Unit 12

• Weightage = 5%

SIDES AND ANGLES OF A TRAINGLE

Student Learning Outcomes (SLOs)

After completing this unit, students will be able to:

- If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it.
- If two angles of a triangle are unequal in measure, the side opposite to the greater angle is longer than the side opposite to the smaller angle.
- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- From a point, outside a line, the perpendicular is the shortest distance from the point to the line.

Introduction

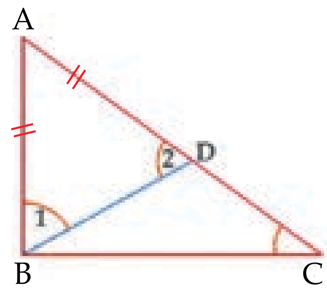
In this unit we will learn the theorems related to the sides and angles of the triangle along with their corollaries and apply them to solve the allied problems.

12.1 Sides and Angles of a Triangle

Theorem 12.1.1

Prove that:

If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it.



Given:

In ΔABC, $m\overline{AC} > m\,\overline{AB}$.

To Prove:

$$m\angle ABC > m\angle ACB$$

Construction:

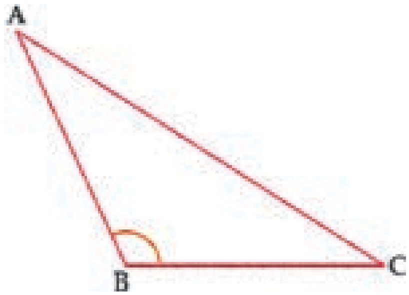
On $\overline{AC}$ take a point D such that $\overline{AD} \cong \overline{AB}$.
Join B to D so that ΔADB is an isosceles triangle.
Label $\angle 1$ and $\angle 2$ as shown in the figure.

Proof:

Statements	Reasons
In ΔABD, $m\angle 1 = m\angle 2$ ... (i)	Angles opposite to congruent sides (construction).
In ΔBCD, $m\angle ACB < m\angle 2$ or $m\angle 2 > m\angle ACB$ ... (ii)	(An exterior angle of a triangle is greater than a non-adjacent interior angles)
$\therefore$ $m\angle 1 > m\angle ACB$ ... (iii)	By (i) and (ii)
But $\angle ABC = m\angle 1 + m\angle DBC$ $\therefore$ $m\angle ABC > m\angle 1$ ... (iv)	Postulate of addition of angles
$\therefore$ $m\angle ABC > m\angle 1 > m\angle ACB$	By (iii) and (iv)
Hence $\angle ABC > m\angle ACB$	(Transitive property of in-equality of real numbers).

Following example will help to understand the above theorem.

Example: Prove that in a scalene triangle, the angle opposite to the largest side is of measure greater than 60°.



Given:

In ΔABC, with, $m\overline{AC} > m\overline{AB}$ and $m\overline{AC} > m\overline{BC}$.

To Prove:

$m\angle B > 60^0$

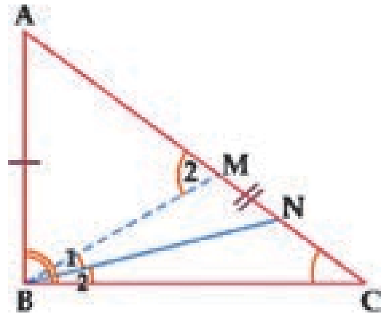
Proof:

Statements	Reasons
In ΔABC.	
We have, $m\angle B > m\angle C$	$m\overline{AC} > m\overline{AB}$
and $m\angle B > m\angle A$	$m\overline{AC} > m\overline{BC}$ } Given
but, $m\angle A + m\angle B + m\angle C = 180^0$	$\angle A$, $\angle B$ and $\angle C$ are the angles of the ΔABC.
$\therefore$ $m\angle B + m\angle B + m\angle B > 180^0$	$m\angle B > m\angle C$ and $m\angle B > m\angle A$
i.e $3\,m\angle B > 180^0$	By addition
$=$ $m\angle B > \frac{180^0}{3}$	Dividing both sides by 3
Thus, $m\angle B > 60^0$	

Q.E.D

Theorem 12.1.2

Prove that if two angles of a triangle are unequal in measure, the side opposite to the greater angle is longer than the side opposite to the smaller angle.



