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Inverse Kinematics for Kuka KR30-L16: Positioning of the wrist

Softwaredevelopment for Industrial Robotics

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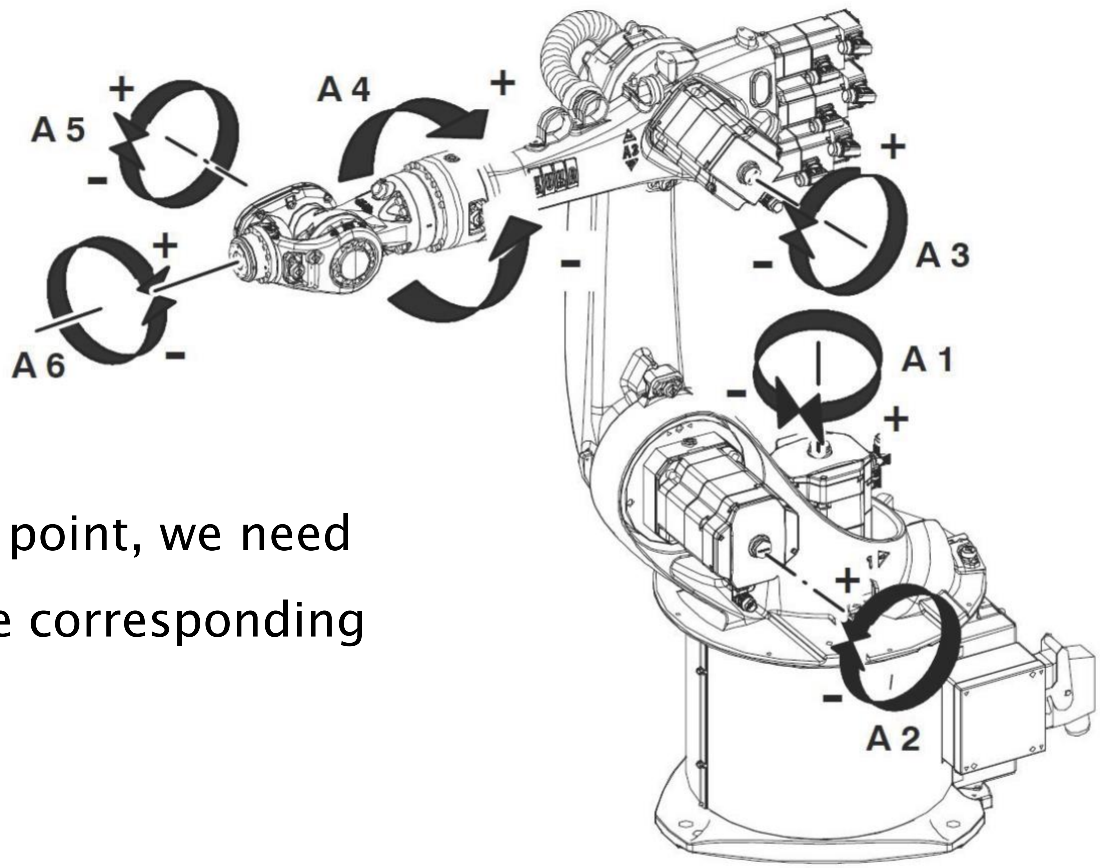
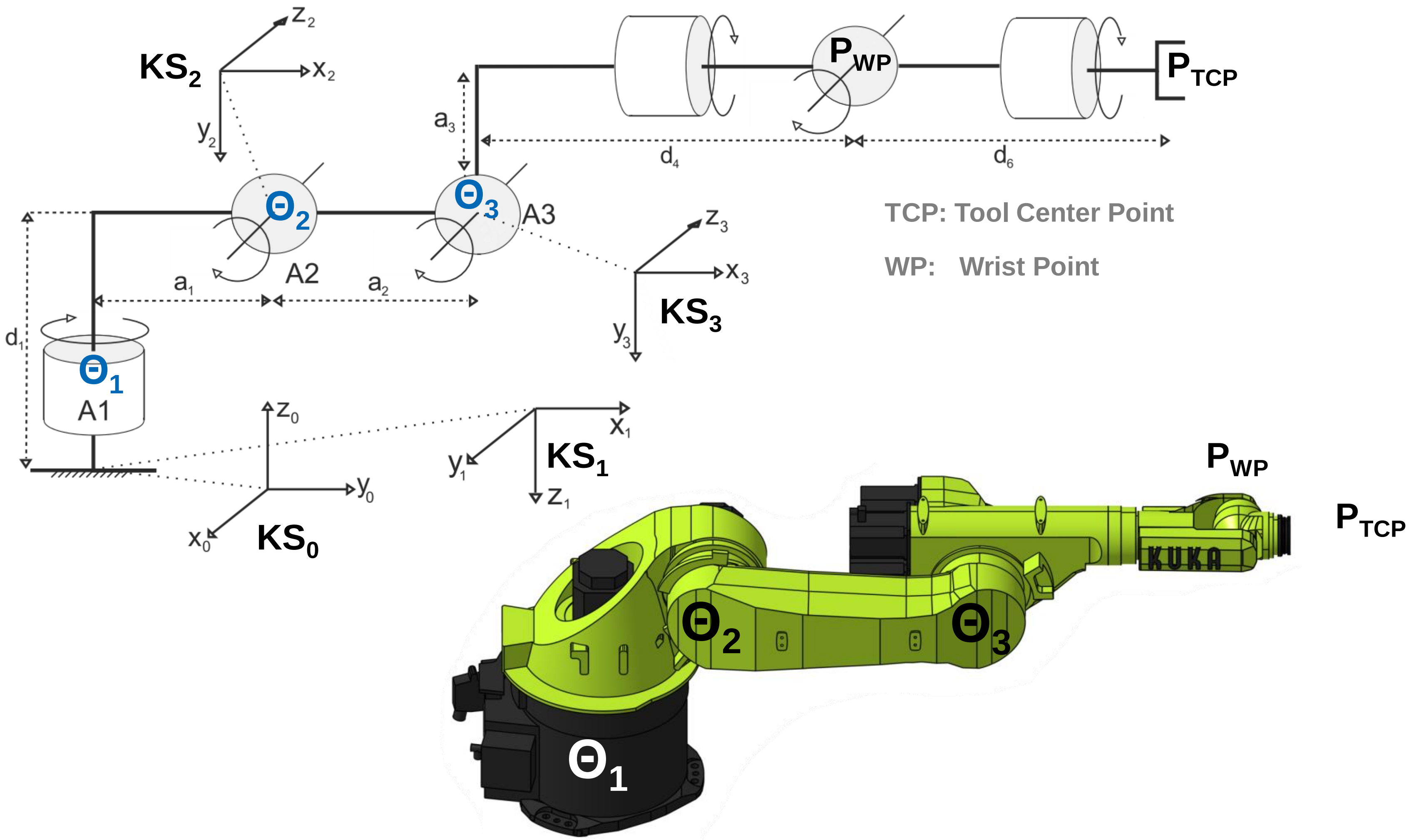


For positioning of wrist point, we need joint **A1**, **A2**, **A3** and the corresponding angles Θ_1, Θ_2, and Θ_3.

