# MODELLING AND SIMULATIONS

## MODELING PRINCIPLES

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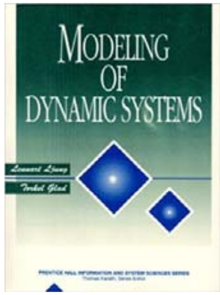
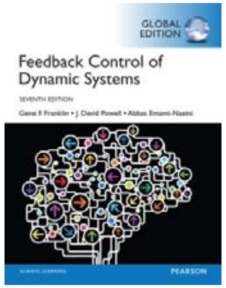
# Materials



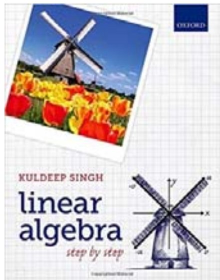

### Main materials

- Ljung, Lennart, and Torkel Glad. "Modeling of dynamic systems." (1994).




### Supporting materials

- Franklin, Gene F., et al. *Feedback control of dynamic systems*. Vol. 3. Reading, MA: Addison-Wesley, 1994.




- Singh, Kuldeep. *Linear algebra: step by step*. OUP Oxford, 2013.

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# Curriculum

- Must have knowledge of the modelling of some typical physical systems, such as mechatronic systems, flow dynamic systems, energy production/transportation/distribution systems, process systems etc., provision of operating conditions
- Must have insight into the theoretical modelling for dynamic systems, including the principles of mass balance, energy balance and momentum balance
- Must have the knowledge of experimental modelling of linear and non-linear dynamic systems, including the experiment design, data collection and pre-filtering, model structure selection, parameter estimation and model validation
- Must have insight of linearization techniques of nonlinear systems,
- Must be able to simulate the obtained mathematical model in some typical simulation environment, such as Matlab/Simulink

# Scope of this course

- Physical modeling
- Transfer functions
- State space models
- Signal analysis
- Filtering
- System identification
- Case studies
    - Process
    - Mechatronic
    - Hydraulic
    - Biologic
- Numerical computing (Matlab/Simulink)

# Content

- What are models for systems and signals?
  - Basic concepts
  - Types of models
- How to build a model for a given system?
  - Physical modeling
  - Experimental modeling

## Definitions & usage

- System is defined as an object or a collection of objects whose properties we want to study
- A model of a system is a tool we use to answer questions about the system without having to do an experiment
  - Mental model
  - Verbal model
  - Physical model
  - Mathematical model

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# Why make a model?

*'A model of a system is a tool we use to answer questions about the system without having to do an experiment' [1]*

- To simulate system behavior without having access to/using the physical system
  - Relevant for large mechanical system e.g. offshore
    - Issues/dangers of doing specific experiment
    - Costly to run experiments
    - Relevant parameters are not measured
- To aid in controller development
  - Behaviors of system *(e.g. in frequency domain)* can be retrieved from model
  - Apply advanced model-based control techniques
  - Test a developed controller on the model *(to do design verification)*

[1] Ljung, Lennart, and Torkel Glad. "Modeling of dynamic systems." (1994)

# What should the model look like?

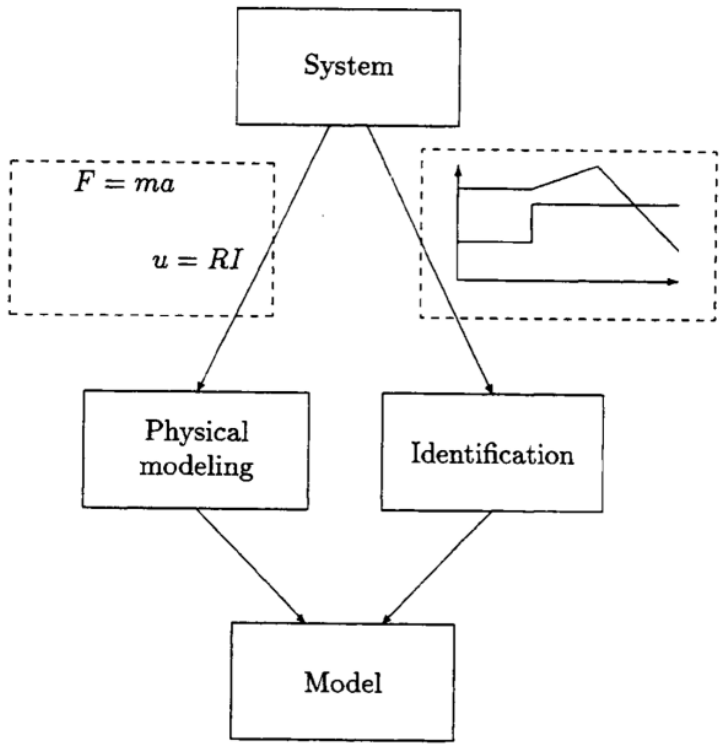
- **What will be modeled?**
  - Choice of what will be included, in the final equations
- **Is a complex model the best?**
  - Are some details too small to be significant, and how can this be determined?
- **When linear and non-linear?**
  - There is a difference in "tools" for linear and nonlinear systems
  - Is the system strongly (very) non-linear or only weakly non-linear ?
  - Does the system behave linearly for the desired operating range?

# Basic principles

- Physical modelling
  - The model has physical relations
  - Equations from physics; e.g. classical mechanics, circuit theory.
  - Not a black-box model

- Identification
  - Input-output relation
  - **Black-box**

- In-between
  - **Grey Box**

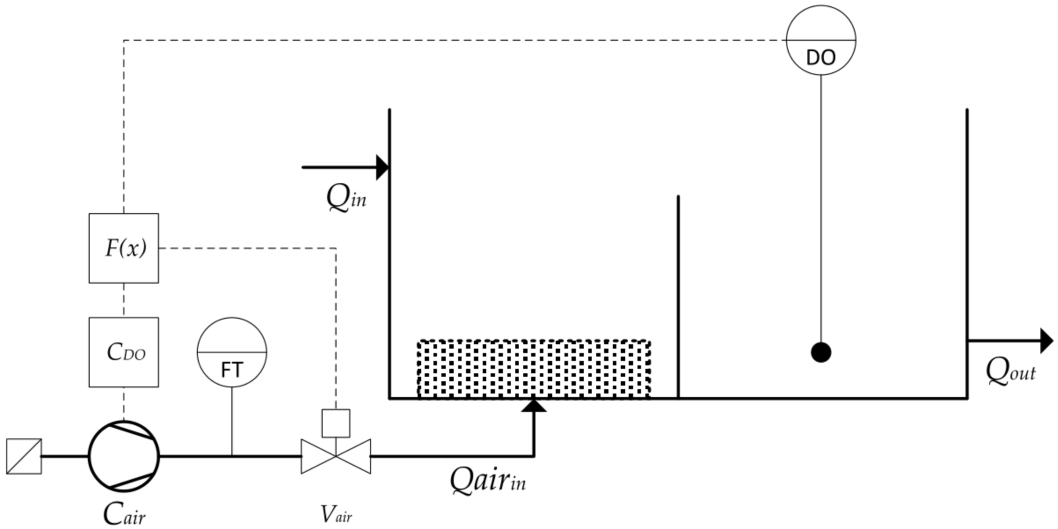


# Principle and Phases

1. Structure the problem:
   - Decomposition (cause and effect, variables) → block diagram
2. Formulate subsystems
   - Usually based on componentization of real system;
     e.g. the propeller, gearbox and generator of a wind turbine
3. Get system model via simplification
   - Selection of which **system behaviors** to include in final model

