Example 03 Find the value of $(ab+bc+ac)$, when $a+b+c=8$ and $a^2+b^2+c^2=20$.

Solution: Given that,

$a+b+c=8$ and $a^2+b^2+c^2=20$,

We know that $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

By substituting the values of $a+b+c=8$ and $a^2+b^2+c^2=20$, in the above formula we get,

$(8)^2=20+2(ab+bc+ac)$

$\Rightarrow \quad 64=20+2(ab+bc+ac)$

$\Rightarrow \quad 64-20=2(ab+bc+ac)$

$\Rightarrow \quad 44=2(ab+bc+ac)$

$\Rightarrow \quad 22=ab+bc+ac$

$\Rightarrow \quad ab+bc+ac=22$

Hence, the value of $(ab+bc+ac)$ is 22.

3.2.3 Know the cubic formulas

(i) $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

or $(a+b)^3=a^3+b^3+3ab(a+b)$

Proof: L.H.S $=(a+b)^3=(a+b)(a+b)^2$

$$=(a+b)(a^2+2ab+b^2)$$

$$=a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$$

$$=a^3+3a^2b+3ab^2+b^3$$

$$=a^3+b^3+3ab(a+b)\ =\text{R.H.S}$$

Hence proved

(ii) $(a-b)^3=a^3-b^3+3ab(a-b)$

or $(a-b)^3=a^3-b^3-3ab(a-b)$

Proof: L.H.S $=(a-b)^3=(a-b)(a-b)^2$

$$=(a-b)(a^2-2ab+b^2)$$

$$=a^3-2a^2b+ab^2-a^2b+2ab^2-b^3$$

$$=a^3-3a^2b+3ab^2-b^3$$

$$=a^3-b^3-3ab(a-b)\ =\text{R.H.S}$$

Hence proved

The following examples are helpful for understanding the application for Cubic formulas.

Example 01 Find the value of $a^3 + b^3$, when $a + b = 4$ and $ab = 5$.

Solution: Given that,

$a + b = 4$ and $ab = 5$

We have to find

$a^3 + b^3$

Since, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$.

By substituting the values of $a + b = 4$ and $ab = 5$, in the above formula we get,

$(4)^3 = a^3 + b^3 + 3(5)(4)$

$\Rightarrow \quad 64 = a^3 + b^3 + 60$

$\Rightarrow \quad 64 - 60 = a^3 + b^3$

$\Rightarrow \quad 4 = a^3 + b^3$

$\Rightarrow \quad \boxed{a^3 + b^3 = 4}$

Hence the value of $(a^3 + b^3)$ is 4.

Example 02 Find the value of ab, when $a^3 - b^3 = 5$ and $a - b = 5$.

Solution: Given that,

$a^3 - b^3 = 5$ and $a - b = 5$

We have to find ab

Since, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$.

Now, substituting the values of $a^3 - b^3 = 5$ and $a - b = 5$, in the above formula, we get,

$(5)^3 = 5 - 3ab(5)$

$\Rightarrow \quad 125 = 5 - 15ab$

$\Rightarrow \quad 125 - 5 = -15ab$

$\Rightarrow \quad 120 = -15ab$

$\Rightarrow \quad -8 = ab$

$\Rightarrow \quad \boxed{ab = -8}$

Example 03 Find the value of $x^3+\frac{1}{x^3}$ when $x+\frac{1}{x}=3$

Solution: Given that

$$x+\frac{1}{x}=3$$

Taking cube on both sides, we have

$$\left(x+\frac{1}{x}\right)^3=3^3$$

$$\Rightarrow \quad x^3+\frac{1}{x^3}+3x\cdot\frac{1}{x}\left(x+\frac{1}{x}\right)=27 \qquad \left[\because (a+b)^3=a^3+b^3+3ab(a+b)\right]$$

