Unit 5

• Weightage = 5%

ALGEBRAIC MANIPULATION

Student Learning Outcomes (SLOs)

After completing this unit, students will be able to:

- Find Highest Common Factor (H.C.F) and Least Common Multiple (L.C.M) of algebraic expressions by factorization method.
- Know the relationship between H.C.F and L.C.M
- Use division method to determine highest common factor and least common multiple.
- Solve real life problems related to HCF and LCM
- Use highest common factor and least common multiple to reduce fractional expressions involving addition, subtraction, multiplication and division.
- Find square root of an algebraic expression by factorization and division methods.
- Solve real life problems related to HCF and LCM.

Introduction:

Algebraic manipulation refers to manipulation of algebraic expressions, often into a simpler form or form which is more easily handled and dealt with. It is one of the most basic, necessary and important skills in a problem solving of algebraic expression.

In this unit, we will discuss HCF, LCM and square root of the algebraic expressions by both factorization and division methods and their applications in daily life.

5.1 Highest Common Factor (HCF) / Greatest Common Division and Least Common Multiple (LCM)

5.1.1 Find Highest Common Factor (HCF) and Least Common Multiple (LCM) of Algebraic expression by Factorization method.

(a) Highest Common Factor (HCF) by Factorization method

For finding the HCF of the given expression, first we find the factors of each polynomial. Then we take the product of their common factors. This product of common factors is known as HCF by factorization.

Notes: 1. In case there is no common factor then HCF is 1.
2. HCF is also called GCD (Greatest Common Divisor).

Example 01 Find the HCF of the following expression by using factorization method.

(i) $x^2 + x - 20$ and $x^2 + 12x + 35$

(ii) $(x + 1)^2$, $x^2 - 1$ and $x^2 + 4x + 3$

Solution (i): We factorize the given expression $x^2+x - 20$ and $x^2+12x+ 35$

The factors are as under:

$$x^2+x - 20 = x^2+5x-4x-20$$
$$= x(x+ 5)-4(x+ 5)$$
$$=(x+ 5)\ (x- 4)$$

and $$x^2+ 12x + 35 = x^2+7x+5x + 35$$
$$= x(x+ 7) +5(x+7)$$
$$= (x+ 5)\ (x+ 7)$$

Common factor in both the expression is $(x+ 5)$

$\therefore$ HCF $= x+ 5$

Solution (ii): We factorize the given expression $(x+1)^2$, x^2-1 and x^2+4x+3

The factors are as under:

$$(x+1)^2 = (x+1)(x+1)$$
$$x^2-1 = (x+1)(x-1)$$

and $x^2+4x+3 = x^2+3x+x+3$
$$= x(x+3)+1(x+3)$$
$$= (x+3)(x+1)$$

Common factor in all the three expression is $(x+1)$

$\therefore$ H.C.F $= x+1$.

Example 02 Find the H.C.F of the following expression by using factorization method.

a^3-b^3 ,a^6-b^6

Solution :

$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$
$$a^6-b^6 = (a^2)^3-(b^2)^3$$
$$= (a^2-b^2)\{(a^2)^2+a^2b^2+(b^2)^2\}$$
$$= (a-b)(a+b)\{(a^2)^2+2a^2b^2+(b^2)^2-a^2b^2\}$$
$$= (a-b)(a+b)\{(a^2+b^2)^2-(ab)^2\}$$
$$= (a-b)(a+b)(a^2+b^2-ab)(a^2+b^2+ab)$$

HCF $= (a-b)(a^2+b^2+ab) = a^3-b^3$

(b) Least Common Multiple (LCM) by Factorization method

Least common multiple (LCM) of two or more polynomials is the expression of least degree which is divisible by the given polynomials. To find LCM by Factorization we use the following formula:

LCM = Common factors × non common factors

Example 01 Find the LCM of x^3-8 and x^2+x-6

Solution: Now find the factors of x^3-8 and x^2+x-6

$\therefore$ $x^3-8 = (x)^3-(2)^3 = (x-2)(x^2+2x+4)$

and $x^2+x-6 = x^2+3x-2x-6 = x(x+3)-2(x+3)$
$$= (x+3)(x-2)$$

Common factor $= (x-2)$

Non common factors $= (x+3)(x^2+2x+4)$

$\because$ LCM= Common factors × non common factors

$\therefore$ LCM$=(x-2)(x^2+2x+4)(x+3)=(x^3-8)(x+3)$

Example 02 Find the L.C.M of x^3-1, x^3-2x^2+x

Solution: Now find the factors of x^3-1 and x^3-2x^2+x

$\therefore$ $x^3-1=(x)^3-(1)^3=(x-1)(x^2+x+1)$

and $x^3-2x^2+x=x(x^2-2x+1)=x(x-1)^2$

$\because$ LCM= Common factors × non common factors

$\therefore$ $\text{LCM}=x(x-1)^2(x^2+x+1)$.

5.1.2 Know the Relationship between HCF and LCM

The relation between HCF and LCM of two polynomials $p(x)$ and $q(x)$ is expressed as under

$$\boxed{\text{HCF} \times \text{LCM} = p(x) \times q(x)}$$

Example 01 Find the HCF and LCM of the polynomials $p(x)$ and $q(x)$ given below, and verify the relation of HCF and LCM.

$p(x) = x^2 - 5x + 6$ and $q(x) = x^2 - 9$.

Solution: First factorize the polynomials $p(x)$ and $q(x)$ into irreducible factors,

We have,

$$\begin{aligned} p(x) &= x^2 - 5x + 6 \\ &= x^2 - 3x - 2x + 6 \\ &= x(x-3) - 2(x-3) \\ &= (x-3)(x-2) \end{aligned}$$

and $q(x) = x^2 - 9 = (x-3)(x+3)$

Thus, H.C.F $= (x-3)$

and L.C.M $= (x-2)(x-3)(x+3) = (x-2)(x^2-9)$

Now, find the product of $p(x)$ and $q(x)$.

so, $p(x) \times q(x) = (x^2-5x+6) \times (x^2-9)$ . . . (i)

LCM × HCF $=(x-2)(x-3)(x+3) \times (x-3)$

$\Rightarrow$ $= (x^2-5x+6) \times (x^2-9)$ . . . (ii)

From results (i) and (ii), we have

LCM × HCF $= p(x) \times q(x)$, Hence, verified.

Example 02 Find the LCM of the following polynomials by using the formula.

$p(x) = x^2+14x+48$ and $q(x) = x^2+8x+12$.

Solution: Now first we have to find the HCF of the $p(x)$ and $q(x)$.

$p(x) = x^2+14x+48$
$= x^2+8x+6x+48$
$= x(x+8)+6(x+8)$
$= (x+6)(x+8)$

and $q(x) = x^2+8x+12$
$= x^2+6x+2x+12$
$= x(x+6)+2(x+6)$
$= (x+2)(x+6)$

so, HCF of $p(x)$ and $q(x)$ is $(x+6)$.

$\because$ $\text{LCM} = \dfrac{p(x)\times q(x)}{\text{HCF}}$

$\therefore$ $\text{LCM} = \dfrac{(x^2+14x+48)\times(x^2+8x+12)}{(x+6)}$

$\Rightarrow$ $\text{LCM} = \dfrac{(x+6)(x+8)(x+2)(x+6)}{(x+6)}$

so, $\boxed{\text{LCM} = (x+2)(x+6)(x+8).}$

Notes:

If $p(x)$, $q(x)$ and $r(x)$ are three polynomials having no common factor to them, then

1. LCM would be their product. i.e $\boxed{\text{LCM} = p(x)q(x)r(x)}$

2. HCF would be unity (one). i.e $\boxed{\text{HCF} = 1}$

5.1.3 Use division method to determine highest common factor and least common multiple .

To find the HCF of two or more algebraic expressions by division method, the following examples will help us to understand the method.

Example 01 Find the HCF by division method of the following polynomials:

$2x^3 + 7x^2 + 4x - 4$ and $2x^3 + 9x^2 + 11x + 2$.

