinate System

- The translation vector **T** describes the relative
- **position** to the base coordinate System

→ Gives us Orientation and Position of the TCP

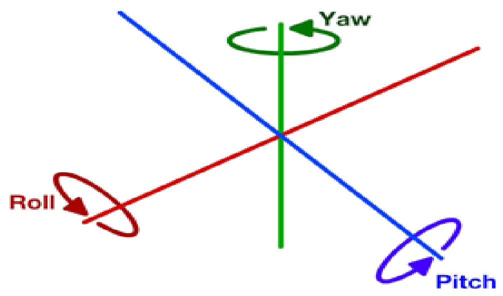
Description of the Orientation

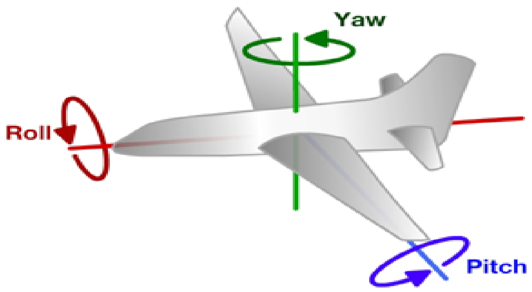
Description by rotation matrices

How can we describe the relative orientation in a less abstract way?

Solution: roll-pitch-yaw angles

Roll Pitch Yaw Angles





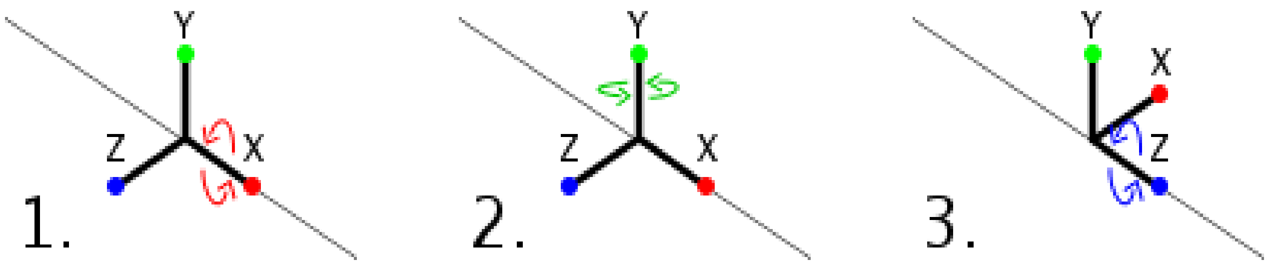
describe orientations by sequential (basic) rotations around x-, y- and z-axis

Extrinsic rotations around a fixed coordinate system: $R_{ex} = R_x(\alpha) \cdot R_y(\beta) \cdot R_z(\gamma)$

Intrinsic rotations around an object coordinate system: $R_{in} = R_z(\gamma) \cdot R_y(\beta) \cdot R_x(\alpha)$

Gimbal Locks

- A rotation of +/-90° around the second rotation axis sets the third rotation axis parallel to the axis of the first rotation



