ED4 Course: Modeling and Simulations (F2018)

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The exam is oral and individual. Students may bring notes or short reports to the exam.

The procedure

Each student will be called in individually according to the schedule. Examiners will ask the students to pick up a random number which corresponds to a specific question set.

First: Each student will be given a time to present his/her answer corresponding to the number.
Second: The student will be asked questions by the examiners.
Third: Examiners will give a grade to individual students based on their performance.

The questions are found on the following pages:

Question One

(A) Given the following system, which is a car with a wheel friction b_2, an input force u. The car is strapped to a wall with a bungee cord which has a spring coefficient k and a damping ratio b_1.

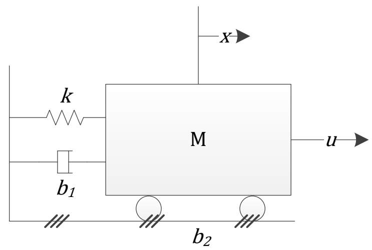


Figure 1: System 1

1. Develop the differential equation of the system
   - HINT: Start by drawing the free body diagram
2. Transform the differential equation into a transfer function
3. Simulate the transfer function with the following two sets of coefficients:
   - Where: $M = 1000kg$, $k = 1kN/m$,$b_1 = 100\ N \cdot sec/m$ and $b_2 = 100\ N \cdot sec/m$
   - Where: $M = 1000kg$, $k = 1kN/m$,$b_1 = 10\ N \cdot sec/m$ and $b_2 = 100\ N \cdot sec/m$
4. Explain why and how changing b_1 changes the system's response?
5. How is this related to the location of the poles of the system?

(B) Given the following step response of a system:

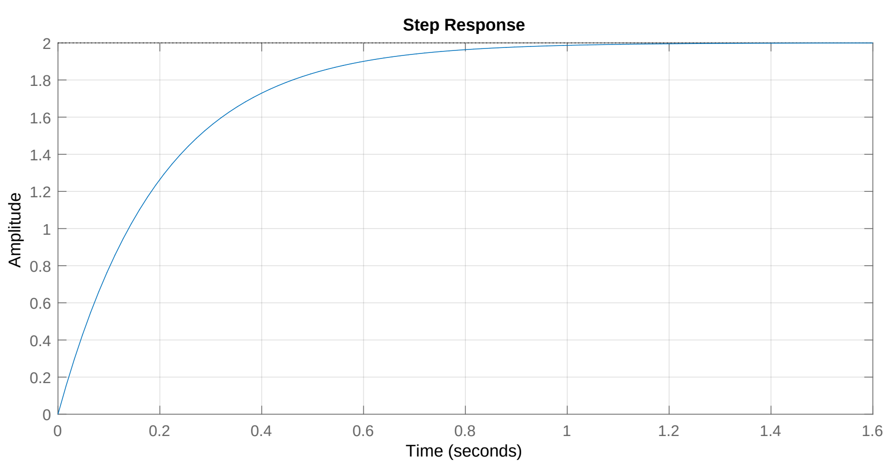


Figure 2: Step response of a system

1. What type of a system is it?
2. Identify the transfer function of the system from the plot
3. Find the poles of the system
4. What can you say about the system from its poles

Question Two

(A) Given the pendulum setup/system seen in figure 3

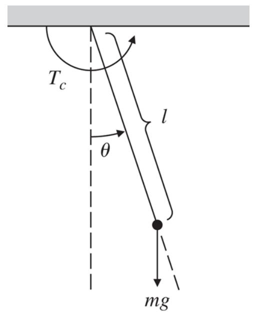


Figure 3: Sketch of an pendulum system

1. If you were to gather data from this pendulum setup, how would you do that? and how would you get the data into Matlab?

If we assume that the system bandwidth of the pendulum is 10 $\frac{rad}{s}$.

2. What sample rate would you choose, if your were to gather data that later would be used to estimate some physical parameters?

