



Computer Systems in Engineering

Software Development for Industrial Robots

3. Homogenous coordinates

- Define some new mapping:

$$\mathbf{V} \rightarrow \mathbf{V} \times \mathbf{R}$$
$$\begin{pmatrix} v_1 \\ \dots \\ v_n \end{pmatrix} \mapsto \begin{pmatrix} v_1 \\ \dots \\ v_n \\ 1 \end{pmatrix}$$

- The new $(n+1)$ -dimensional vector space is called homogenous coordinates

- Benefit: Rotations and translations may be described as matrix operations

- Example: Rotation in 2D

$$A_{\alpha} := \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Example: Translation in 2D

$$A_{\begin{pmatrix} x \\ y \end{pmatrix}} := \begin{pmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix}$$

- In general:

$$T(\alpha, \vec{p}): V \rightarrow V$$

$$v \mapsto \begin{pmatrix} R_\alpha & \vec{p} \\ 0 & 1 \end{pmatrix} v = \begin{pmatrix} \cos \alpha & -\sin \alpha & p_1 \\ \sin \alpha & \cos \alpha & p_2 \\ 0 & 0 & 1 \end{pmatrix} v$$

- Inverse of this mapping:

$$T^{-1}(\alpha, \vec{p}): V \rightarrow V$$

$$v \mapsto \begin{pmatrix} R_\alpha^{-1} & -R_\alpha^{-1} \vec{p} \\ 0 & 1 \end{pmatrix} v = \begin{pmatrix} \cos \alpha & \sin \alpha & -p_1 \cos \alpha - p_2 \sin \alpha \\ -\sin \alpha & \cos \alpha & p_1 \sin \alpha - p_2 \cos \alpha \\ 0 & 0 & 1 \end{pmatrix} v$$

Definition: Co-Sys Transformation

Let CS' be some coordinate system defined by some translation-rotation $T\left(\alpha, \begin{pmatrix} x \\ y \end{pmatrix}\right)$ in CS . Then some point P with coordinate vector v' in CS' , can also be described by the coordinate vector

$$v = T\left(\alpha, \begin{pmatrix} x \\ y \end{pmatrix}\right)v'$$

in CS .

Corollar:

Consequently, a point P with coordinate vector v' in CS' , can also be described by the coordinate vector

$$v' = T^{-1} \left(\alpha, \begin{pmatrix} x \\ y \end{pmatrix} \right) v$$

in CS' .

Consequence:

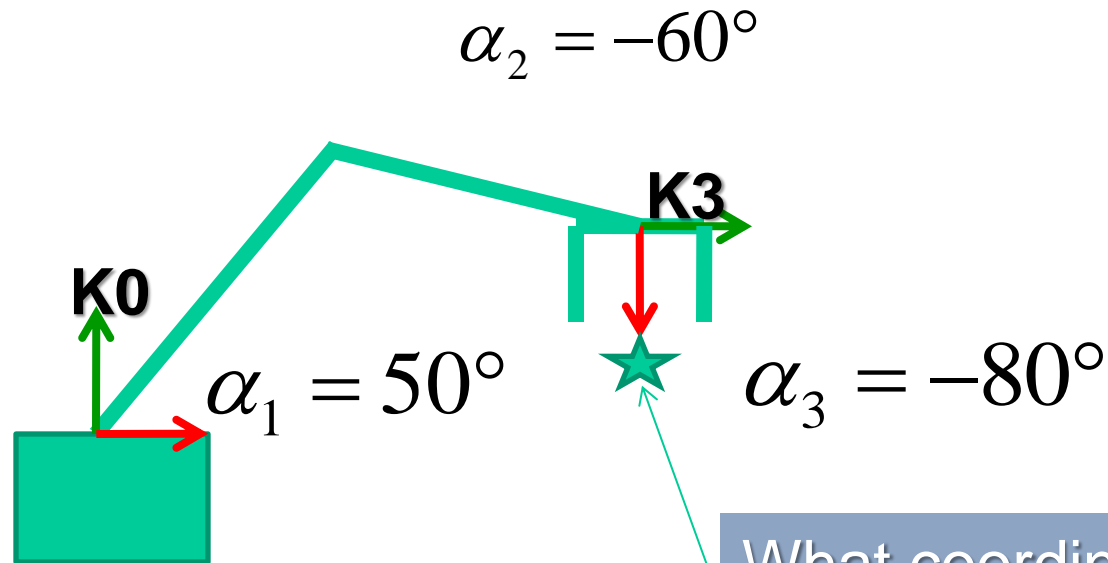
Switching coordinate systems is very easy!

Enough theory!