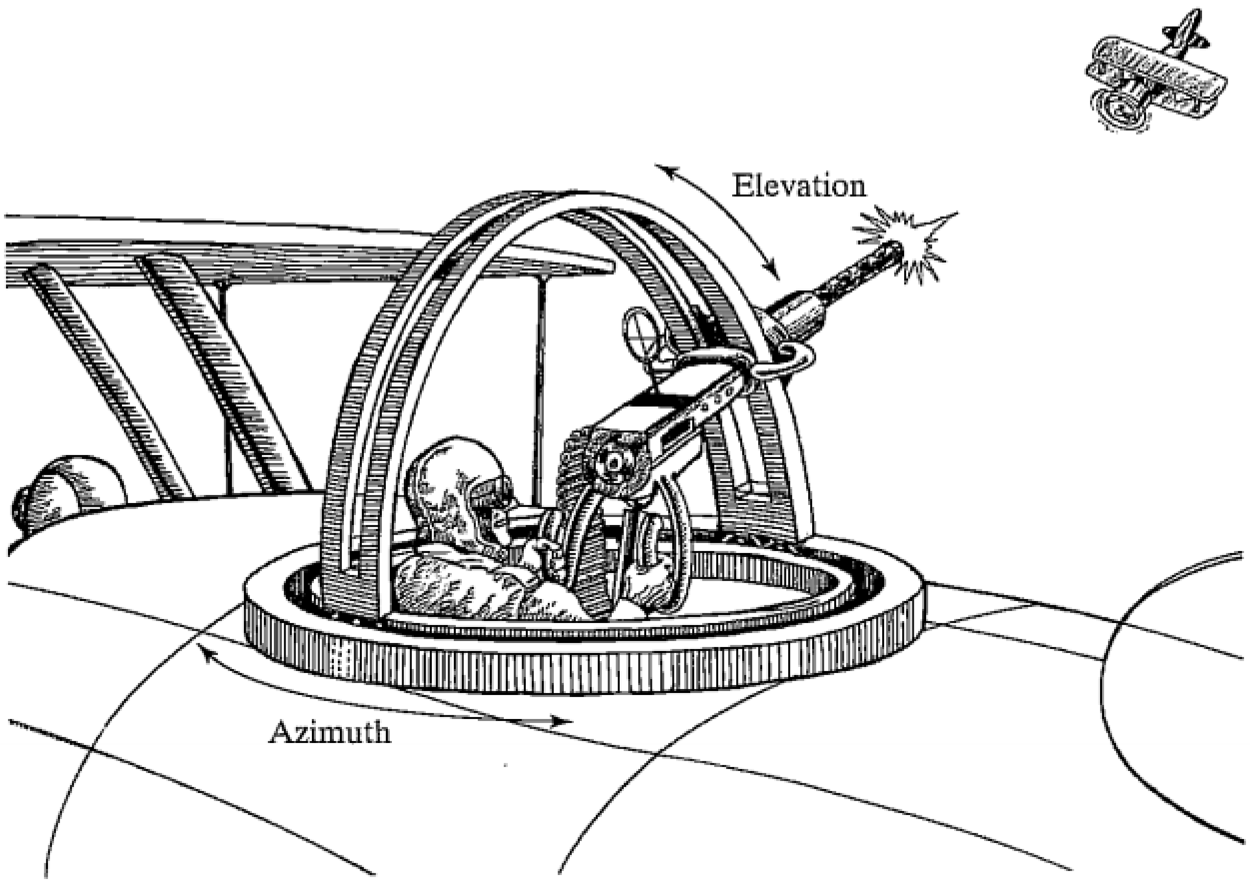
→ X and Z share the **same** rotation axis XZ

→ the third dimension of the rotation system is lost

→ Infinite rotation solutions arise:

$$R_{XZ}(\alpha + \gamma) = R_X(\alpha) + R_Z(\gamma)$$

Gimbal Locks (2)



Roll-Pitch-Yaw

R = $$\begin{pmatrix} \cos(\alpha)\cos(\beta) & \cos(\alpha)\sin(\beta)\sin(\gamma)-\sin(\alpha)\cos(\gamma) & \cos(\alpha)\sin(\beta)\cos(\gamma)+\sin(\alpha)\sin(\gamma) \\ \sin(\alpha)\cos(\beta) & \sin(\alpha)\sin(\beta)\sin(\gamma)+\cos(\alpha)\cos(\gamma) & \sin(\alpha)\sin(\beta)\cos(\gamma)-\cos(\alpha)\sin(\gamma) \\ -\sin(\beta) & \cos(\beta)\sin(\gamma) & \cos(\beta)\cos(\gamma) \end{pmatrix}$$

$$\boxed{R = R_x(\gamma)\cdot R_y(\beta)\cdot R_z(\alpha) = \begin{pmatrix} r_{00} & r_{01} & r_{02} \\ r_{10} & r_{10} & r_{12} \\ r_{20} & r_{21} & r_{22} \end{pmatrix}}$$

pitch:

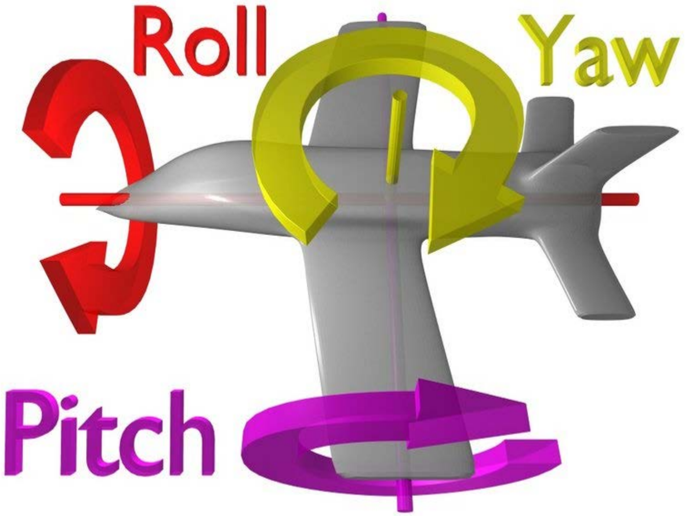
$$\boxed{\beta = atan2(-r_{20}, \sqrt{r_{00}^2 \cdot r_{10}^2})}$$