1.Overview: Zero Configuration of KR30-L16

Angles Θ_1, Θ_2, and Θ_3 = 0°



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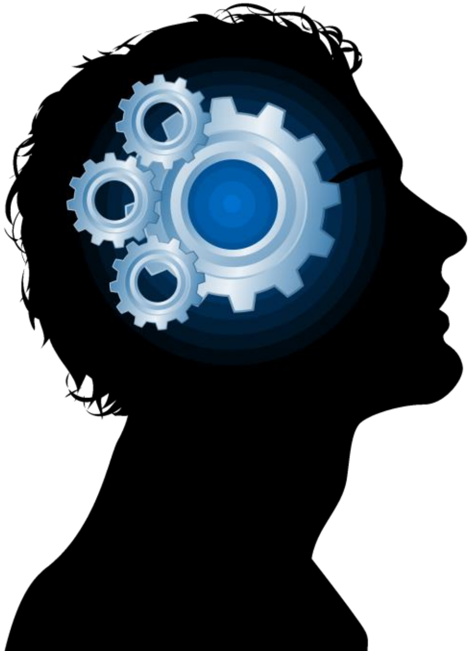
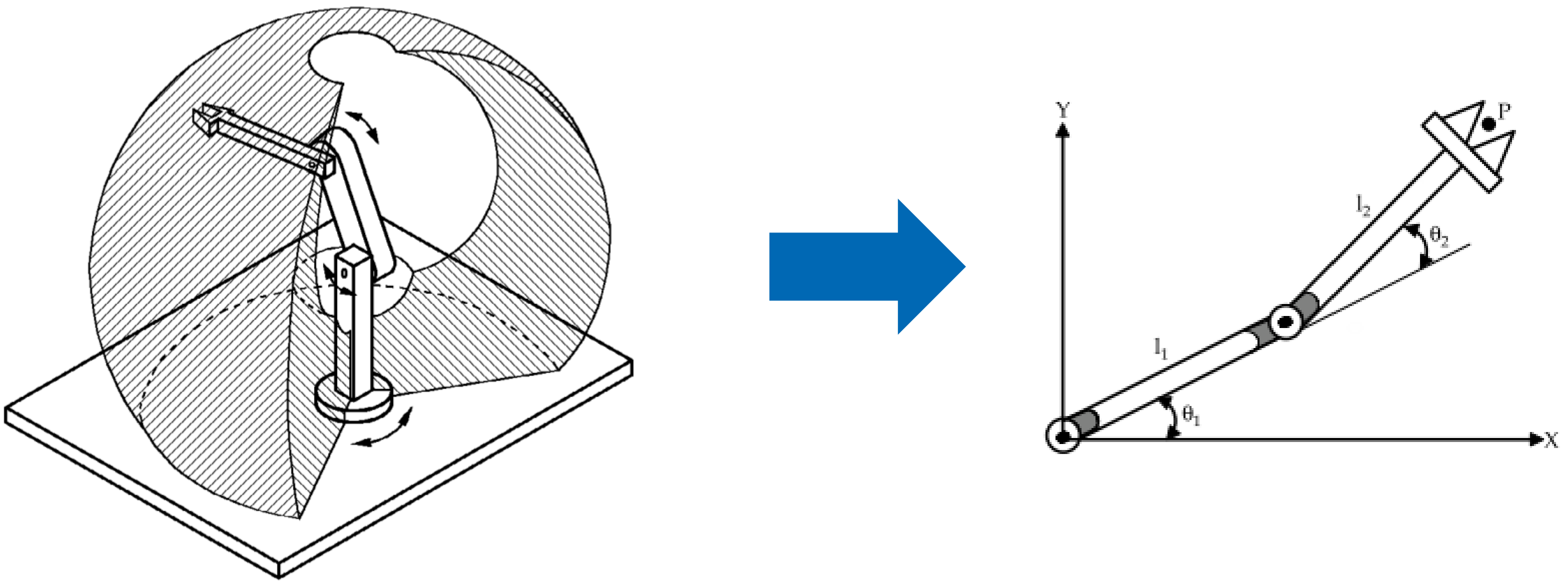


2. Solution statement

Split 3D problem of inverse kinematics in several 2D problems:



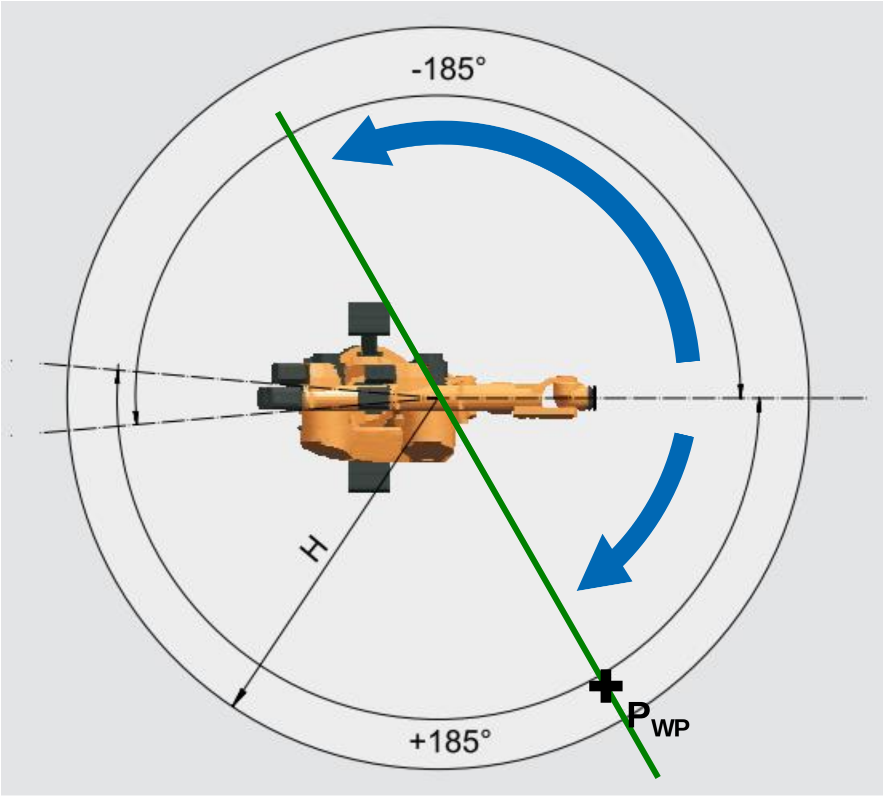
2. Solution statement [2]

Task: Reach the target point P_{WP} with the wrist of the robot

<u>First 2D problem in topview:</u>

Rotation of the robot arm to the plane of the wrist point (P_{WP}) [Θ_1].

→ „**WP Plane**“

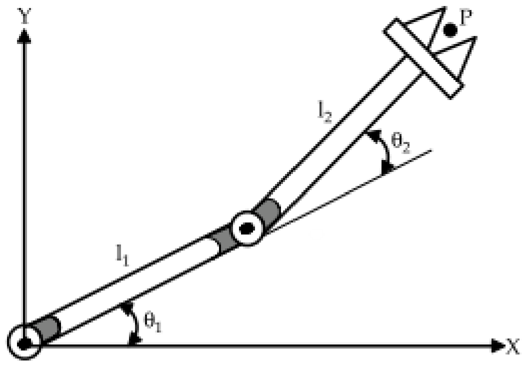


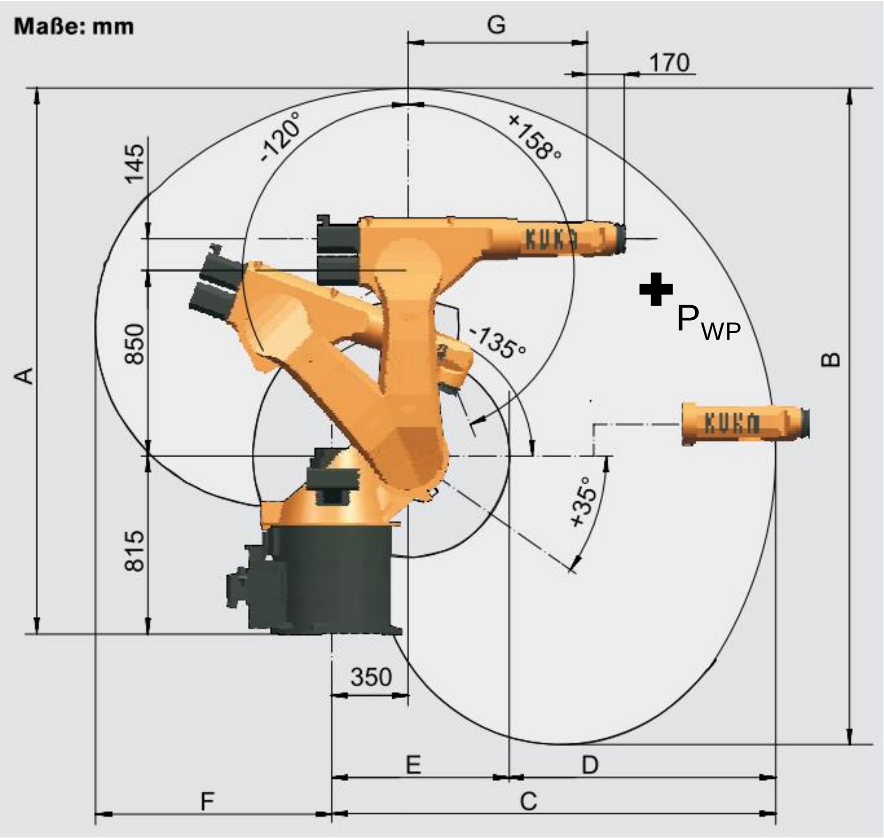
2. Solution statement [3]