$$\Rightarrow \quad x^3+\frac{1}{x^3}+3(3)=27$$

$$\Rightarrow \quad x^3+\frac{1}{x^3}+9=27$$

$$\Rightarrow \quad x^3+\frac{1}{x^3}=27-9$$

$$\Rightarrow \quad \boxed{x^3+\frac{1}{x^3}=18}$$

Example 04 Find $8x^3-\frac{1}{x^3}$, when $2x-\frac{1}{x}=4$

Solution: Given that

As $2x-\frac{1}{x}=4$

Cubing on both sides, we set

$$(2x-\frac{1}{x})^3=(4)^3$$

$$(2x)^3-(\frac{1}{x})^3-3(2x)(\frac{1}{x})(2x-\frac{1}{x})=64$$

$$8x^3-\frac{1}{x^3}-6(4)=64$$

$$8x^3-\frac{1}{x^3}-24=64 \qquad \Rightarrow 8x^3-\frac{1}{x^3}=88$$

3.2.4 Know the formula $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$.

(i) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Proof: R.H.S $= (a+b)(a^2 - ab + b^2)$

$= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$

$= a^3 + b^3 =$ L.H.S

Hence proved

(ii) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Proof: R.H.S $= (a-b)(a^2 + ab + b^2)$

$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$

$= a^3 - b^3 =$ L.H.S

Hence proved

Example 01 Find the product of $\left(x + \frac{1}{x}\right)$ and $\left(x^2 + \frac{1}{x^2} - 1\right)$

Solution: $\left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right)$

$\left(x + \frac{1}{x}\right)\left((x)^2 - (x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2\right)$

$\because (a+b)(a^2 - ab + b^2) = a^3 + b^3$

Thus, $\left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right) = x^3 + \frac{1}{x^3}$

Example 02 Find the product of $\left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right) = x^3 - \frac{1}{x^3}$

Solution: $\left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right)$

$\left(x - \frac{1}{x}\right)\left((x)^2 + (x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2\right)$

$\because (a-b)(a^2 + ab + b^2) = a^3 - b^3$

Thus, $\left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right) = x^3 - \frac{1}{x^3}$

Example 03 Find the continued product of:

$(x+y)(x^2 - xy + y^2)(x-y)(x^2 + xy + y^2)$

Solution: $(x+y)(x^2 - xy + y^2)(x-y)(x^2 + xy + y^2)$

$= (x^3 + y^3)(x^3 - y^3)$ $[\because (a \pm b)(a^2 \mp ab + b^2) = a^3 \pm b^3]$

$= (x^3)^2 - (y^3)^2$

$= x^6 - y^6$

Exercise 3.2

Find the value of

1. $a^2 + b^2$ and ab, when $a + b = 8$ and $a - b = 6$.
2. $a^2 + b^2$ and ab, when $a + b = 5$ and $a - b = 3$.
3. $a^2 + b^2 + c^2$, when $a + b + c = 9$ and $ab + bc + ac = 13$.
4. $a^2 + b^2 + c^2$, when $a + b + c = \dfrac{1}{3}$ and $ab + bc + ac = \dfrac{-2}{9}$.
5. $a + b + c$, when $a^2 + b^2 + c^2 = 29$ and $ab + bc + ac = 10$.
6. $a + b + c$, when $a^2 + b^2 + c^2 = 0.9$ and $ab + bc + ac = 0.8$.
7. $ab + bc + ac$, when $a + b + c = 10$ and $a^2 + b^2 + c^2 = 20$.
8. $a^3 + b^3$, when $a + b = 4$ and $ab = 3$.
9. ab, when $a^3 - b^3 = 5$ and $a - b = 5$.
10. ab, when $a^3 - b^3 = 16$ and $a - b = 4$.
11. $a^3 - b^3$, when $a - b = 5$ and $ab = 7$.
12. $125x^3 + y^3$ when $5x + y = 13$ and $xy = 10$.
13. $216a^3 - 343b^3$, when $6a - 7b = 11$ and $ab = 8$.
14. $x^3 + \dfrac{1}{x^3}$, when $x + \dfrac{1}{x} = 7$.
15. $x^3 - \dfrac{1}{x^3}$, when $x - \dfrac{1}{x} = 11$
16. Find product of