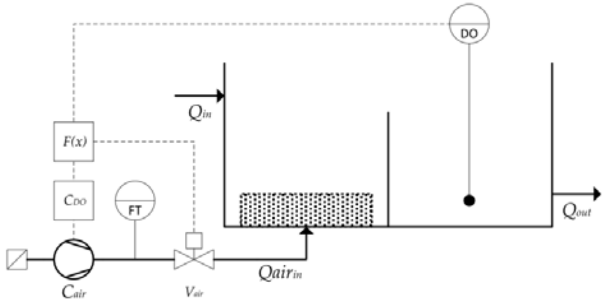
# Validity of models

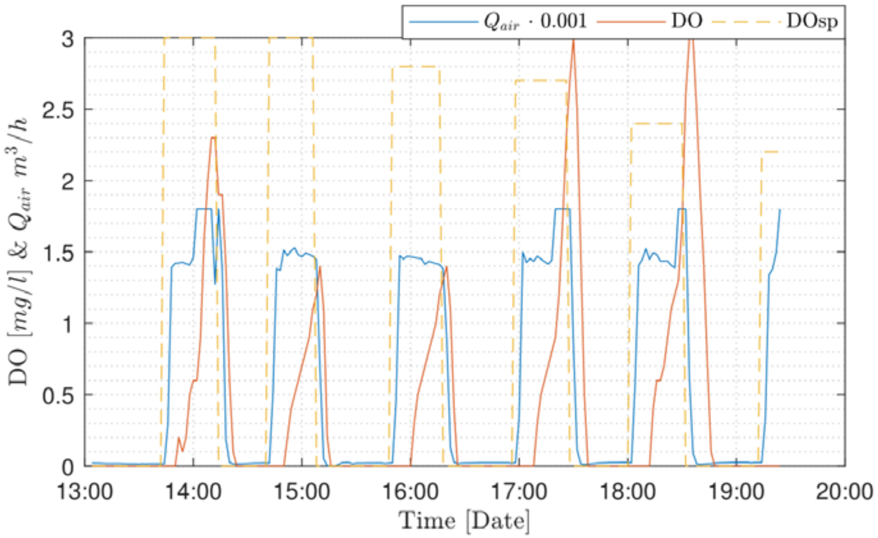
- Example with a WWTP
- Simple diagram
- Complex process



# Validity of models

- Example with a WWTP
- Simple diagram
- Complex process
- **Real data collected**

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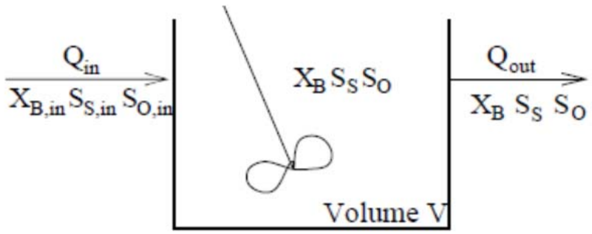
# Validity of models

- Example with a WWTP
- Simple diagram
- Complex process
- Real data collected
- **Simplified model designed**




$$\dot{X}_B(t) = \frac{Q_{in}}{V} \cdot X_{B,in} - \frac{Q_{out}}{V} \cdot X_B(t) + \frac{\mu \cdot S_S(t)}{K_S + S_S(t)} \cdot X_B(t) - b \cdot X_B(t)$$

$$\dot{S}_S(t) = \frac{Q_{in}}{V} \cdot S_{S,in} - \frac{Q_{out}}{V} \cdot S_S(t) - \frac{1}{Y} \cdot \frac{\mu \cdot S_S(t)}{K_S + S_S(t)} \cdot X_B(t)$$

$$\dot{S}_O(t) = \frac{Q_{in}}{V} \cdot S_{O,in} - \frac{Q_{out}}{V} \cdot S_O(t) - \frac{1-Y}{Y} \cdot \frac{\mu \cdot S_S(t)}{K_S + S_S(t)} \cdot X_B(t) - b \cdot X_B(t)$$

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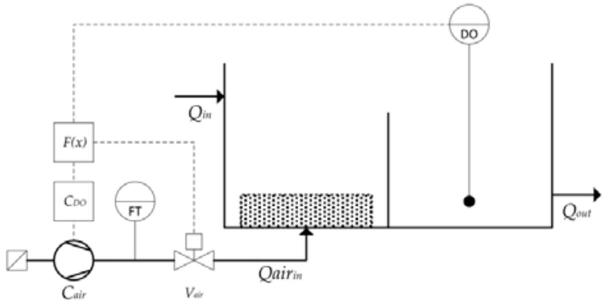
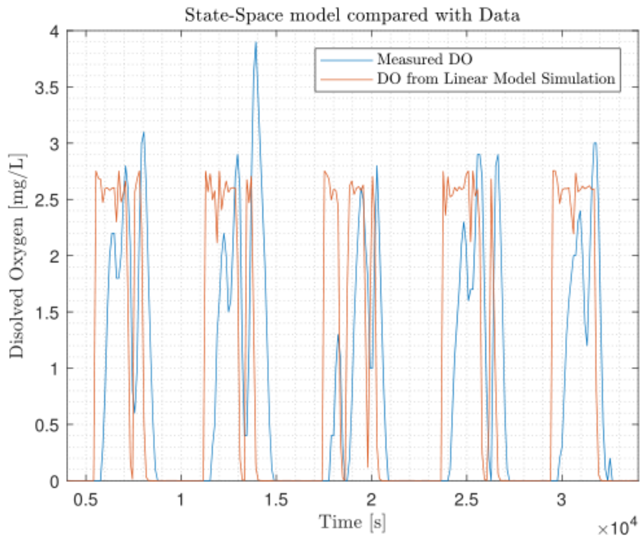
# Validity of models

- Example with a WWTP
- Simple diagram
- Complex process
- Real data collected
- Simplified model designed
- **Comparison with simulations**
  - **Validation results???**

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Newtonian physics

# PHYSICAL MODELING

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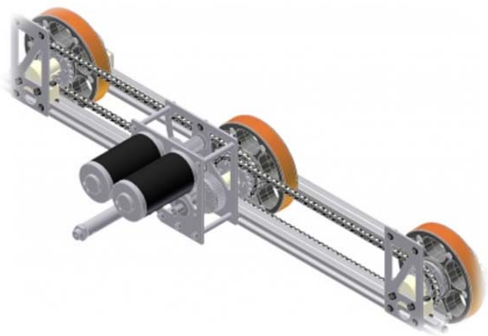
# Classical mechanics - translation & rotation

- There are two main forms of motion that are prevalent in modeling physical systems
  - Translation: means movement of mass in a direction
  - Rotation: means circular motion of a mass about an axis

- Also known as *Newtonian mechanics*



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# Newton's 2 law - Translation

