

**Example 03** Find the value of  $(ab+bc+ac)$ , when  $a+b+c=8$  and  $a^2+b^2+c^2=20$ .

**Solution:** Given that,

$$a+b+c=8 \text{ and } a^2+b^2+c^2=20,$$

$$\text{We know that } (a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

By substituting the values of  $a+b+c=8$  and  $a^2+b^2+c^2=20$ ,  
in the above formula we get,

$$(8)^2=20+2(ab+bc+ac)$$

$$\Rightarrow 64=20+2(ab+bc+ac)$$

$$\Rightarrow 64-20=2(ab+bc+ac)$$

$$\Rightarrow 44=2(ab+bc+ac)$$

$$\Rightarrow 22=ab+bc+ac$$

$$\Rightarrow ab+bc+ac=22$$

Hence, the value of  $(ab+bc+ac)$  is 22.

### 3.2.3 Know the cubic formulas

$$(i) \quad (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\text{or } (a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

**Proof:** L.H.S =  $(a+b)^3 = (a+b)(a+b)^2$

$$= (a+b)(a^2 + 2ab + b^2)$$

$$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$$

$$= a^3 + 3a^2b + 3ab^2 + b^3$$

$$= a^3 + b^3 + 3ab(a+b) = \text{R.H.S}$$

Hence proved

$$(ii) \quad (a-b)^3 = a^3 - b^3 + 3ab(a-b)$$

$$\text{or } (a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

**Proof:** L.H.S =  $(a-b)^3 = (a-b)(a-b)^2$

$$= (a-b)(a^2 - 2ab + b^2)$$

$$= a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3$$

$$= a^3 - 3a^2b + 3ab^2 - b^3$$

$$= a^3 - b^3 - 3ab(a-b) = \text{R.H.S}$$

Hence proved

The following examples are helpful for understanding the application for Cubic formulas.

**Example 01** Find the value of  $a^3 + b^3$ , when  $a + b = 4$  and  $ab = 5$ .

**Solution:** Given that,

$$a + b = 4 \text{ and } ab = 5$$

We have to find

$$a^3 + b^3$$

$$\text{Since, } (a + b)^3 = a^3 + b^3 + 3ab(a + b).$$

By substituting the values of  $a + b = 4$  and  $ab = 5$ , in the above formula we get,

$$(4)^3 = a^3 + b^3 + 3(5)(4)$$

$$\Rightarrow 64 = a^3 + b^3 + 60$$

$$\Rightarrow 64 - 60 = a^3 + b^3$$

$$\Rightarrow 4 = a^3 + b^3$$

$$\boxed{a^3 + b^3 = 4}$$

Hence the value of  $(a^3 + b^3)$  is 4.

**Example 02** Find the value of  $ab$ , when  $a^3 - b^3 = 5$  and  $a - b = 5$ .

**Solution:** Given that,

$$a^3 - b^3 = 5 \text{ and } a - b = 5$$

We have to find  $ab$

$$\text{Since, } (a - b)^3 = a^3 - b^3 - 3ab(a - b).$$

Now, substituting the values of  $a^3 - b^3 = 5$  and  $a - b = 5$ , in the above formula, we get,

$$(5)^3 = 5 - 3ab(5)$$

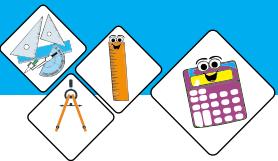
$$\Rightarrow 125 = 5 - 15ab$$

$$\Rightarrow 125 - 5 = -15ab$$

$$\Rightarrow 120 = -15ab$$

$$\Rightarrow -8 = ab$$

$$\boxed{ab = -8}$$



**Example 03** Find the value of  $x^3 + \frac{1}{x^3}$  when  $x + \frac{1}{x} = 3$

**Solution:** Given that

$$x + \frac{1}{x} = 3$$

Taking cube on both sides, we have

$$\left(x + \frac{1}{x}\right)^3 = 3^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) = 27 \quad \left[\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)\right]$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3(3) = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 9 = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 27 - 9$$