Given:

In ΔABC, $m\angle B > m\angle C$

To Prove:

$m\overline{AC} > m\overline{AB}$

Construction:

Make $\angle ABM \cong \angle C$. Draw $\overline{BN}$, the bisector of $\angle MBC$, i.e. $m\angle 1 = m\angle 2$.

Proof:

Statements	Reasons
$\angle ANB$ is the exterior $\angle$ of ΔCBN	By definition of exterior $\angle$
$\therefore \quad m\angle ANB = m\angle C + m\angle 2$	
$= m\angle C + m\angle 1$	$\therefore \quad m\angle 2 = m\angle 1$ (Construction)
$= m\angle ABM + m\angle 1$	$\therefore \quad m\angle C = m\angle ABM$ (Construction)
$= m\angle ABN$	By angle addition postulate
$\therefore \quad \overline{AB} \cong \overline{AN}$	$\therefore \quad m\angle ANB = m\angle ABN$ (Proved above)
$\therefore \quad m\overline{AC} > m\overline{AB}$	$\therefore \quad m\overline{AC} > m\overline{AN}$

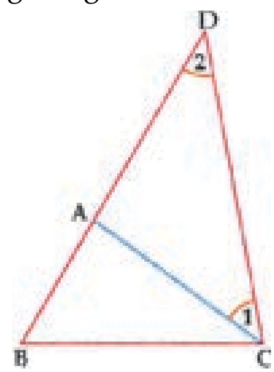
Q.E.D

Corollaries:

1. The hypotenuse of a right angle is longer than each of the other two sides.
2. In an obtuse angled triangle, the side opposite to the obtuse angle is longer than each of the other two sides.

Theorem 12.1.3

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



Given:

ΔABC

To Prove:

i) $m\overline{AB}+m\overline{AC}>m\overline{BC}$

ii) $m\overline{AB}+m\overline{BC}>m\overline{CA}$

iii) $m\overline{AC}+m\overline{BC}>m\overline{AB}$

Construction:

Produce $\overrightarrow{BA}$ to D, making $\overline{AD}\cong\overline{AC}$. Draw $\overline{DC}$

Proof:

Statements	Reasons
In ΔADC, $\overline{AD}\cong\overline{AC}$	Construction
$\therefore\ m\angle 1=m\angle 2$	Angles opposite to congruent sides
But $m\angle BCD>m\angle 1$	$m\angle BCD=m\angle BCA+m\angle 1$
$\therefore\ m\angle BCD>m\angle 2$	Transitive property of inequality
$\therefore$ In ΔBDC, $m\overline{BD}>m\overline{BC}$ ...(i)	Grater angle has greater side opposite to it.
But $m\overline{BD}=m\overline{AB}+m\overline{AD}$	By construction
$=m\overline{AB}+m\overline{AC}$	$m\overline{AD}=m\overline{AC}$
$\therefore\ m\overline{AB}+m\overline{AC}>m\overline{BC}$	Putting value of $\overline{BD}$ in (i)
Similarly, we can prove that:	
$m\overline{AB}+m\overline{BC}>m\overline{AC}$	By the above process
and $m\overline{BC}+m\overline{AC}>m\overline{AB}$	

Q.E.D

The following example will help to understand the above theorem.

Example 01 Which of the following sets of lengths of the sides form a triangle:

(i) 3 cm, 4 cm and 5 cm **(ii)** 4 cm, 5 cm and 4.5 cm

(iii) 60 mm, 80 mm and 10 cm **(iv)** 3 cm, 4 cm and 10 cm

Solution:

(i) 3 cm, 4 cm and 5 cm

Since, $3 + 4 > 5$, $3 + 5 > 4$ and $4 + 5 > 3$

$\therefore$ the sum of the two sides of greater than the 3rd side.

Thus, the given set of lengths form a triangle.

(ii) 4 cm, 5 cm and 4.5 cm

Since, $4 + 5 > 4.5$, $5 + 4.5 > 4$ and $4.5 + 3 > 4$

Thus, the given set of lengths form a triangle.

(iii) 60 mm, 80 mm and 10 cm

Since, 10 mm = 1 cm so, 60 mm = 6 cm and 80 mm = 8 cm

Now, $6 + 8 > 10$, $6 + 10 > 8$ and also $8 + 10 > 6$

Thus, the given set of lengths form a triangle.

