MODELLING AND SIMULATIONS

SIMULATION

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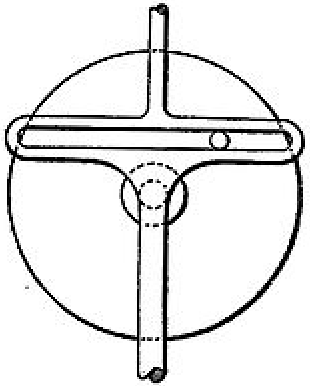
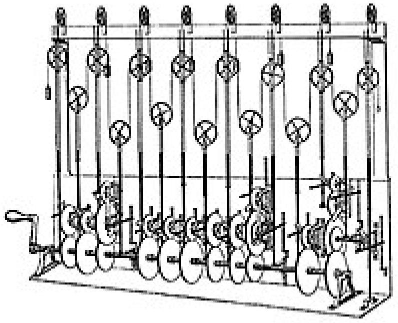
Simulation

- Solve the equation that we have found
- Present the results
- This is referred to as Simulation
- Equations are solved using numeric tools (Matlab)

- Originally done mechanically, eg. For sinusoidal function:
  - Expresses $A_1 \cos(\omega_1 t + \phi_1)$





https://en.wikipedia.org/wiki/Tide-predicting_machine

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Numeric Methods

- Many versions exist:
- Simplest is the Euler's method

Euler's method

Approximating $\dot{x}(t)$, with a difference ratio

$$\frac{\boldsymbol{x_{n+1} - x_n}}{\boldsymbol{h}} \approx \dot{\boldsymbol{x}}(\boldsymbol{t_n}) = \boldsymbol{f}(\boldsymbol{t_n}, \boldsymbol{x_n}), \qquad \boldsymbol{where\ h = t_{n+1} - t_n}$$

Giving the following equation:

$$\boldsymbol{x_{n+1} = x_n + hf(t_n, x_n)}$$

An explicit one step method

- as x_{x+1} is not included in the expression and we calculate one step ahead
- This method is not that effective
  - It has a small region of stability
  - It has a lower accuracy than other available methods, *'large local error'*

Numeric Methods

- Many versions exist:
- Simplest is the Euler's method

Euler's method

Approximating $\dot{x}(t)$, with a difference ratio

$$\frac{\boldsymbol{x_{n+1} - x_n}}{\boldsymbol{h}} \approx \dot{\boldsymbol{x}}(\boldsymbol{t_n}) = \boldsymbol{f}(\boldsymbol{t_n}, \boldsymbol{x_n}), \qquad \boldsymbol{where\ h = t_{n+1} - t_n}$$

Giving the following equation:

$$\boldsymbol{x_{n+1} = x_n + hf(t_n, x_n)}$$

- This method is not that effective
- We use other methods such as: Runge Kutta, Adam's Method, Variable Step Length…
  - More info in section 11.6 (Modeling of Dynamic Systems, Ljung)

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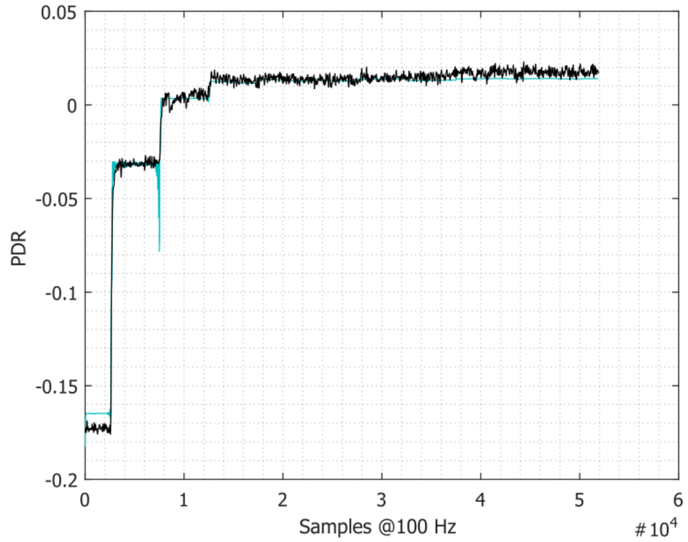
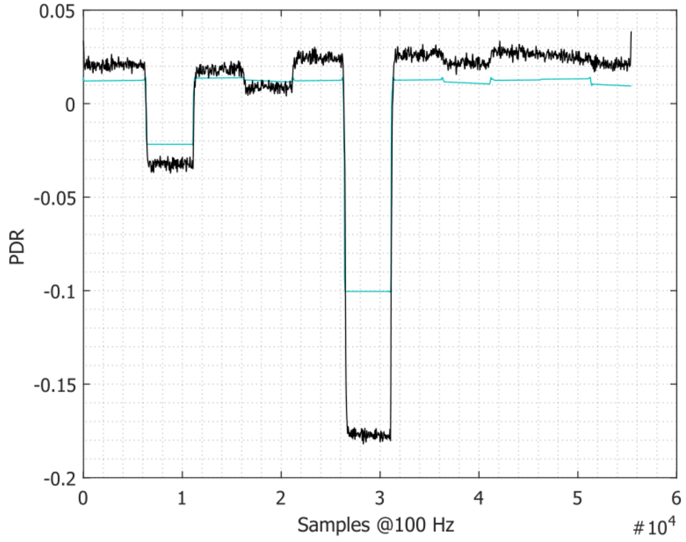
Presentation of the Simulations

- Plot of the model simulations
  - Remember to add offset
- We can do it in time or frequency domain
- We can choose to show multiple dimensions