Solution: Now, by actual division, we have,

$$\begin{array}{r|l} & 1 \\ 2x^3+7x^2+4x-4 & 2x^3+9x^2+11x+2 \\ & \underline{-2x^3 \pm 7x^2 \pm 4x \quad 4} \\ & \qquad 2x^2 + 7x + 6 \end{array}$$

Again,

$$\begin{array}{r|l} & x \\ 2x^2+7x+6 & 2x^3+7x^2+4x-4 \\ & \underline{-2x^3 \pm 7x^2 \pm 6x + 0} \\ & \qquad -2x-4 \end{array}$$

We take common factor -2 from $-2x - 4$ and omit it.

$$\begin{array}{r|l} & 2x+3 \\ x+2 & 2x^2+7x+6 \\ & \underline{-2x^2 \pm 4x + 0} \\ & \qquad 3x+6 \\ & \qquad \underline{-3x \pm 6} \\ & \qquad 0 \quad 0 \end{array}$$

Required HCF is $x + 2$.

Example 02 Find the HCF by division method of the following polynomials:

$x^2 + 2x+1$, $x^2 - 1$ and $x^2+ 4x+ 3$.

Solution: First we find the HCF of any two expression then its HCF with third

Now, by actual division, we have,

$$\begin{array}{r|l} & 1 \\ x^2+2x+1 & x^2+4x+3 \\ & \underline{x^2 \pm 2x \pm 1} \\ & \qquad 2x+2 \end{array}$$

We take common factor $+2$ from $2x+2$ and omit it.

Again,

$$\begin{array}{r|l} & x+1 \\ \hline x+1 & x^2+2x+1 \\ & -x^2 \pm x+0 \\ \hline & \quad x+1 \\ & \quad -x \pm 1 \\ \hline & \quad 0 \quad 0 \end{array}$$

$\therefore$The HCF of x^2+2x+1 and x^2+4x+3 is $x+1$

Now we find HCF of $x+1$ and x^2-1

$$\begin{array}{r|l} & x-1 \\ \hline x+1 & x^2-1 \\ & -x^2 \pm x \\ \hline & -x-1 \\ & \quad x \quad 1 \\ \hline & 0 \quad 0 \end{array}$$

The HCF of all the three polynomials is$(x+1)$.

LCM by Division Method

To find the LCM of two or more algebraic expressions (Polynomials) by division method, the following formula is used

$$\boxed{\text{LCM}=\frac{\text{Product of two polynomials}}{\text{HCF of two polynomials}}}$$

Example Find the LCM of $x^3-6x^2+11x-6$ and x^3-4x+3.

Solution: $x^3-6x^2+11x-6$ and x^3-4x+3

Now, find HCF by actual division,

$$\begin{array}{r|l} & 1 \\ \hline x^3-4x+3 & x^3-6x^2+11x-6 \\ & -x^3 \qquad \mp 4x \pm 3 \\ \hline & \quad -6x^2+15x-9 \end{array}$$

$-6x^2+15x-9=-3(2x^2-5x+3)$, we omit -3.

Now multiply x^3-4x+3 by 2

$$\begin{array}{r|l} & x+5 \\ \hline 2x^2-5x+3 & 2x^3-8x+6 \\ & -2x^3 \pm 3x \mp 5x^2 \\ \hline & \quad 5x^2-11x+6 \end{array}$$

Multiplying by 2, i.e., $10x^2-22x+12$,

$$\begin{array}{r|l} & \quad\quad\quad 5 \\ 2x^2-5x+3 & 10x^2-22x+12 \\ & \underline{-10x^2 \quad 25x \pm 15} \\ & \quad\quad 3x-3 \end{array}$$

$3x-3=3(x-1)$ omit 3 and then again by division

$$\begin{array}{r|l} & \quad 2x-3 \\ x-1 & 2x^2-5x+3 \\ & \underline{-2x^2 \quad 2x} \\ & \quad -3x+3 \\ & \quad \underline{3x \pm 3} \\ & \quad\quad 0 \quad 0 \end{array}$$

$\therefore$ HCF = $x-1$.