→ Let's use this stuff!

4. (2D) Direct kinematics

- Example: 2-arm robot (in some 2D disc world)



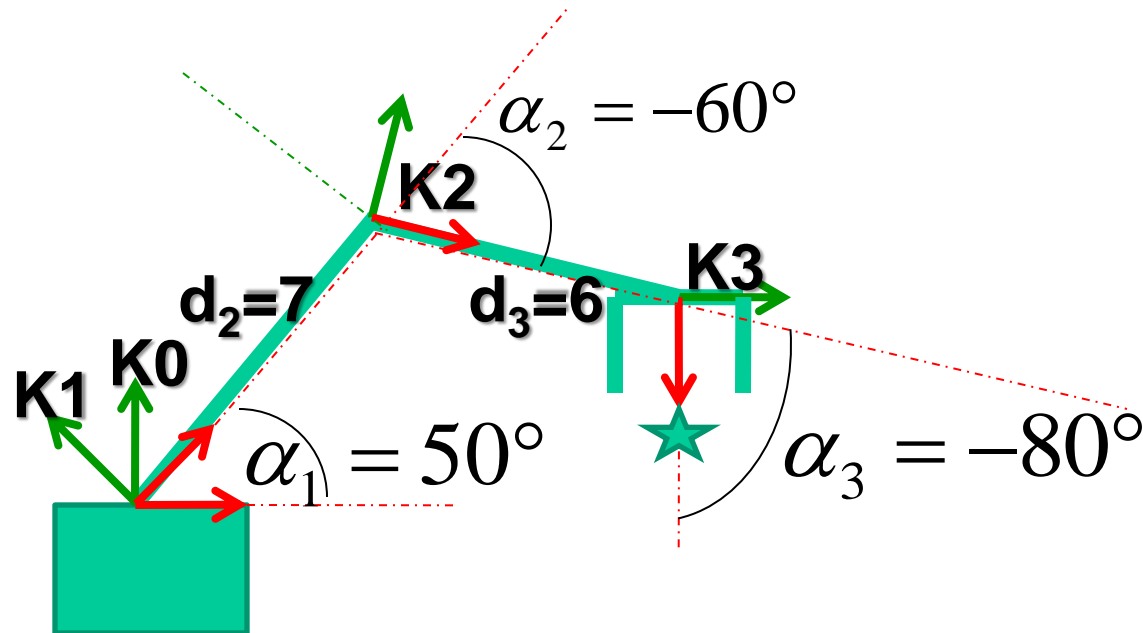
What coordinates does this point have in CS?

$$(x''', y''') = (2, 0)$$

- **Definition:**
The direct kinematics of a robot is an operation which takes joint angles as input and calculates the position of the robot's hand.
- **Core idea:**
Direct kinematics is simply a set of well-defined coordinate system transformations