<u>Second 2D problem in lateral view:</u>

Calculation of the other

angles [Θ_2,Θ_3] in the „**WP Plane**“



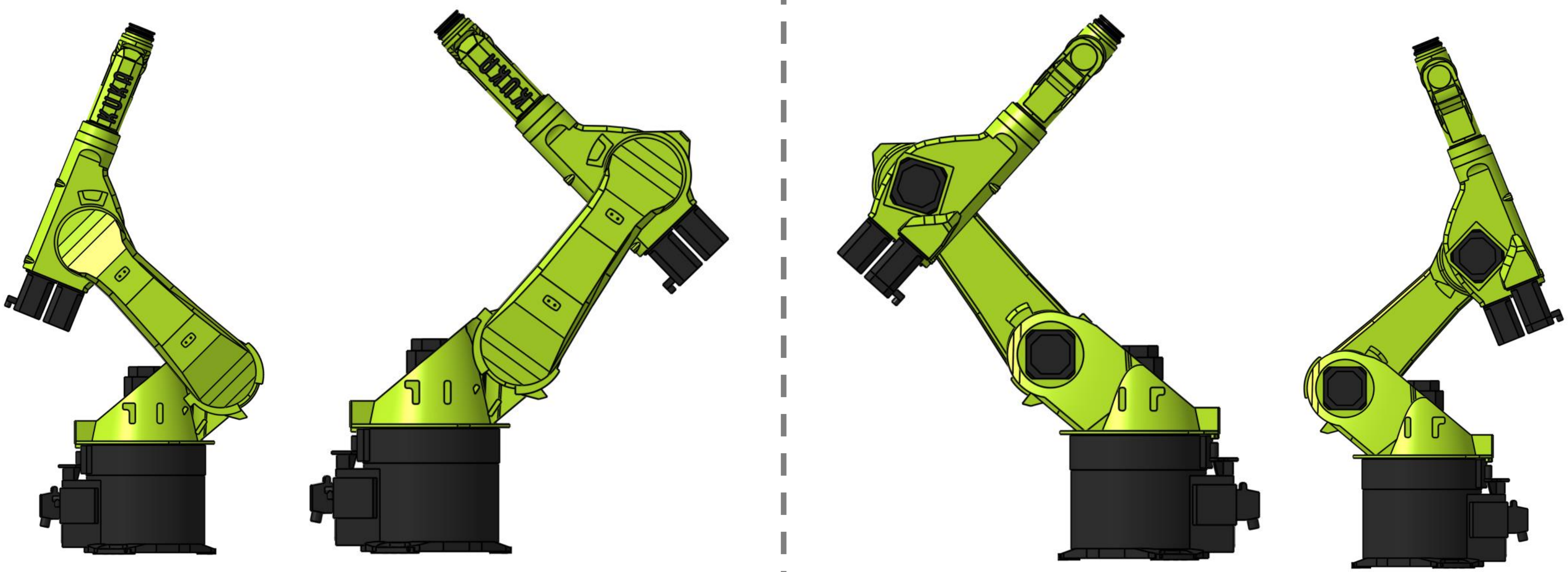


2. Solution statement: Possible Solutions

4 possible solutions:

Shoulder: right → up, down | left → down, up

Elbow: up | down | down | up



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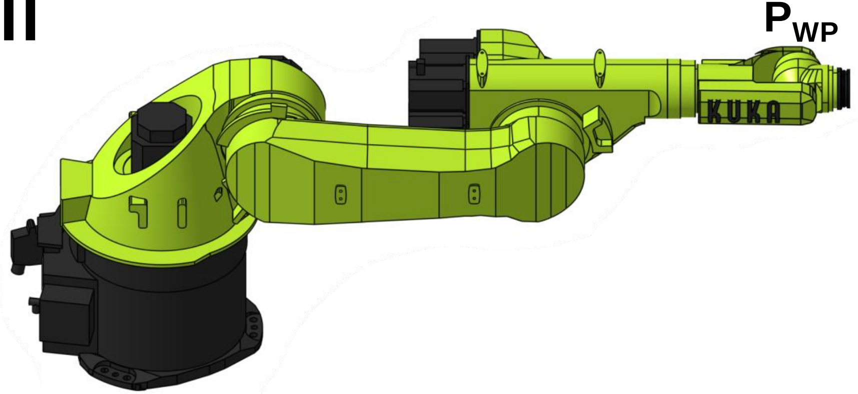
3.3 Calculation of Θ_2

3.4 Calculation of Θ_3

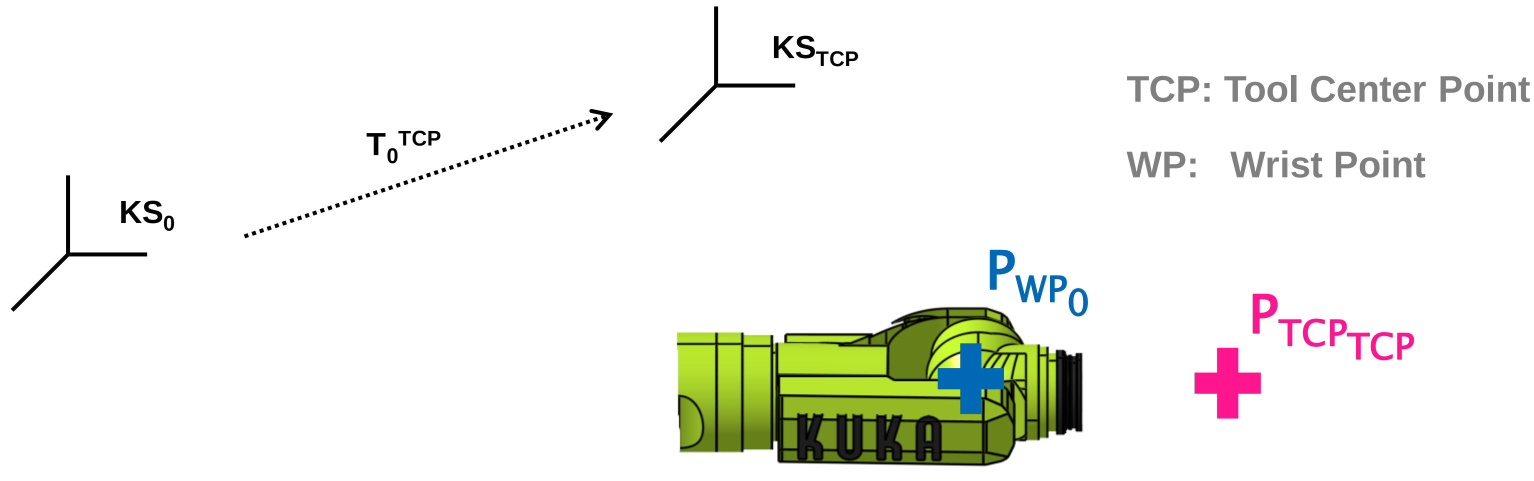


4. Conclusion

5. Appendix

3.1 P_{WP}: Calculation of Wrist Point

Input: T_0^{TCP} (transformation of KS_0 to KS_{TCP})



$P_{WP_{TCP}} = (0, 0, -d_6)$ … (depends on how KS_{TCP} looks like)

$P_{WP_0} = T_0^{TCP} \cdot (0, 0, -d_6)$

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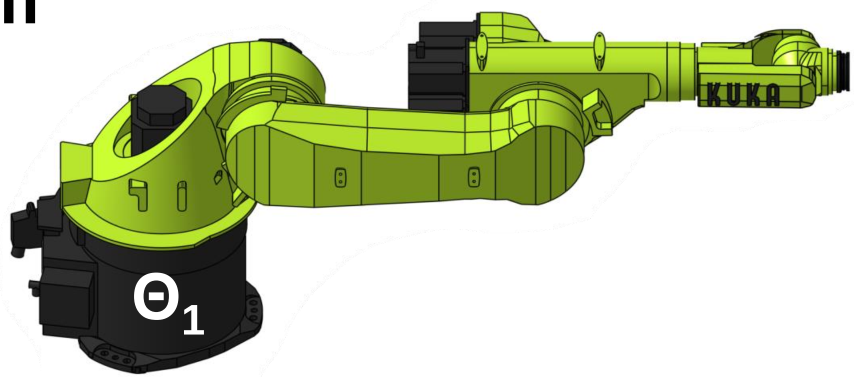

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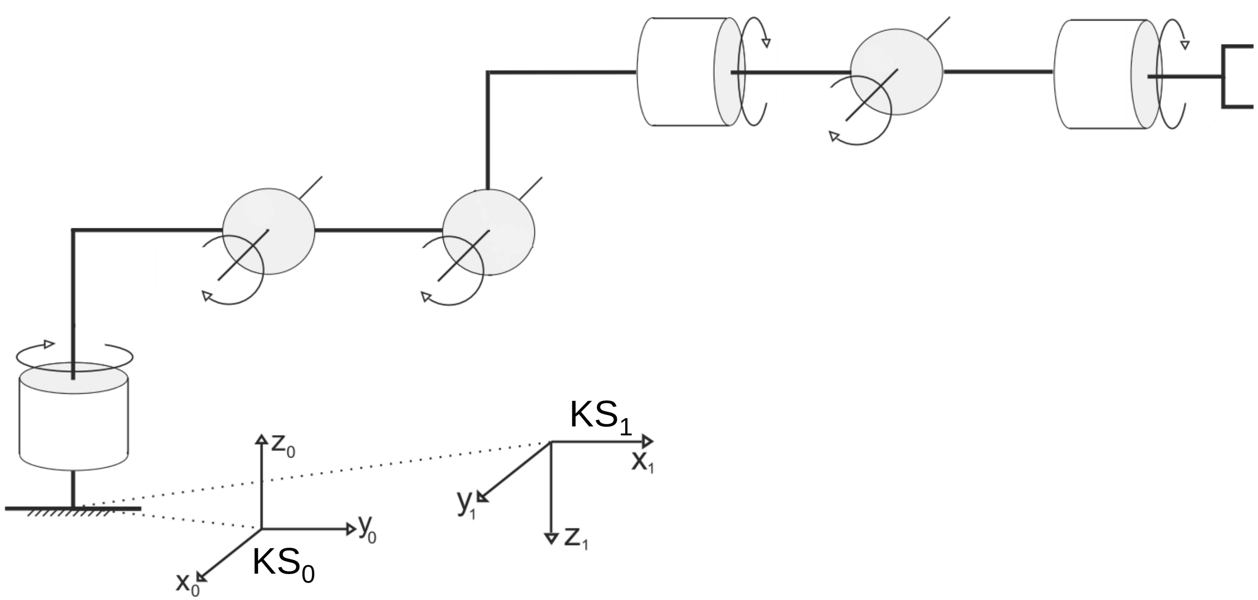
3.2 Θ_1: Transformation of P_{WP_0} to KS_1