$$\Rightarrow \boxed{x^3 + \frac{1}{x^3} = 18}$$

**Example 04** Find  $8x^3 - \frac{1}{x^3}$ , when  $2x - \frac{1}{x} = 4$

**Solution:** Given that

$$\text{As } 2x - \frac{1}{x} = 4$$

Cubing on both sides, we set

$$(2x - \frac{1}{x})^3 = (4)^3$$

$$(2x)^3 - (\frac{1}{x})^3 - 3(2x)(\frac{1}{x})(2x - \frac{1}{x}) = 64$$

$$8x^3 - \frac{1}{x^3} - 6(4) = 64$$

$$8x^3 - \frac{1}{x^3} - 24 = 64 \quad \Rightarrow 8x^3 - \frac{1}{x^3} = 88$$



### 3.2.4 Know the formula $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$ .

(i)  $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

**Proof:** R.H.S  $= (a+b)(a^2 - ab + b^2)$   
 $= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$   
 $= a^3 + b^3 = \text{L.H.S}$  Hence proved

(ii)  $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

**Proof:** R.H.S  $= (a-b)(a^2 + ab + b^2)$   
 $= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$   
 $= a^3 - b^3 = \text{L.H.S}$  Hence proved

**Example 01** Find the product of  $\left(x + \frac{1}{x}\right)$  and  $\left(x^2 + \frac{1}{x^2} - 1\right)$

**Solution:**  $\left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right)$   
 $\quad \quad \quad \left(x + \frac{1}{x}\right)\left(x^2 - (x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2\right)$   
 $\therefore (a+b)(a^2 - ab + b^2) = a^3 + b^3$

Thus,  $\left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right) = x^3 + \frac{1}{x^3}$

**Example 02** Find the product of  $\left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right) = x^3 - \frac{1}{x^3}$

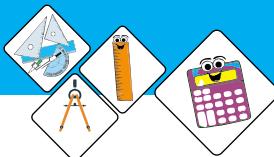
**Solution:**  $\left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right)$   
 $\quad \quad \quad \left(x - \frac{1}{x}\right)\left(x^2 + (x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2\right)$   
 $\therefore (a-b)(a^2 + ab + b^2) = a^3 - b^3$

Thus,  $\left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right) = x^3 - \frac{1}{x^3}$

**Example 03** Find the continued product of:

$$(x+y)(x^2 - xy + y^2)(x-y)(x^2 + xy + y^2)$$

**Solution:**  $(x+y)(x^2 - xy + y^2)(x-y)(x^2 + xy + y^2)$   
 $= (x^3 + y^3)(x^3 - y^3) \quad [\because (a \pm b)(a^2 \mp ab + b^2) = a^3 \pm b^3]$   
 $= (x^3)^2 - (y^3)^2$   
 $= x^6 - y^6$



### Exercise 3.2

**Find the value of**

1.  $a^2 + b^2$  and  $ab$ , when  $a + b = 8$  and  $a - b = 6$ .
2.  $a^2 + b^2$  and  $ab$ , when  $a + b = 5$  and  $a - b = 3$ .
3.  $a^2 + b^2 + c^2$ , when  $a + b + c = 9$  and  $ab + bc + ac = 13$ .
4.  $a^2 + b^2 + c^2$ , when  $a + b + c = \frac{1}{3}$  and  $ab + bc + ac = \frac{-2}{9}$ .
5.  $a + b + c$ , when  $a^2 + b^2 + c^2 = 29$  and  $ab + bc + ac = 10$ .
6.  $a + b + c$ , when  $a^2 + b^2 + c^2 = 0.9$  and  $ab + bc + ac = 0.8$ .
7.  $ab + bc + ac$ , when  $a + b + c = 10$  and  $a^2 + b^2 + c^2 = 20$ .
8.  $a^3 + b^3$ , when  $a + b = 4$  and  $ab = 3$ .
9.  $ab$ , when  $a^3 - b^3 = 5$  and  $a - b = 5$ .
10.  $ab$ , when  $a^3 - b^3 = 16$  and  $a - b = 4$ .
11.  $a^3 - b^3$ , when  $a - b = 5$  and  $ab = 7$ .
12.  $125x^3 + y^3$  when  $5x + y = 13$  and  $xy = 10$ .
13.  $216a^3 - 343b^3$ , when  $6a - 7b = 11$  and  $ab = 8$ .

