INF

Wrist orientation

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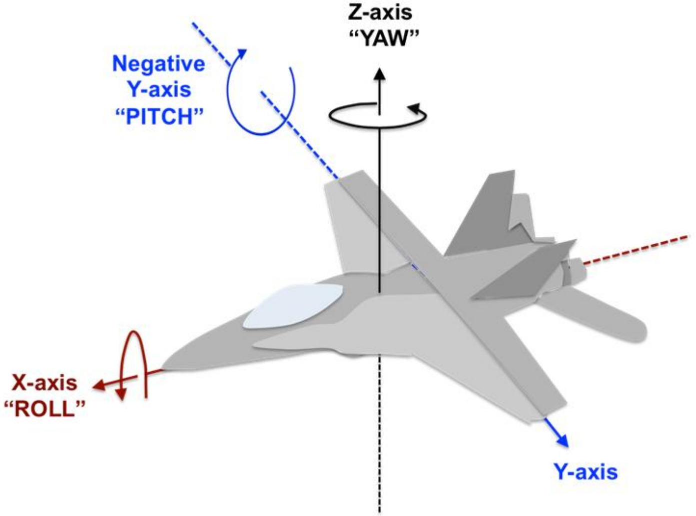
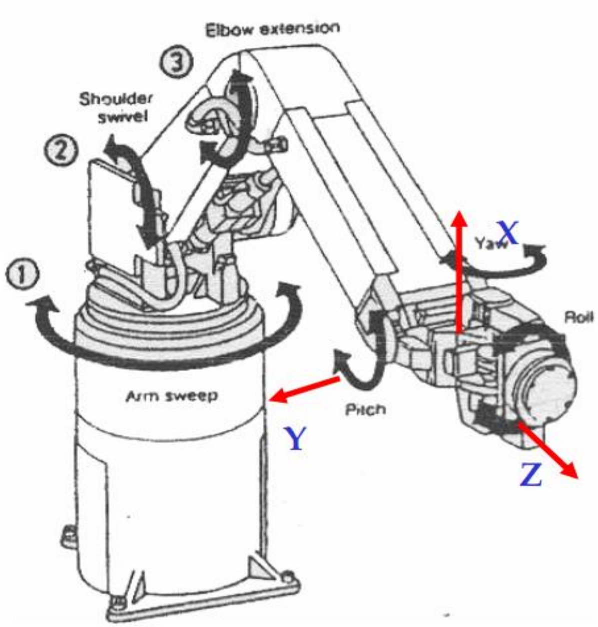
Agenda

1. Introduction of robot wrist kinematics for orientation
2. RPY - Roll Pitch Yaw
3. Euler angles
4. Difference between RPY and Euler angles
5. Computing the wrist angles