For Calculation of Θ_1 in KS_1 → Transformation of P_{WP_0} to P_{WP_1}

$$\mathbf{T_0^1} = Rot_{z_0}(\Theta_0) \cdot Trans_{z_0}(d_0) \cdot Trans_{x_1}(a_0) \cdot Rot_{x_1}(\alpha_0)$$

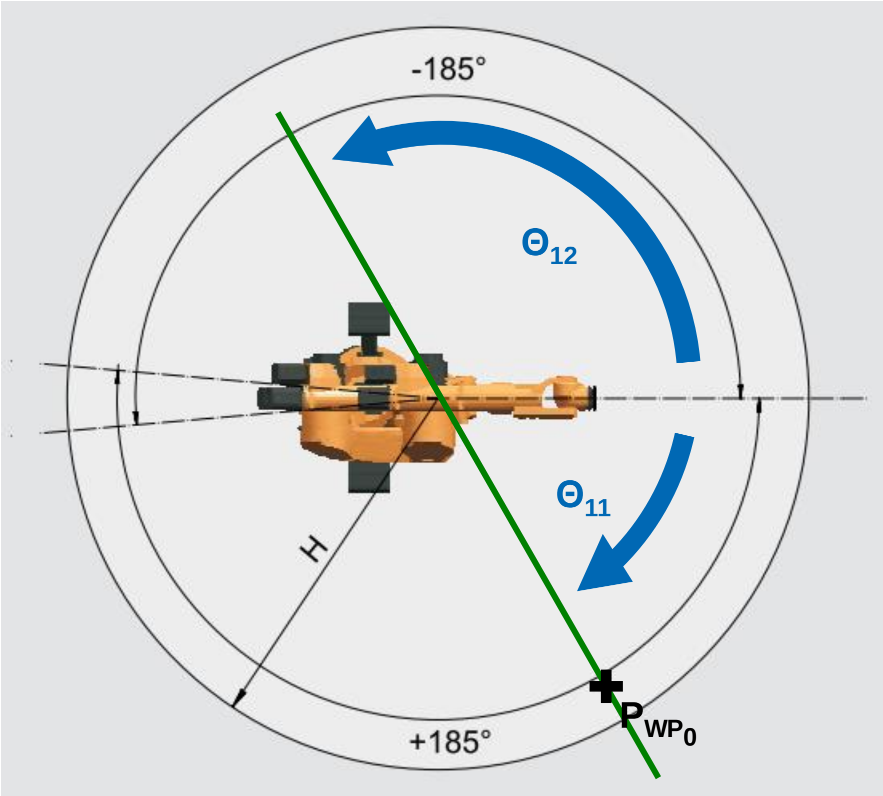
$$\mathbf{P_{WP_0}} = T_0^1 \cdot P_{WP_1} \quad \rightarrow \quad \mathbf{P_{WP_1}} = (T_0^1)^{-1} \cdot P_{WP_0}$$

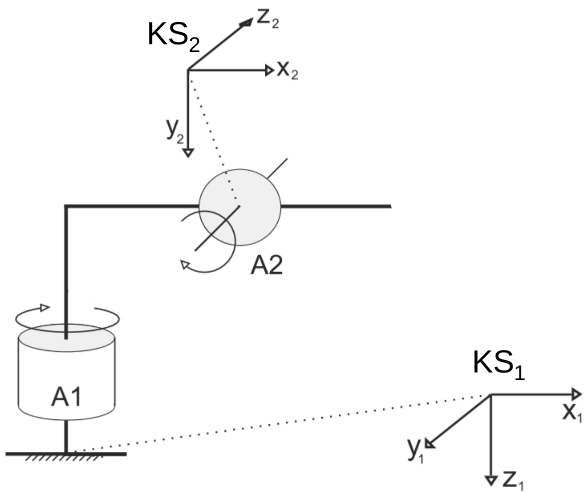
$\Theta_0 = \pi/2$
$d_0 = 0$
$a_0 = 0$
$\alpha_0 = \pi$



3.2 Θ_1: Calculation of Θ_1

Calculation of Θ_1 (top view) in KS_1





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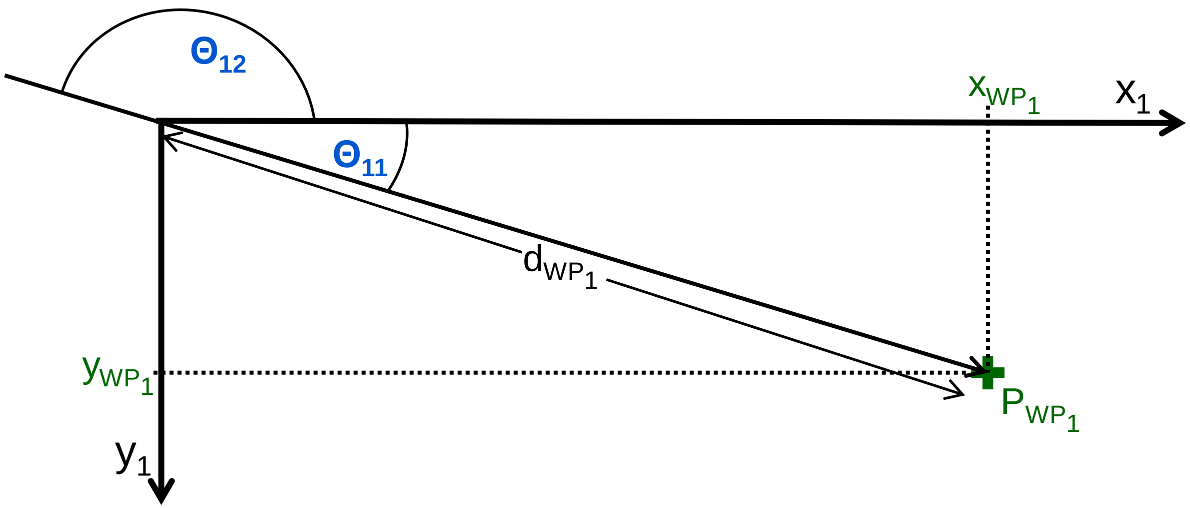
3.2 Θ_1: Calculation of Θ_1

Calculation of Θ_1 (topview) in KS_1

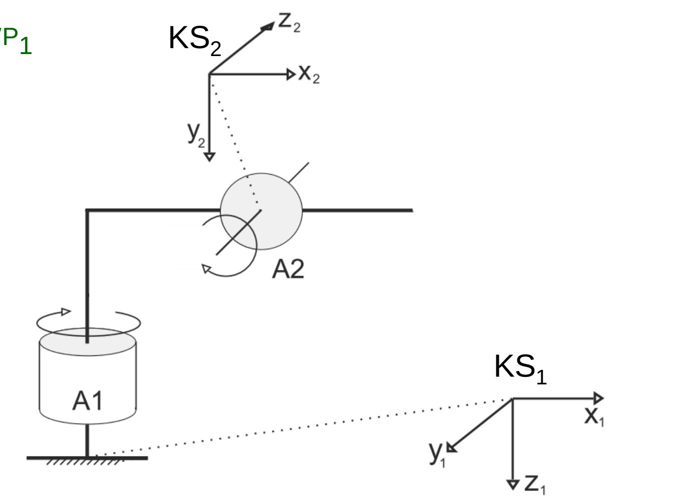


$$\tan(\Theta_{11}) = \frac{\sin(\Theta_{11})}{\cos(\Theta_{11})}$$

$$\sin(\Theta_{11}) = \frac{y_{WP_1}}{d_{WP_1}}$$

$$\cos(\Theta_{11}) = \frac{x_{WP_1}}{d_{WP_1}}$$

$$d_{WP_1} = \sqrt{x_{WP_1}^{\ 2} + y_{WP_1}^{\ 2}}$$



$\Theta_{11} = \text{atan2}(x_{WP_1}, y_{WP_1})$

if $(\Theta_{11} < 0)$ then $\Theta_{12} = \Theta_{11} + \pi$

else $\Theta_{12} = \Theta_{11} - \pi$

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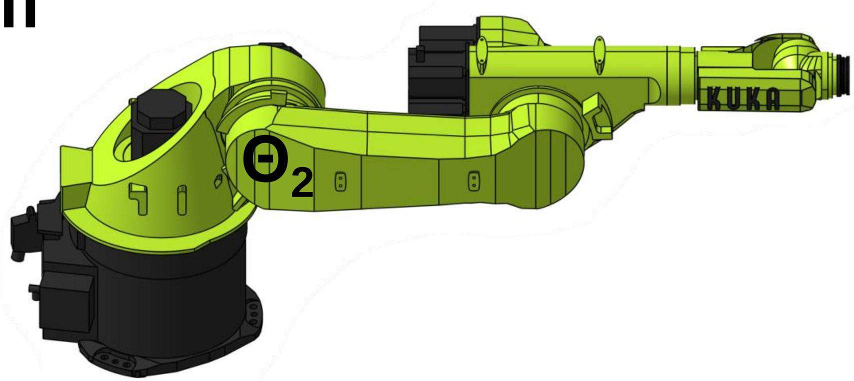

