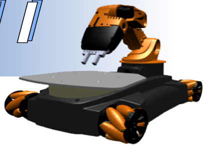
EduFill Advanced Curriculum - Manipulation module

Lecture 1



Summary

Advanced: Affine transformations

An *affine transformation* or *map* is a composition of two functions:

- Translation
- Linear map (In this context, the *rotation* map).

Vector algebra uses matrix multiplication to apply linear maps, while vector addition represents translations. In order to represent both the translation and the linear map, the concept of *augmented matrix* is introduced.

Augmented matrix

Given:

- A linear map R which describes the rotation of a frame $i+1$ in relation to a frame i;
- a translation vector p which describes the translation of a frame $i+1$ in relation to a frame i;

The Augmented matrix T which represents the affine transform of frame $i+1$ in relation to frame i is defined as:

$$T^i_{i+1} = \left[\begin{array}{c|c} R^i_{i+1} & p^i_{i+1} \\ \hline 0,\dots,0 & 1 \end{array}\right]$$

Augmented matrix

This transform can be applied to a position vector to change its reference frame coordinates[1];

$$\begin{bmatrix} p^i \\ \hline 1 \end{bmatrix} = T^i_{i+1} \begin{bmatrix} p^{i+1} \\ \hline 1 \end{bmatrix}$$

Or, more generally, one can concatenate transforms:

$$p^{i-1} = \quad T^{i-1}_{i+1}\, p^{i+1} \quad = \quad (T^{i-1}_{i}\; T^{i}_{i+1})\; p^{i+1}$$

[1] The apexes are not numerical exponents, but, as described by the transform definition, they stands for the fixed reference frame indexes.

Denavit-Hartenberg parameters

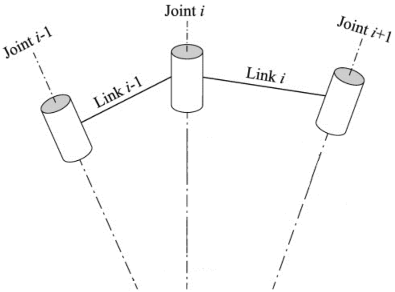
The building of the kinematic chain of a manipulator involves the use of different frames of reference and the corresponding affine transformations between them.

In an open-link kinematic chain, transformations can be described as combinations of rotations and translations for each of every coordinate axis.

Ideally, the best approach would be to describe reference frame transforms by using only a subset of all the possible transform combinations; Denavit-Hartenberg convention adopts this approach.

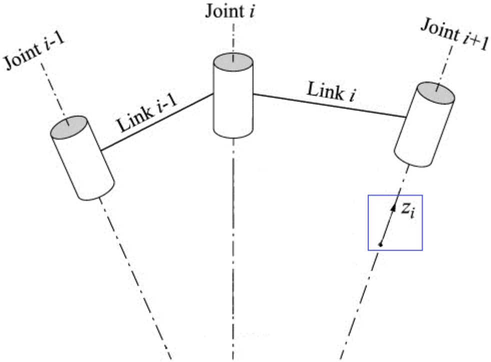
Denavit-Hartenberg procedure - I

Denavit-Hartenberg convention describes transforms by considering *rotations* and *translations* on *x* and *z* axes.



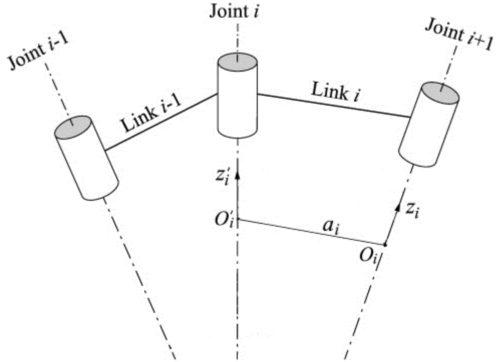
Assuming as i the joint axis which connects the link $i-1$ to the link i, as depicted in the above image, the procedure is the following:

Denavit-Hartenberg procedure - II



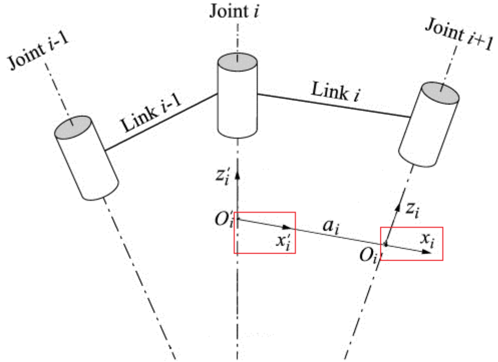
- The z_i axis is defined as laying on the axis of the $i+1$ joint;

Denavit-Hartenberg procedure - III



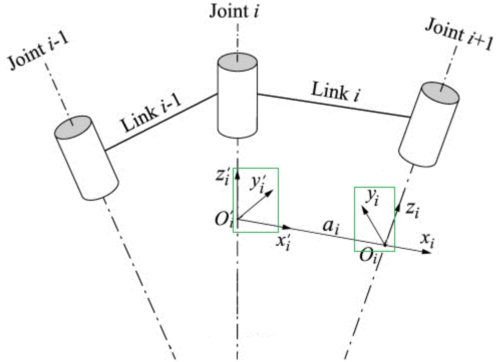
- O_i is identified as the intersection of the z_i axis with the common normal a_i between z_{i-1} (here called z_i') and z_i axes;
- O_i' is identified as the intersection of the common normal with z_{i-1};

Denavit-Hartenberg procedure - IV



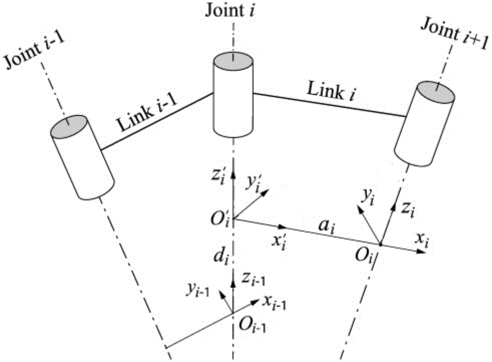
- The x_i axis is identified along the common normal a_i, with positive direction from joint i to $i+1$;
- x_{i-1} axis (named z_i') lays on the same a_i, with O_i' as its origin;

Denavit-Hartenberg procedure - V



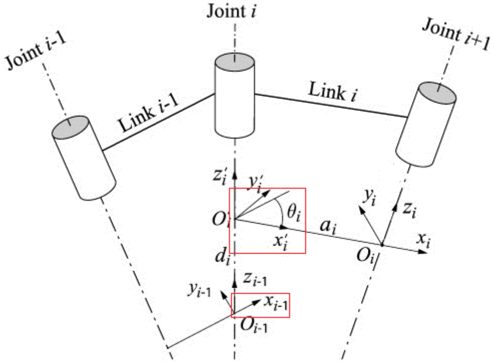
- y_i axis is chosen in order to build a right-handed frame of reference;
- The same applies to find y_{i-1} (depicted in the picture as y_i') axis;

Denavit-Hartenberg procedure - VI



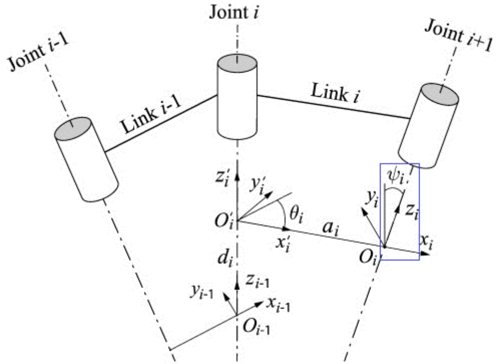
- Considering O_{i-1}:
  d_i is the coordinate of O_i' along z_{i-1};