- Basis for derivations of equations from free body diagrams

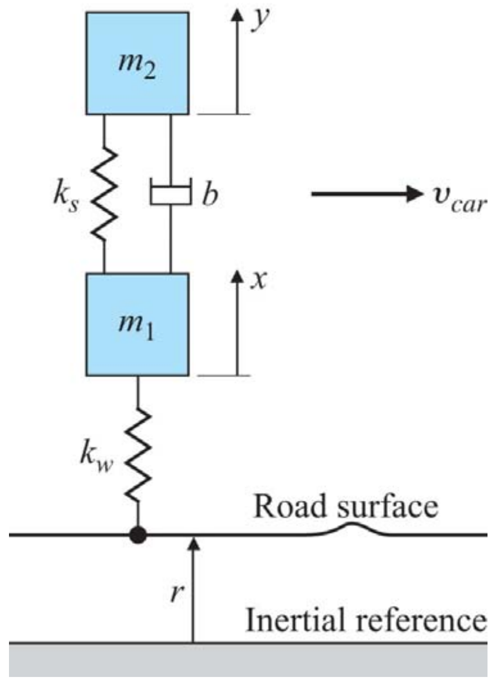
$$\sum F = ma$$

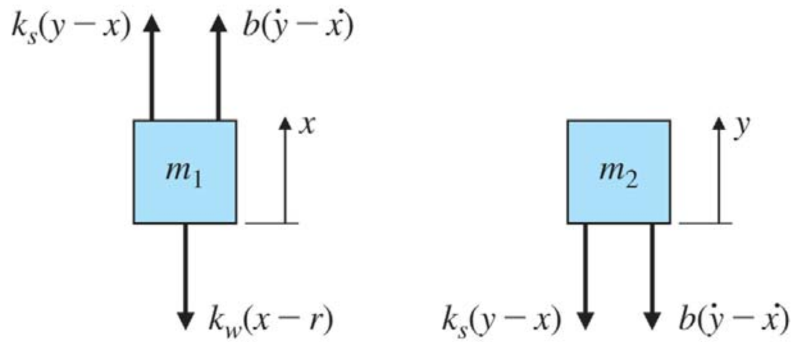
- To derive differential (dynamic) equations remember that acceleration is the double derivative of position

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

equivalently:

$$\iint a\,dt = \int v\,dt = p$$



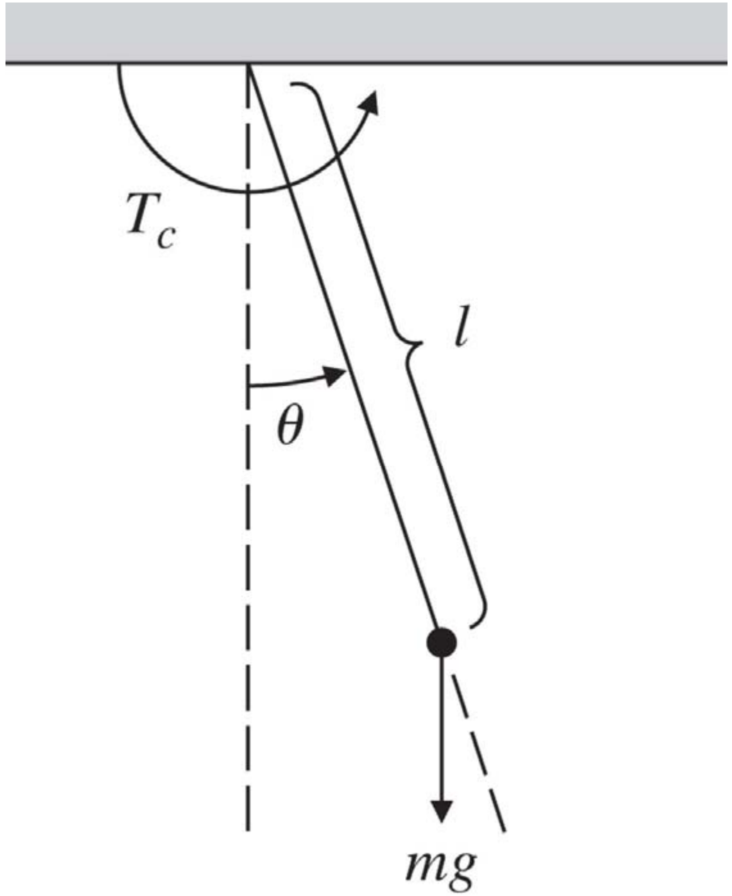



# Force

- A force can change a velocity $[v]$ of a object with a mass $[m]$:
$$F = a \cdot m$$
- Gravity acting on an object creates a force
$$F_g = g \cdot m$$
- Thus $F_g$ can change the position of the object

- The rate of change is depending on the magnitude of $m$ and $g$ and thus $F_g$

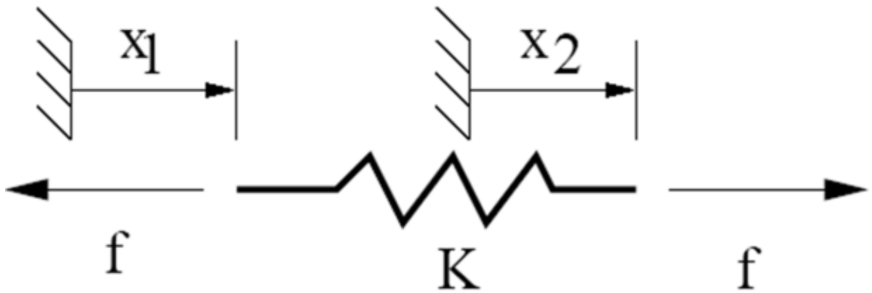
- The SI unit for torque is the newton meter (N·m)



# Energy storage; linear

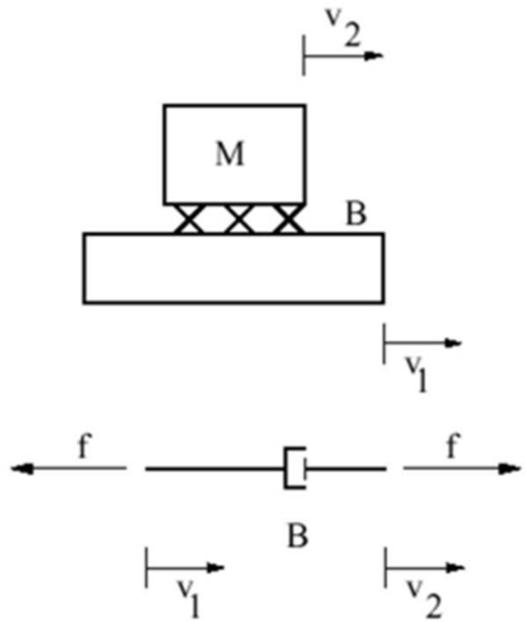
**Spring**

$$f = -K(x_2 - x_1)$$




**Friction**

$$f = -B(V_2 - V_1)$$



# Newton's 2 law – **Rotational Motion**

- Basis for derivations of equations from free body diagrams

$$\sum \tau = I \cdot \alpha$$

Where

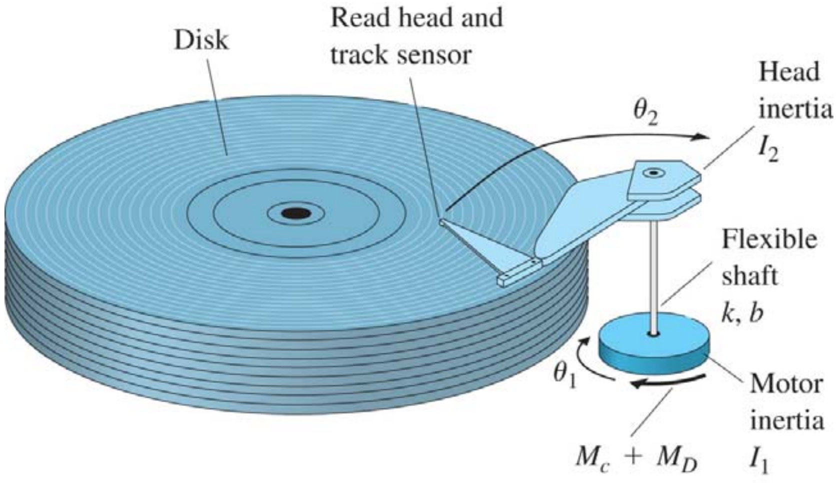
$$I \;= Inertia = m \cdot r^2$$

- To derive differential (dynamic) equations remember that acceleration is the double derivative of position