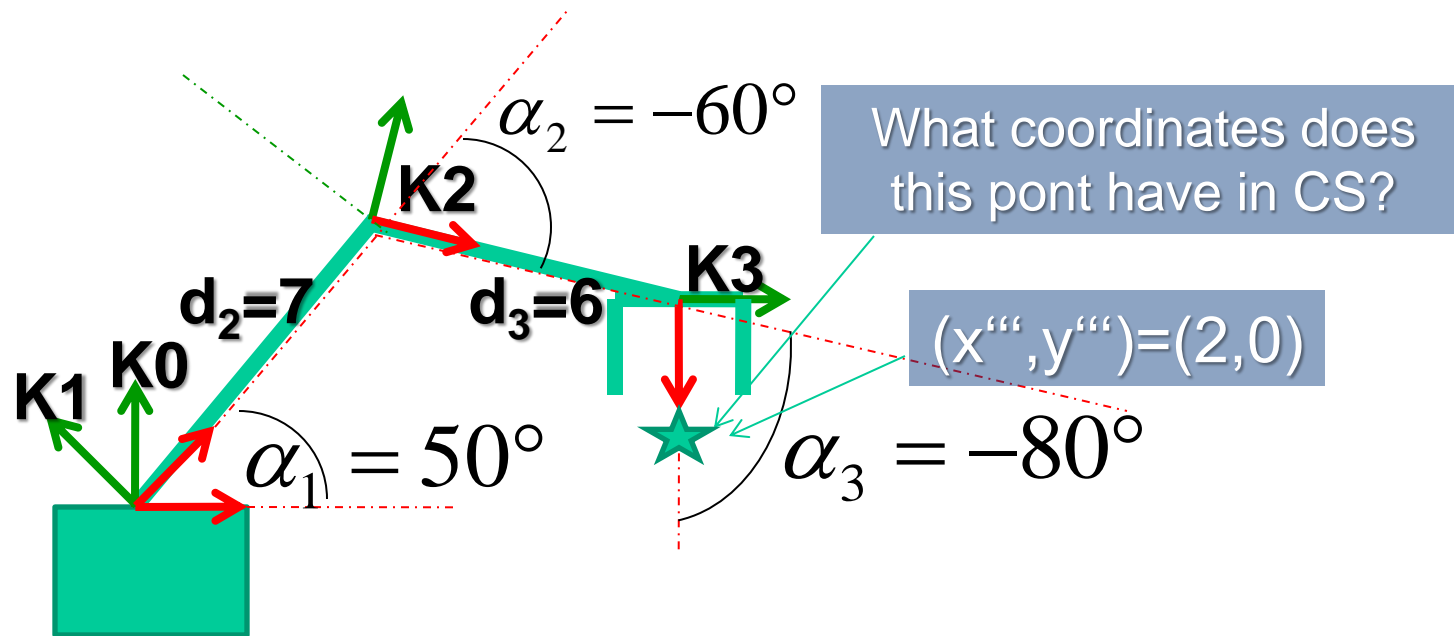
Chains of coordinate systems

- Convention: Put one coordinate system into each joint, such that the X axis is pointing towards the next joint.



Chains of coordinate systems

- Now it's simple to express any point in K0!

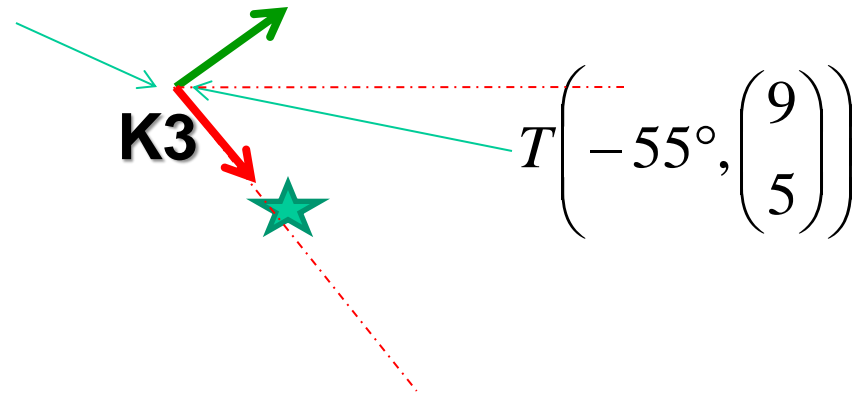
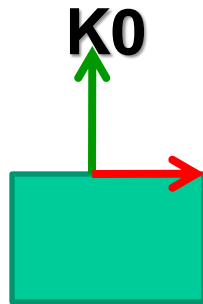


$$v = T\left(50^\circ, \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right) T\left(-60^\circ, \begin{pmatrix} 7 \\ 0 \end{pmatrix}\right) T\left(-80^\circ, \begin{pmatrix} 6 \\ 0 \end{pmatrix}\right) \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

- The direct kinematics of a robot can be understood as a simple chain of coordinate system transformations.
- This works for
 - 2D and 3D analogously
 - Rotational as well as translational joints
- Question:
Which types of robot kinematics may not be solved with this approach?

Inverse kinematics

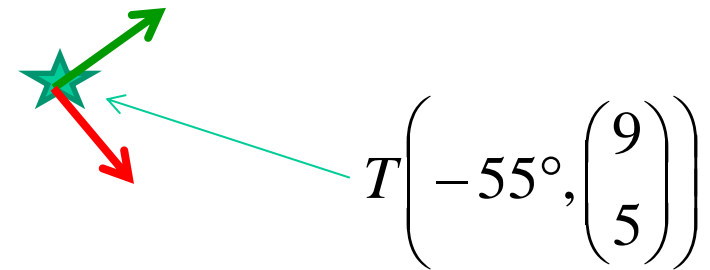
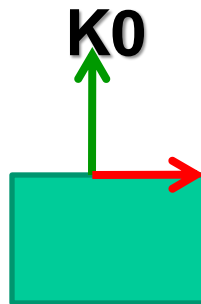
Which angles place the robot's hand to the desired point?