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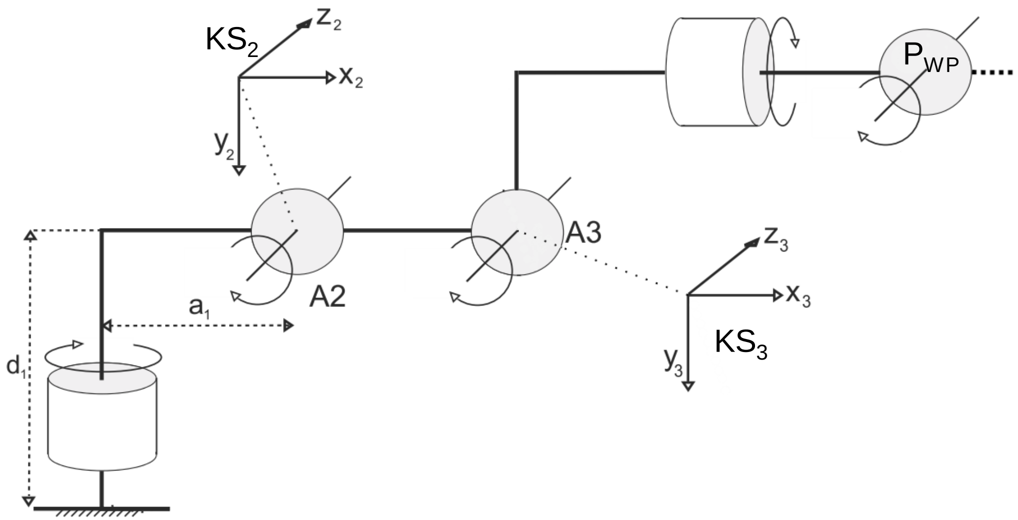
3.3 Θ_2: Distinction of cases for Θ_1

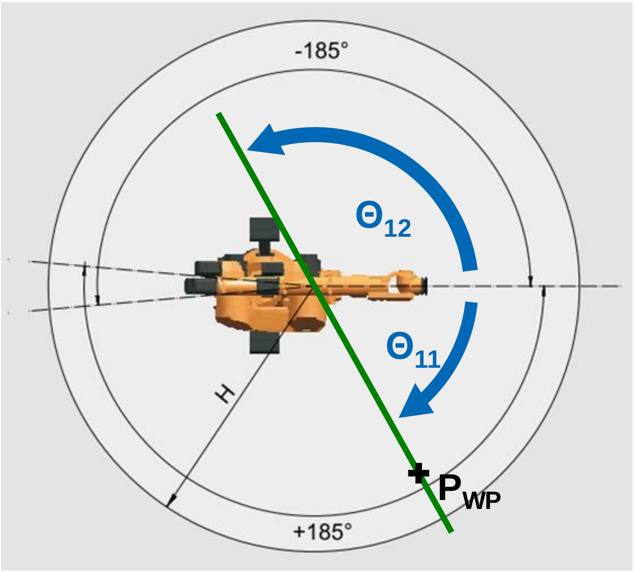
For Calculation of Θ_2 in KS_2 → Transformation of P_{WP_1} to P_{WP_2}

$$\mathbf{T_1}^2 = Rot_{z_1}(\Theta_1) \cdot Trans_{z_1}(d_1) \cdot Trans_{x_2}(a_1) \cdot Rot_{x_2}(\alpha_1)$$

$$\mathbf{P_{WP_1}} = T_1{}^2 \cdot P_{WP_2} \quad \rightarrow \quad \mathbf{P_{WP_2}} = (T_1{}^2)^{-1} \cdot P_{WP_1}$$

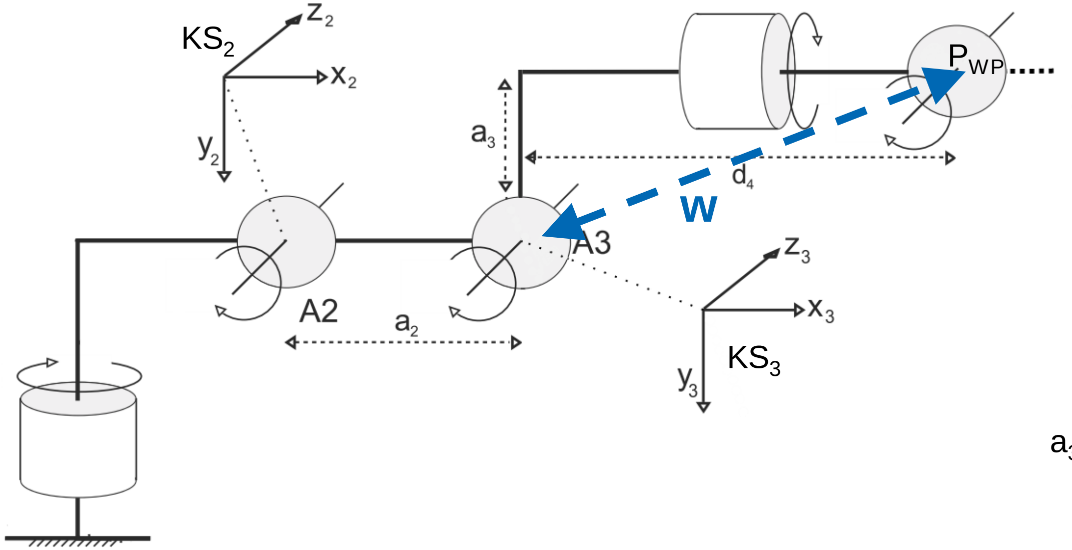
Distinction of cases for Θ_{11} and Θ_{12} → $\mathbf{P_{WP_{21}}}$ and $\mathbf{P_{WP_{22}}}$