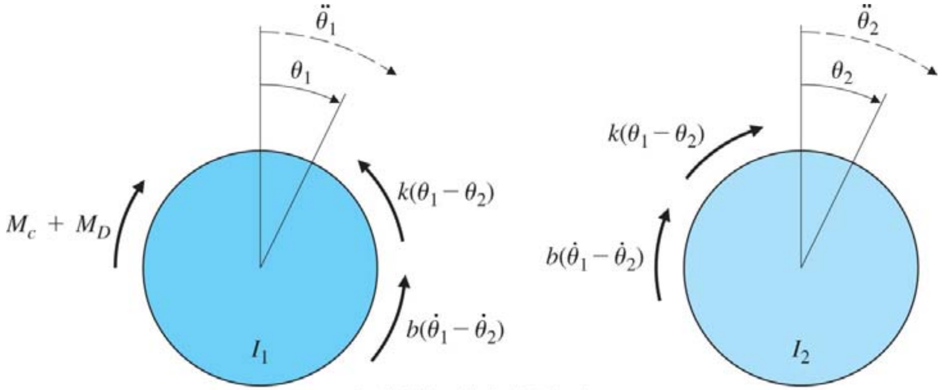
$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

equivalently:

$$\iint \alpha\, dt = \int \omega\, dt = \theta$$









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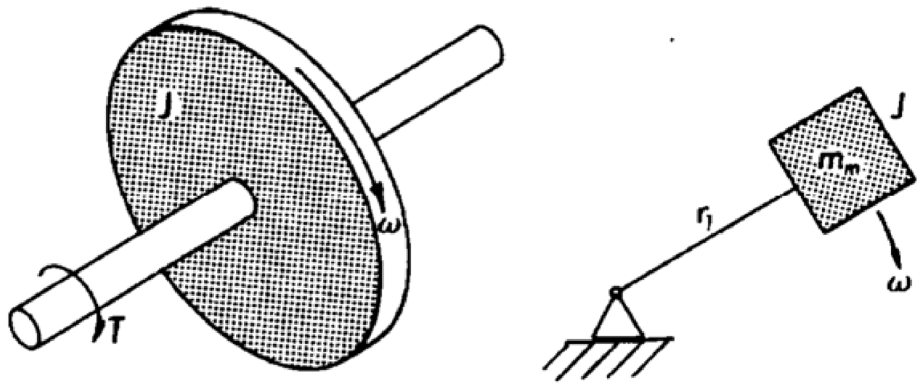
# Moment of inertia

## Definition

- The moment of inertia of an object about a given axis describes how difficult it is to change its angular motion about that axis

- Plays the same role (or is equivalent to) mass in linear (translational) systems

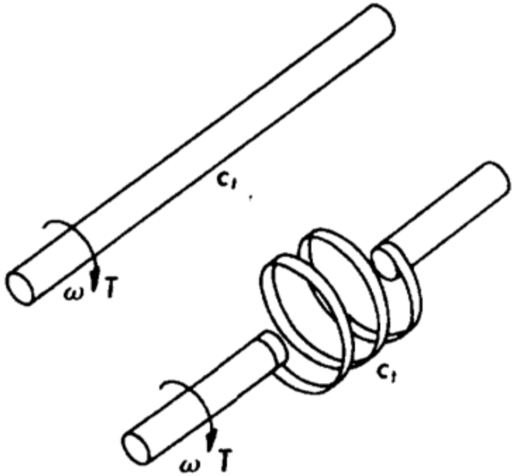
## Examples



# Rotation

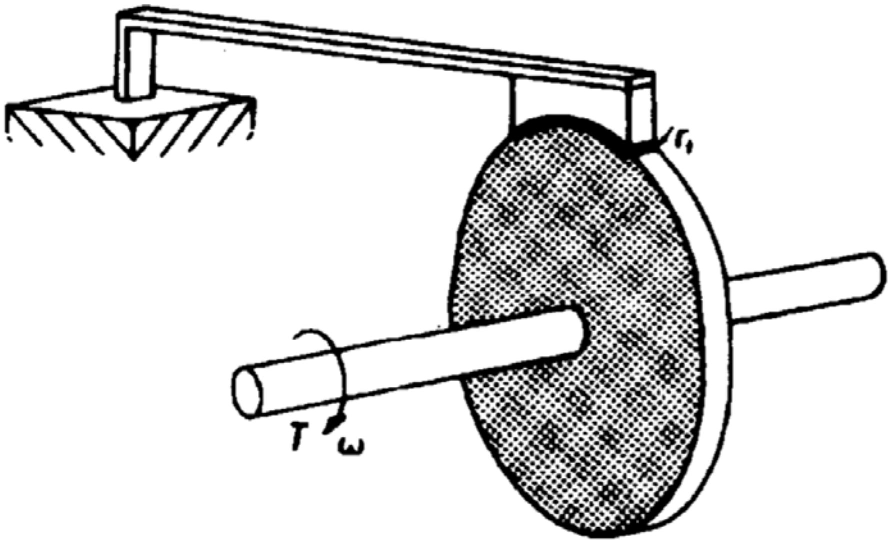
## Torsional springs

$$-\tau = \frac{1}{c_t} \cdot \theta$$




## Friction

$$-\tau = r \cdot \omega$$

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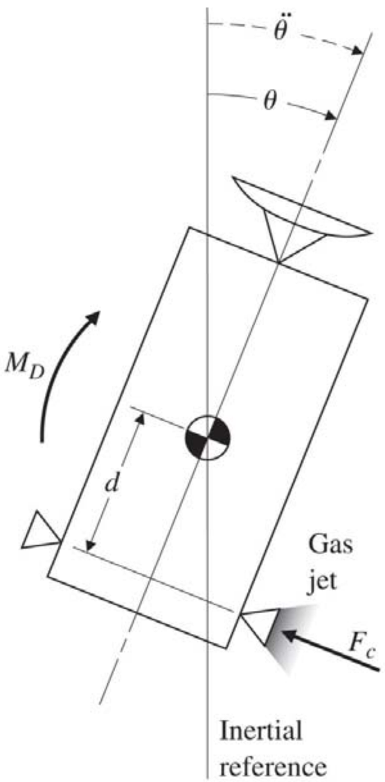
# Rotational Motion Example

## Satellite Attitude Control

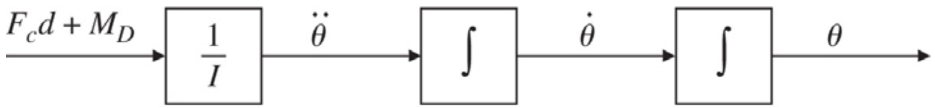
- Satellite position
- Angular position as output $\Theta$
- Double integration system from the force inputs

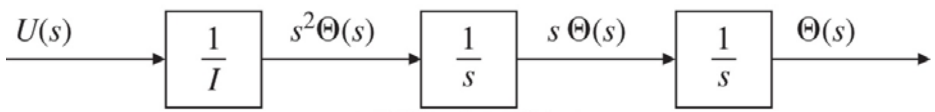
$$F_c d + M_d = I\,\ddot{\theta}$$

- System has following block diagram





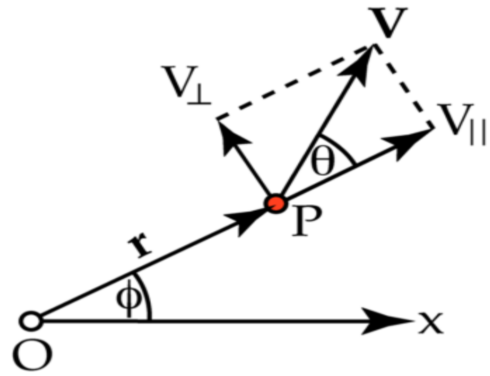
# Angular Velocity & Position

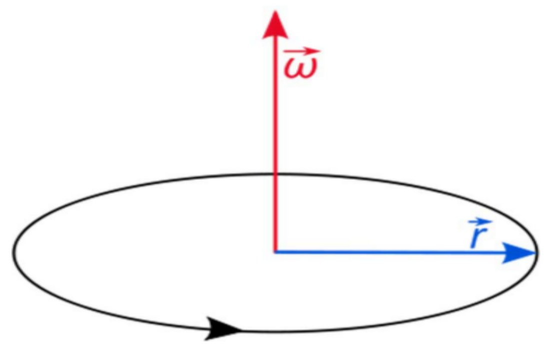
- The angular velocity of the particle at $P$ with respect to the origin $O$ is determined by the perpendicular component of the velocity vector v.