Introduction of robot wrist kinematics for orientation

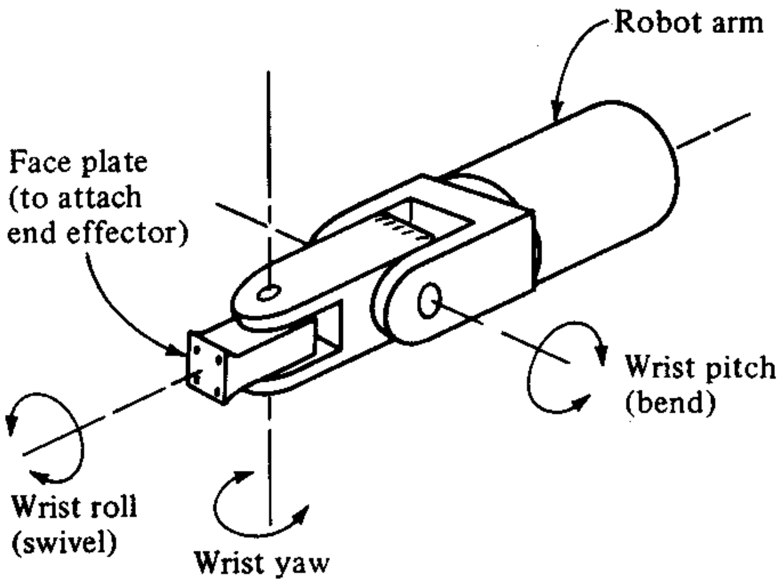


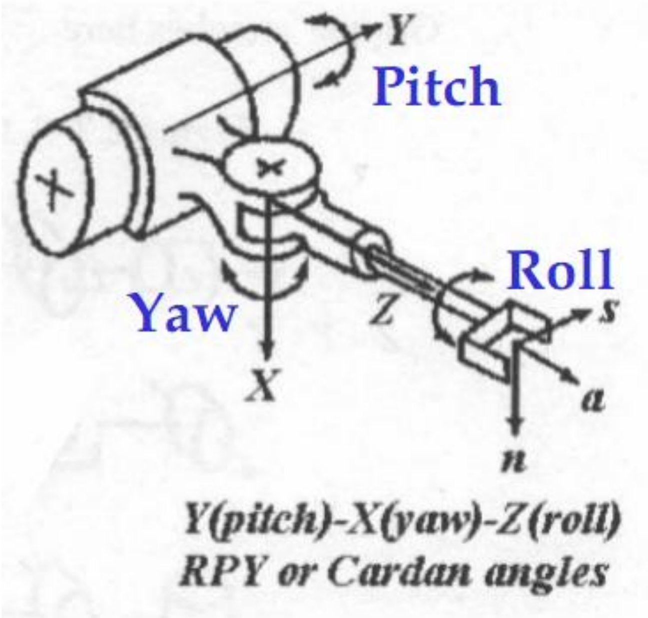


Wrist orientation

RPY - Roll Pitch Yaw

- Roll Pitch Yaw

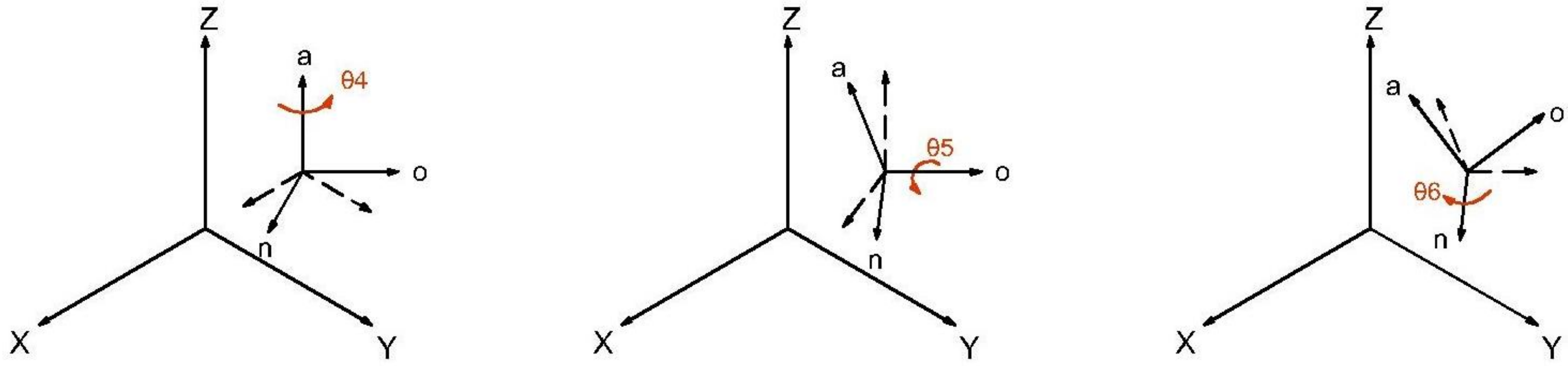




RPY - Roll Pitch Yaw

- Kinematics of RPY wrist

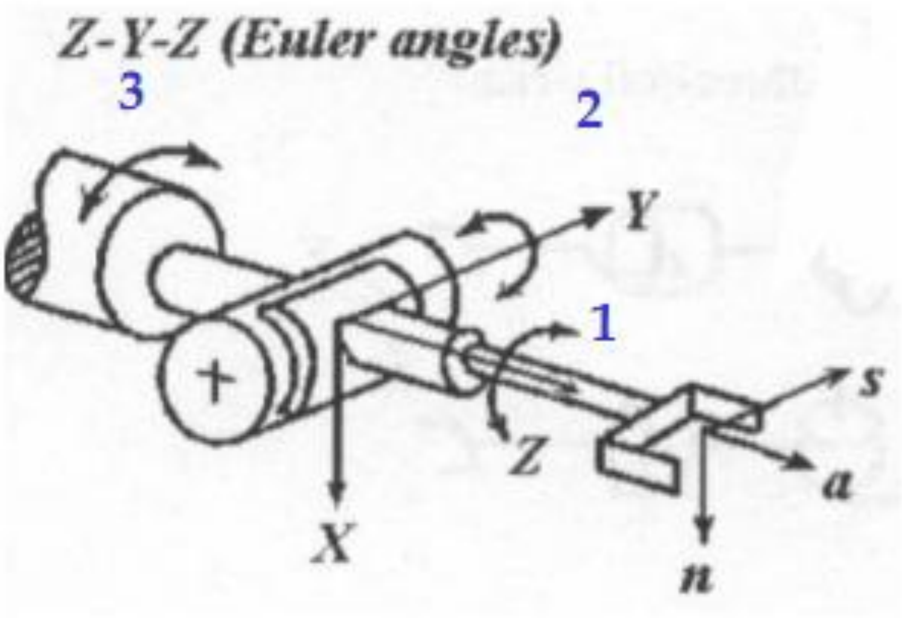
Roll **Pitch** **Yaw**



$$RPY(\theta_4, \theta_5, \theta_6) = Rot(a, \theta_4)\ Rot(o, \theta_5)\ Rot(n, \theta_6)$$