    (i) $\left(\dfrac{3}{2}b + \dfrac{2}{3b}\right)\left(\dfrac{9b^2}{4} + \dfrac{4}{9b^2} - 1\right)$ (ii) $\left(\dfrac{7y^2}{9} + \dfrac{9}{7y^2}\right)\left(\dfrac{49y^4}{81} + \dfrac{81}{49y^4} - 1\right)$

    (iii) $\left(\dfrac{x^4}{12} + \dfrac{12}{x^4}\right)\left(\dfrac{x^8}{144} + \dfrac{144}{x^8} + 1\right)$ (iv) $\left(c^2 - \dfrac{1}{c^2}\right)\left(c^4 + \dfrac{1}{c^4} + 1\right)$

17. Find the continued product by using the relevant formulas.

    (i) $(2x^2 + 3y^2)(4x^4 - 6x^2y^2 + 9y^4)$

    (ii) $(2x^2 - 3y^2)(4x^4 + 6x^2y^2 + 9y^4)$

    (iii) $(x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$.

    (iv) $(2x + 3y)(2x - 3y)(4x^2 + 9y^2)(16x^4 + 81y^4)$

3.3 Surds and their Applications

3.3.1 Recognize the surds and their applications.

Surd: An expression is called a surd which has at least one term contain radical term in its simplified form.

For examples, $\sqrt{2}, \sqrt{a-4}, \sqrt[3]{\frac{5}{10}}, \left(\frac{1}{3}+\sqrt{3}\right), \left(\sqrt[5]{2}-\frac{1}{2}\right)$ are surds.

All surds are irrational numbers.

If $\sqrt[n]{a}$ is an irrational number and 'a' is not a perfect n^{th} power then it is called a surd of n^{th} order. The result of $\sqrt[n]{a}$ is an irrational number. It is also called an irrational radical with rational radicand.

For examples: $\sqrt{\frac{5}{7}}, \sqrt[3]{5}, \sqrt[4]{6}, \sqrt[5]{2}, \sqrt[7]{10}$ are surds of order 2^{nd}, 3^{rd}, 4^{th}, 5^{th} and 7^{th} respectively. But $\sqrt[3]{27}$ and $\sqrt{\frac{1}{4}}$ are not surds because they represent the number 3 and $\frac{1}{2}$ respectively.

3.3.2 Explain the surds of the second order use basic operations on surds of second order to rationalize the denominators and to evaluate them.

(a) Surds of the second order:

(i) A surd which contains a single term is called a monomial surds.

For examples, $\sqrt{53}, \sqrt{a-9}, \sqrt{\frac{4}{5}}$ etc. are monomials and of 2^{nd} orders.

(ii) A surd which contains sum or difference of two monomial surds or sum of a monomial surd and a rational number is called a binomial surd.

For examples, $\sqrt{17}+\sqrt{11}, \sqrt{2}-13, \sqrt{3}-35$ etc. are binomial surds and of 2^{nd} order.

(iii) Conjugate of Binomial Surds

Expressions of the type

(a) $\left(\sqrt{a}+c\sqrt{b}\right)$ and $\left(\sqrt{a}-c\sqrt{b}\right)$ are conjugate surds of each other.

(b) $a+\sqrt{b}$ and $a-\sqrt{b}$ are conjugate surds of each other.

(b) Basic operations on surds of second order to rationalize the denominators and to evaluate them.

(i) Addition and subtraction of Surds.

The addition and subtraction of surds can be done by using following law.