roll:

$$\boxed{\alpha = \begin{cases} 0 & \text{if } |\beta| = \pm\frac{\Pi}{2} \\ atan2(\frac{r_{10}}{\cos\beta}, \frac{r_{00}}{\cos\beta}) & \text{if } |\beta| \neq \pm\frac{\Pi}{2} \end{cases}}$$

pitch:

$$\boxed{\gamma = \begin{cases} \frac{\beta}{|\beta|} atan2(r_{01}, r_{11}) & \text{if } |\beta| = \pm\frac{\Pi}{2} \\ atan2(\frac{r_{21}}{\cos\beta}, \frac{r_{22}}{\cos\beta}) & \text{if } |\beta| \neq \pm\frac{\Pi}{2} \end{cases}}$$

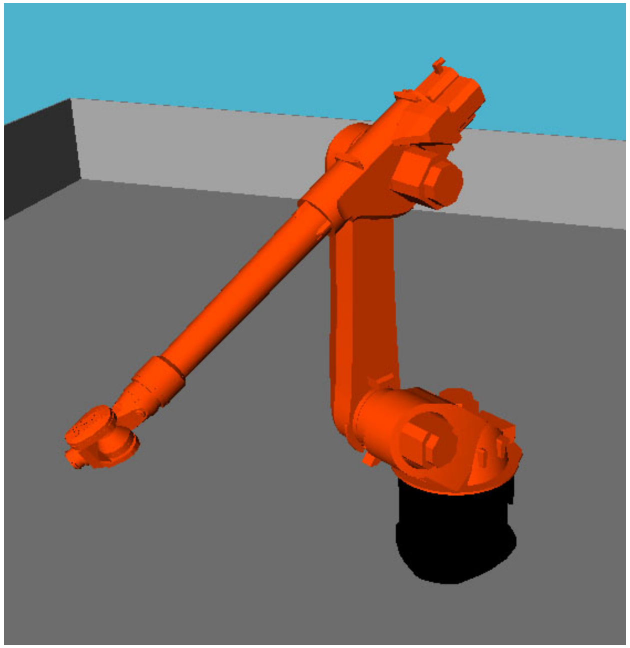


- There may be more than one solution

Direct Kinematic

Example:

$$q_0 = 0°$$
$$q_1 = 30°$$
$$q_2 = 90°$$
$$q_3 = -120°$$
$$q_4 = 90°$$
$$q_5 = -15°$$
$$q_6 = 0°$$



TransformationMatrix for 0 to 1:

$$T_1^0 = T\begin{pmatrix} q_0 + \theta_0 \\ d_0 \\ \alpha_0 \\ a_0 \end{pmatrix}$$

$$T_1^0 = T\begin{pmatrix} q_0 + 90 \\ 0.0 \\ 180 \\ 0.0 \end{pmatrix}$$

$$T_1^0(q_0 = 0) = \left(\begin{array}{ccc|c} 0.0 & 1.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -1.0 & 0.0 \\ \hline 0.0 & 0.0 & 0.0 & 1.0 \end{array}\right)$$

roll: 180.0 pitch: 0.0 yaw: 90.0

TransformationMatrix for 1 to 2:

$$T_2^1 = T\begin{pmatrix} q_1 + \theta_1 \\ d_1 \\ \alpha_1 \\ a_1 \end{pmatrix}$$

$$T_2^1 = T\begin{pmatrix} q_1 + 0 \\ -0.815 \\ 90 \\ 0.35 \end{pmatrix}$$

$$T_2^1(q_0 = 0) = \left(\begin{array}{ccc|c} 1.0 & 0.0 & 0.0 & 0.35 \\ 0.0 & 0.0 & -1.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & -0.815 \\ \hline 0.0 & 0.0 & 0.0 & 1.0 \end{array}\right)$$

roll: 90.0 pitch: 0.0 yaw: 0.0

$$T_2^1(q_0 = 30) = \left(\begin{array}{ccc|c} 0.866 & 0.0 & 0.5 & 0.303 \\ 0.5 & 0.0 & -0.866 & 0.175 \\ 0.0 & 1.0 & 0.0 & -0.815 \\ \hline 0.0 & 0.0 & 0.0 & 1.0 \end{array}\right)$$

roll: 90.0 pitch: 0.0 yaw: 30.0

TransformationMatrix for 2 to 3:

$$T_3^2 = T\begin{pmatrix} q_2+\theta_2 \\ d_2 \\ \alpha_2 \\ a_2 \end{pmatrix}$$