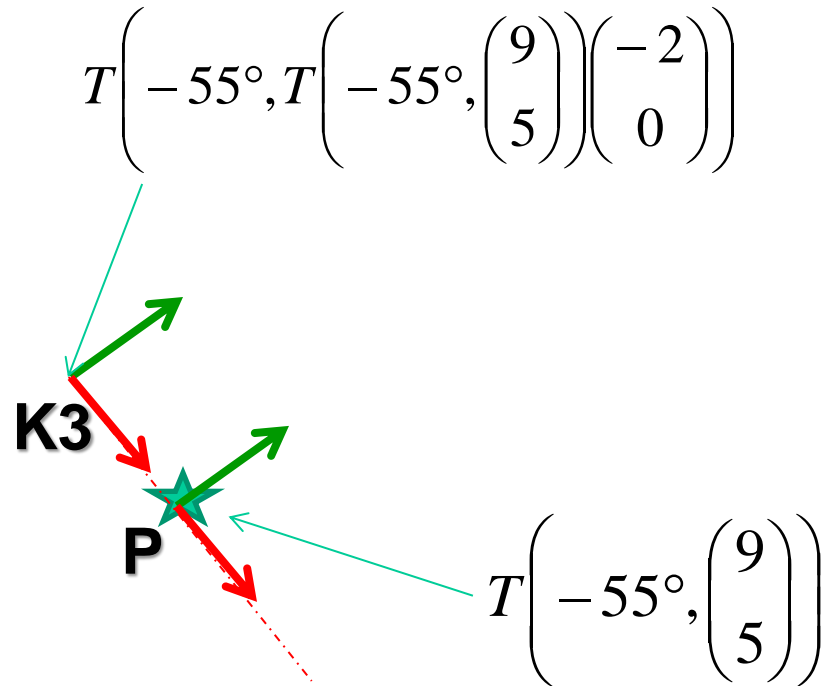
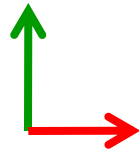
5. (2D) Inverse kinematics

- **Definition:**
The inverse kinematics of a robot is an operation which takes the position of the robot's hand as input and calculates joint angles.
- **Possible strategies for solution:**
 - **Analytical:** take the direct kinematics formula and isolate all angles
 - **Numerical:** use some approximation algorithm for computing the solution
 - **Geometric:** use geometric understanding of the robot's layout

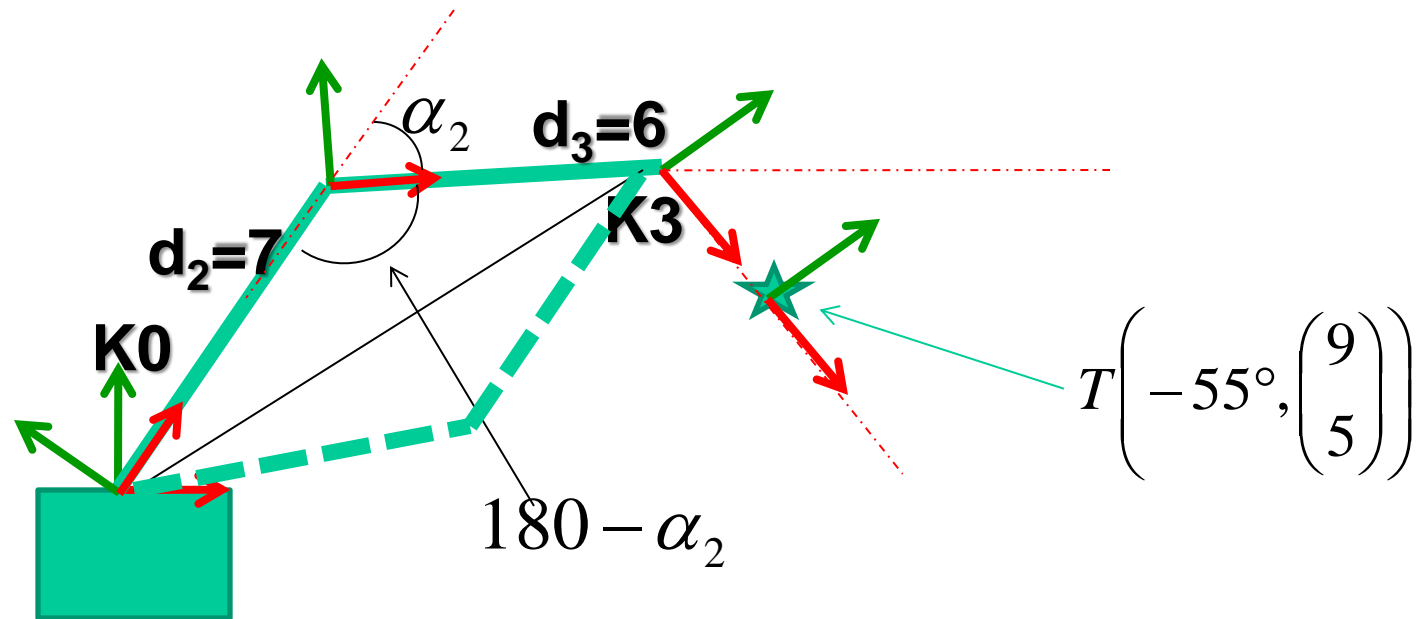
Core idea: Use the first two joints for bringing the robot's wrist to a desired position, then use the third axis to find the correct orientation!