$$\omega = \frac{|v| \sin(\theta)}{|r|}$$

$$\vec{\omega} = \frac{\vec{r} \times \vec{v}}{|\vec{r}|^2}$$

- Angular velocity describes the speed of rotation and the orientation of the instantaneous axis about which the rotation occurs.
  - The direction of the angular velocity pseudo-vector will be along the axis of rotation; in this case (counter-clockwise rotation) the vector points up.

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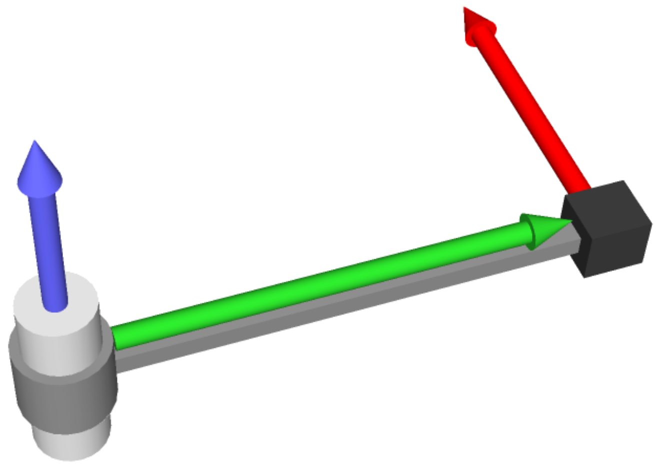
# Torque

- Torque, moment or moment of force, is the tendency of a force to rotate an object about an axis
- The magnitude of torque depends on three quantities: the **force** applied, the **length** of the lever arm connecting the axis to the point of force application, and the **angle** between the force vector and the lever arm.

$$\boldsymbol{\tau} = r \times \boldsymbol{F}$$

$$\tau = r \cdot F \cdot sin(\theta)$$

- Where $\boldsymbol{\tau}$ Is torque vector, $\tau$ is the magnitude of the torque, **F** is the force vector, F is the magnitude of the force and $\theta$ is the angle between the force vector and the leaver arm vector.



$$\tau = \mathbf{r} \times \mathbf{F}$$

# Gear

Gears have a proportional relationship to angular velocity and torque through the gear-ratio:

$$\tau_2 = \frac{r_2}{r_1} \cdot \tau_1$$

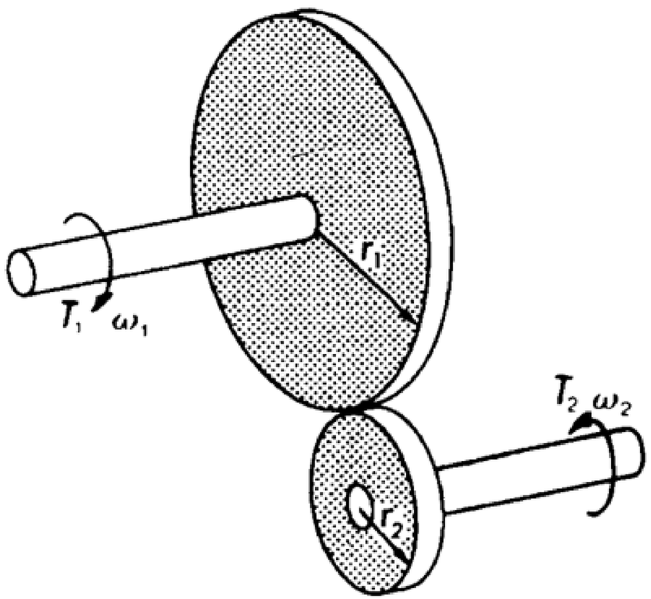
$$\omega_2 = \frac{r_1}{r_2} \omega_1$$

Gearing affect on Inertia

$$\frac{\tau_1}{\omega_1} = \left(\frac{r_1}{r_2}\right)^2 \cdot \frac{\tau_2}{\omega_2}$$

$$J_1 = \left(\frac{r_1}{r_2}\right)^2 \cdot J_2$$

**Figure**

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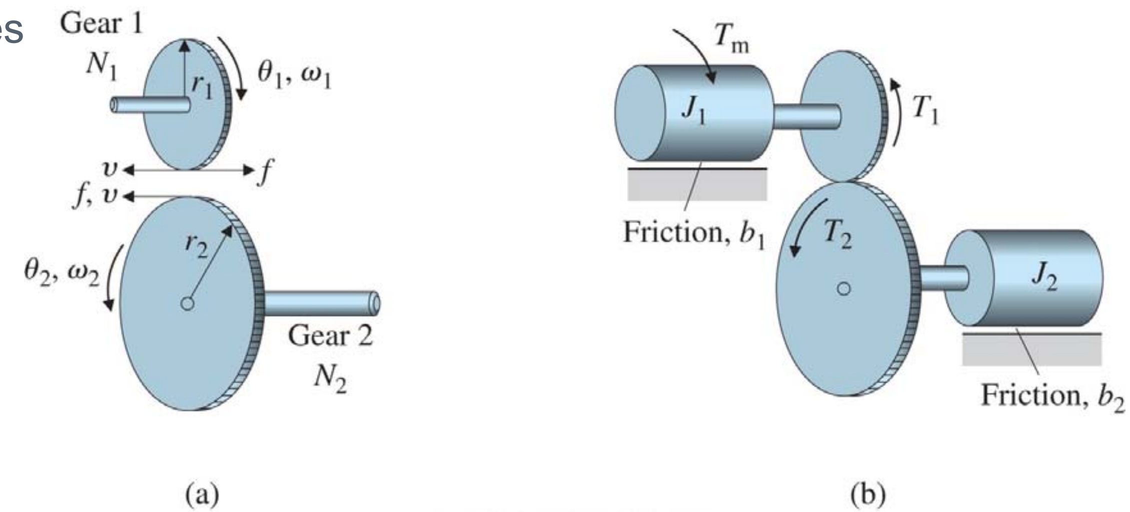
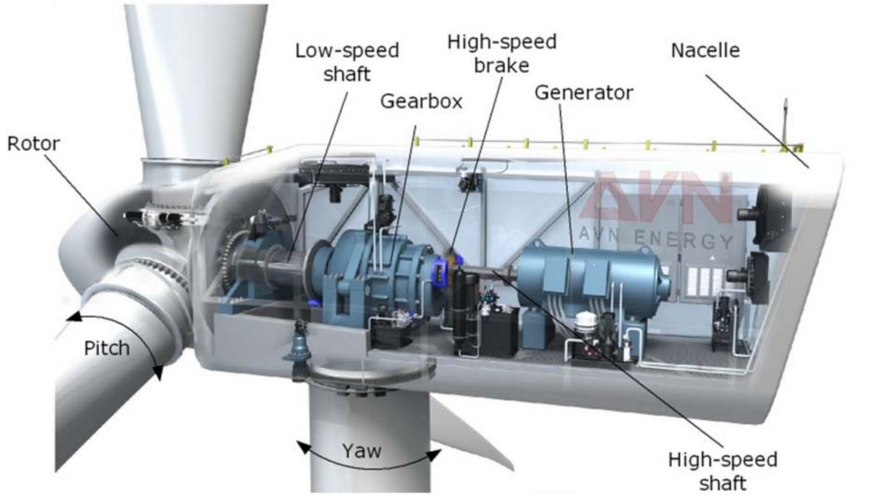
# Translation of angular to linear motion