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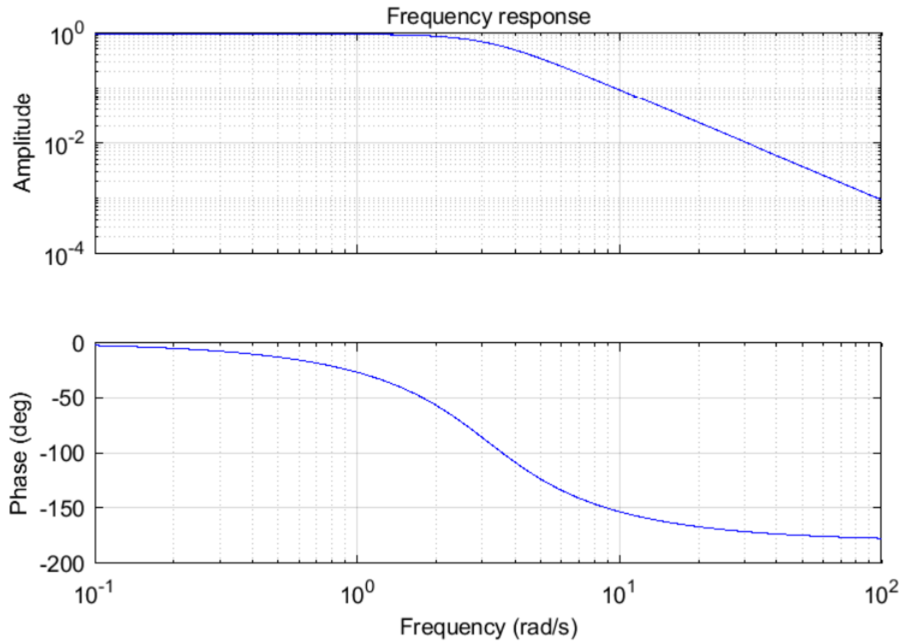

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Presentation of the Simulations

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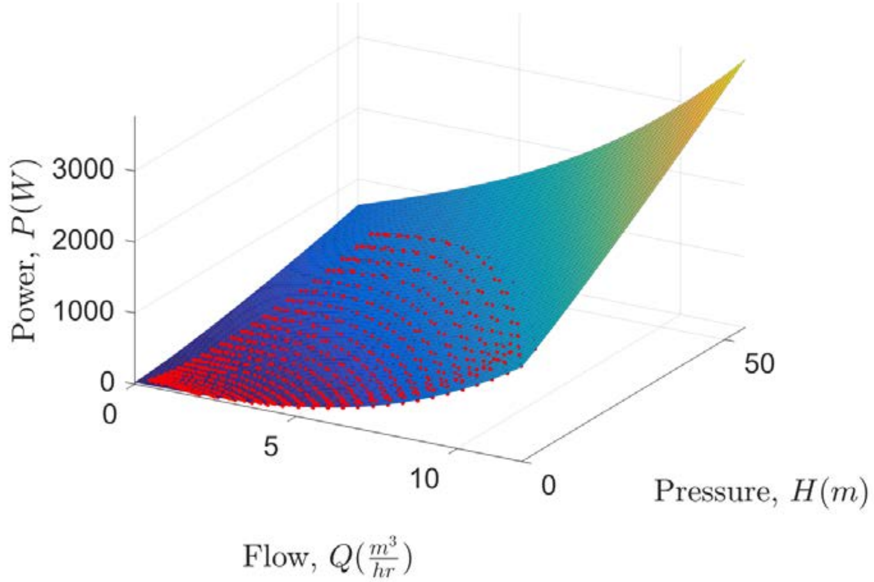

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MODELLING AND SIMULATIONS

MODEL VALIDATION AND MODEL USE

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Model Validation

- What is a Valid model?
- For our purpose it can yield good result
    - But it may be wrong
- We need to test the model’s validity

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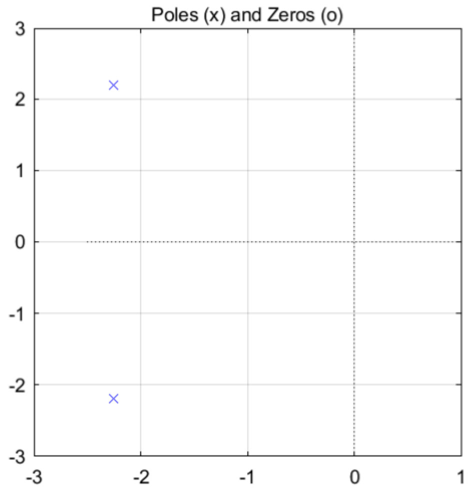
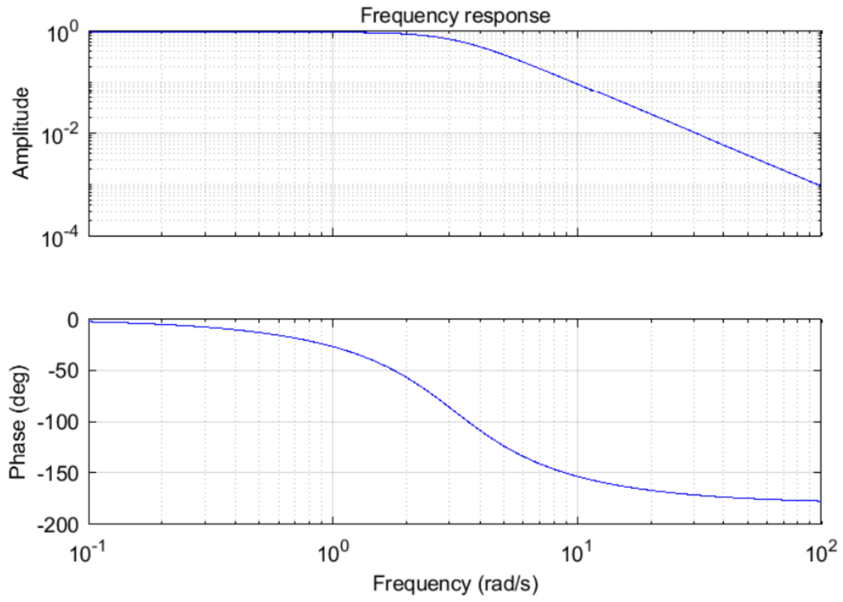
    - But it may be wrong
- We need to test the model’s validity