For example, $a\sqrt{c}+b\sqrt{c}=(a+b)\sqrt{c}$ and $a\sqrt{c}-b\sqrt{c}=(a-b)\sqrt{c}$

Example 01 Simplify: $\sqrt{343}-3\sqrt{7}-2\sqrt{7}$

Solution:

$$\sqrt{343}-3\sqrt{7}-2\sqrt{7}$$
$$=\sqrt{7\times 7\times 7}-3\sqrt{7}-2\sqrt{7}$$
$$=7\sqrt{7}-3\sqrt{7}-2\sqrt{7}$$
$$=(7-3-2)\sqrt{7}$$
$$=(7-5)\sqrt{7}$$
$$=2\sqrt{7}$$

Example 02 Simplify: $\sqrt{32}+5\sqrt{2}+\sqrt{128}+7\sqrt{2}$

Solution:

$$\sqrt{32}+5\sqrt{2}+\sqrt{128}+7\sqrt{2}$$
$$=\sqrt{16\times 2}+5\sqrt{2}+\sqrt{64\times 2}+7\sqrt{2}$$
$$=\sqrt{(4)^2\times 2}+5\sqrt{2}+\sqrt{(8)^2\times 2}+7\sqrt{2}$$
$$=4\sqrt{2}+5\sqrt{2}+8\sqrt{2}+7\sqrt{2}$$
$$=(4+5+8+7)\sqrt{2}$$
$$=24\sqrt{2}$$

(ii) Multiplication and Division of Surds.

The Multiplication and division of the surds can be simplified by using the following laws:

(a) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ (b) $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$, provided $a > 0$ and $b > 0$.

Example 01 Simplify: $\sqrt{125} \times \sqrt{48}$

Solution: Simplification

$$\sqrt{125} \times \sqrt{48}$$
$$= \sqrt{(5)^2 \times 5} \times \sqrt{(4)^2 \times 3}$$
$$= 5\sqrt{5} \times 4 \times \sqrt{3}$$
$$= (5 \times 4)\left(\sqrt{5} \times \sqrt{3}\right)$$
$$= 20\sqrt{15}$$

Example 02 Simplify: $\dfrac{\sqrt{162}}{\sqrt{144}}$

Solution:

$$\frac{\sqrt{162}}{\sqrt{144}}$$
$$= \frac{\sqrt{2 \times 81}}{\sqrt{12 \times 12}}$$
$$= \frac{\sqrt{2 \times (9)^2}}{\sqrt{(12)^2}} = \frac{9\sqrt{2}}{12} = \frac{3\sqrt{2}}{4}$$

Exercise 3.3

1. Simplify

(i) $\sqrt[4]{81x^{-8}z^{4}}$ (ii) $\sqrt[3]{256a^{6}b^{12}c^{9}}$ (iii) $\sqrt[7]{128}$ (iv) $\sqrt{7776}$

(v) $\dfrac{\sqrt[3]{(125)^2\times 8}}{\sqrt{(2\times 32)^2}}$ (vi) $\dfrac{\sqrt{21}\times\sqrt{28}}{\sqrt{121}}$ (vii) $\sqrt{\dfrac{(216)^{\frac{2}{3}}\times(125)^{2}}{(0.04)^{-3}}}$

(viii) $\dfrac{\sqrt[6]{4}\times\sqrt[3]{27}\times\sqrt{60}}{\sqrt{180}\times\sqrt[3]{0.25}\times\sqrt[4]{9}}$

2. Find the conjugate of

(i) $\left(8-4\sqrt{3}\right)$ (ii) $\left(6\sqrt{6}+2\sqrt{3}\right)$ (iii) $\left(8\sqrt{12}+\sqrt{8}\right)$

(iv) $\left(2-\sqrt{3}\right)$

3. Simplify

(i) $\left(6\sqrt{2}+4\sqrt{2}+7\sqrt{128}\right)$ (ii) $\sqrt{5}+\sqrt{125}+7\sqrt{5}$

(iii) $\left(13+15\sqrt{3}\right)+\left(7-6\sqrt{3}\right)$ (iv) $\sqrt{250}+\sqrt{490}+3\sqrt{10}$