- Different gear ratios equal to different distances traveled or their derivatives $[\omega, \alpha]$
- The same occurs for the transferred torque

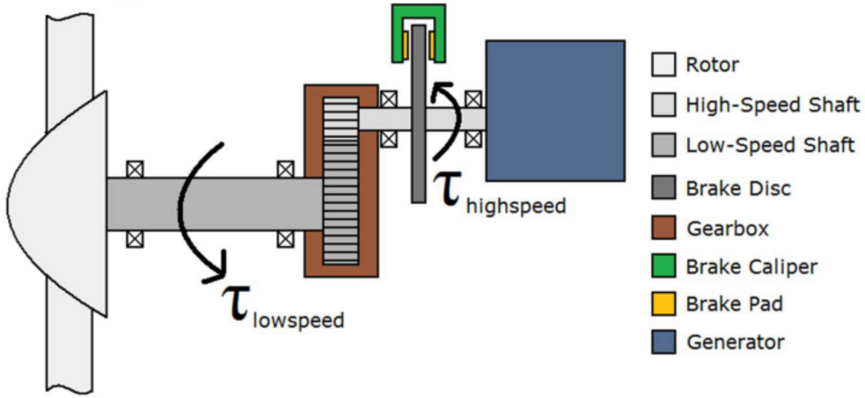


# Example of an electromechanical system

- Physical system illustration




- Schematic

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# Example of non-linearity: Friction

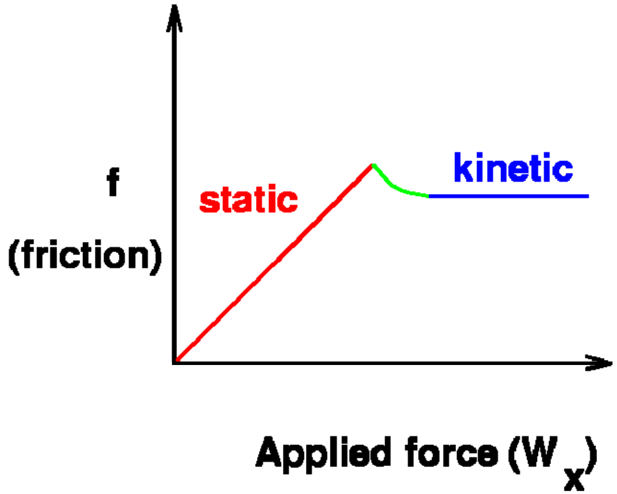
- Dry friction:
  - Surfaces rub against each other
  - Static
  - Dynamic

- Viscous friction:
  - In fluids (liquids or gasses), e.g. lubricated
  - Normally linear with speed, can be quadratic at high speed

$$F = B \cdot (V_2 - V_1)$$

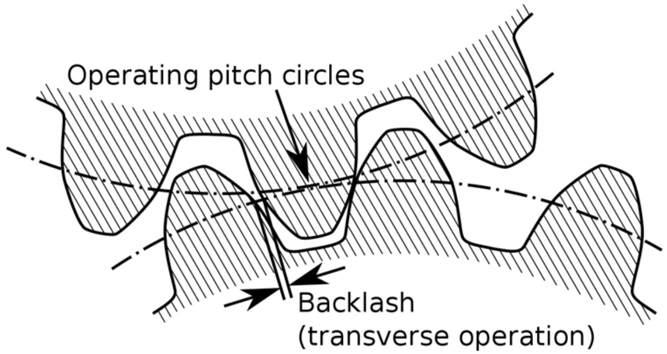
or

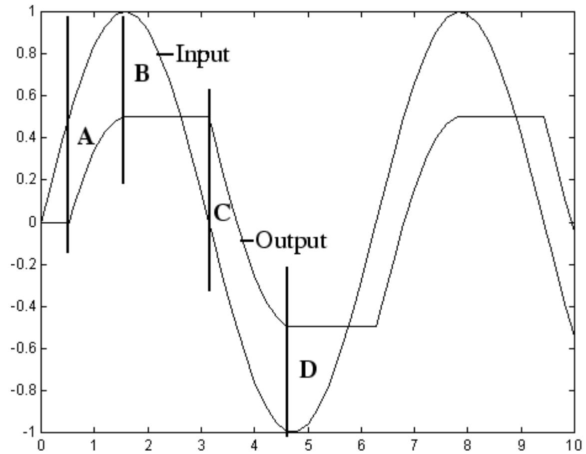
$$F = B \cdot (V_2 - V_1)^2$$



# Nonlinear friction ‘Backlash’

- Dead-band
- Output is distorted
- Non-linear



Electrical circuits

# PHYSICAL MODELING

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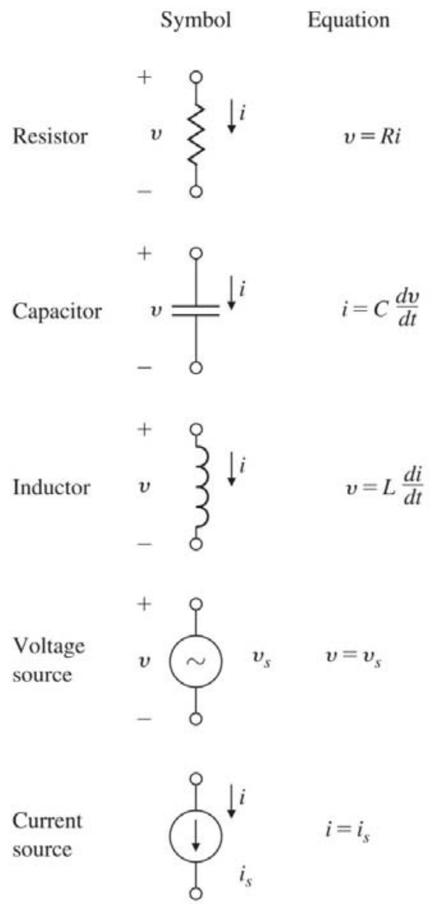
# Electric circuits

- Basic electric circuits and their euqations



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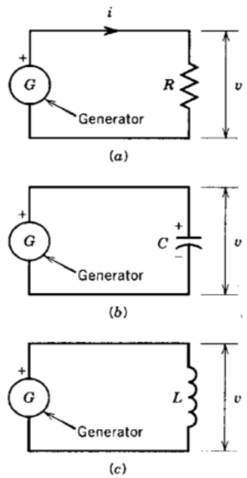
# Passive Electronics

- Passivity is a property of engineering systems, used in a variety of engineering disciplines, but most commonly found in analog electronics and control systems.
- A passive component, depending on field, may be either a component that consumes (but does not produce) energy (thermodynamic passivity), or a component that is incapable of power gain (incremental passivity).




| Energy: | Current/voltage relation |
| --- | --- |
| $w = \int_0^V Cv\,dv = \frac{1}{2}CV^2$ | $i = C\frac{dv}{dt}$ |
| $w = \int_0^I Li\,di = \frac{1}{2}LI^2$ | $v = L\frac{di}{dt}$ |