Model Validation

1. Model Quality
    1. Stability
        1. Poles and Zeros
        2. Bode analysis, frequency function



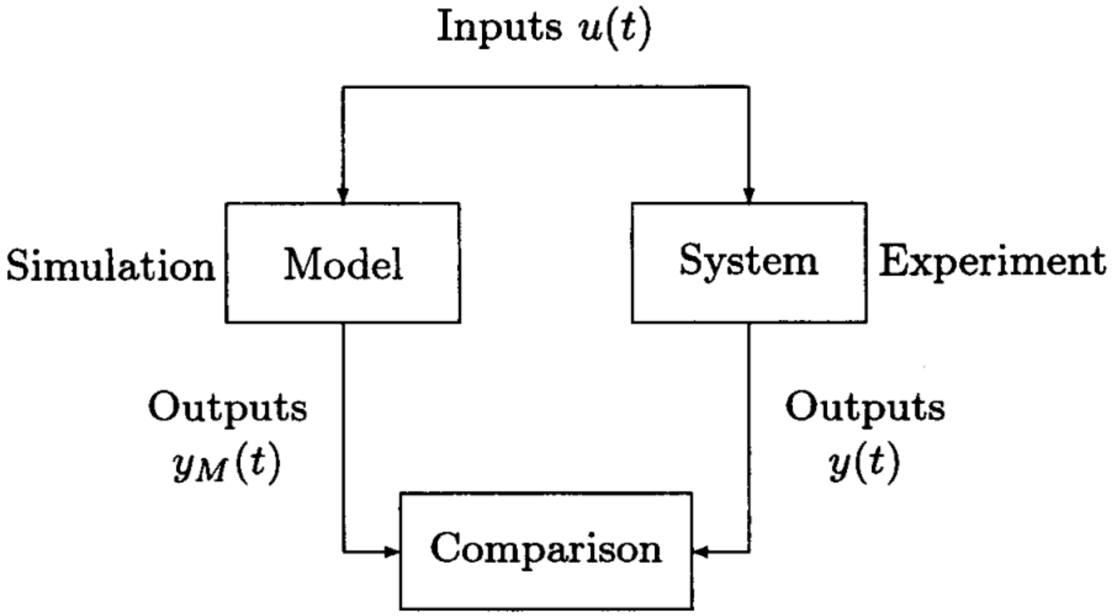


Model Validation

1. Model Quality
    1. Stability (Bode analysis, frequency function)
    2. Ability to reproduce system behavior
        1. Input output behavior comparison (simulated to real(new data set))
        2. k-step prediction (prediction horizon k larger than τ)
            1. Can be done using the residuals (`resid` ‘1 step ahead prediction errors’)

Test of Validity

- *Compare the **model's** output with the **experimental** results*
- *The **comparison** of the two **outputs** must be **small***



Test of Validity

- *Compare the **model's** output with the **experimental** results*
- *The **comparison** of the two **outputs** must be **small***

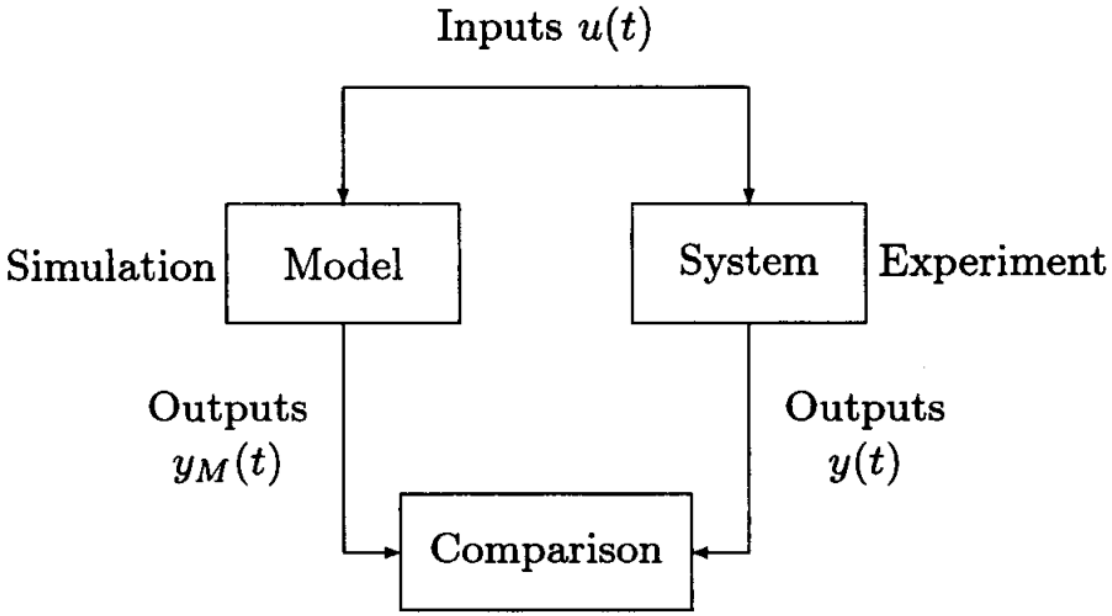
How to determine what is small enough?