14.  $x^3 + \frac{1}{x^3}$ , when  $x + \frac{1}{x} = 7$ .

15.  $x^3 - \frac{1}{x^3}$ , when  $x - \frac{1}{x} = 11$

16. Find product of

(i)  $\left(\frac{3}{2}b + \frac{2}{3b}\right)\left(\frac{9b^2}{4} + \frac{4}{9b^2} - 1\right)$       (ii)  $\left(\frac{7y^2}{9} + \frac{9}{7y^2}\right)\left(\frac{49y^4}{81} + \frac{81}{49y^4} - 1\right)$

(iii)  $\left(\frac{x^4}{12} + \frac{12}{x^4}\right)\left(\frac{x^8}{144} + \frac{144}{x^8} + 1\right)$       (iv)  $\left(c^2 - \frac{1}{c^2}\right)\left(c^4 + \frac{1}{c^4} + 1\right)$

17. Find the continued product by using the relevant formulas.

- (i)  $(2x^2 + 3y^2)(4x^4 - 6x^2y^2 + 9y^4)$
- (ii)  $(2x^2 - 3y^2)(4x^4 + 6x^2y^2 + 9y^4)$
- (iii)  $(x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$ .
- (iv)  $(2x + 3y)(2x - 3y)(4x^2 + 9y^2)(16x^4 + 81y^4)$



### 3.3 Surds and their Applications

#### 3.3.1 Recognize the surds and their applications.

**Surd:** An expression is called a surd which has at least one term contain radical term in its simplified form.

For examples,  $\sqrt{2}$ ,  $\sqrt{a-4}$ ,  $\sqrt[3]{\frac{5}{10}}$ ,  $\left(\frac{1}{3} + \sqrt{3}\right)$ ,  $\left(\sqrt[5]{2} - \frac{1}{2}\right)$  are surds.

All surds are irrational numbers.

If  $\sqrt[n]{a}$  is an irrational number and 'a' is not a perfect  $n^{\text{th}}$  power then it is called a surd of  $n^{\text{th}}$  order. The result of  $\sqrt[n]{a}$  is an irrational number. It is also called an irrational radical with rational radicand.

For examples:  $\sqrt[2]{\frac{5}{7}}$ ,  $\sqrt[3]{5}$ ,  $\sqrt[4]{6}$ ,  $\sqrt[5]{2}$ ,  $\sqrt[7]{10}$  are surds of order 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, 5<sup>th</sup> and 7<sup>th</sup> respectively. But  $\sqrt[3]{27}$  and  $\sqrt[4]{1}$  are not surds because they represent the number 3 and  $\frac{1}{2}$  respectively.

#### 3.3.2 Explain the surds of the second order use basic operations on surds of second order to rationalize the denominators and to evaluate them.

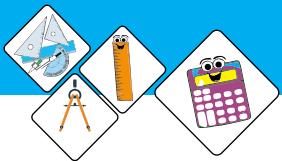
##### (a) Surds of the second order:

(i) A surd which contains a single term is called a monomial surds.

For examples,  $\sqrt{53}$ ,  $\sqrt{a-9}$ ,  $\sqrt{\frac{4}{5}}$  etc. are monomials and of 2<sup>nd</sup> orders.

(ii) A surd which contains sum or difference of two monomial surds or sum of a monomial surd and a rational number is called a binomial surd.

For examples,  $\sqrt{17} + \sqrt{11}$ ,  $\sqrt{2} - 13$ ,  $\sqrt{3} - 35$  etc. are binomial surds and of 2<sup>nd</sup> order.



(iii) Conjugate of Binomial Surds

Expressions of the type

(a)  $(\sqrt{a} + c\sqrt{b})$  and  $(\sqrt{a} - c\sqrt{b})$  are conjugate surds of each other.

(b)  $a + \sqrt{b}$  and  $a - \sqrt{b}$  are conjugate surds of each other.

**(b) Basic operations on surds of second order to rationalize the denominators and to evaluate them.**

### (i) Addition and subtraction of Surds.

The addition and subtraction of surds can be done by using following law.

