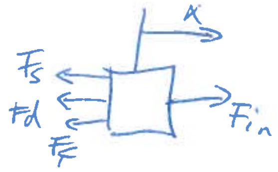
SE 1

1.)

A) Find system equation



Spring Damp Friction

$$a = m\ddot{x} = \sum F = F_S + F_d + F_F + F_{in}$$

$F_S = -kx$

$F_d = -b_1 \dot{x}$

$F_f = -b_2 \dot{x}$

$F_{in} = u$

$$m\ddot{x} = -kx - (b_1+b_2)\dot{x} + u$$

2.) Find system TF

$$m X(s) s^2 = -kX(s) - (b_1+b_2) X(s) s + U(s)$$

$$H(s) = \frac{X(s)}{U(s)} = \frac{1}{ms^2 + (b_1+b_2)s + k}$$

3.) Simulate with values

$$H_1(s) = \frac{1}{1000s^2 + 200s + 1000}$$

$$H_2(s) = \frac{1}{1000s^2 + 110s + 1000}$$

4.) What does b_1 change?

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

B_1 changes the damping ratio.

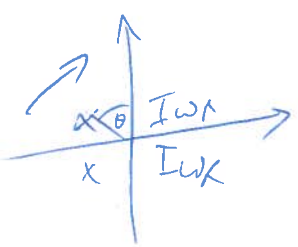
5.) How does it affect the poles?

Poles H_1 $s_1 = -0{,}1 + 0{,}995i$

$s_2 = -0{,}1 - 0{,}995i$

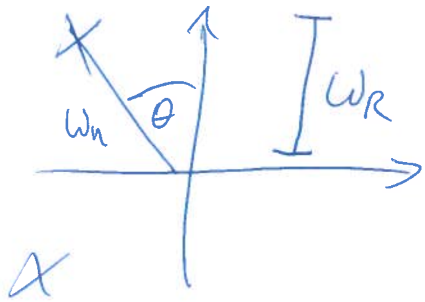
Poles H_2 $s_1 = -0{,}055 + 0{,}998i$

$s_2 = -0{,}055 - 0{,}998i$



$\theta = \sin^{-1}\xi$

$\omega_R = \omega_n \sqrt{1-\xi^2}$



MSE 1 B

1. No overshoot $\Rightarrow$ 1st order

2.

$$H(s) = \frac{k}{\tau s + 1}$$

$K = DC = 2$

τ at $0{,}63 \cdot DC = 1{,}26$

$\tau = 0{,}2s$

$$H(s) = \frac{2}{0{,}2s + 1}$$

3.)

$$0{,}2s + 1 = 0$$

$$s = \frac{-1}{0{,}2} = -5 \Rightarrow \text{pole at } (-5|0)$$

4.)

Stable

MSE2A

1.) Encoder on port

Matlab

Arduino

2.) Band width

~~$\omega = \frac{2\pi}{T} = 2\pi f \qquad f = 2\pi\omega = 2\pi 10 =$~~

$f = \frac{\omega}{2\pi} = \qquad \omega = 2\pi f$

$= \frac{10}{2\pi} = 1{,}59\ Hz$

Nyquist

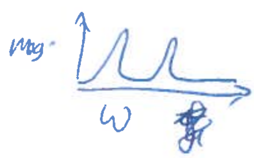
$f_s \geq 2 f = 3{,}18 Hz$

3.)

Noise → Measure spikes by hand $\Delta t = T_n = 30ms$

$f_n = \frac{1}{T_n} = \frac{1}{0{,}03s} = 33\ Hz$

→ Look Spectrum analysis / Fourier transformation

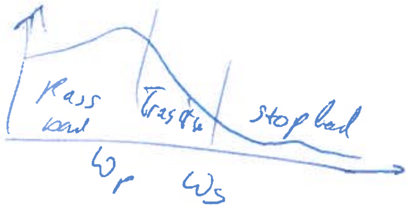


signal 1,59 Hz

Noise 30-40 Hz

MSE 2 A

4. Low pass



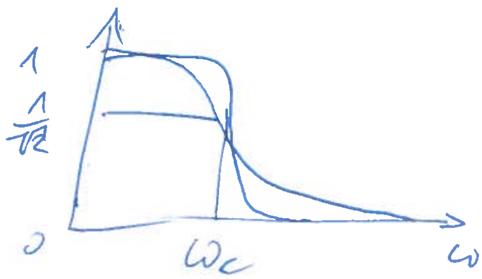
First order low pass

$$H(s) = \frac{1}{\tau s + 1} \qquad \tau = \frac{1}{\omega}$$

Butterworth filter

$$H(j\omega) = \frac{1}{1 + \left(\frac{j\omega}{j\omega_c}\right)^{2N}}$$

N = Filtering coefficient
(higher more filter, more delay)



Butterworth

MSE 2B

2nd order gain

1. Damping ratio ~~Rdg~~ b

general form 2nd order

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

2. Physical:

Friction: lube

bearing

air resistance

MSE 3.1

A

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underset{A}{\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}} \begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underset{B}{\begin{bmatrix}0\\ 1\end{bmatrix}} u \;; \qquad y = \underset{C}{\begin{bmatrix}1 & 0\end{bmatrix}} \begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underset{D}{0}$$