We know that LCM = $\dfrac{p(x).q(x)}{\text{HCF}}$

$\therefore$ LCM = $\dfrac{(x^3-6x^2+11x-6)(x^3-4x+3)}{x-1}$.

Now divide $x^3-6x^2+11x-6$ by $x-1$, we have,

$$\begin{array}{r|l} & \quad x^2-5x+6 \\ x-1 & x^3-6x^2+11x-6 \\ & \underline{-x^3 \quad x^2} \\ & \quad -5x^2+11x \\ & \quad \underline{-5x^2 \quad \pm 5x} \\ & \quad\quad\quad 6x-6 \\ & \quad\quad\quad \underline{-6x \pm 6} \\ & \quad\quad\quad\quad 0 \quad 0 \end{array}$$

Therefore: $\boxed{\text{LCM} = (x^3-4x+3)(x^2-5x+6).}$

5.1.4 Solve real life problems related to HCF and LCM

Example 01 Rida has two pieces of cloth one piece is 45 inches wide and other piece is 90 inches wide. She wants to cut both the strips of equal width. How wide should she cut the strips?

Solution: This problem can be solved using HCF because she is cutting or dividing the cloth for widest possible strips.

So,

HCF of 45 and 90

$$45 = 3\times 3\times 5$$

$$90 = 2\times 3\times 3\times 5$$

$$\text{HCF} = \text{Product of common factors}$$

$$\text{HCF} = 3\times 3\times 5$$

$$\text{HCF} = 45$$

So, Rida should cut each piece to be 45 inches wide.

Example 02 Sarfraz exercises every 8 days and Imran every 4 days. Sarfraz and Imran both exercised today. After how many days they will exercise together again?

Solution: This problem can be solved using least common multiple because we are trying to find out the time they will exercise, time that it will occurs out the same time.

LCM of 8 and 4 is

$$8 = 2\times 2\times 2$$

$$4 = 2\times 2$$

$$\text{LCM} = \text{Product of common factors}\times\text{Product of non common factors}$$

$$\text{LCM} = 2\times 2\times 2$$

$$\text{LCM} = 8$$

So, They will exercise together again after 8 days.

Exercise 5.1

1. **Find the HCF of the following expressions by factorization method:**
   (i) $72x^4y^5z^2$ and $120x^3y^6z^8$ (ii) $18r^3s^4t^5$, $120r^4s^3t^8$ and $210r^7s^7t^3$
   (iii) $x^2-3x-18$ and x^2+5x+6 (iv) $4x^2-9$ and $2x^2-5x+3$
   (v) $(2a^2-8b^2)$, $(4a^2+4ab-24b^2)$ and $(2a^2-12ab+16b^2)$
   (vi) x^3+x^2+x+1 and x^3+3x^2+3x+1

2. **Find the HCF of the following expressions by division method:**
   (i) x^2+3x+2 and $3x^2-3x-6$
   (ii) $2x^3+15x^2+31x+12$ and $6x^3+46x^2+100x+48$
   (iii) $x^3-5x^2+10x-8$, x^3-4x^2+7x-6
   (iv) $x^4+3x^3+2x^2+3x+1$, x^3+4x^2+4x+1 and x^3+5x^2+7x+2

3. **Find the LCM of the following expressions by factorization method:**
   (i) $27a^4b^5c^2$ and $81ab^2c^8$ (ii) $24p^2q^3r^4$, $100p^5q^4r^5$ and $300p^3qr^8$
   (iii) $21x^2-14x$ and $3x^2-5x+2$ (iv) $x^2+11x+28$ and x^2+x-12
   (v) $6x^2+11x+3$, $3x^2-2x-1$ and $3x^2-2x-1$
   (vi) x^2-y^2, x^3-y^3 and $x^4+x^2y^2+y^4$

4. **Find the LCM by Division method:**
   (i) $x^2-25x+100$ and x^2-x-20
   (ii) $3x^2+14x+8$ and $6x^2+x-2$
   (iii) $x^2-y^2-z^2-2yz$ and $y^2-z^2-x^2-2xz$
   (iv) $3x^3+9x^2-84x$ and $4x^4-24x^3+32x^2$

5. If the HCF of the $x^2-11x+24$ and x^2-6x+6 is $(x-3)$. Find the **LCM.**

6. The **HCF** and **LCM** of two expressions are $(x+3)$ and $(x^3+7x^2+7x-15)$, respectively. If one expression is $x^2+8x+15$. Find the second expression.