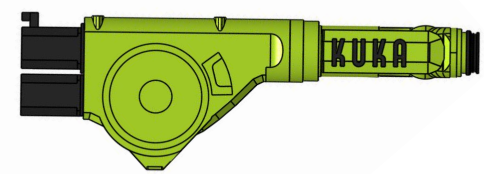


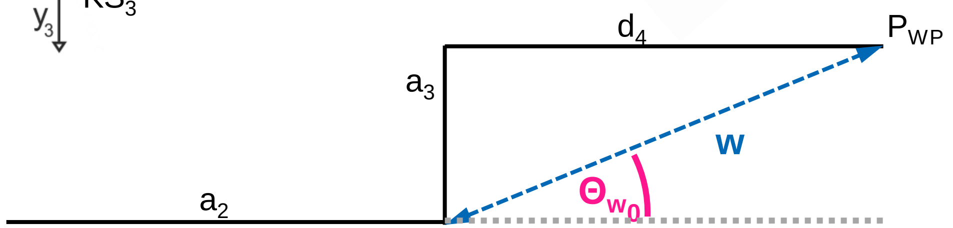


3.3 Θ_2: Simplification of the problem

For simplification: uniquely calculation of w and Θ_{w_0}





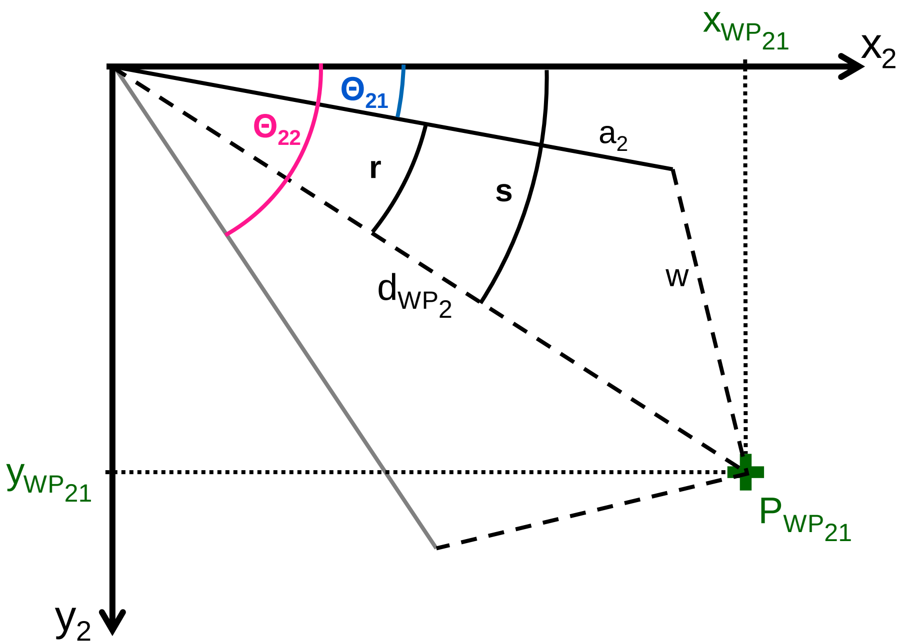


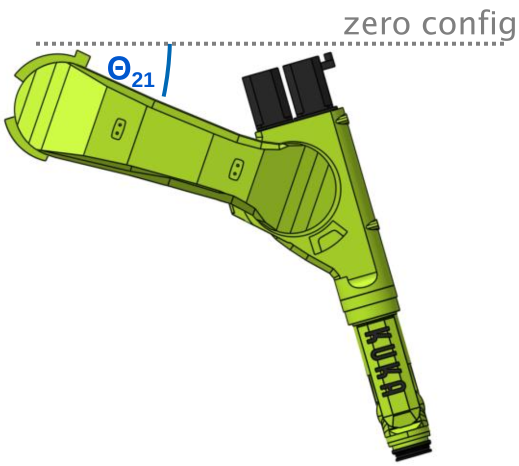
$$\mathbf{w} = \sqrt{a_3^2 + d_4^2} \quad \rightarrow \quad \text{Calculation of } \Theta_2$$

$$\Theta_{w_0} = \operatorname{atan2}(d_4, a_3) \quad \rightarrow \quad \text{Calculation of } \Theta_3$$

3.3 Θ_2: Calculation of Θ_2

Calculation of Θ_{21} and Θ_{22} with $P_{WP_{21}}$ in x_2–y_2–plane (KS_2)

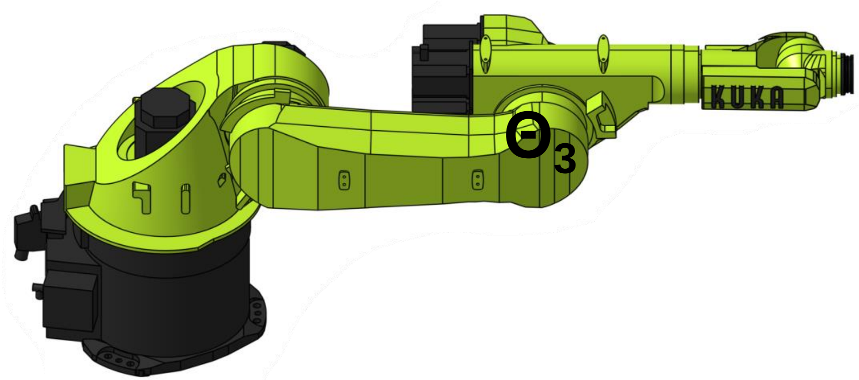




$\Theta_{21} = s - r$ $\quad$ $\Theta_{22} = s + r$ $\quad$ $s = \operatorname{atan2}(x_{WP_{21}}, y_{WP_{21}})$

cosine rule $\rightarrow$ $w^2 = {d_{WP_{21}}}^2 + {a_2}^2 - 2 \cdot d_{WP_{21}} \cdot a_2 \cdot \cos(r)$

conversion $\rightarrow$ $\mathbf{r} = \operatorname{acos}\left(\dfrac{w^2 - {d_{WP_{21}}}^2 - {a_2}^2}{-2 \cdot d_{WP_{21}} \cdot a_2}\right)$


KUKA
Θ_3

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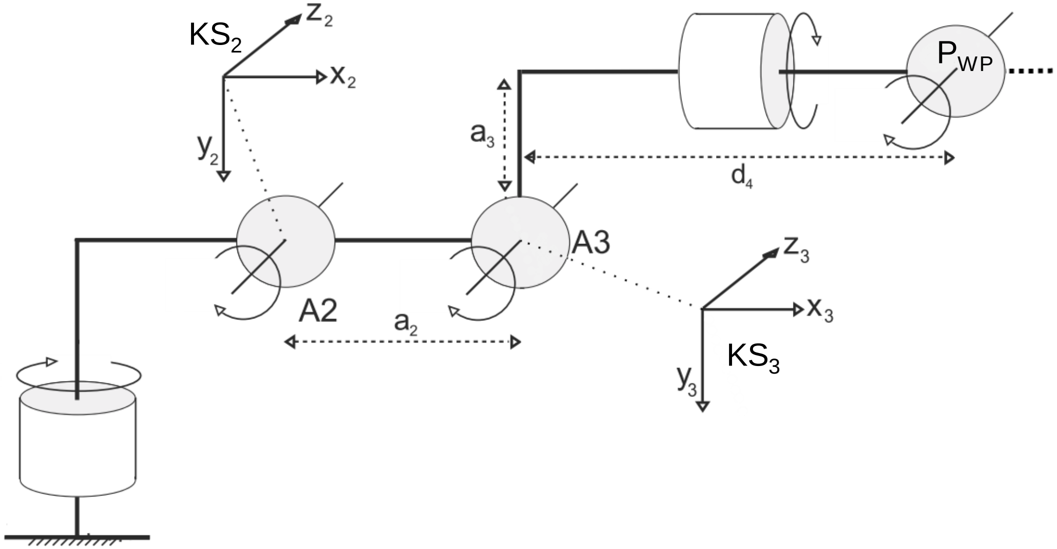
3.4 Θ_3: Distinction of cases for Θ_2

For Calculation of Θ_3 in KS_3 → Transformation of P_{WP_2} to P_{WP_3}

$$T_2{}^3 = Rot_{z_2}(\Theta_2) \cdot Trans_{z_2}(d_2) \cdot Trans_{x_3}(a_2) \cdot Rot_{x_3}(\alpha_2)$$

$$\mathbf{P_{WP_2}} = T_2{}^3 \cdot P_{WP_3} \rightarrow \mathbf{P_{WP_3}} = (T_2{}^3)^{-1} \cdot P_{WP_2}$$

Distinction of cases for Θ_{21} and Θ_{22} → $\mathbf{P_{WP_{31}}}$ and $\mathbf{P_{WP_{32}}}$