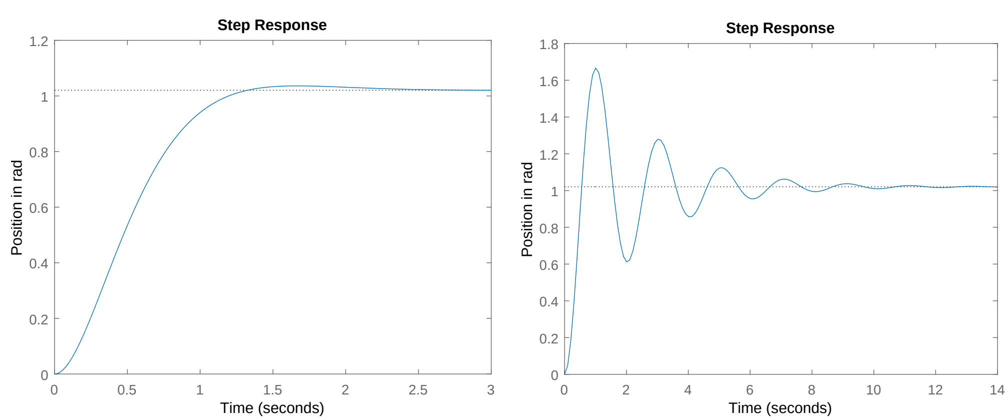
An experiment with a sample rate of $100Hz$ was carried out, and the data from that experiment can be found as the file named: "*pendulum_raw_data.mat*".

3. Analyze the frequency of the signal
4. Design a filter to remove noise from the experiment data, argue for choice of filter and the cutoff frequency.

(B) The transfer function model of the pendulum is given by equation 1. Figure 4 shows 2 different step responses, made with the same model structure, but with one parameter changed.

$$\frac{\theta}{T_c} = \frac{\frac{1}{ml^2}}{s^2 + b \cdot s + \frac{g}{l}} \tag{1}$$

1. What parameter was changed from figure 4a to figure 4b?
2. Explain how this is connected to the physical system.



(a) Response 1 (b) Response 2

Figure 4: Pendulum response with different system parameters

Question Three

(A) Given the state space equation:

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}u \tag{2}$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + 0 \tag{3}$$

1. Derive the transfer function by hand
   - Do not use a standard Matlab function
2. Find the poles of the state space function
   - Do not use a standard Matlab function such as:

     `pole(sys)`
3. Find the poles of the transfer function
   - Do not use a standard Matlab function such as:

     `pole(sys)`
4. Derive the state space model from the transfer function using the control canonical form, show your progress and explain the procedure.
5. Simulate both the systems in Matlab and compare the result
   - Does the response compare well to the poles location?

(B) We would like to know the water level h in a tank, shown in figure 5 with the cross sectional area $A = w * d = 2m^2$, where the outgoing flow F_{out} is restricted by a reduction in pipe diameter and can be calculated as $F_{out} = C\sqrt{h}$, where C is a constant and F_{out} is the output flow and the input is the flow into the tank F_{in}.

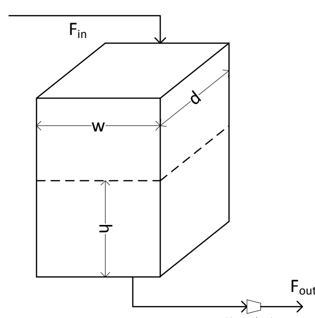


Figure 5: Tank of exercise 3

1. If none of the flows are measurable, how would it be possible to determine the inlet flow F_{in} through experiments; give an idea of the experiment design and requirements?
2. If the level and the flows were measurable, how would you set up an experiment and identify the coefficient C?

For this specific scenario, the inlet flow is given as $F_{in} = 1\frac{m^3}{s}$ and the coefficient C is given by $C = 0.15$

3. Write by hand the (non-linear) dynamic model of the water level h.
4. Linearize the model around an operating point of $h = 0.15m$
5. Would the linear model deviate form the non-linear model?