0. Compute wrist position



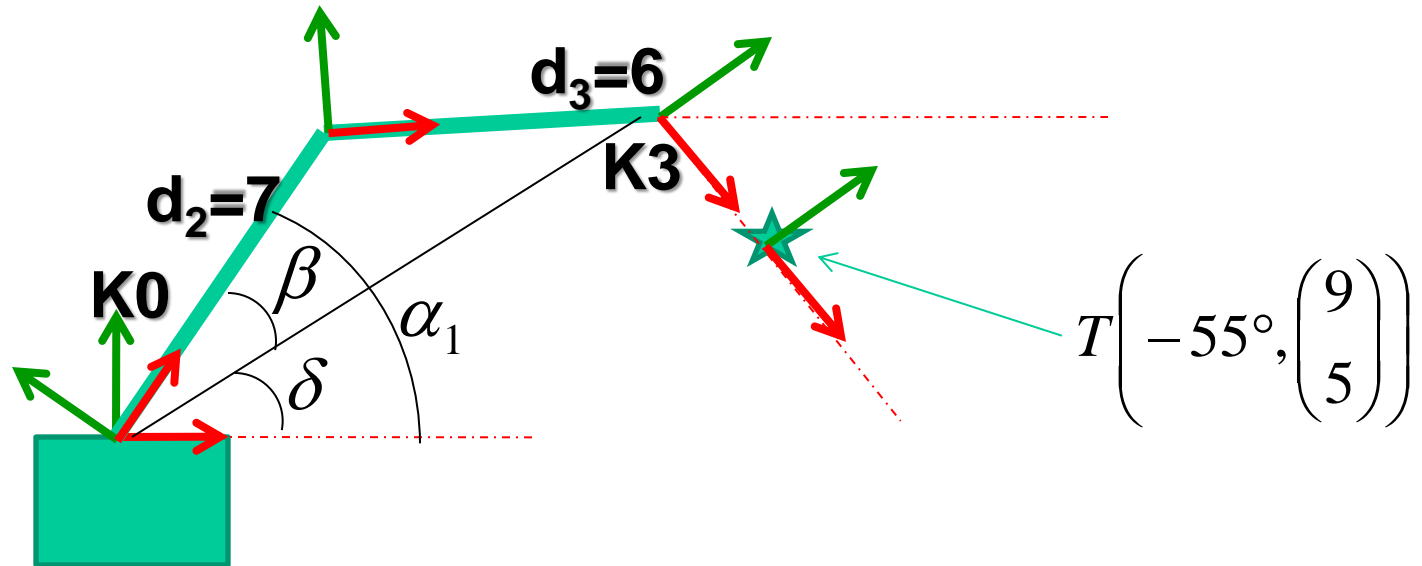
1. Compute angle α_2



$$\left. \begin{aligned} \cos \rho &= -\cos(\pi - \rho) \\ c^2 &= a^2 + b^2 - 2ab \cos \gamma \end{aligned} \right\} \Rightarrow$$