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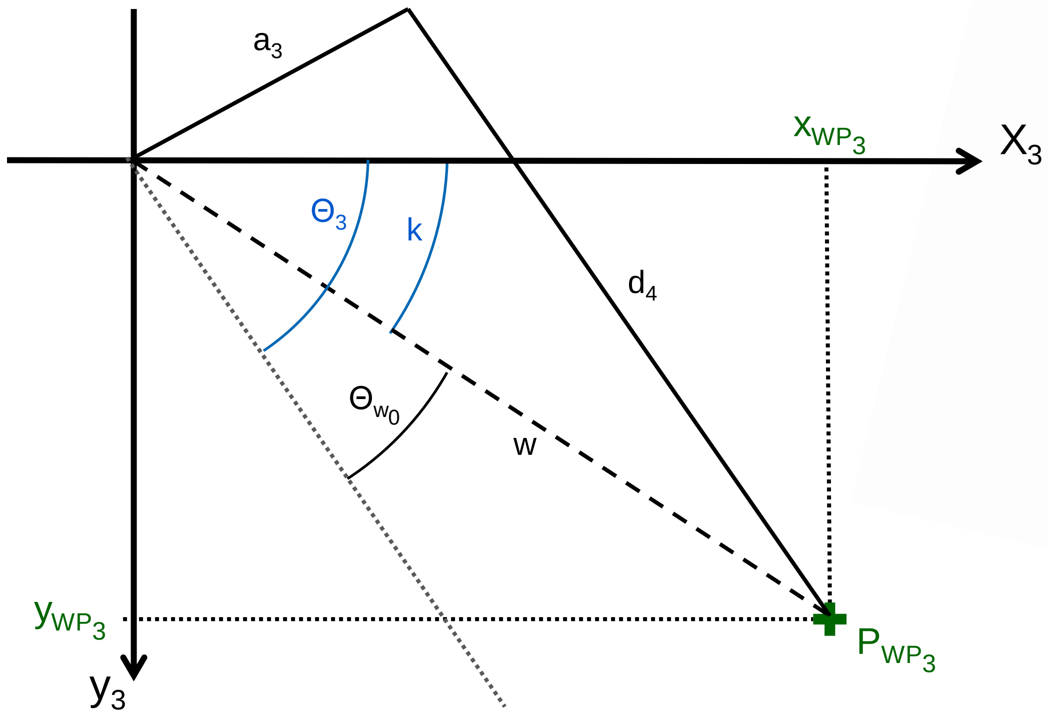
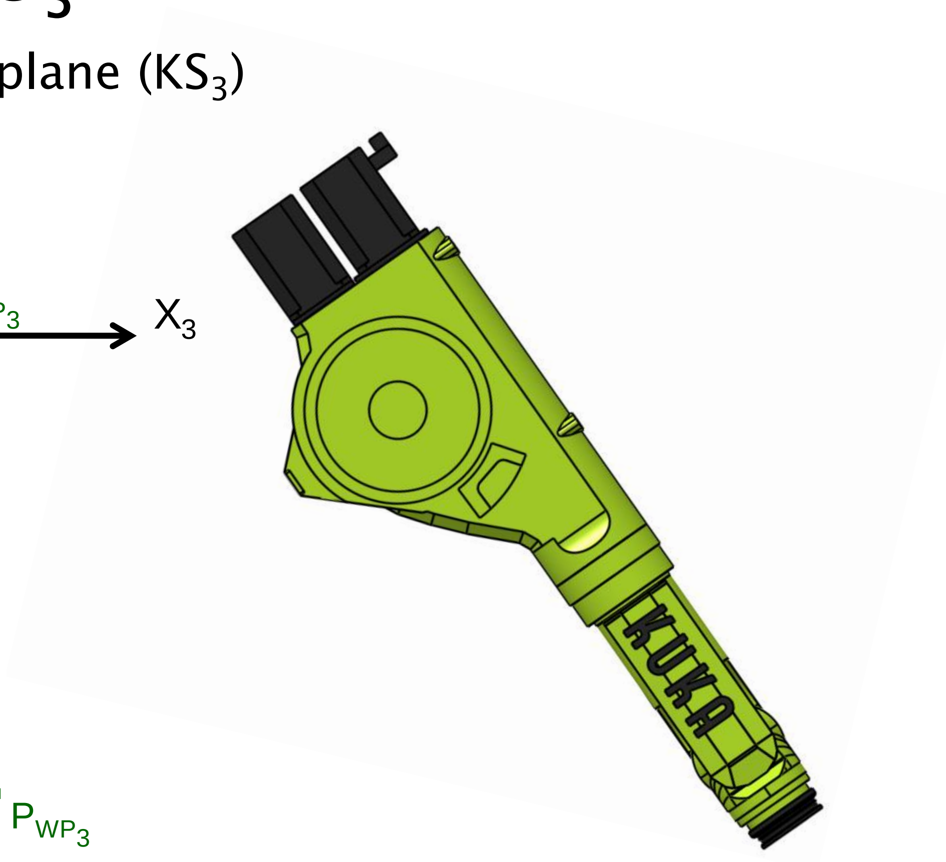
3.4 Θ_3: Calculation of Θ_3

Calculation of Θ_3 with P_{WP_3} in x_3–y_3–plane (KS_3)





$$\Theta_3 = \Theta_{w_0} + k$$

$$k = \mathrm{atan2}(x_{WP_3}, y_{WP_3})$$

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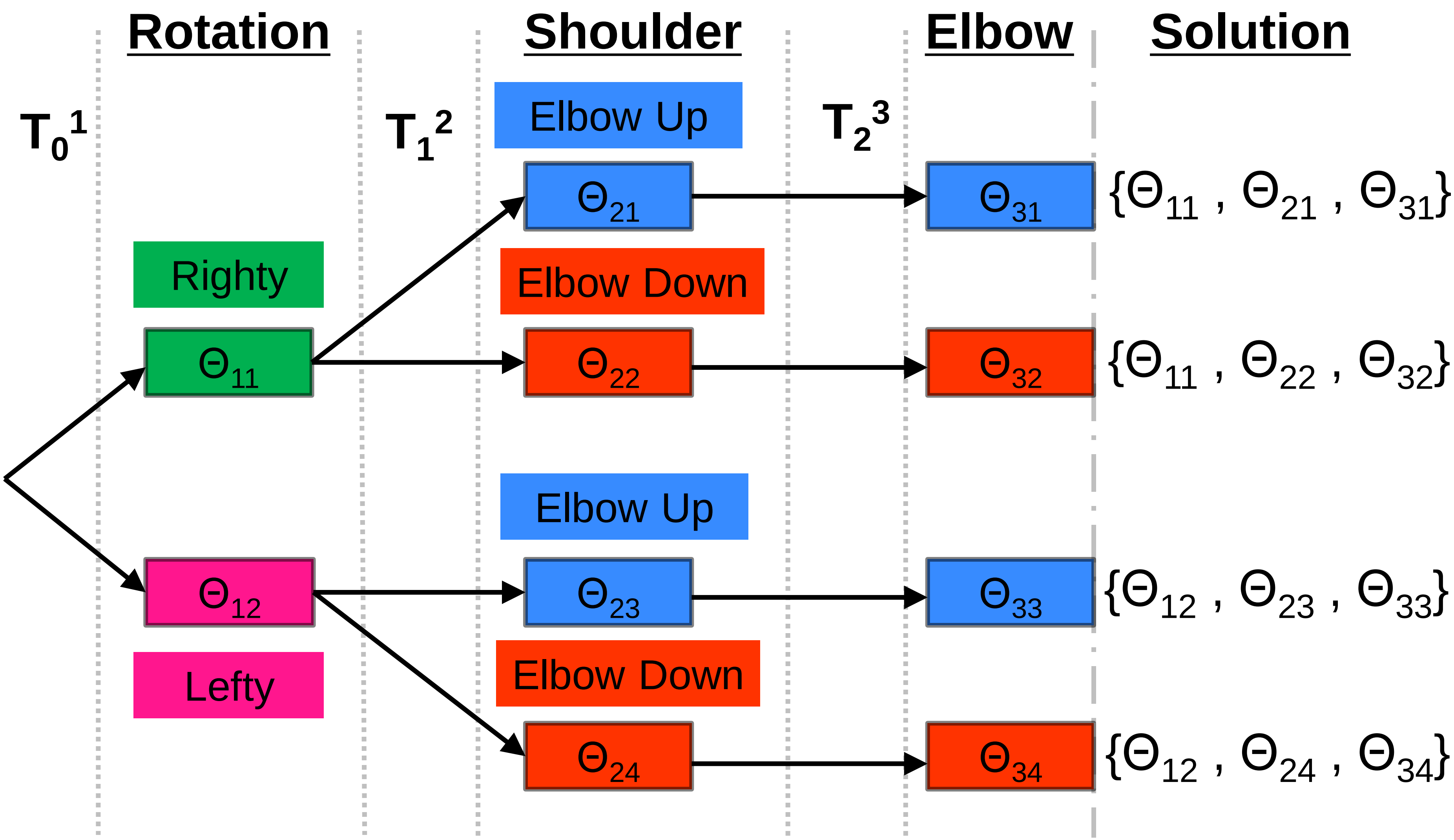
4. Conclusion: Overview of Solutions



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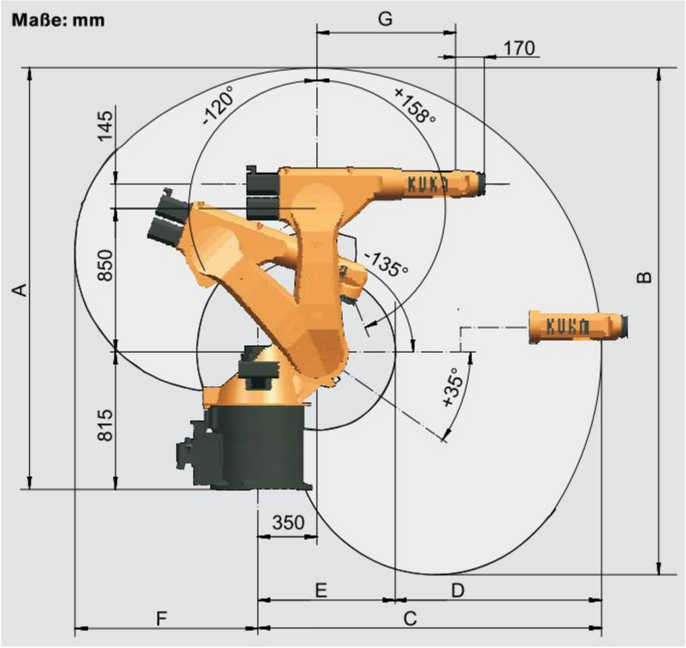
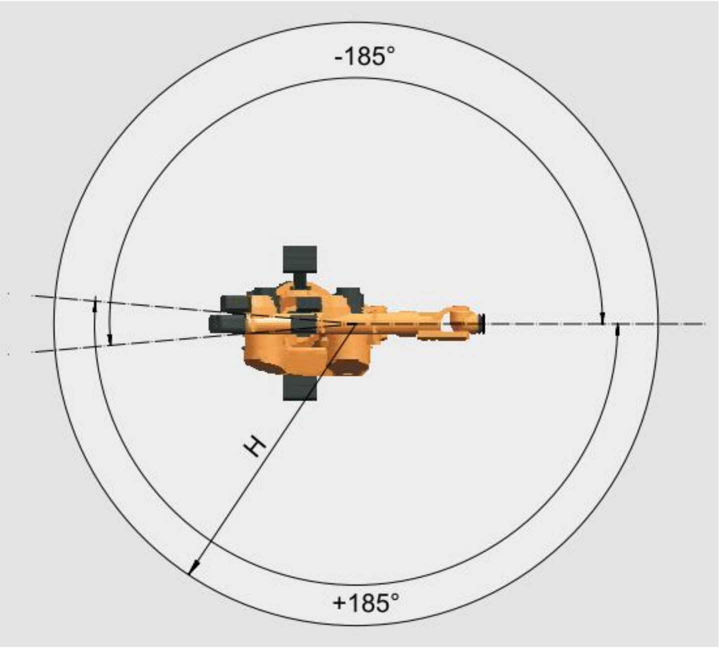
4. Conclusion: Control of joint constraints