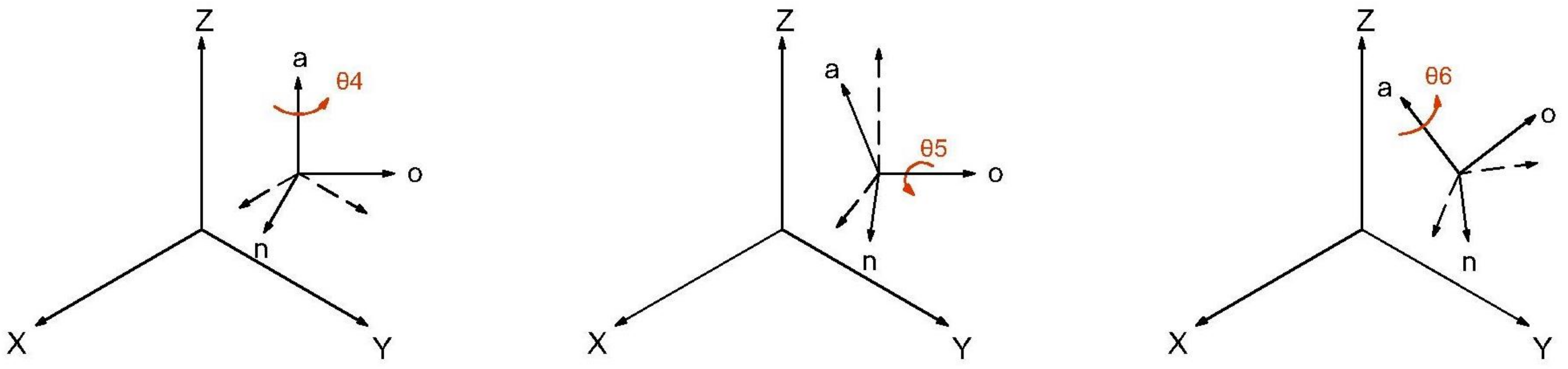
Euler angles

- Euler angles
- The most common angles for robotic wrists are the ZY'Z'' angles



Euler angles

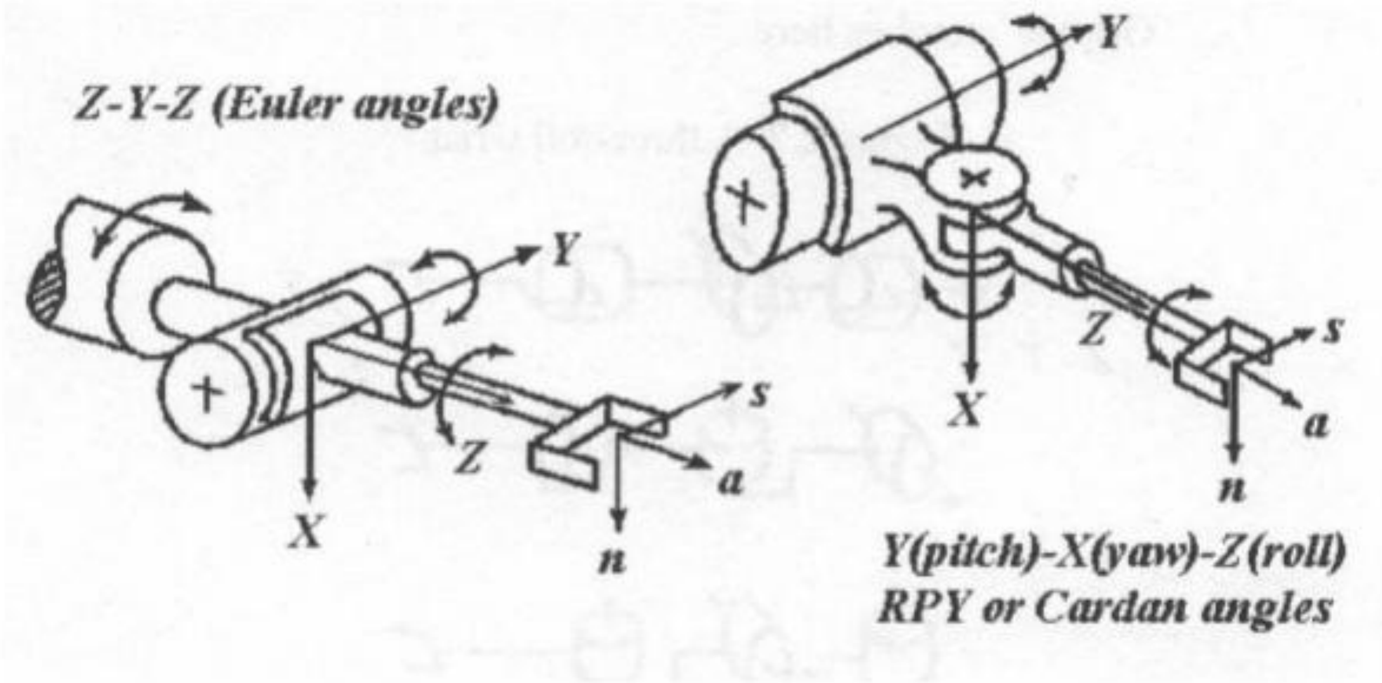
- Kinematics of Euler angles