(v) $\sqrt{245}+\sqrt{625}-\sqrt{45}$ (vi) $10\sqrt{11}-\sqrt{396}-3\sqrt{11}$

(vii) $\sqrt{17}\left(10\sqrt{17}-2\sqrt{17}\right)$ (viii) $\dfrac{3}{2}\left(\sqrt{18}+\sqrt{32}-\sqrt{50}\right)$

(ix) $\left(\dfrac{\sqrt{2}}{\sqrt{3}}+\dfrac{1}{\sqrt{3}}\right)\left(\dfrac{\sqrt{2}}{\sqrt{3}}-\dfrac{1}{\sqrt{3}}\right)$ (x) $\left(\sqrt{13}+\sqrt{11}\right)\left(\sqrt{13}-\sqrt{11}\right)$

(xi) $\left(3\sqrt{6}-4\sqrt{5}\right)^2$ (xii) $\left(2\sqrt{3}+3\sqrt{2}\right)^2$

3.4 Rationalization

3.4.1 Explain rationalization (with precise meaning) of real numbers on surds of the types $\frac{1}{a+b\sqrt{x}}$, $\frac{1}{\sqrt{x}+\sqrt{y}}$ and their combinations, where x,y are natutal number and a and b are integer.

- If the product of two surds is a rational number, then each surd is called the rationalizing factor of the other.
  For example, $\left(35+\sqrt{31}\right)$ and $\left(35-\sqrt{31}\right)$ are rationalizing factor of each other.
- The process of multiplying a given surd by its rationalizing factor to get a rational number as product is called rationalization of the given surd. The product of the conjugate surds is a rational number.

Example 01 Find the product of $\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)$

Solution: $\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)$

$=\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2$

$=3-2=1$ which is a rational number.

Rationalization of denominator

Keeping the above discussion in mind, we observe that, in order to rationalize a denominator of the form $(a+b\sqrt{x})$ or $(a-b\sqrt{x})$, we multiply both numerator and denominator by the conjugate factor $(a-b\sqrt{x})$ or $(a+b\sqrt{x})$, by doing this we eliminate the radical and thus obtain a denominator free of the surd.

Rationalization of real numbers of the Types.

(i) $\frac{1}{a+b\sqrt{x}}$ (ii) $\frac{1}{\sqrt{x}+\sqrt{y}}$

For the expressions $\frac{1}{a+b\sqrt{x}}$ and $\frac{1}{\sqrt{x}+\sqrt{y}}$ also their rationalization, where $x,y\in\mathbf{N}$ and $a,b\in\mathbf{Z}$. The following examples will help to understand the concept of rationalization.

Example 01 Rationalize: $\dfrac{1}{5+2\sqrt{3}}$

Solution: $\dfrac{1}{5+2\sqrt{3}}$

Multiplying and dividing by conjugate of denominator, we have

$$= \frac{1}{5+2\sqrt{3}} \times \frac{5-2\sqrt{3}}{5-2\sqrt{3}}$$
$$= \frac{5-2\sqrt{3}}{(5)^2-(2\sqrt{3})^2}$$
$$= \frac{5-2\sqrt{3}}{25-12}$$
$$= \frac{5-2\sqrt{3}}{13}$$

Observe that the denominator has been obtained free from radical sign due to rationalization. Hence, we obtain a rational number in the denominator. This process is said to be rationalization

Example 02 Rationalize: $\dfrac{5}{\sqrt{3}+\sqrt{2}}$

Solution:

$$\frac{5}{\sqrt{3}+\sqrt{2}} = \frac{5}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$
$$= \frac{5\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$
$$= \frac{5\left(\sqrt{3}-\sqrt{2}\right)}{3-2}$$
$$= \frac{5\left(\sqrt{3}-\sqrt{2}\right)}{1}$$
$$= 5\left(\sqrt{3}-\sqrt{2}\right)$$

Example 03 If $x = 2-\sqrt{3}$ then find the value of $x^2-\dfrac{1}{x^2}$ and $x^2+\dfrac{1}{x^2}$