# Transformer

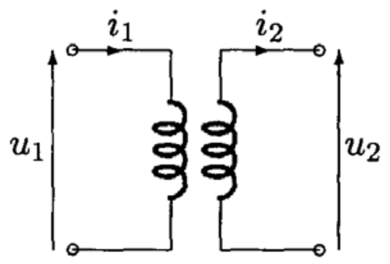
## Equations

*Transforms $v$ and $i$ with a constant product.*

- $u_1 \cdot i_1 = u_2 \cdot i_2$
- $u_1 = \alpha u_2$
- $i_1 = \frac{1}{\alpha} i_2$

Where $\alpha$ is the ratio of turns in the transformer

## Diagram

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# Electric circuits

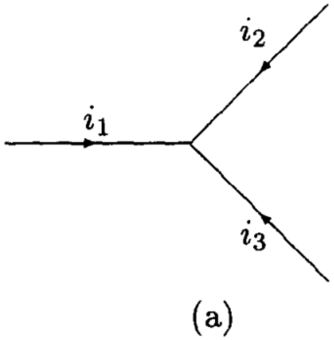
Kirchhoff's laws:

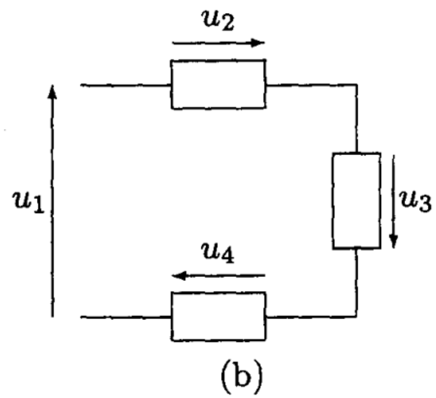
- KCL

The algebraic sum of currents leaving a junction or node equals the algebraic sum of currents entering that node

- KVL

The algebraic sum of all voltages taken around a closed path in a circuit is zero



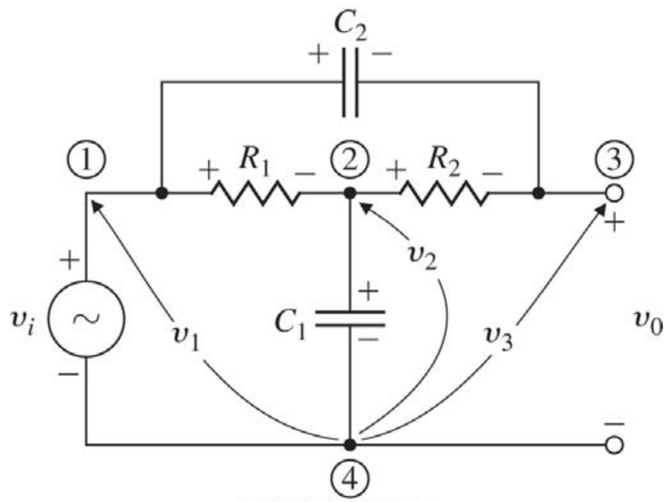
# Electric circuits: Node Analysis

Use KVL to find the equation of the circuit to the right,

1. Select 4 nodes, with 4 as reference
2. Voltages $v_1, v_2$ and $v_3$ are unknowns

Node 2:

–



# BLOCK DIAGRAMS

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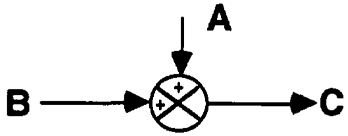
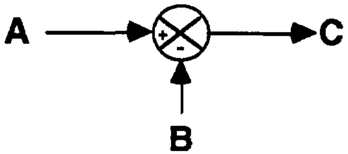
## 1. Summer




$\Rightarrow C = A + B$

## 2. Comparator

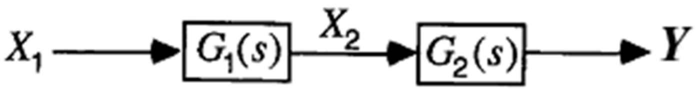



$\Rightarrow C = A - B$

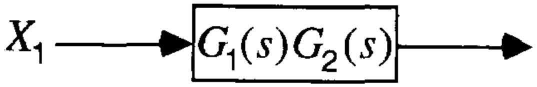
## 3. Block


$$\therefore Y(s) = G(s)X(s)$$

- **Blocks in <u>Series</u>**




**are equivalent to...**

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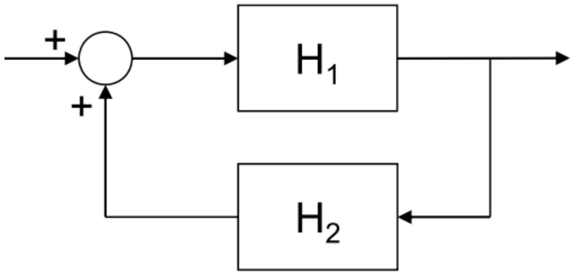
# Block Diagram and Transfer function

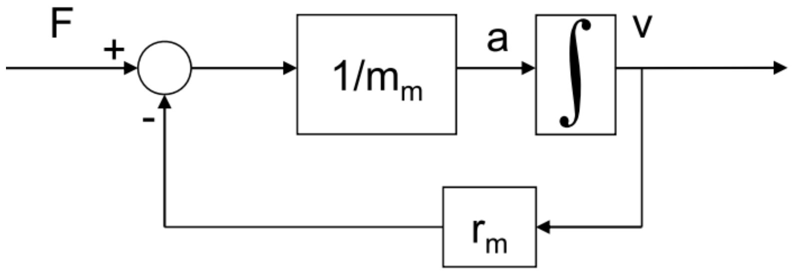
## Masons rule

$$H = \frac{Openloop}{1 - Openloop \cdot Feedback} = \frac{H_1}{1 - H_1 \cdot H_2}$$




## Example

$$H = \frac{\frac{1}{m_m} \cdot \frac{1}{s}}{1 - \left(\frac{1}{m_m} \cdot \frac{1}{s}\right) \cdot (-r_m)} = \frac{\frac{1}{m_m \cdot s}}{1 + \left(\frac{1}{m_m \cdot s}\right) \cdot r_m} = \frac{\frac{1}{m_m}}{s + \left(\frac{r_m}{m_m}\right)}$$

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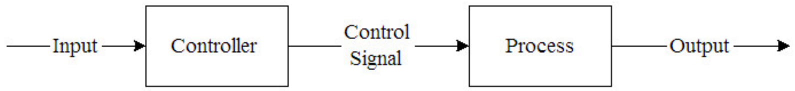
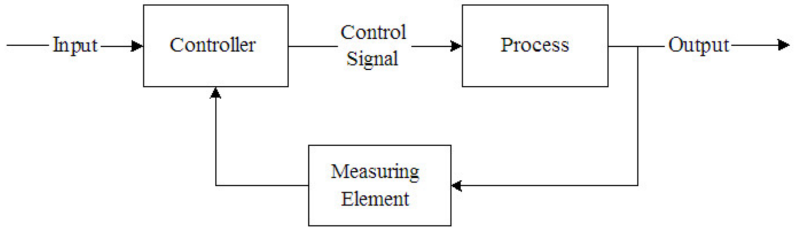
# Open- and closed-loop

- **Open-loop Control:** A control process which does not utilize the feedback mechanism, i.e., the output(s) has no effect upon the control input(s)
- **Closed-loop Control:** A control process which utilizes the feedback mechanism, i.e., the output(s) does have effect upon the control input(s)