Depends on:

- The purpose of the model
- Disturbances influencing the model

Which parameters influence the system most

  - Pay attention to these and update



Model Validation – Residual Analysis

- Goodness of fit between test and reference data
    - The goodness of fit is calculated using the normalized root mean square error as the cost function

`goodnessOfFit`

Model Validation

1. Model Quality
2. Residual Analysis

Model Validation – Residual Analysis

This is a guess, or a prediction of $y(t)$ at time $(t-1)$

$$\hat{y}(t|\theta)$$

Where θ is the parameter vector which contains the parameters of the identified model, where parameters of θ are to be adjusted to collected data.

Then we can calculate the residuals, *(the parts of the data that the model could not reproduce)*

$$\epsilon(t) = \epsilon\left(t, \hat{\theta}_n\right) = y(t) - \hat{y}(t|\hat{\theta}_n)$$

In Matlab you can also call

```
resid(Data,sys)
```

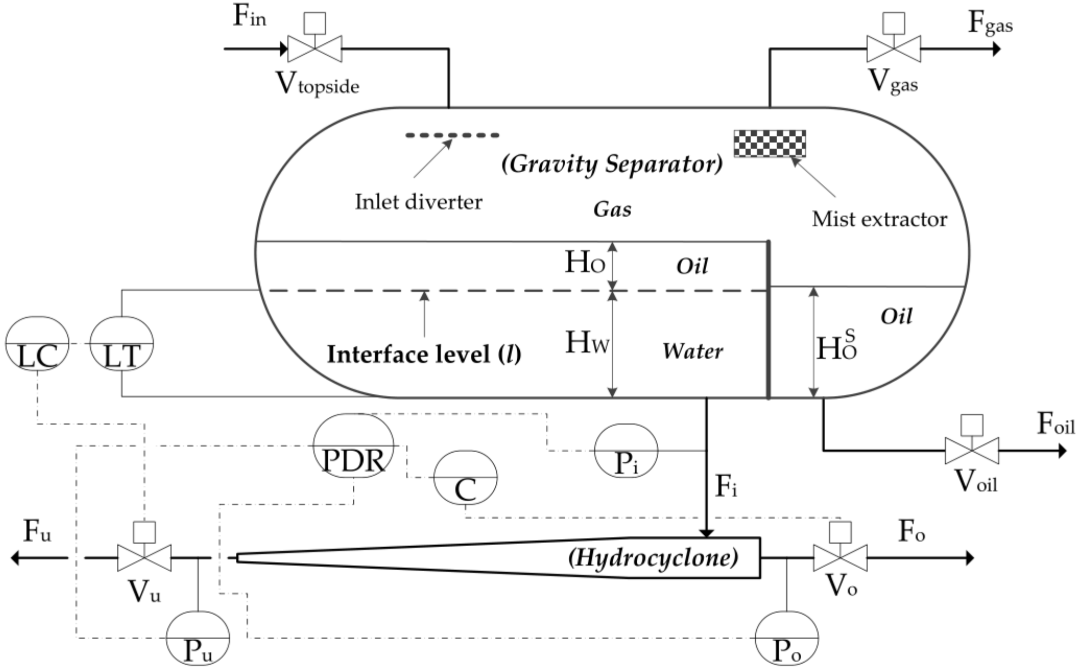
Domain of Validity

- Operating range
  - Right operating conditions
  - Must assure that

Domain of Validity

- Example with a model of a valve governing the PDR of a hydrocyclone
- Valve V_o is the input
- The pressure drop ratio (PDR) is the output
  - $\mathrm{PDR} = \frac{P_i - P_o}{P_i - P_u}$