Solution: As $x = 2-\sqrt{3}$

$$\therefore \quad \frac{1}{x} = \frac{1}{2-\sqrt{3}}$$
$$\frac{1}{x} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$
$$\frac{1}{x} = \frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})}$$
$$\frac{1}{x} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$$
$$\therefore \quad x+\frac{1}{x} = (2-\sqrt{3})+(2+\sqrt{3})$$
$$x+\frac{1}{x} = 4$$
$$\therefore \quad x-\frac{1}{x} = (2-\sqrt{3})-(2+\sqrt{3})$$
$$x-\frac{1}{x} = \cancel{2}-\sqrt{3} - \cancel{2}-\sqrt{3}$$
$$x-\frac{1}{x} = -2\sqrt{3}$$

Now,

$$x^2-\frac{1}{x^2} = \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$
$$x^2-\frac{1}{x^2} = 4\left(-2\sqrt{3}\right)$$
$$x^2-\frac{1}{x^2} = -8\sqrt{3}$$

Also, $\left(x+\dfrac{1}{x}\right)^2 = (4)^2$

$$\Rightarrow \quad x^2+2+\frac{1}{x^2} = 16$$
$$\Rightarrow \quad x^2+\frac{1}{x^2} = 16-2 = 14$$

Exercise 3.4

1. Rationalize the denominator of the following.

(i) $\dfrac{1}{2+\sqrt{3}}$ (ii) $\dfrac{1}{3+2\sqrt{2}}$ (iii) $\dfrac{1}{4\sqrt{3}-5\sqrt{2}}$

(iv) $\dfrac{16}{2\sqrt{3}+\sqrt{11}}$ (v) $\dfrac{9-\sqrt{2}}{9+\sqrt{2}}$ (vi) $\dfrac{\sqrt{13}+3}{\sqrt{13}-3}$

2. (i) If $x=8-3\sqrt{7}$, find the value of $\left(x+\dfrac{1}{x}\right)^2$

(ii) If $\dfrac{1}{x}=2\sqrt{28}-11$, find the value of x.

(iii) If $x=3-2\sqrt{2}$

find the value of: $x+\dfrac{1}{x}, x-\dfrac{1}{x}, x^2+\dfrac{1}{x^2}, x^2-\dfrac{1}{x^2}$ and $x^4+\dfrac{1}{x^4}$

3. If $x=\sqrt{5}+2$, find the value of $x^4+\dfrac{1}{x^4}$.

4. If $\dfrac{1}{y}=2+\sqrt{3}$, find the value of $y^4+\dfrac{1}{y^4}$.

5. If $\dfrac{1}{z}=7-4\sqrt{3}$, find the value of z^2-z^{-2}.

Review Exercise 3

1. Encircle the correct answer.

(i) Every polynomial is:
(a) an irrational expression (b) a rational expression
(c) a sentence (d) none of these

(ii) A surd which contains sum of two monomial surds is called
(a) Trinomial surd (b) Binomial surd
(c) Conjugate surd (d) Monomial surd

(iii) $3x + 2y - 3$ is an algebraic
(a) Expression (b) Equation
(c) Sentence (d) In–equation

(iv) The degree of the $3x^2y + 5y^4 - 10$ is
(a) 4 (b) 5 (c) 6 (d) 3

(v) $\sqrt{7}$ is an example of
(a) Monomial surd (b) Trinomial surd
(c) Binomial surd (d) Conjugate surd

(vi) Quotient $\dfrac{p(x)}{q(x)}$ of two polynomials $p(x)$ and $q(x)$, where $q(x) \neq 0$ is called
(a) Rational expression (b) Irrational expression
(c) Polynomial (d) Conjugate

(vii) $\dfrac{1}{x-y} - \dfrac{1}{x+y}$ is equal to
(a) $\dfrac{2x}{x^2-y^2}$ (b) $\dfrac{2y}{x^2-y^2}$ (c) $\dfrac{-2x}{x^2-y^2}$ (d) $\dfrac{-2y}{x^2-y^2}$