7. The HCF and LCM of two polynomial of the second degree are $3x-2$ and $3x^3+7x^2-4$ respectively. Find the product of two polynomial.

8. Verify the relationship between HCF and LCM.
   i.e.(HCF×LCM = $p(x)q(x)$) for the polynomial $p(x)=x^2-8x-20$ and $q(x)=x^2-15x+50$

9. A carpenter got some free wooden planks. Some are 12cm long and some are 18cm. He wants to cut them so that he has equal size planks to make using them easier. What size planks should he cut them into to avoid wasting any wood?
10. Train A and train B stops at Hyderabad as 10:30am. Train A stops every 12 minutes and train B stops every 14 minutes. when do they both stop together?

5.2 Basic operations on Algebraic Fractions

If $p(x)$ and $q(x)$ are algebraic expressions and $q(x) \neq 0$ then $\dfrac{p(x)}{q(x)}$ is called an Algebraic Fraction.

Simplest form of algebraic fraction is a fraction in which there is no common factor except 1 in numerator and denominator. In algebraic fraction fundamental operations $(+,-,\div,\times)$ are carried out in the same way as in common fractions.

In the following examples we shall explain the use of highest common factor and least common multiple to reduce fractional expressions in simplest form involving fundamental operations.

5.2.1 Use highest common factor and least common multiple to reduce fractional expressions involving addition, subtraction, multiplication and division.

Example 01 Simplify: $\dfrac{x^2-x-6}{2x^2-5x-3}+\dfrac{1}{4x^2-1}$

Solution:

$$\frac{x^2-x-6}{2x^2-5x-3}+\frac{1}{4x^2-1} = \frac{x^2-3x+2x-6}{2x^2-6x+x-3}+\frac{1}{(2x-1)(2x+1)}$$

$$= \frac{x(x-3)+2(x-3)}{2x(x-3)+1(x-3)}+\frac{1}{(2x-1)(2x+1)}$$

$$= \frac{(x-3)(x+2)}{(x-3)(2x+1)}+\frac{1}{(2x-1)(2x+1)} \qquad \text{where } x \neq 3$$

$$= \frac{(x+2)}{(2x+1)} + \frac{1}{(2x-1)(2x+1)}$$

$$= \frac{(x+2)(2x-1)+1}{(2x-1)(2x+1)}$$

$$= \frac{2x^2 - x + 4x - 2 + 1}{(2x-1)(2x+1)} = \frac{2x^2 + 3x - 1}{4x^2 - 1}$$

Example 02 Simplify: $\dfrac{2}{x+2} - \dfrac{x-4}{2x^2+x-6}$

Solution:

$$\frac{2}{x+2} - \frac{x-4}{2x^2+x-6}$$

$$= \frac{2}{x+2} - \frac{x-4}{2x^2+4x-3x-6}$$

$$= \frac{2}{x+2} - \frac{x-4}{2x(x+2)-3(x+2)}$$

$$= \frac{2}{x+2} - \frac{x-4}{(x+2)(2x-3)}$$

$$= \frac{2(2x-3)-(x-4)}{(x+2)(2x-3)}$$

$$= \frac{4x-6-x+4}{(x+2)(2x-3)}$$

$$= \frac{3x-2}{2x^2+x-6}$$

Example 03 Simplify: $\dfrac{ab^2+2a}{ab-6+2b-3a} \times \dfrac{b^2-6b+9}{b^3+2b}$