$$\mathrm{Euler}(\theta_4, \theta_5, \theta_6) = \mathrm{Rot}(a, \theta_4)\ \mathrm{Rot}(o, \theta_5)\ \mathrm{Rot}(a, \theta_6)$$

Difference between RPY and Euler angles

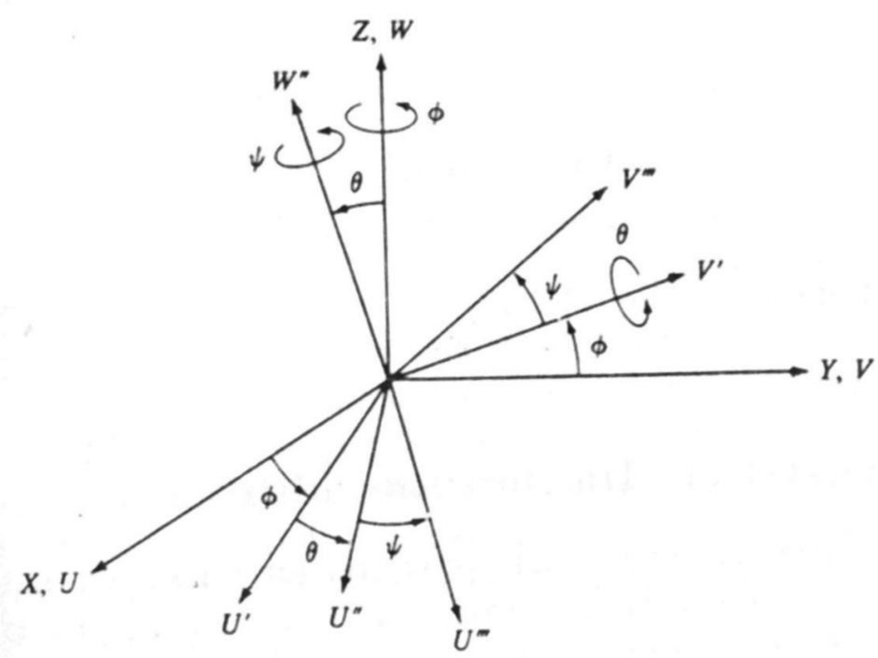
- What is the difference between RPY angles and Euler angles?