$$\frac{21}{84} = \cos 180 - \alpha_2 \Rightarrow \alpha_2 = \pm 73^\circ$$

2. Compute angle α_1

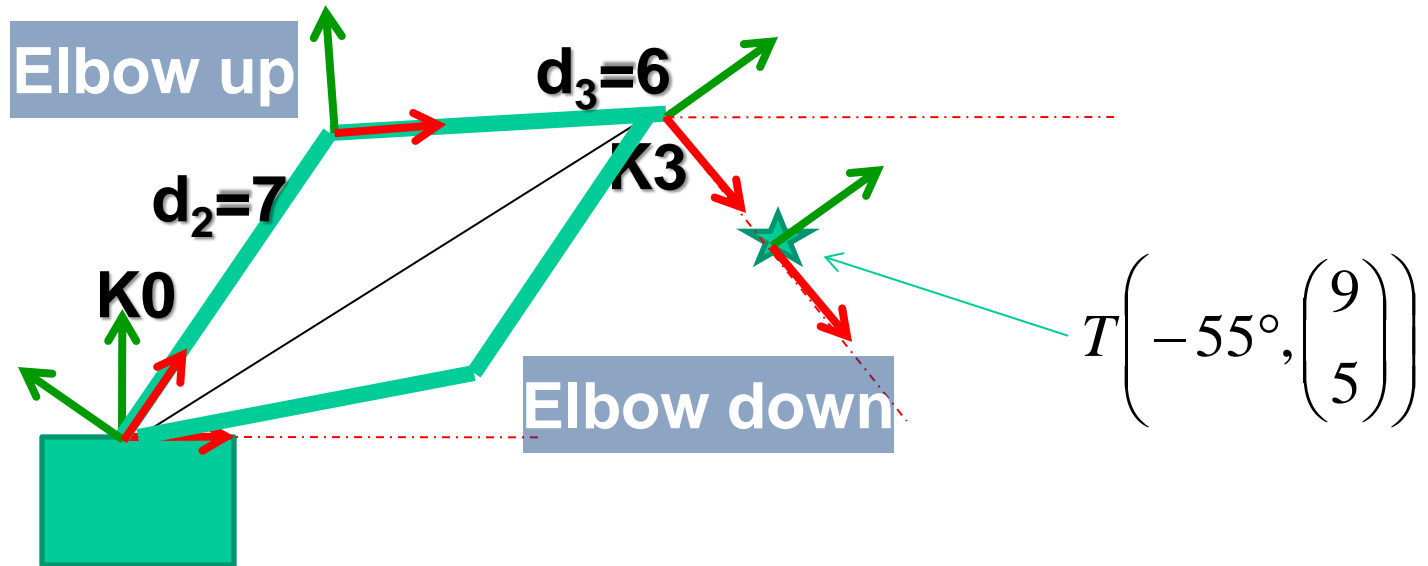


$$\delta = \arctan 2(9;5)$$

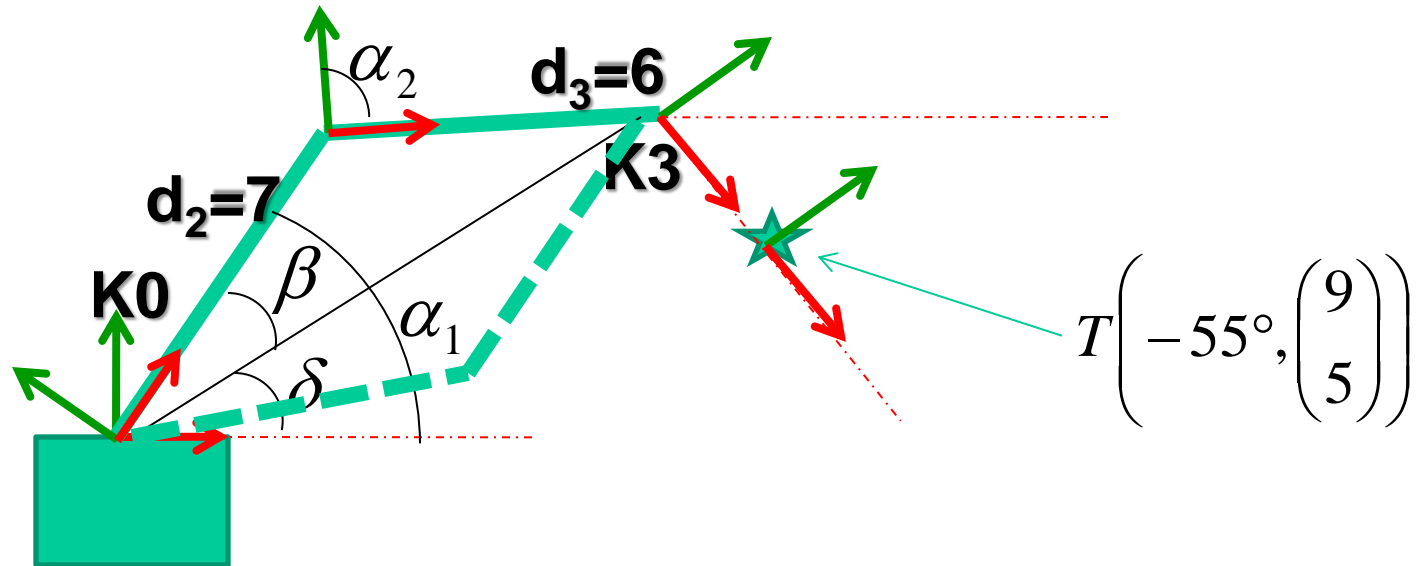
$$d_3^2 = d_2^2 + (x^2 + y^2) - 2d_2\sqrt{x^2 + y^2} \cos \beta$$