Solution:

$$\frac{ab^2+2a}{ab-6+2b-3a} \times \frac{b^2-6b+9}{b^3+2b}$$

$$\frac{ab^2+2a}{ab-6+2b-3a} \times \frac{b^2-6b+9}{b^3+2b} = \frac{a(b^2+2)}{b(a+2)-3(a+2)} \times \frac{b^2-3b-3b+9}{b(b^2+2)}$$

$$= \frac{a}{(a+2)(b-3)} \times \frac{b(b-3)-3(b-3)}{b}$$

$$= \frac{a}{(a+2)(b-3)} \times \frac{(b-3)(b-3)}{b}$$

$$= \frac{a}{(a+2)(b-3)} \times \frac{(b-3)(b-3)}{b}$$

$$= \frac{a(b-3)}{b(a+2)} \qquad b - 3 \neq 0$$

Example 04 Simplify: $\dfrac{p^2-q^2}{r^2+2rs+s^2} \div \dfrac{2(p+q)}{3(r+s)s}$

Solution: Simplification $\dfrac{p^2-q^2}{r^2+2rs+s^2} \div \dfrac{2(p+q)}{3(r+s)s}$

$$= \frac{(p+q)(p-q)}{(r+s)^2} \div \frac{2(p+q)}{3(r+s)s}$$

$$= \frac{(p+q)(p-q)}{(r+s)^2} \times \frac{3(r+s)s}{2(p+q)} \qquad \text{provided } p+q \neq 0 \text{ and } r+s \neq 0$$

$$= \frac{3s(p-q)}{2(r+s)}$$

Exercise 5.2

Simplify the following

(i) $\dfrac{4x}{x^2+2x+1} + \dfrac{3}{x+1}$

(ii) $\dfrac{3}{x(2x+1)} + \dfrac{6x+7}{3x(x+1)}$

(iii) $\dfrac{3x-1}{x^2+2x+1} - \dfrac{4x^2-1}{x^2-2x-3}$

(iv) $\dfrac{1}{x+1} - \dfrac{2}{x+2} + \dfrac{3}{x+3}$

(v) $\dfrac{x^2+4x+3}{5} \times \dfrac{10}{x+1}$

(vi) $\dfrac{x^2-4x-21}{x^2-6x-7} \div \dfrac{x+9}{x+1}$

(vii) $\left[\dfrac{3}{x+1} + \dfrac{1}{x+2}\right] \div \left[\dfrac{2}{x+3} - 1\right]$

(viii) $\left(\dfrac{1}{x^2-9}\right) \div \left(\dfrac{1}{x+3}\right) - \dfrac{3}{x-2}$

(ix) $\dfrac{1}{x^2+8x+15} + \dfrac{1}{x^2+7x+12} - \dfrac{1}{x^2+x-12}$

(x) $2\left(\dfrac{x^2+7x+12}{x^2-16} + \dfrac{x^2+x-2}{x^2+4x+4}\right) \times \dfrac{x^2-2x-8}{8x^2+2x+4}$.

5.3 Square Root of an Algebraic Expressions

5.3.1 Find square root of an algebraic expression by Factorization and Division

We shall discuss two methods to find the square roots of the algebraic expressions.

(a) By Factorization Method (b) By Division Method

(a) Square Root by Factorization method.

Example 01 Find square root of the expression $49x^2+126xy+81y^2$ by factorization method.

Solution: $49x^2+126xy+81y^2$

$= (7x)^2+2(7x)(9y)+(9y)^2$

$= (7x+9y)^2$

Therefore $\sqrt{49x^2+126xy+81y^2} = \sqrt{(7x+9y)^2}$

$= 7x+9y$

(b) Square Root by Division Method.

Example 01 Find square root of the expression $4x^4+12x^3-19x^2-42x+49$ by division method.

Solution: Method is illustrated below,

$$
\begin{array}{r|l}
 & 2x^2+3x-7 \\
\hline
2x^2 & 4x^4+12x^3-19x^2-42x+49 \\
+2x^2 & \pm 4x^4 \\
\hline
4x^2+3x & 12x^3-19x^2-42x+49 \\
+3x & \pm 12x^3 \pm 9x^2 \\
\hline
4x^2+6x-7 & -28x^2-42x+49 \\
-7 & \mp 28x^2 \mp 42x \pm 49 \\
\hline
4x^2+6x-14 & 0 \quad 0 \quad 0
\end{array}
$$

Therefore: $\sqrt{4x^4+12x^3-19x^2-42x+49} = 2x^2+3x-7.$

Example 02 For what value of a the expression $36x^4 + 36x^3 + 57x^2 + 24x + a$ will be the perfect square?