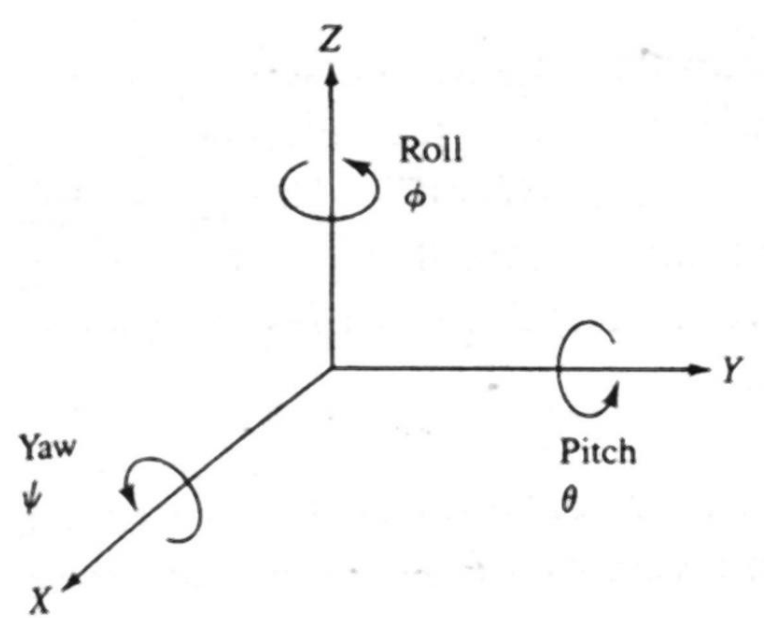


Difference between RPY and Euler angles

$\underline{EU}(\Phi,\Theta,\Psi)=\underline{Rot}(z,\Phi)\underline{Rot}(v,\Theta)\underline{Rot}(w,\Psi)$

$\underline{RPY}(\phi,\theta,\psi)=\underline{Rot}(z,\phi)\underline{Rot}(y,\theta)\underline{Rot}(x,\psi)$





Computing the wrist angles

- Rotate the reference frame by the angle θ_4 about axis z
  - $R_z = A_4 = \begin{pmatrix} \cos\theta_4 & \sin\theta_4 & 0 \\ -\sin\theta_4 & \cos\theta_4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
- Rotate the reference frame by the angle θ_5 about axis y'
  - $R_{y'} = A_5 = \begin{pmatrix} \cos\theta_5 & 0 & -\sin\theta_5 \\ 0 & 1 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 \end{pmatrix}$
- Rotate the reference frame by the angle θ_6 about axis z''
  - $R_{z''} = A_6 = \begin{pmatrix} \cos\theta_6 & \sin\theta_6 & 0 \\ -\sin\theta_6 & \cos\theta_6 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Computing the wrist angles