$$\alpha_1 = \delta + \beta = 29^\circ \pm 34^\circ$$

2 possible solutions



3. Compute axis α_3



$$\alpha_3 = -55 - \alpha_2 - \alpha_1$$

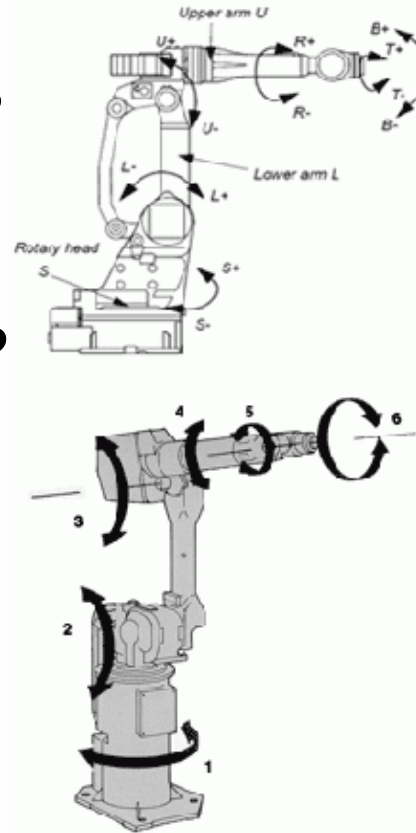
$$\alpha_{3,elbow_up} = -45^\circ \quad \alpha_{3,elbow_down} = -123^\circ$$

- Orthonormal (right-handed) coordinate systems differ only in a translation and one rotation.
- For kinematics computation, it is very useful to place one coordinate system into each joint.
- A simple geometric solution to the inverse kinematics problem is to compute first position of the robot's wrist and then it's orientation.

4. Basic concept of industrial robots

How are industrial robots programmed?

- Programming languages?
- Atomic operations?
- Specific challenges?
- Process of programming?



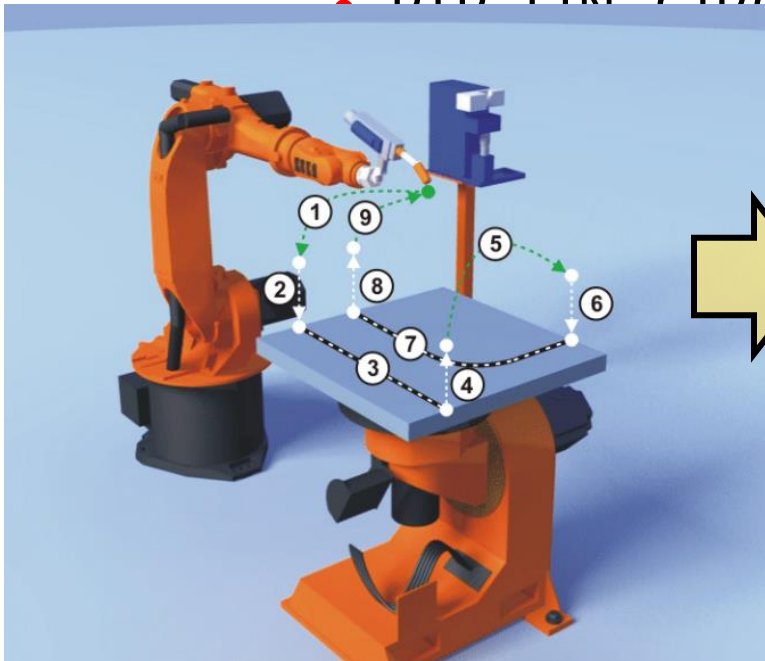
- Programming languages?
- Atomic operations?
- Specific challenges/programming layers?
- Programming process?

- Programming languages?
 - Feedback, Events, Responses
 - Schleifen, Verzweigung, Rekursion, OO
 - Regel-basiert / deklarativ / Unterspezifikation
- Atomic operations?
 - Sollwerte für Gelenke setzen + Zeitinformation
 - Zielpunkt in Koordinaten angeben
 - LIN (kartesisch), SPLINE
 - Atomic commands sollen "thread safe"
- Specific challenges/programming layers?
 - Low level: HW-Gelenke, Top level: Geometrie
 - Aufteilung: Hersteller-spezifisch vs. anwendungs-spezifisch
- Programming process?
 - Testen ist schwieriger
 - Auswahl HW, Auswahl Progsprache in Abhängigkeit von Aufgabe
 - Modularisierung der SW / SW-Architektur
 - Mischen von Offline-Programmierung mit Teachen

- Often people only describe movements by pathes (e.g. move on a straight line). A path (ger. **Bahn**) is a subset of 3D (resp. 6D) space.
- However, in most applications not pathes but trajectories (ger. **Trajektorie**) are intended. A trajectory is a mapping of time to a path.