angle	min value	max value
Θ_1	-185°	+185°
Θ_2	-135°	+35°
Θ_3	-120°	+158°





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4. Formula summery

$P_{WP_0} = T_0^{TCP} \cdot (0, 0, -d_6)$	Calculation of wrist point (KS_0)
$P_{WP_1} = (T_0^{1})^{-1} \cdot P_{WP_0}$	Transformation of wrist point to KS_1
$\Theta_{11} = \mathrm{atan2}(x_{WP_1}, y_{WP_1})$ $\rightarrow \Theta_{12} = \Theta_{11} +/- \pi$	Calculation of Θ_1 in KS_1 [independent]
$P_{WP_2} = (T_1^{2})^{-1} \cdot P_{WP_1}$	Transformation of P_{WP_1} with Θ_1 to KS_2
$\Theta_2 = \mathrm{atan2}(x_{WP_{21}}, y_{WP_{21}})$ $+/- \mathrm{acos}\left(\frac{w^2 - d_{WP_{21}}^{\ 2} - a_2^{\ 2}}{-2 \cdot d_{WP_{21}} \cdot a_2}\right)$	Calculation of Θ_2 in KS_2 [depends on Θ_1]
$P_{WP_3} = (T_2^{3})^{-1} \cdot P_{WP_2}$	Transformation of P_{WP_2} with Θ_2 to KS_3
$\Theta_3 = \mathrm{atan2}(d_4, a_3)$ $+ \mathrm{atan2}(x_{WP_3}, y_{WP_3})$	Calculation of Θ_3 in KS_3 [depends on Θ_2]

Many thanks for your attention!

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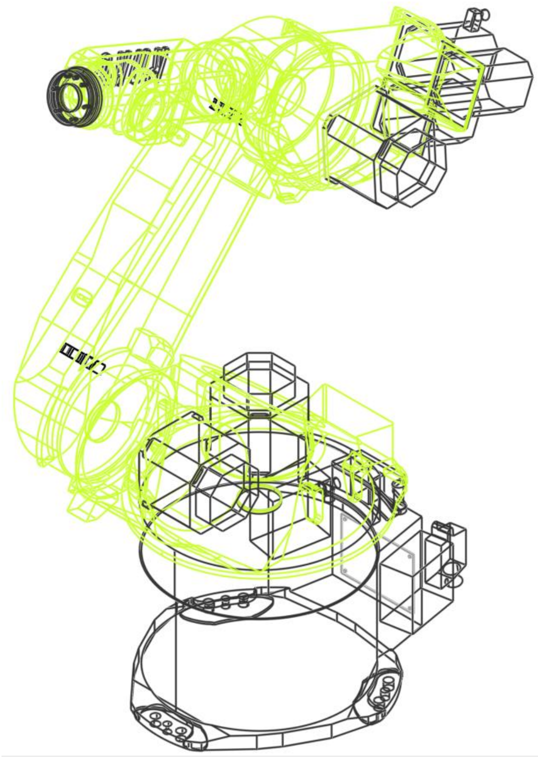

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5. Appendix: DH parameters

$$T_i^{i+1} = Rot_{z_i}(\Theta_i) \cdot Trans_{z_i}(d_i) \cdot Trans_{x_{i+1}}(a_i) \cdot Rot_{x_{i+1}}(\alpha_i)$$

i	Θ	d	a	α
0	$\pi/2$	0	0	π
1	Θ_1	$-d_1$	a_1	$\pi/2$
2	Θ_2	0	a_2	0
3	$\pi/2 + \Theta_3$	0	a_3	$-\pi/2$
4	Θ_4	$-d_4$	0	$\pi/2$

Compute Θ_1, Θ_2, and Θ_3

Θ_4 is not relevant for positioning problem

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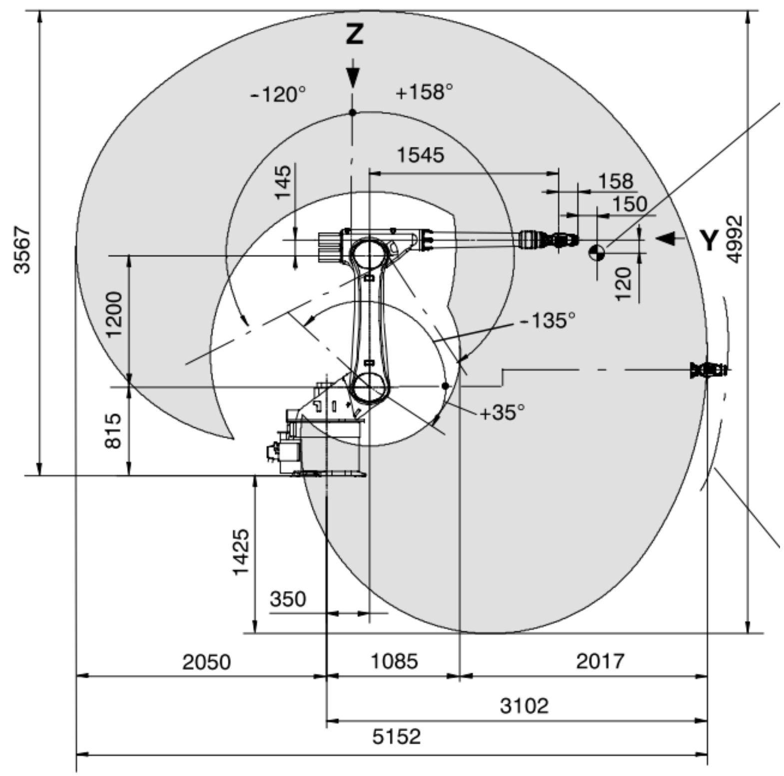
5. Appendix: Dimensions of KR30–L16

Constants	Value in mm
d_1	815
a_1	350
a_2	1200
a_3	145
d_4	1545
d_6	158



Selfmade parameters	Calculation	Value
w	$\sqrt{a_3{}^2 + d_4{}^2}$	ca. 1551,789mm
Θ_{w0}	atan2(d_4, a_3)	ca. 5,362°

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5. Appendix: Why we use *atan2(x, y)*?

- Two arguments: Information on the signs of inputs
  - → Distinction between opposite directions in KS
- Programming: No need to take care of KS quadrants
  - → less programm logic
- atan2(y, x) is the angle in radians between the positive x-axis of a plane and the point given by the coordinates (x, y)

$$\operatorname{atan2}(x, y) = \begin{cases} \arctan\left(\frac{y}{x}\right) & x > 0 \\ \arctan\left(\frac{y}{x}\right) + \pi & y \geq 0, x < 0 \\ \arctan\left(\frac{y}{x}\right) - \pi & y < 0, x < 0 \\ +\frac{\pi}{2} & y > 0, x = 0 \\ -\frac{\pi}{2} & y < 0, x = 0 \\ \text{undefined} & y = 0, x = 0 \end{cases}$$

$y > 0$: upper half-plane
$y < 0$: lower half-plane

Quelle: Wikipedia

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bibliography

1) Lee, Ziegler: *A geometric approach in solving the inverse kinematics of puma robots*. The University of Michigan, 1983.
2) Linnemann: *Robotortechnik* (Vorlesung). Beuth Hochschule für Technik Berlin, 2013.
3) Kuka Roboter GmbH: *KUKA KR30-3 Datenblatt*. Gefunden auf http://www.kuka-robotics.com/res/sps/f776ebab-f613-4818-9feb-527612db8dc4_PF0042_KR_30-3_KR_60-3_de.pdf. Abgerufen am 09.11.13.
4) Bjerkeng: *Coordinated Control with Obstacle Avoidance for Robot Manipulators - Chapter 3*. Norwegian University of Science and Technology, 2010.
5) Suchý: *Grundlagen der Robotik* (Vorlesung). Technische Universität Chemniz, 2010.