$$T_3^2 = T\begin{pmatrix} q_2+0 \\ 0.0 \\ 0 \\ 1.2 \end{pmatrix}$$

$$T_3^2(q_0=0) = \left(\begin{array}{ccc|c} 1.0 & 0.0 & 0.0 & 1.2 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ \hline 0.0 & 0.0 & 0.0 & 1.0 \end{array}\right)$$

roll: 0.0 pitch: 0.0 yaw: 0.0

$$T_3^2(q_0=90) = \left(\begin{array}{ccc|c} 0.0 & -1.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 1.2 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ \hline 0.0 & 0.0 & 0.0 & 1.0 \end{array}\right)$$

roll: 0.0 pitch: 0.0 yaw: 90.0

Direct Kinematic

TransformationMatrix for 4 to 5:

$$T_5^4 = T\begin{pmatrix} q_4+\theta_4 \\ d_4 \\ \alpha_4 \\ a_4 \end{pmatrix}$$

$$T_5^4 = T\begin{pmatrix} q_4+0 \\ -1.545 \\ 90 \\ 0.0 \end{pmatrix}$$

$$T_5^4(q_0=0) = \left(\begin{array}{ccc|c} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -1.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & -1.545 \\ \hline 0.0 & 0.0 & 0.0 & 1.0 \end{array}\right)$$

roll: 90.0 pitch: 0.0 yaw: 0.0

$$T_5^4(q_0=90) = \left(\begin{array}{ccc|c} 0.0 & 0.0 & 1.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & -1.545 \\ \hline 0.0 & 0.0 & 0.0 & 1.0 \end{array}\right)$$

roll: 90.0 pitch: 0.0 yaw: 90.0

Direct Kinematic

TransformationMatrix for 5 to 6:

$$T_6^5 = T\begin{pmatrix} q_5+\theta_5 \\ d_5 \\ \alpha_5 \\ a_5 \end{pmatrix}$$

$$T_6^5 = T\begin{pmatrix} q_5+0 \\ 0.0 \\ -90 \\ 0.0 \end{pmatrix}$$

$$T_6^5(q_0=0) = \left(\begin{array}{ccc|c} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & -1.0 & 0.0 & 0.0 \\ \hline 0.0 & 0.0 & 0.0 & 1.0 \end{array}\right)$$

roll: -90.0 pitch: 0.0 yaw: 0.0

$$T_6^5(q_0=-15) = \left(\begin{array}{ccc|c} 0.966 & 0.0 & 0.259 & 0.0 \\ -0.259 & 0.0 & 0.966 & 0.0 \\ 0.0 & -1.0 & 0.0 & 0.0 \\ \hline 0.0 & 0.0 & 0.0 & 1.0 \end{array}\right)$$

roll: -90.0 pitch: 0.0 yaw: -15.0

TransformationMatrix for 6 to 7:

$$T_7^6 = T\begin{pmatrix} q_6+\theta_6 \\ d_6 \\ \alpha_6 \\ a_6 \end{pmatrix} \qquad T_7^6(q_0=0) = \left(\begin{array}{ccc|c} -1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -1.0 & -0.158 \\ \hline 0.0 & 0.0 & 0.0 & 1.0 \end{array}\right)$$

roll: 180.0 pitch: 0.0 yaw: 180.0

$$T_7^6 = T\begin{pmatrix} q_6+180 \\ -0.158 \\ 180 \\ 0.0 \end{pmatrix} \qquad T_7^6(q_0=0) = \left(\begin{array}{ccc|c} -1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -1.0 & -0.158 \\ \hline 0.0 & 0.0 & 0.0 & 1.0 \end{array}\right)$$

roll: 180.0 pitch: 0.0 yaw: 180.0

RESULTS:

$$T_7^0(q_i = 0, 0, 0, 0, 0, 0, 0) = \left(\begin{array}{ccc|c} 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 3.253 \\ 1.0 & 0.0 & 0.0 & 0.96 \\ \hline 0.0 & 0.0 & 0.0 & 1.0 \end{array}\right)$$

roll: 0.0 pitch: -90.0 yaw: -90.0

$$T_7^0(q_i = 0, 30, 90, -120, 90, -15) = \left(\begin{array}{ccc|c} -0.949 & -0.25 & 0.194 & 0.838 \\ 0.289 & -0.433 & 0.854 & 1.534 \\ -0.129 & 0.866 & 0.483 & 0.589 \\ \hline 0.0 & 0.0 & 0.0 & 1.0 \end{array}\right)$$

roll: 60.853 pitch: 7.435 yaw: 163.064

What we wanted you to know

- Introduction into the KUKA 30
- Kinematic Chain
- DH Parameters
- Position and orientation in forward kinematics