Postmutiplication of the matrices

$$T_3^6 = A_4(\theta_4) \cdot A_5(\theta_5) \cdot A_6(\theta_6)$$

$$T_3^6 = \begin{pmatrix} c\,\theta_4 \cdot c\,\theta_5 \cdot c\,\theta_6 - s\,\theta_4 \cdot s\,\theta_6 & -c\,\theta_4 \cdot c\,\theta_5 \cdot s\,\theta_6 - s\,\theta_4 \cdot c\,\theta_6 & c\,\theta_4 \cdot s\,\theta_5 \\ s\,\theta_4 \cdot c\,\theta_5 \cdot c\,\theta_6 + c\,\theta_4 \cdot s\,\theta_6 & -s\,\theta_4 \cdot c\,\theta_5 \cdot s\,\theta_6 + c\,\theta_4 \cdot c\,\theta_6 & s\,\theta_4 \cdot s\,\theta_5 \\ -s\,\theta_5 \cdot c\,\theta_6 & s\,\theta_5 \cdot s\,\theta_6 & c\,\theta_5 \end{pmatrix}$$

(The notations $c\,\theta$ and $s\,\theta$ are the abbreviations for $\cos\theta$ and $\sin\theta$)

Computing the wrist angles

- We already know:

    1. The position of the wrist (Group 2)
        - $T_0^3 = A_1(\theta_1) \cdot A_2(\theta_2) \cdot A_3(\theta_3)$

    2. The position and orientation of the tool center point
        - A_{TCP} (Matrix with fixed values depending on the tool)

    3. The desired position and orientation of the end-effector
        - $A_{intended}$ (Input)

Computing the wrist angles

- We are looking for the orientation of the wrist (T_3^6)

- Basically, we have this equation

$$A_{intended} = A_1(\theta_1) \cdot A_2(\theta_2) \cdot A_3(\theta_3) \cdot A_4(\theta_4) \cdot A_5(\theta_5) \cdot A_6(\theta_6) \cdot A_{TCP}$$

- As shown before, we can simplify the equation

$$A_{intended} = T_0^3 \cdot T_3^6 \cdot A_{TCP}$$

- Solving the equation for T_3^6

$$T_3^6 = {T_0^3}^{-1} \cdot A_{intended} \cdot {A_{TCP}}^{-1}$$

Computing the wrist angles

- Now we have $T_3^6 = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$

- T_3^6 contains real values from r11 to r33
- Remember that our rotation is based on the matrix below

$$\begin{pmatrix} c\,\theta_4 \cdot c\,\theta_5 \cdot c\,\theta_6 - s\,\theta_4 \cdot s\,\theta_6 & -c\,\theta_4 \cdot c\,\theta_5 \cdot s\,\theta_6 - s\,\theta_4 \cdot c\,\theta_6 & c\,\theta_4 \cdot s\,\theta_5 \\ s\,\theta_4 \cdot c\,\theta_5 \cdot c\,\theta_6 + c\,\theta_4 \cdot s\,\theta_6 & -s\,\theta_4 \cdot c\,\theta_5 \cdot s\,\theta_6 + c\,\theta_4 \cdot c\,\theta_6 & s\,\theta_4 \cdot s\,\theta_5 \\ -s\,\theta_5 \cdot c\,\theta_6 & s\,\theta_5 \cdot s\,\theta_6 & c\,\theta_5 \end{pmatrix}$$

- With this information we can compute the angles using inverse trigonometric functions