Solution: By division method, we have,

	$6x^2 + 3x + 4$
$6x^2$	$36x^4 + 36x^3 + 57x^2 + 24x + a$
$+6x^2$	$\pm 36x^4$
$12x^2 + 3x$	$36x^3 + 57x^2 + 24x + a$
$+ 3x$	$\pm 36x^3 \pm 9x^2$
$12x^2 + 6x + 4$	$48x^2 + 24x + a$
$+4$	$\pm 48x^2 \pm 24x \pm 16$
$12x^2 + 6x + 8$	$a - 16$

Given expression will be a perfect square if
$a - 16 = 0 \Rightarrow a = 16$,

Therefore, $a = 16$, will make the given expression a perfect square.

Example 03 What should be added to the expression $x^4 + 4x^3 + 10x^2 + 5$, so that it may be a perfect square

Solution: By division method, we have,

	$x^2 + 2x + 3$
x^2	$x^4 + 4x^3 + 10x^2 + 5$
x^2	$\pm x^4$
$2x^2 + 2x$	$4x^3 + 10x^2 + 5$
$+ 2x$	$\pm 4x^3 \pm 4x^2$
$2x^2 + 4x + 3$	$6x^2 + 5$
$+3$	$\pm 6x^2 \pm 12x$
$2x^2 + 4x + 6$	$-12x - 4$ or $-(12x + 4)$

The given expression would be perfect square if remainder vanishes, which is only possible when $(12x+4)$ is added.

Exercise 5.3

1. **Find the square root of the following algebraic expressions by factorization method.**

(i) $36x^2-60xy+25y^2$

(ii) $9x^2+\frac{1}{x^2}+6$

(iii) $4x^4y^4-\frac{12x^3y^3}{z^2}+\frac{9x^2y^2}{z^4}$

(iv) $36(3-2x)^2-48(3-2x)y+16y^2$

(v) $\left(x^2+\frac{1}{x^2}\right)+2\left(x+\frac{1}{x}\right)+3$

(vi) $(4x^2-4x+1)(9x^2-54x+81)$

(vii) $(x^2-2x+1)(x^2-6x+9)$

(viii) $(x^2+8x+15)(x^2+7x+10)(x^2+5x+6)$

2. **Find the square root of the following algebraic expressions by division method.**

(i) $x^4+2x^3+3x^2+2x+1$

(ii) $25x^4+40x^3+26x^2+8x+1$

(iii) $4x^4+8x^3+20x^2+16x+16$

(iv) $\frac{x^2}{y^2}+\frac{y^2}{x^2}+47-\frac{14y}{x}+\frac{14x}{y}$

(v) $x^2-2x+3-\frac{2}{x}+\frac{1}{x^2}$

(vi) $x^2+\frac{y^2}{9}+9z^2+\frac{2xy}{3}+2yz+6xz$

(vii) $(x^2+\frac{1}{x^2})^2-8(x^2+\frac{1}{x^2})+16$

(viii) $x^6+\frac{1}{x^6}-4\left(x^3+\frac{1}{x^3}\right)+6,\ x\neq 0$

3. What should be added to $4x^4+4x^3+17x^2+8x+9$ to make it perfect square?

4. What should be subtracted from $9x^6-12x^5+4x^4-18x^3-12x^2+18$ to make it a perfect square?

5. For what value of 'm', $9x^4+12x^3+34x^2+mx+25$ will be the perfect square?

6. For what value of 'p' and 'q', the expression $x^4+8x^3+30x^2+px+q$ will be the perfect square?

7. For what values of 'a' and 'b', $x^4+4x^3+10x^2+ax+b$ will be the perfect square?

Review Exercise 5

1. True and false questions

Read the following sentences carefully and encircle 'T' in case of true and 'F' in case of false statement.