Open Loop System




Closed Loop System

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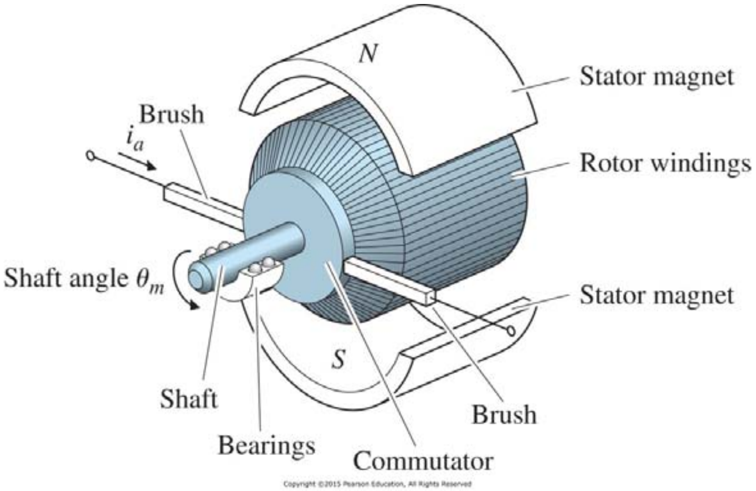
# DC motor

## Diagram




## Equations

- Torque

$$T = K_t i_a$$

- Back emf

$$e = K_e \dot{\theta}_m$$

Where $K_t = K_e$

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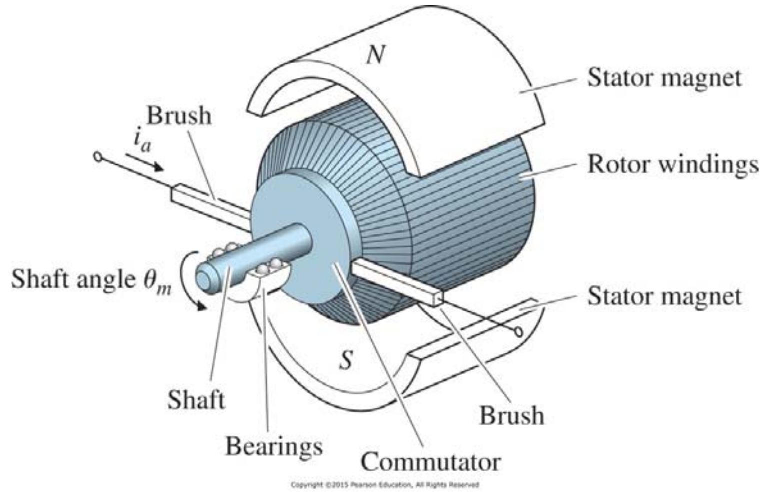
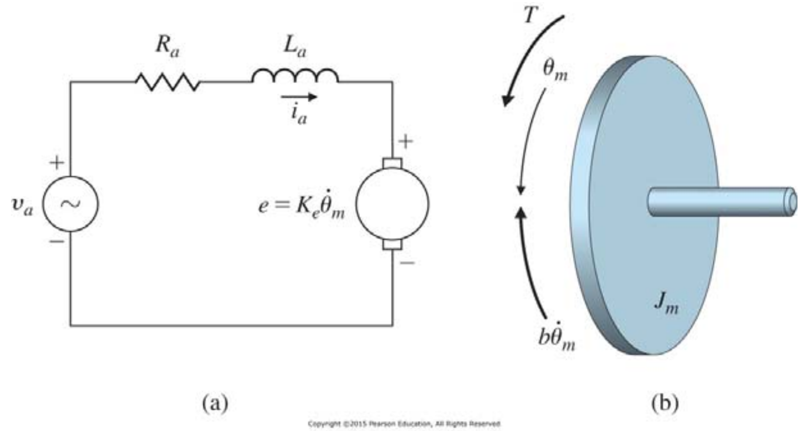
# DC motor model

## Equations

- Mechanical part equation, with inertia and friction
$$J_m \cdot \ddot{\theta}_\mathrm{m} + b \cdot \dot{\theta}_\mathrm{m} = K_t \cdot i_a$$
- Electrical part equation from KVL (circuit)
$$L_a \cdot \frac{di_a}{dt} + R_a \cdot i_a = v_a - K_e \cdot \dot{\theta}_\mathrm{m}$$

## Diagram

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# DC motor model

**Diagram**

**Equations**

Mechanical: $J_m \cdot \ddot{\theta}_{\mathrm{m}} + b \cdot \dot{\theta}_{\mathrm{m}} = K_t \cdot i_a$

Electrical: $L_a \cdot \frac{di_a}{dt} + R_a \cdot i_a = v_a - K_e \cdot \dot{\theta}_{\mathrm{m}}$

- As the relative effect of the Inductance is neglible in many cases, we can simplify and combined the two equations:

$$J_m \cdot \ddot{\theta}_{\mathrm{m}} + \left(b + \frac{\mathrm{K_t} \cdot K_e}{R_a}\right) \cdot \dot{\theta}_{\mathrm{m}} = \frac{K_t}{R_a} \cdot v_a$$

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# EXERCISES

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# Exercise

Implement the following equation in Matlab:

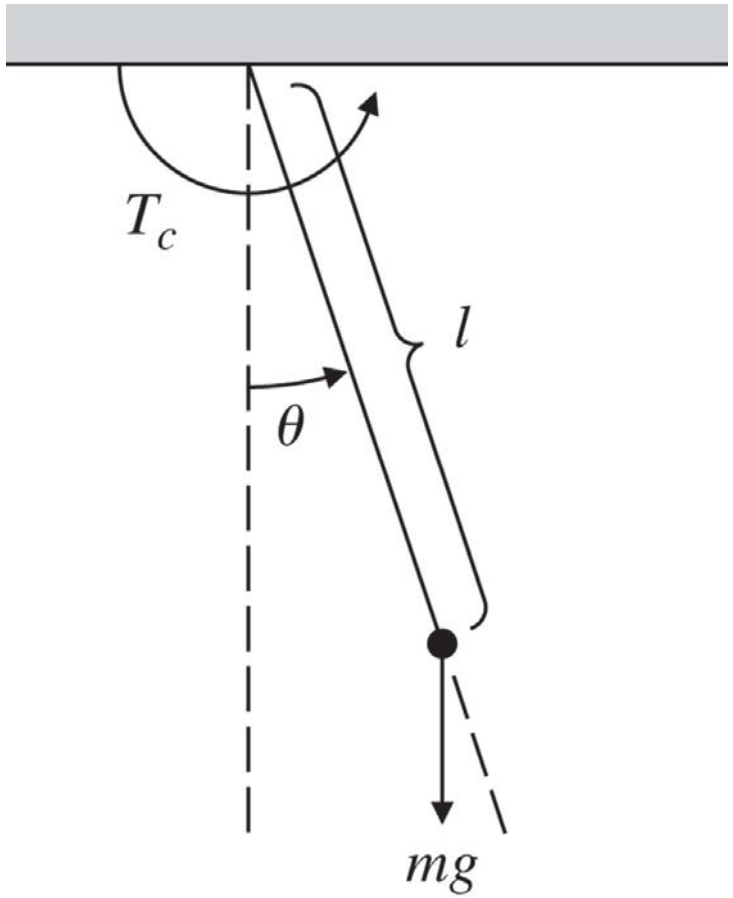
$$\frac{\Theta(s)}{T_c(s)} = \frac{\frac{1}{ml^2}}{s^2+\frac{g}{l}}$$

Using the following parameters:

$\mathrm{m} = 10\ (kg)$

$l = 10\ (m)$

- Try and plot the system step response
- Try to change the length of the pendulum to 100 and compare the step response
- Compare the poles of the two systems



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