For example,  $a\sqrt{c} + b\sqrt{c} = (a+b)\sqrt{c}$  and  $a\sqrt{c} - b\sqrt{c} = (a-b)\sqrt{c}$

**Example 01** Simplify:  $\sqrt{343} - 3\sqrt{7} - 2\sqrt{7}$

**Solution:**

$$\begin{aligned}
 & \sqrt{343} - 3\sqrt{7} - 2\sqrt{7} \\
 &= \sqrt{7 \times 7 \times 7} - 3\sqrt{7} - 2\sqrt{7} \\
 &= 7\sqrt{7} - 3\sqrt{7} - 2\sqrt{7} \\
 &= (7 - 3 - 2)\sqrt{7} \\
 &= (7 - 5)\sqrt{7} \\
 &= 2\sqrt{7}
 \end{aligned}$$

**Example 02** Simplify:  $\sqrt{32} + 5\sqrt{2} + \sqrt{128} + 7\sqrt{2}$

**Solution:**

$$\begin{aligned}
 & \sqrt{32} + 5\sqrt{2} + \sqrt{128} + 7\sqrt{2} \\
 &= \sqrt{16 \times 2} + 5\sqrt{2} + \sqrt{64 \times 2} + 7\sqrt{2} \\
 &= \sqrt{(4)^2 \times 2} + 5\sqrt{2} + \sqrt{(8)^2 \times 2} + 7\sqrt{2} \\
 &= 4\sqrt{2} + 5\sqrt{2} + 8\sqrt{2} + 7\sqrt{2} \\
 &= (4+5+8+7)\sqrt{2} \\
 &= 24\sqrt{2}
 \end{aligned}$$

**(ii) Multiplication and Division of Surds.**

The Multiplication and division of the surds can be simplified by using the following laws:

$$(a) \sqrt{a} \times \sqrt{b} = \sqrt{ab} \quad (b) \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}, \text{ provided } a > 0 \text{ and } b > 0.$$

**Example 01** Simplify:  $\sqrt{125} \times \sqrt{48}$

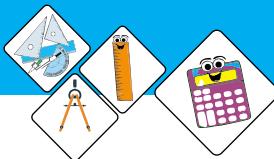
**Solution:** Simplification

$$\begin{aligned} & \sqrt{125} \times \sqrt{48} \\ &= \sqrt{(5)^2 \times 5} \times \sqrt{(4)^2 \times 3} \\ &= 5\sqrt{5} \times 4 \times \sqrt{3} \\ &= (5 \times 4)(\sqrt{5} \times \sqrt{3}) \\ &= 20\sqrt{15} \end{aligned}$$

**Example 02** Simplify:  $\frac{\sqrt{162}}{\sqrt{144}}$

**Solution:**

$$\begin{aligned} & \frac{\sqrt{162}}{\sqrt{144}} \\ &= \frac{\sqrt{2 \times 81}}{\sqrt{12 \times 12}} \\ &= \frac{\sqrt{2 \times (9)^2}}{\sqrt{(12)^2}} = \frac{9\sqrt{2}}{12} = \frac{3\sqrt{2}}{4} \end{aligned}$$



### Exercise 3.3

#### 1. Simplify

(i)  $\sqrt[4]{81x^{-8}z^4}$

(ii)  $\sqrt[3]{256a^6b^{12}c^9}$

(iii)  $\sqrt[7]{128}$

(iv)  $\sqrt{7776}$

(v)  $\frac{\sqrt[3]{(125)^2 \times 8}}{\sqrt{(2 \times 32)^2}}$

(vi)  $\frac{\sqrt{21} \times \sqrt{28}}{\sqrt{121}}$

(vii)  $\sqrt{\frac{(216)^{\frac{2}{3}} \times (125)^2}{(0.04)^{-3}}}$

(viii)  $\frac{\sqrt[6]{4} \times \sqrt[3]{27} \times \sqrt{60}}{\sqrt{180} \times \sqrt[3]{0.25} \times \sqrt[4]{9}}$