Denavit-Hartenberg procedure - VII



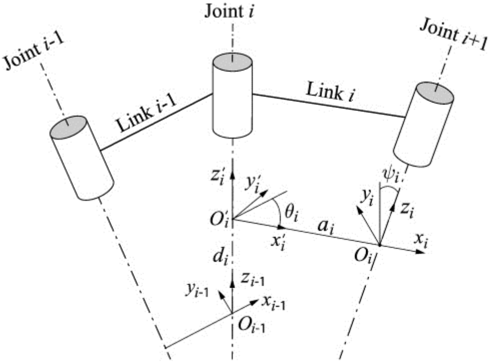
- θ_i is the angle around z_{i-1} axis between x_{i-1} and x_i axes;

Denavit-Hartenberg procedure - VIII



- ψ_i is the angle around x_i axis between z_{i-1} and z_i axes;

Denavit-Hartenberg procedure - IX



- All other reference frames transforms parameters (θ, ψ, d, a)[2] can be obtained in this way, recursively.

[2]Depending on nomenclature varition, α is usually found instead of ψ.

Summary

Computation with Denavit-Hartenberg table

By having the parameters, it is possible to express the coordinate transform which links the i reference frame to the $i-1$ reference frame. Starting from a reference frame corresponding to the $i-1$ frame, it can be obtained by following these steps:

1. Translate it of d_i along z_{i-1} axis, rotating it by θ_i around z_{i-1}.
   This operation brings the reference frame exactly on the i' reference frame; we can describe this transform as $A_{i'}^{i-1}$.
2. Translate it of a_i along x_i' axis, rotating it by ψ_i around x_i'.
   This operation bring the reference frame exactly on the i reference frame, and we can describe this transform as $A_i^{i'}$.

Computation with Denavit-Hartenberg table

The overall transformation is obtained multiplying the previous transforms as it follows:

$$A_i^{i-1}(q_i) = A_{i'}^{i-1}\ A_i^{i'} = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\psi_i} & s_{\theta_i}s_{\psi_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\psi_i} & -c_{\theta_i}s_{\psi_i} & a_i s_{\theta_i} \\ 0 & s_{\psi_i} & c_{\psi_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In order to compute the forward kinematics of a manipulator, this matrix must be evaluated for every joint i.
The forward kinematics, which links the frame 0 to the n frame, is computed by concatenating the single transforms:

$$T_n^0(q) = A_1^0 \dots A_n^{n-1}$$

Summary

1. Kinematic chain representation
2. Advanced: Forward Kinematics
3. Examples

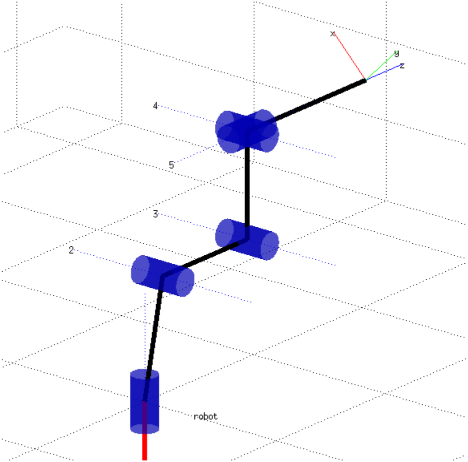
DH parameters for youBot manipulator

Here follows the Denavit-Hartenberg parameters table for the youBot manipulator. This is a minimal parameter table for the arm.

- Exactly one row is sufficient to describe the transform from one frame to anoter;
- An offset is added to the fourth joint variable, in order to have conventionally the arm straight horizontal, when all joint values are set to zero.

Link	θ	d	a	α
1	q_1	0.147	0.0330	$\frac{\pi}{2}$
2	q_2	0	0.1550	0
3	q_3	0	0.1350	0
4	$q_4 + \frac{\pi}{2}$	0	0	$\frac{\pi}{2}$
5	q_5	0.2175	0	0

Arm visualization



Generated structure for joint values $[0,\ 0,\ \frac{\pi}{2},\ -\frac{\pi}{2},\ -\frac{\pi}{4}]$