1) Find TF

$$H(s) = C(sI - A)^{-1}B + D$$

Laplace of $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + Du(t)$

Solve $Y(s) \to U(s)$, solve $X(s)$

$X(s) = (sI - A)^{-1}$

in output $Y(s)$

$$sI - A = \begin{bmatrix}s & 0\\ 0 & s\end{bmatrix} - \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix} = \begin{bmatrix}s & -1\\ 2 & s+3\end{bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix}a & b\\ c & d\end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix} =$$

$$= \frac{1}{s(s+3)+2}\begin{bmatrix}s+3 & 1\\ -2 & s\end{bmatrix} =$$

$$C(sI-A)^{-1}B = \begin{bmatrix}1 & 0\end{bmatrix} \cdot (sI - A)^{-1} \cdot \begin{bmatrix}0\\ 1\end{bmatrix} = \frac{1}{s^2+3s+2}$$

2)

Poles from SS (Eigenvalues)

$$\det(A - \lambda I) = \det\left(\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix} - \begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\right) = \det\left(\begin{bmatrix}-\lambda & 1\\ -2 & -3-\lambda\end{bmatrix}\right) =$$

$$= -2 - ((-\lambda)(-3-\lambda)) = -2 - (3\lambda + \lambda^2) = -\lambda^2 - 3x + 2$$

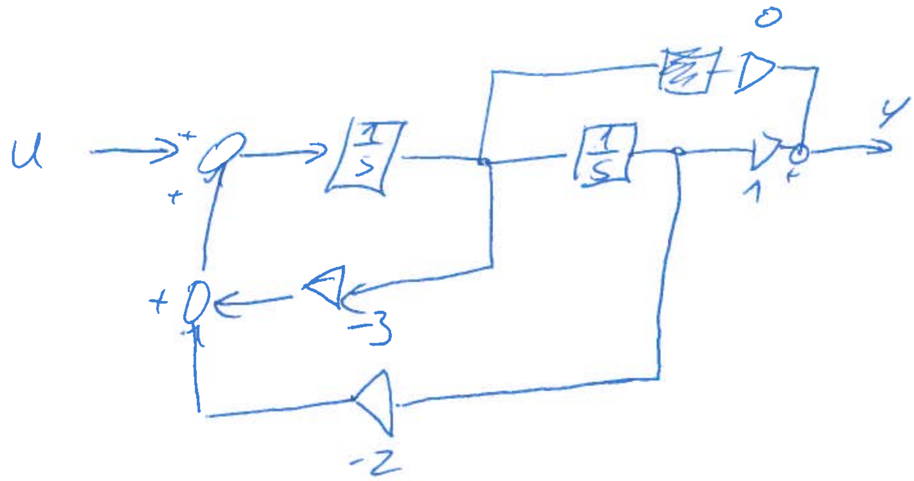
$\underline{\lambda_1 = -2}$, $\underline{\lambda_2 = -1}$

3) Poles from TF

$s^2 + 3s + 2 = 0$ $\qquad s_1 = -1$; $s_2 = -2$

MSE 3A

4.



$$A_c = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_n \\ 1 & 0 & & \\ \vdots & 1 & \ddots & \\ 0 & 0 & \cdots 1 & 0 \end{bmatrix} \qquad B_c = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$C_c = [b_1 \ \ b_2 \ \ \ldots b_n]$$

$$A_c = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} \quad B_c = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C_c = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

5.)

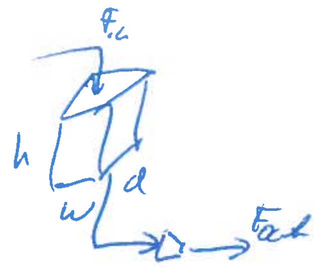
results of SS and TF same

Both plan [illegible] / Stable

USE 3 D

1.) Find F_{in}

$$A \cdot \dot{h} = F_{in} - F_{out} = F_{in} - C\sqrt{h}$$



$$F_{in} = A\dot{h} + C\sqrt{h}$$

2.) Find C

$$F_{out} = C\sqrt{h} \qquad C = \frac{F_{out}}{\sqrt{h}}$$

3.)

$$\dot{h} = \frac{1}{A}(F_{in} - F_{out}) = \frac{1}{A}(F_{in} - C\sqrt{h}) =$$

$$= \frac{1}{2}(1 - 0{,}15\sqrt{h}) = \frac{1}{2} - 0{,}075\sqrt{h}$$

4.)

$$f(h) = \sqrt{h} \qquad h_0 = 0{,}15\,m$$

$$f'(h) = \frac{1}{2\sqrt{h}}$$

$$T = f(h_0) + f'(h_0)\cdot(h - h_0) = 1{,}29h + 0{,}20$$

$$\dot{h} = 0{,}486 - 0{,}095h$$