#### 2. Find the conjugate of

(i)  $(8 - 4\sqrt{3})$

(ii)  $(6\sqrt{6} + 2\sqrt{3})$

(iii)  $(8\sqrt{12} + \sqrt{8})$

(iv)  $(2 - \sqrt{3})$

#### 3. Simplify

(i)  $(6\sqrt{2} + 4\sqrt{2} + 7\sqrt{128})$

(ii)  $\sqrt{5} + \sqrt{125} + 7\sqrt{5}$

(iii)  $(13 + 15\sqrt{3}) + (7 - 6\sqrt{3})$

(iv)  $\sqrt{250} + \sqrt{490} + 3\sqrt{10}$

(v)  $\sqrt{245} + \sqrt{625} - \sqrt{45}$

(vi)  $10\sqrt{11} - \sqrt{396} - 3\sqrt{11}$

(vii)  $\sqrt{17}(10\sqrt{17} - 2\sqrt{17})$

(viii)  $\frac{3}{2}(\sqrt{18} + \sqrt{32} - \sqrt{50})$

(ix)  $\left(\frac{\sqrt{2}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right) \left(\frac{\sqrt{2}}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right)$

(x)  $(\sqrt{13} + \sqrt{11})(\sqrt{13} - \sqrt{11})$

(xi)  $(3\sqrt{6} - 4\sqrt{5})^2$

(xii)  $(2\sqrt{3} + 3\sqrt{2})^2$



### 3.4 Rationalization

**3.4.1 Explain rationalization (with precise meaning) of real numbers on surds of the types  $\frac{1}{a+b\sqrt{x}}$ ,  $\frac{1}{\sqrt{x}+\sqrt{y}}$  and their combinations, where  $x,y$  are natural number and  $a$  and  $b$  are integer.**

- If the product of two surds is a rational number, then each surd is called the rationalizing factor of the other.  
For example,  $(35 + \sqrt{31})$  and  $(35 - \sqrt{31})$  are rationalizing factor of each other.
- The process of multiplying a given surd by its rationalizing factor to get a rational number as product is called rationalization of the given surd. The product of the conjugate surds is a rational number.

**Example 01** Find the product of  $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$

**Solution:** 
$$\begin{aligned} & (\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) \\ &= (\sqrt{3})^2 - (\sqrt{2})^2 \\ &= 3 - 2 = 1 \text{ which is a rational number.} \end{aligned}$$

#### Rationalization of denominator

Keeping the above discussion in mind, we observe that, in order to rationalize a denominator of the form  $(a + b\sqrt{x})$  or  $(a - b\sqrt{x})$ , we multiply both numerator and denominator by the conjugate factor  $(a - b\sqrt{x})$  or  $(a + b\sqrt{x})$ , by doing this we eliminate the radical and thus obtain a denominator free of the surd.

#### Rationalization of real numbers of the Types.

$$(i) \quad \frac{1}{a+b\sqrt{x}} \qquad (ii) \quad \frac{1}{\sqrt{x}+\sqrt{y}}$$

For the expressions  $\frac{1}{a+b\sqrt{x}}$  and  $\frac{1}{\sqrt{x}+\sqrt{y}}$  also their rationalization,

where  $x,y \in \mathbb{N}$  and  $a,b \in \mathbb{Z}$ . The following examples will help to understand the concept of rationalization.



**Example 01** Rationalize:  $\frac{1}{5+2\sqrt{3}}$

**Solution:**  $\frac{1}{5+2\sqrt{3}}$

Multiplying and dividing by conjugate of denominator, we have

$$\begin{aligned} &= \frac{1}{5+2\sqrt{3}} \times \frac{5-2\sqrt{3}}{5-2\sqrt{3}} \\ &= \frac{5-2\sqrt{3}}{(5)^2 - (2\sqrt{3})^2} \\ &= \frac{5-2\sqrt{3}}{25-12} \\ &= \frac{5-2\sqrt{3}}{13} \end{aligned}$$

Observe that the denominator has been obtained free from radical sign due to rationalization. Hence, we obtain a rational number in the denominator. This process is said to be rationalization

**Example 02** Rationalize:  $\frac{5}{\sqrt{3}+\sqrt{2}}$

**Solution:**

$$\begin{aligned} \frac{5}{\sqrt{3}+\sqrt{2}} &= \frac{5}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} \\ &= \frac{5(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2 - (\sqrt{2})^2} \\ &= \frac{5(\sqrt{3}-\sqrt{2})}{3-2} \\ &= \frac{5(\sqrt{3}-\sqrt{2})}{1} \\ &= 5(\sqrt{3}-\sqrt{2}) \end{aligned}$$