(i) HCF of $y^2 - 4$ and $y - 2$ is $y - 2$. T/F

(ii) HCF of $a^3 - 1$ and $a^2 - 1$ is $a + 1$. T/F

(iii) LCM of $x^3 + 1$ and $x + 1$ is $x^3 + 1$. T/F

(iv) LCM of $x^4 - y^4$ and $x^2 - y^2$ is $x^2 + y^2$. T/F

(v) HCF of $a^2 + 4a + 3$ and $a^2 + 5a + 6$ is $a + 3$. T/F

2. Fill in the blanks.

(i) There are ______ methods for finding the HCF of polynomials.

(ii) LCM × HCF of two polynomials $p(x)$ and $q(x)$ = ___________.

(iii) HCF of $y^2 - 5y$, $y + 6$ and $y - 2$ is ________.

(iv) LCM of y^2+3y+2 and y^2+5y+6 is ________.

(v) HCF of $y^2 - \frac{1}{y^2}$ and $y + \frac{1}{y}$ is ________.

3. Tick (✓) the correct answers

(i) HCF of $x^3 - 8y^3$ and $x^2 - 4xy + 4y^2$ is:

(a) $x - 4y$ (b) $x^2 + 2xy + y^2$

(c) $x + 2y$ (d) $x - 2y$

(ii) LCM of $(2y+3z)^5$ and $(2y+3z)^3$ is:

(a) $(2y+3z)^8$ (b) $(2y+3z)^3$

(c) $(2y+3z)^2$ (d) $(2y+3z)^5$

(iii) HCF of $x^3 - y^3$ and x^2+xy+y^2 is:

(a) $x+y$ (b) x^2+xy+y^2

(c) $x-y$ (d) $(x-y)^2$

(iv) LCM of $(x-y)^4$ and $(x-y)^3$ is:

(a) $(x-y)$ (b) $(x-y)^3$

(c) $(x-y)^4$ (d) $(x-y)^7$

(v) Simplified form of $\frac{1}{x+y}+\frac{y}{x^2-y^2}$ is:

(a) $\frac{y+1}{x^2-y^2}$ (b) $\frac{x}{x^2-y^2}$

(c) $\frac{y}{x^2-y^2}$ (d) $\frac{x+y}{x^2-y^2}$

(vi) Simplified form of $\frac{y}{25x^2-y^2}-\frac{1}{5x-y}$ is:

(a) $\frac{5x}{25x^2-y^2}$ (b) $\frac{5x}{5x-y}$

(c) $\frac{-5x}{5x+y}$ (d) $\frac{-5x}{25x-y^2}$

(vii) $\frac{a^3x^3+a^3y^3}{a^2(x+y)}$ =______________:

(a) ax^2+ay^2 (b) x^2+y^2

(c) $a(x^2-xy+y^2)$ (d) $a(x^2+xy+y^2)$

(viii) $\frac{a}{a-b}-\frac{b}{a+b}$ =______________:

(a) $\frac{a^2+b^2}{a-b}$ (b) $\frac{a^2+b^2}{a^2-b^2}$

(c) $\frac{a+b}{a^2-b^2}$ (d) $\frac{a-b}{a^2-b^2}$

(ix) LCM =____________________ , given that p and q are any two polynomials.

(a) $\frac{\text{HCF}}{p\times q}$ (b) $\frac{p\times q}{\text{HCF}}$

(c) $\frac{p}{\text{HCF}}$ (d) $\frac{q}{\text{HCF}}$

(x) LCM of x^2-x+1 and x^3+1 is:

(a) $x+1$ (b) x^2-x+1 (c) x^3+1 (d) x^2+x+1

Summary

- There are two methods to be used to find the HCF and LCM of algebraic expression
  (i) Factorization method (ii) Division method
- LCM × HCF of two polynomials = product of polynomials
- With help of LCM and HCF, addition, multiplication, subtraction and division of algebraic expression can be found.
- There are two methods to be used to find the square root of algebraic expression
  (i) Square root by Factorization method
  (ii) Square root Division method