Example 02 By using the idea of the above theorem decide, which of the following sets of lengths of the sides form a triangle:

(i) 2 cm, 4 cm and 7 cm **(ii)** 5.5 cm, 5 cm and 9.5 cm

Solution:

(i) 2 cm, 4 cm and 7 cm

Since, $2 + 4 < 7$, $4 + 7 > 2$ and $7 + 2 > 4$

Thus, this type of set of lengths cannot form a triangle.

(ii) 5.5 cm, 5 cm and 9.5 cm

Since, $5.5 + 5 > 9.5$, $5 + 9.5 > 5.5$ and $9.5 + 5.5 > 5$

Thus, the given set of lengths form a triangle.

Activity

If $a = 3$ cm

$b = 4$ cm

$c = 5$ cm

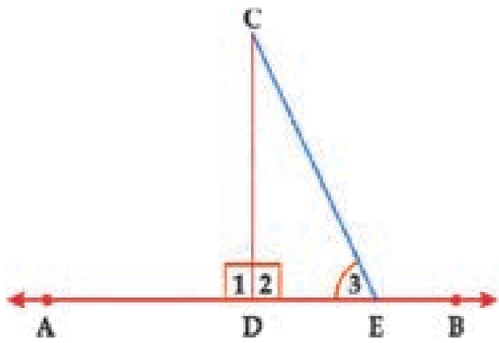
then ΔABC can be formed or not.

Theorem 12.1.4: Prove that:

From a point, outside a line, the perpendicular is the shortest distance from the point to the line.

Given:

From a point C, $\overline{CD}$ is drawn perpendicular to $\overleftrightarrow{AB}$ meeting it in D and $\overline{CE}$ is any other segment meeting $\overleftrightarrow{AB}$ in E.



To Prove:

$m\overline{CD} < m\overline{CE}$

Proof:

Statements	Reasons
$\angle 1$ is an exterior $\angle$ of ΔCDE	By definition of exterior $\angle$
$\therefore \quad m\angle 1 > m\angle 3$	Exterior $\angle$ is greater than non-adjacent interior $\angle$
$\therefore \quad m\angle 2 > m\angle 3$	$m\angle 1 = m\angle 2$ (right $\angle$s)
$\therefore \quad m\overline{CE} > m\overline{CD}$	Side opposite to greater angle
Similarly, it can be proved that $m\overline{CD}$ is less than any other segment drawn from C to $\overleftrightarrow{AB}$	By the above process

Q.E.D

Corollaries:

1. The distance between a line and point (on a line) is zero.

Exercise 12.1

1. **O** is an interior point of the ΔABC.

Show that: $m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2}(m\overline{AB} + m\overline{BC} + m\overline{CA})$.

2. In ΔABC, $m\angle B = 70^o$ and $m\angle C = 45^o$. Which of the sides of the triangle is longest?

3. In ΔABC, $m\angle A = 55^o$ and $m\angle B = 65^o$, which of the side of the triangle is smallest?

Review Exercise 12

1. **Tick (✓) True or False from the following statements.**

(i) Sum of the two sides of a triangle is greater than the third side. **T /F**

(ii) The difference of two sides of a triangle is larger than the third side. **T /F**

(iii) Perpendicular distance from a point to line is the longest distance between them. **T /F**

(iv) In a right angled triangle the largest angle is of 100^o. **T /F**

(v) A perpendicular on a line always makes and angle of 90^o. **T /F**

2. **Fill in the blanks to make the sentences true sentences.**

(i) In any right angled triangle, _______ is the longest side of the triangle.

(ii) In a right angled triangle, sum of the measures of the sides containing right angles is _______ than the measure of the hypotenuse.

(iii) In ΔABC, $m\angle A = 50^o$ and $m\angle B = 30^o$. Side _______ will be longer than its other sides.

(iv) Length of diagonal of any quadrilateral is __________ than the sum of the measures of its two adjacent sides.

3. Tick (✓) the correct answer.

(i) Measure of one side of an equilateral triangle is 6 cm, then the length of its median is ______ 9cm.

a) less than b) greater than

c) equal to d) none of the above

(ii) Perimeter of a rectangle is 22cm, then the length of its diagonal is _____ 11cm.

a) equal to b) greater than

c) less than d) none of the above

Summary

- If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it.
- If two angles of a triangle are unequal in measure, the side opposite to the greater angle is longer than the side opposite to the smaller angle.
- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- From a point, outside a line, the perpendicular is the shortest distance from the point to the line.