**Example 03** If  $x = 2-\sqrt{3}$

then find the value of

$$x^2 - \frac{1}{x^2}$$

**Solution:** As  $x = 2-\sqrt{3}$

$$\therefore \frac{1}{x} = \frac{1}{2-\sqrt{3}}$$

$$\frac{1}{x} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$\frac{1}{x} = \frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})}$$

$$\frac{1}{x} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$$

$$\therefore x + \frac{1}{x} = (2-\sqrt{3}) + (2+\sqrt{3})$$

$$x + \frac{1}{x} = 4$$

$$\therefore x - \frac{1}{x} = (2-\sqrt{3}) - (2+\sqrt{3})$$

$$x - \frac{1}{x} = 2\sqrt{3} - 2\sqrt{3}$$

$$x - \frac{1}{x} = -2\sqrt{3}$$

Now,

$$x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$x^2 - \frac{1}{x^2} = 4(-2\sqrt{3})$$

$$x^2 - \frac{1}{x^2} = -8\sqrt{3}$$

$$\text{Also, } \left(x + \frac{1}{x}\right)^2 = (4)^2$$

$$\Rightarrow x^2 + 2 + \frac{1}{x^2} = 16$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 16 - 2 = 14$$



### Exercise 3.4



**1.** Rationalize the denominator of the following.

(i)  $\frac{1}{2+\sqrt{3}}$

(ii)  $\frac{1}{3+2\sqrt{2}}$

(iii)  $\frac{1}{4\sqrt{3}-5\sqrt{2}}$

(iv)  $\frac{16}{2\sqrt{3}+\sqrt{11}}$

(v)  $\frac{9-\sqrt{2}}{9+\sqrt{2}}$

(vi)  $\frac{\sqrt{13}+3}{\sqrt{13}-3}$

**2.** (i) If  $x = 8 - 3\sqrt{7}$ , find the value of  $\left(x + \frac{1}{x}\right)^2$

(ii) If  $\frac{1}{x} = 2\sqrt{28} - 11$ , find the value of  $x$ .

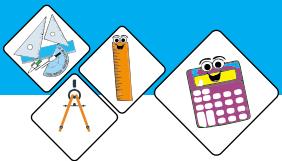
(iii) If  $x = 3 - 2\sqrt{2}$

find the value of:  $x + \frac{1}{x}, x - \frac{1}{x}, x^2 + \frac{1}{x^2}, x^2 - \frac{1}{x^2}$  and  $x^4 + \frac{1}{x^4}$

**3.** If  $x = \sqrt{5} + 2$ , find the value of  $x^4 + \frac{1}{x^4}$ .

**4.** If  $\frac{1}{y} = 2 + \sqrt{3}$ , find the value of  $y^4 + \frac{1}{y^4}$ .

**5.** If  $\frac{1}{z} = 7 - 4\sqrt{3}$ , find the value of  $z^2 - z^{-2}$ .



## Review Exercise 3

**1.** Encircle the correct answer.

- (i) Every polynomial is:
 

(a) an irrational expression	(b) a rational expression
(c) a sentence	(d) none of these
- (ii) A surd which contains sum of two monomial surds is called
 

(a) Trinomial surd	(b) Binomial surd
(c) Conjugate surd	(d) Monomial surd
- (iii)  $3x + 2y - 3$  is an algebraic
 

(a) Expression	(b) Equation
(c) Sentence	(d) In-equation
- (iv) The degree of the  $3x^2y + 5y^4 - 10$  is
 

(a) 4	(b) 5	(c) 6	(d) 3
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- (v)  $\sqrt{7}$  is an example of
 

(a) Monomial surd	(b) Trinomial surd
(c) Binomial surd	(d) Conjugate surd
- (vi) Quotient  $\frac{p(x)}{q(x)}$  of two polynomials  $p(x)$  and  $q(x)$ , where  $q(x) \neq 0$  is called
 

(a) Rational expression	(b) Irrational expression
(c) Polynomial	(d) Conjugate
- (vii)  $\frac{1}{x-y} - \frac{1}{x+y}$  is equal to
 

(a) $\frac{2x}{x^2 - y^2}$	(b) $\frac{2y}{x^2 - y^2}$	(c) $\frac{-2x}{x^2 - y^2}$	(d) $\frac{-2y}{x^2 - y^2}$
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- (viii) Conjugate of  $2 - \sqrt{3}$  is
 