- Different pathes:
 - Linear movement
 - Point-to-point movement
 - Circular movement
 - Modern: (cubic) spline movement
- Different trajectories:
 - Velocity profiles
 - Acceleration profiles
 - Total movement time

- Very simple programming languages
 - No recursion
 - No object
 - Only global variables
 - PTP, LIN, CIRC



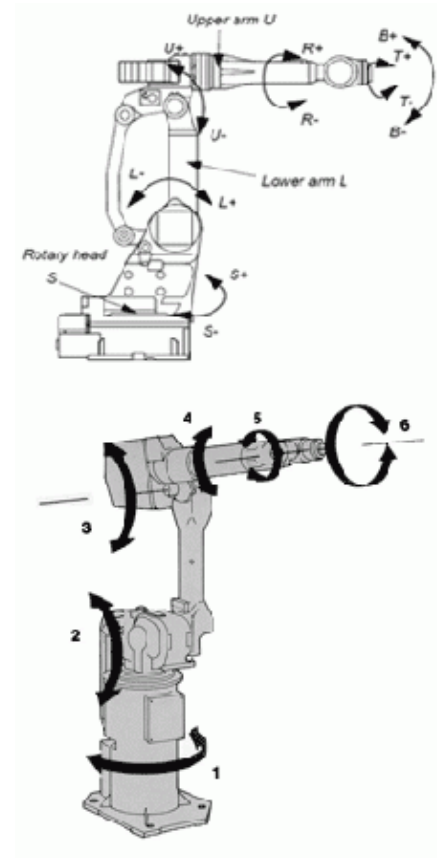
```

1  INI
2  DEF ...
3  PTP HOME VEL= 100 % DEFAULT
...
4  PTP P1 VEL= 100 % PDAT 1 Tool[1] Base[0]
5  PTP P2 VEL=100 % PDAT2 Tool[1] Base[0]
...
6  ...1()
7  PTP P7 Vel= 100 % PDAT
8  PTP P8 Vel= 100 % PDAT
...2()
...
10 PTP P15 Vel= 100 % PDAT1
11 PTP P16 Vel= 60 % PDAT
12 PTP HOME VEL= 100 % DEFAULT
...
13 GLOBAL DEF ..._EHS(ConsiderCERr:IN,Motion:IN,
    Service:IN;MsgType:IN,SeamState:OUT,ErrStrategy2Counthax:IN,
    TargetState:IN,CallFrom[]):OUT)
...
14 CASE #PartServiceRoutine01
...
15 CASE #PartServiceRoutine02
...
16 END
    
```

- **Electrical control:**
“How do I need to control voltage, such that the arm reaches/holds a position without tremor?”
- **Realtime control:**
“How can I guarantee, that intended values are always available in time?”
- **Robot control/path planing:**
“How do I calculate (sequences of) intended values for all axis of a robot?”
- **High-level functionalities:**
“How can I specify and calculate a collision-free trajektory efficiently?”

5. Industrial 6 axis robots

“Standard-” industrial robots



“Standard-” industrial robots

- Design:
 - 6 rotational joints
 - Anthropomorphic hand (i.e. the last three axis intersect in one point)
- Our goal:
Design and implementation of a control for such a robot.

- Control should – in theory – be capable of control any standard robot
- Necessary commands:
 - Movement in axis space
 - Movement in Cartesian space: LIN, PTP
 - Velocity profiles
 - Synchronuous and asynchronuous movements

- Identification of necessary subtasks
- Solving these subtask in separate groups
 - We will give support to each group during the next two weeks
 - Result: Presentation of necessary concepts, such that everybody else understands it (and can implement it)
- Important:
 - Early specification is important for
 - More easy system integration
 - Better understanding of you problems



Computer Systems in Engineering

What subtasks do we need to solve?

1. Positioning of the wrist
Grotzke
2. Orientation of the wrist
Stellmacher,
3. Standardized coordinate systems / layouts of robots
Gonschorek, Bahl, Sulkowski
4. Computation of trajectories / velocity profiles
Belov, Baier, Hagemann
5. Selection of solution /singularities
Schubert,
6. Software architecture
7. Robot programming framework (openRAVE)
Meuschke, Pauksch, Hettig

- Eine Anforderung wurde vergessen!
- Wer macht die Vorwärtskinematik?
- Entscheiden Sie selbst, welche Gruppe am Sinnvollsten dieses Zusatzproblem mit bearbeitet!