(viii) Conjugate of $2 - \sqrt{3}$ is
(a) $2+\sqrt{3}$ (b) $-2-\sqrt{3}$ (c) $\sqrt{2}+3$ (d) $\sqrt{3}-4$

(ix) $a^3 - 3ab(a-b) - b^3$ is equal to
(a) $(a-b)^3$ (b) $(a+b)^3$ (c) $a^3 + b^3$ (d) $a^3 - b^3$

(x) If $a+b=5$ and $a-b=3$, then the value of ab is
(a) 4 (b) 5 (c) 3 (d) 6

(xi) $\left(5+\sqrt{15}\right)\left(5-\sqrt{15}\right)$ is equal to

(a) 10 (b) 15 (c) 25 (d) 30

(xii) $a^2+b^2+c^2+2ab+2bc+2ca$ is equal to

(a) $(a+b-c)^2$ (b) $(a+b+c)^2$

(c) $(a-b+c)^2$ (d) $(a+b+c)^3$

2. Fill in the blanks.

(i) Degree of any polynomial is __________ .

(ii) Conjugate of surd $2-\sqrt{3}$ is __________ .

(iii) Degree of polynomial $2x^3+x^2-4x^4+7x-9$ is __________ .

(iv) $\dfrac{\sqrt{x}}{3x+5}$ is a/an __________ expression.

(v) $(x-y)(x+y)(x^2+y^2)=$__________ .

Summary

- A polynomial expression (simply say polynomial) in one variable x can be written as: $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+a_3x^{n-3}+\cdots+a_{n-1}x^1+a_n$
  A polynomial is usually denoted by $p(x)$.
- An algebraic expression which can be written in the form $\dfrac{p(x)}{q(x)}$, where $q(x)\neq 0$, and $p(x)$, $q(x)$ are both polynomials, called **rational expression** in x.
- An algebraic expression which cannot be written in form of $\dfrac{p(x)}{q(x)}$, where $q(x)\neq 0$, and $p(x)$, $q(x)$ are both polynomials, called irrational expression
- A polynomial expression consisting of only single term is called monomial.
- A polynomial expression consisting of two terms is called binomial.
- A polynomial expression consisting of three terms is called trinomial.
- Polynomial expression consisting two or more than two terms is called multinomial.

- The rational expression $\dfrac{p(x)}{q(x)}$, is said to be in its lowest form, if $p(x)$ and $q(x)$ are polynomials with integral coefficients and have no common factor.
- $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$ and $(a+b)^2 - (a-b)^2 = 4ab$.
- $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$.
- $(a+b)^3 = a^3+b^3+3ab(a+b)$ and $(a-b)^3 = a^3-b^3-3ab(a-b)$
- $a^3+b^3 = (a+b)(a^2-ab+b^2)$.
- $a^3-b^3 = (a-b)(a^2+ab+b^2)$.
- An expression is called a surd which has at least one term involving a radical sign. For example, $\sqrt{2}$, $\sqrt{5}$, $\sqrt{\dfrac{3}{10}}$ are surds.
- If $\sqrt[n]{a}$ is an irrational number and **'*a*'** is not a perfect n^{th} power then it is called a surd of n^{th} order.
- A surd which contains a single term is called a monomial.
- A surd which contains sum or difference of two surds or sum of monomial surd and a rational number is called binomial surd.
- Expressions of the type $\left(\sqrt{a}+c\sqrt{b}\right)$ and $\left(\sqrt{a}-c\sqrt{b}\right)$ are conjugate surds of each other.
- $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ and $\dfrac{\sqrt{a}}{\sqrt{b}}=\sqrt{\dfrac{a}{b}}$, provided $a>0$ and $b>0$.
- If the product of two surds is a rational number, then each surd is called the rationalizing factor of the other.
- The process of multiplying a given surd by its rationalizing factor to get a rational number as product is called rationalization of the given surd.