(a) $2 + \sqrt{3}$	(b) $-2 - \sqrt{3}$	(c) $\sqrt{2} + 3$	(d) $\sqrt{3} - 4$
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- (ix)  $a^3 - 3ab(a-b) - b^3$  is equal to
 

(a) $(a-b)^3$	(b) $(a+b)^3$	(c) $a^3 + b^3$	(d) $a^3 - b^3$
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- (x) If  $a+b=5$  and  $a-b=3$ , then the value of  $ab$  is
 

(a) 4	(b) 5	(c) 3	(d) 6
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- (xi)  $(5 + \sqrt{15})(5 - \sqrt{15})$  is equal to  
 (a) 10      (b) 15      (c) 25      (d) 30
- (xii)  $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$  is equal to  
 (a)  $(a+b-c)^2$       (b)  $(a+b+c)^2$   
 (c)  $(a-b+c)^2$       (d)  $(a+b+c)^3$

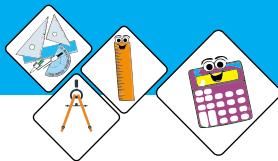
**2. Fill in the blanks.**

- (i) Degree of any polynomial is \_\_\_\_\_ .
- (ii) Conjugate of surd  $2 - \sqrt{3}$  is \_\_\_\_\_ .
- (iii) Degree of polynomial  $2x^3 + x^2 - 4x^4 + 7x - 9$  is \_\_\_\_\_ .
- (iv)  $\frac{\sqrt{x}}{3x+5}$  is a/an \_\_\_\_\_ expression.
- (v)  $(x-y)(x+y)(x^2+y^2) =$  \_\_\_\_\_ .



## Summary

- ◆ A polynomial expression (simply say polynomial) in one variable  $x$  can be written as:  $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + \dots + a_{n-1}x^1 + a_n$ .  
 A polynomial is usually denoted by  $p(x)$ .
- ◆ An algebraic expression which can be written in the form  $\frac{p(x)}{q(x)}$ , where  $q(x) \neq 0$ , and  $p(x), q(x)$  are both polynomials, called **rational expression** in  $x$ .
- ◆ An algebraic expression which cannot be written in form of  $\frac{p(x)}{q(x)}$ , where  $q(x) \neq 0$ , and  $p(x), q(x)$  are both polynomials, called **irrational expression**.
- ◆ A polynomial expression consisting of only single term is called **monomial**.
- ◆ A polynomial expression consisting of two terms is called **binomial**.
- ◆ A polynomial expression consisting of three terms is called **trinomial**.
- ◆ Polynomial expression consisting two or more than two terms is called **multinomial**.



- ◆ The rational expression  $\frac{p(x)}{q(x)}$ , is said to be in its lowest form, if  $p(x)$  and  $q(x)$  are polynomials with integral coefficients and have no common factor.
- ◆  $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$  and  $(a+b)^2 - (a-b)^2 = 4ab$ .
- ◆  $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$ .
- ◆  $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$  and  $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- ◆  $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ .
- ◆  $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ .
- ◆ An expression is called a surd which has at least one term involving a radical sign. For example,  $\sqrt{2}$ ,  $\sqrt{5}$ ,  $\sqrt{\frac{3}{10}}$  are surds.
- ◆ If  $\sqrt[n]{a}$  is an irrational number and 'a' is not a perfect  $n^{\text{th}}$  power then it is called a surd of  $n^{\text{th}}$  order.
- ◆ A surd which contains a single term is called a monomial.
- ◆ A surd which contains sum or difference of two surds or sum of monomial surd and a rational number is called binomial surd.
- ◆ Expressions of the type  $(\sqrt{a} + c\sqrt{b})$  and  $(\sqrt{a} - c\sqrt{b})$  are conjugate surds of each other.
- ◆  $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$  and  $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ , provided  $a > 0$  and  $b > 0$ .
- ◆ If the product of two surds is a rational number, then each surd is called the rationalizing factor of the other.
- ◆ The process of multiplying a given surd by its rationalizing factor to get a rational number as product is called rationalization of the given surd.