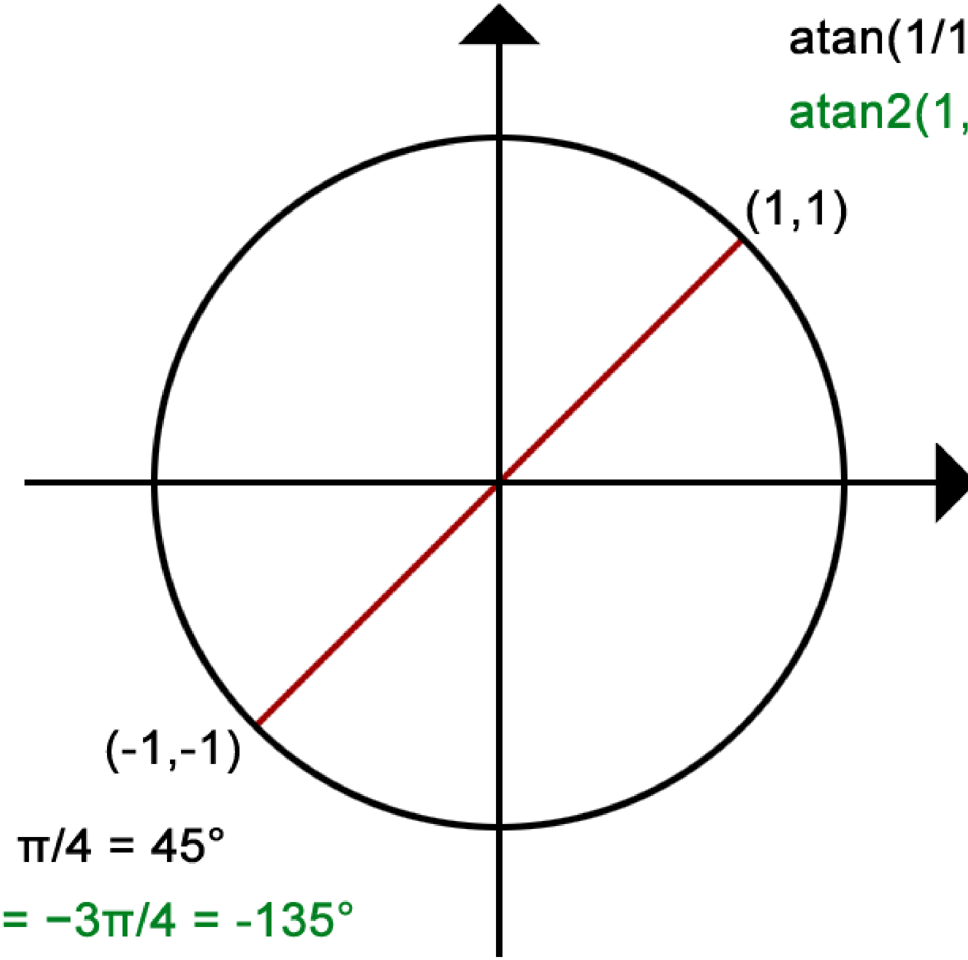
Two-argument arctangent function

- For the computation of the angles we need the inverse trigonometric tangent function
- It is used to get the angle in Radian for a given angle function value
- The one-argument arctangent function can not distinguish between diametrically opposite directions
- To achieve full accuracy for all angles, Atan2 should be used

Two-argument arctangent function

atan(1/1) = π/4 = 45°

atan2(1,1) = π/4 = 45°

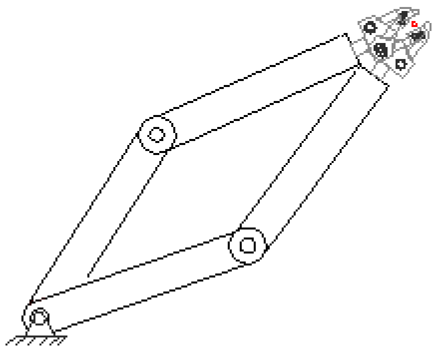


atan(-1/-1) = π/4 = 45°

atan2(-1,-1) = −3π/4 = -135°

Computing the wrist angles

- Computing the angles there are two possible solutions depending on θ_5



- θ_5 can range from $(0, \pi)$ or from $(-\pi, 0)$

Computing the wrist angles

- Getting the first solution for θ_5 from $(0, \pi)$

- Computing the first wrist angle

$$\theta_4 = \mathrm{Atan2}(r_{23}, r_{13})$$

- Computing the second wrist angle

$$\theta_5 = \mathrm{Atan2}\left(\sqrt{r_{13}^2 + r_{23}^2}\,,\ r_{33}\right)$$

- Computing the third wrist angle

$$\theta_6 = \mathrm{Atan2}(r_{32}, -r_{31})$$

Computing the wrist angles

- In order to get the second solution with θ_5 from (-π, 0) we use following equations

$$\theta_4 = \mathrm{Atan2}(-r_{23}\,, -r_{13})$$

$$\theta_5 = \mathrm{Atan2}\left(-\sqrt{r_{13}^2 + r_{23}^2}\;,\; r_{33}\right)$$

$$\theta_6 = \mathrm{Atan2}(-r_{32}\,, r_{31})$$

The end…

Thank you for your attention!

Sources

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