- A linear model is generated from data collected



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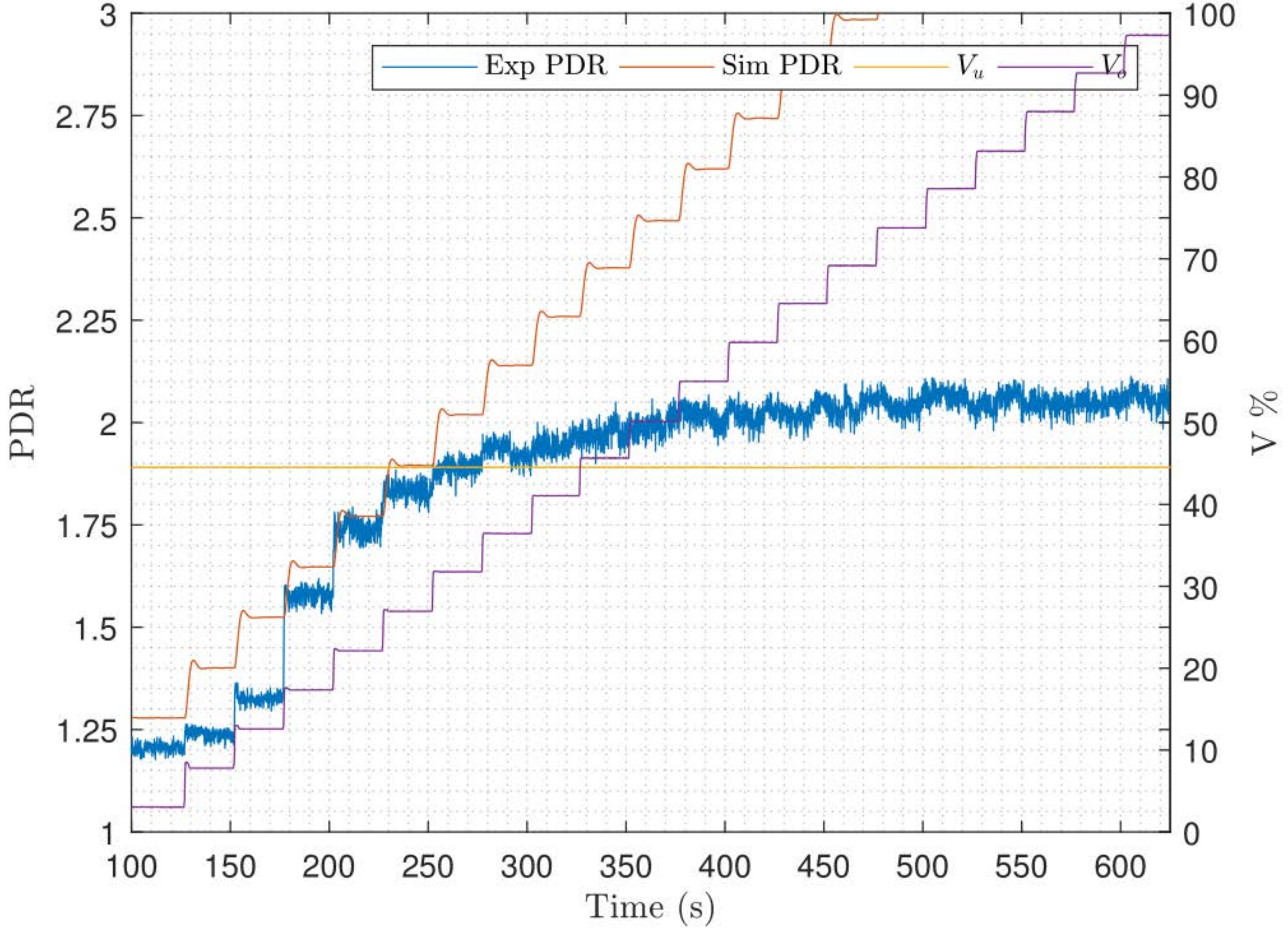
Domain of Validity

- Example with a model of a **valve** governing the **PDR** of a hydrocyclone
- The linear model is identified around a **operating point** of:
  - **PDR = 1.8%**
  - **Pressure 7 bars**
- Moving **too far** away from this point results in deviation from real data
- Thus the mode is only **valid** within a certain operating point



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Models Validity and the Critical View

- A model may be invalid for changing operating conditions
  - Such as PDR, pressure, valve opening…
- A model cannot be made *Perfect*
  - *i.e. it cannot have a 100% fit for all conditions*
- Make sure that the model fits well for your use and operating conditions
- Be prepared to modify the model to fit to new operating conditions if necessary
- Do not over complicate the model, model the necessary dynamics

MODELLING AND SIMULATIONS

CASE STUDY: PROCESS DYNAMICS MODELING

MODELLING A TANK’S LEVEL

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Hydraulic system Modeling

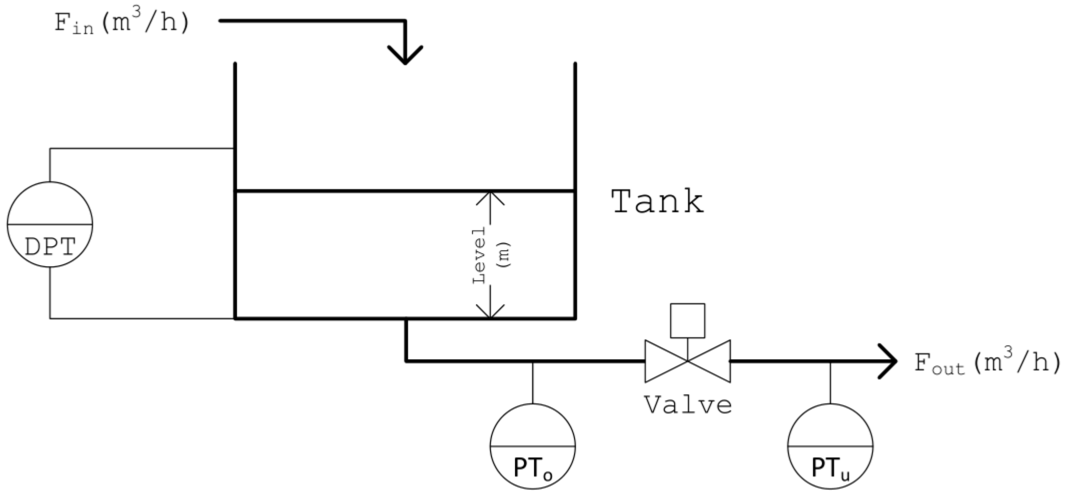
- Consider only non-compressible fluids
- Main parameters to consider
  - Pressure $\left[P = \frac{N}{m^2}\right]$
  - Flow $\left[F = \frac{m^3}{s}\right]$ (Volumetric flow)
  - Mass flow (multiply with density ρ)
  - Density $\left[\rho = \frac{m}{V}\right]$
  - Gravity $\left[g = 9.84\,\frac{m}{s^2}\right]$

Modeling of A Hydraulic System: Level of a tank

One of the most **common** problems in **industrial control**

Model the **level** of a **tank** which is controlled by a **downstream valve**

Sparse measurements: *level*, *pressure* ***(no flow)***



Modeling of A Hydraulic System: Level of a tank

We are given the following system:

Unknown flow into the tank

$$F_{in}\ [m^3/h]$$

Flow out of the system

$$F_{out}\ [m^3/h]$$

A valve that governs the *flow* out of the system

$$V$$

A delta pressure transmitter (DPT) to measure the *level*

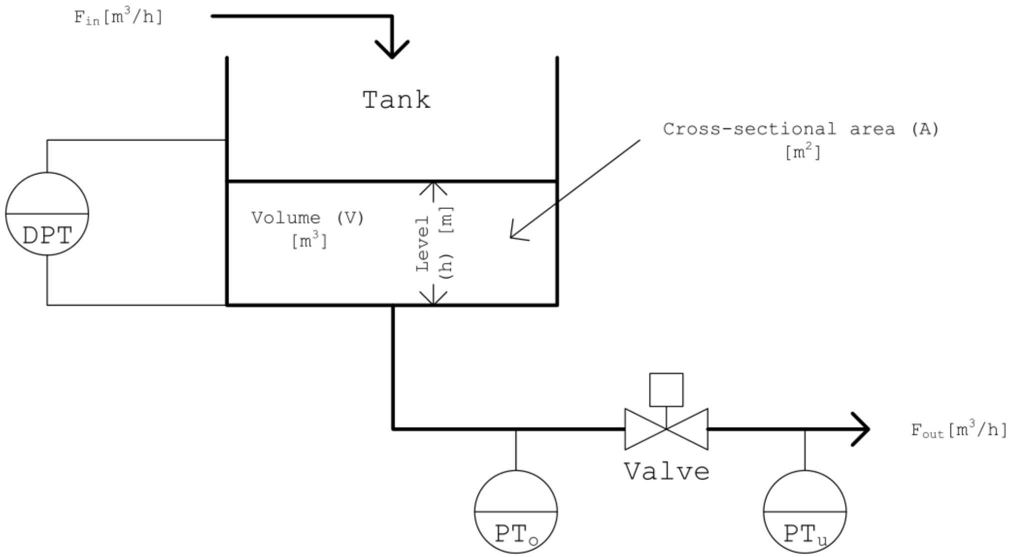
$$l\ [m]$$

Pressure upstream the *valve*

$$PT_o[bar]$$

Pressure downstream the *valve*, assumed to be atmospheric

$$PT_u[bar] = 1bar$$



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Conservation Laws

- Conservation of mass

$$\begin{Bmatrix} rate\ of\ mass \\ accumulation \end{Bmatrix} = \begin{Bmatrix} rate\ of \\ mass\ in \end{Bmatrix} - \begin{Bmatrix} rate\ of \\ mass\ out \end{Bmatrix}$$

Or

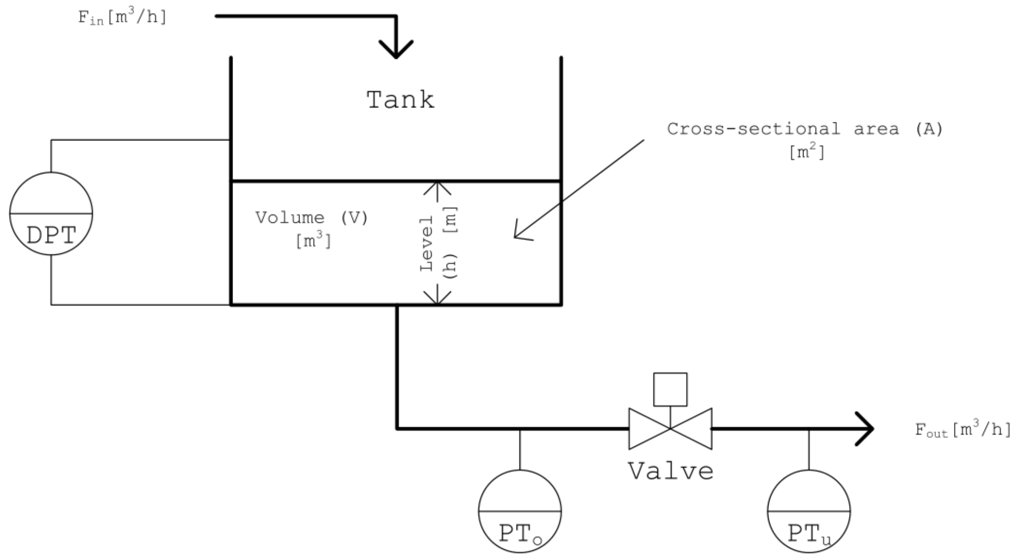
$$\{Volume\} = \begin{Bmatrix} rate\ of \\ mass\ in \end{Bmatrix} - \begin{Bmatrix} rate\ of \\ mass\ out \end{Bmatrix}$$

Which can be written as

$$\frac{d(\rho V)}{dt} = \rho \cdot F_{in}(t) - \rho \cdot F_{out}(t) = 0$$

Where ρ is the liquid's **density**, here **assumed constant,** we now have:

$$\frac{d(\rho V)}{dt} = F_{in}(t) - F_{out}(t) = 0$$



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Relationship between height (level) and the volume

The volume is equal to:

$$V = A \cdot h$$

Thus we can write:

$$A\frac{dh(t)}{dt} = F_{in}(t) - F_{out}(t)$$

*Again we must assume **constant** ρ as **volume** is not **constant** for **fluids***

Further we have that the volume of the tank is the integral of the flow, i.e. conservation of energy:

$$V = \int F\,dt$$

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Flow balance

Some important aspects of flow:

The sum of flow connected in a junction is zero

$$\sum_i F_i(t) \equiv 0$$

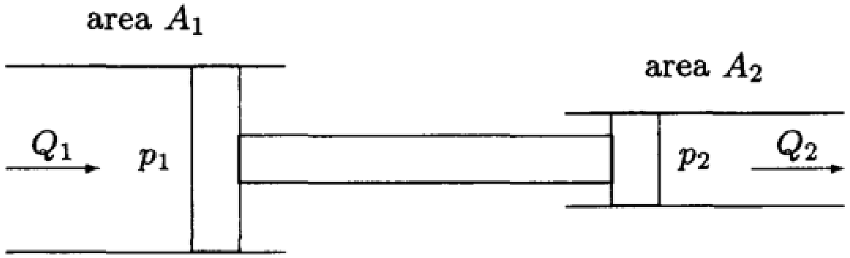
The total pressure over a series of connections is equal to the sum of pressure drops

$$p_{r+1} = \sum_{i=1}^{r} p_i$$

In addition if incompressible fluid is considered we have that

$$p_1 F_1 = p_2 F_2$$

Where $Q = F$



Flow Rates

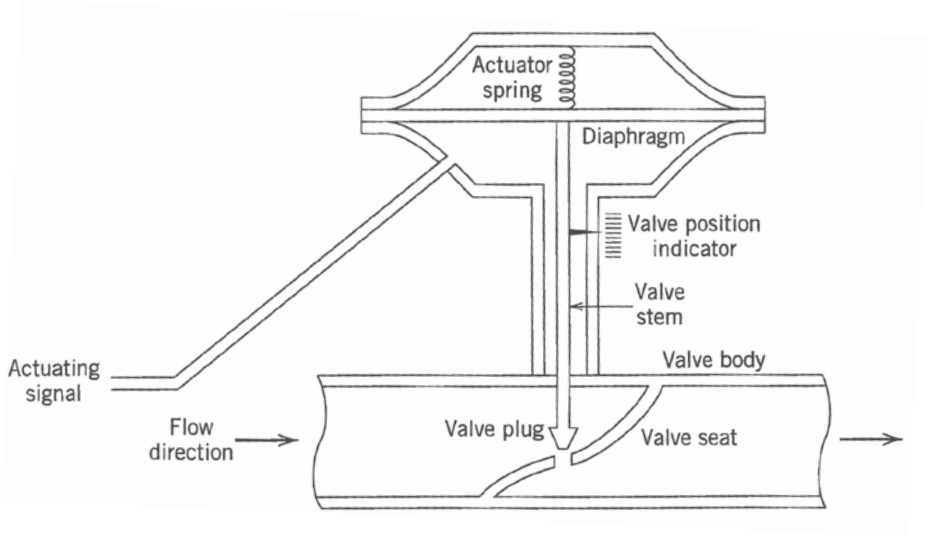
The inlet flow rate is a unknown disturbance

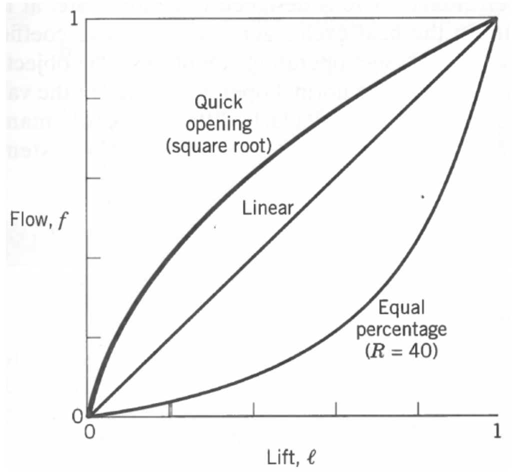
The outlet flow **‘assuming that the tank has a valve on the outlet’** is unknown but governed by the **valves opening degree**

Control Valves

- Control the flow rate by altering the head loss
- Desirable to have close to **zero** head loss at **fully open**

- Consider:
  - Flow characteristic and the fluid properties
  - Actuator properties, *topworks*





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Valve's Equation

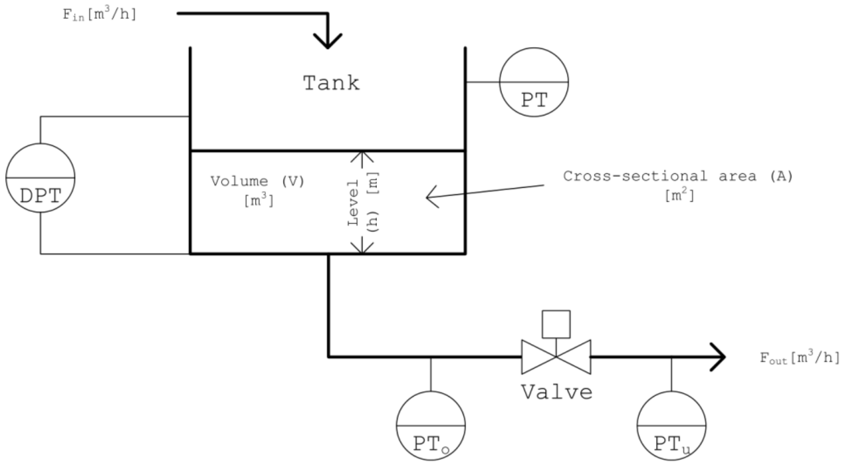
The flow through a fixed valve, or an orifice is facilitated by the pressure drop

For a fixed valve at the bottom of the tank we have the following pressure drop equation:

$$\Delta P = P_b - P_u$$

Where P_b is the pressure at the bottom of the tank and the P_u the pressure downstream the valve *(assumed turbulent flow)*

In addition P_u is assumed ambient



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Valve's Equation (Bernoulli equation)

Considering the valve as an orifice, we can derive the following equation under the following assumptions:

- *Non-compressible flow*

$$F = C_v^* \sqrt{\frac{P_b - P_u}{\rho}} = C_v \sqrt{\frac{P_b - P_u}{\rho}} f(u)$$

Where:

$F =$ is the flow out of the tank ($\mathrm{F_{out}}$)
$C_v^* =$ is the valve constant depending on the opening degree of the valve
$\rho =$ is the density of the liquid
$f(u) =$ represents the valve's characteristics of the openness area related to the openness percentage u

Which is a *mechanical energy balance*, or the *Bernoulli equation*

i.e. the valve can only be actuated in one direction, since $\sqrt{-1} = 1i$

Estimate the unknown parameter

- In the equation for the valve we have an unknown parameter C_v

$$F_{in} = C_v \sqrt{\frac{P_b - P_u}{\rho}} f(u)$$

- It can be estimated from data
  - F_{in}, u, P_b, P_u
- Use least squares estimate (LSE) to find the coefficient C_v

$$r_i = y_i - \hat{y}_i$$

The summed square of residuals

$$S = \sum_{i=1}^{n} r_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Estimate the unknown parameter in the valve equation

- In the equation for the valve we have an unknown parameter C_v

$$F_{in} = C_v \sqrt{\frac{P_b - P_u}{\rho}} f(u)$$

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  - F_{in}, u, P_b, P_u
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$$r_i = y_i - \hat{y}_i$$

The summed square of residuals

$$S = \sum_{i=1}^{n} r_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

For our case it can be written as, **(under the assumption that C_v is proportional to the valve opening)**:

$$\min_{C_v} \sum_i \left| F_{in}(i) - C_v u(i) \sqrt{\frac{P_b(i) - P_u(i)}{\rho}} \right|^2$$

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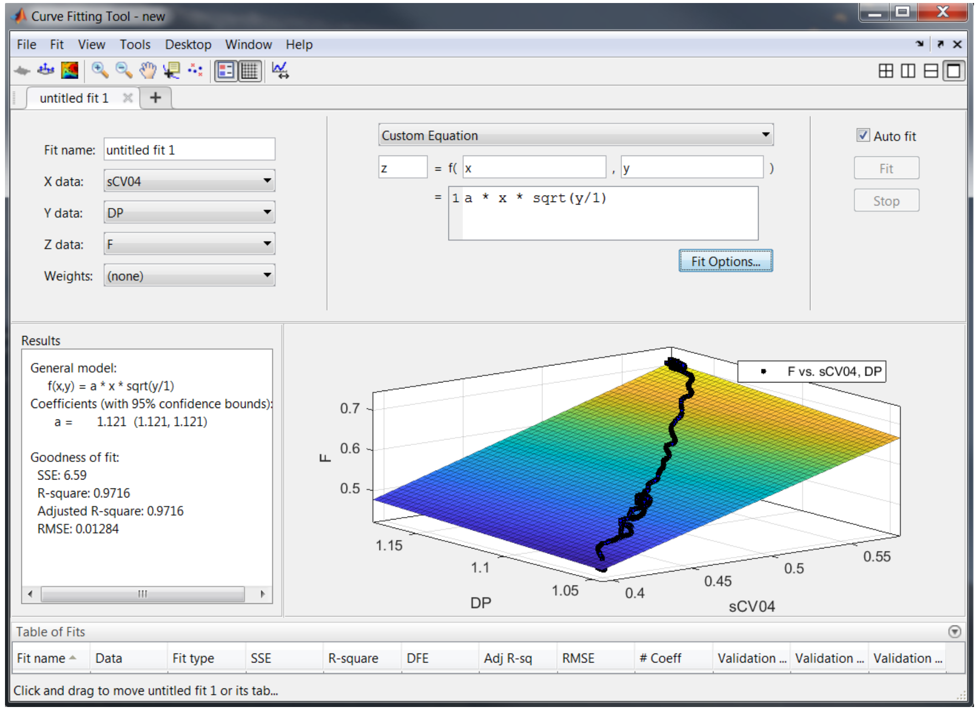
Estimate the unknown parameter

- Matlab can be used to solve such problems, using e.g. the **curve fitting toolbox `[cftool]`**.



Demo in Matlab:

calculating the unknown parameter a using **least squares Method** using collected data.

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Valve as a Flow Resistance

- If we see the valve as a flow resistance, we have:

$$h = F_{out} \cdot \mathrm{R}_v$$

Where

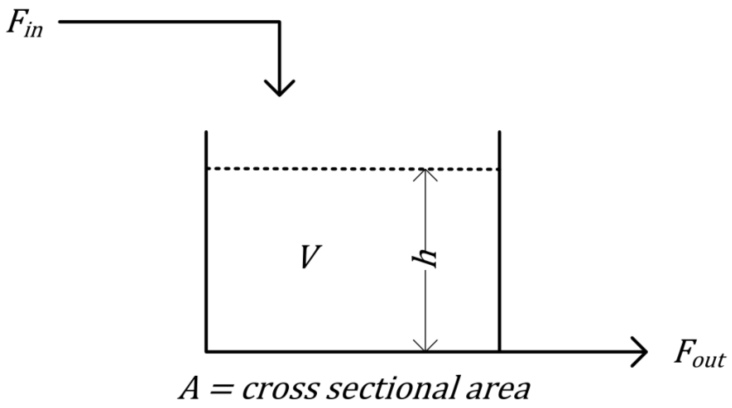
$F =$ is the flow,

$h =$ is the head or the liquid level,

$R_v =$ is the resistance of the line induced by the valve.

We can rearrange this into the flow-head equation:

$$F_{out} = \frac{1}{R_v} h$$



Valve's Equation (Hydrostatic Pressure)

The driving force of the flow F is the pressure.
Assume no pressure at the head of the tank, then,
The **pressure** at the **bottom** of the tank is **related** to the **level height** of the water h through the force balance:

$$P_b = \frac{\rho g}{A} \cdot V = P_u + \frac{\rho g}{g_c} h$$

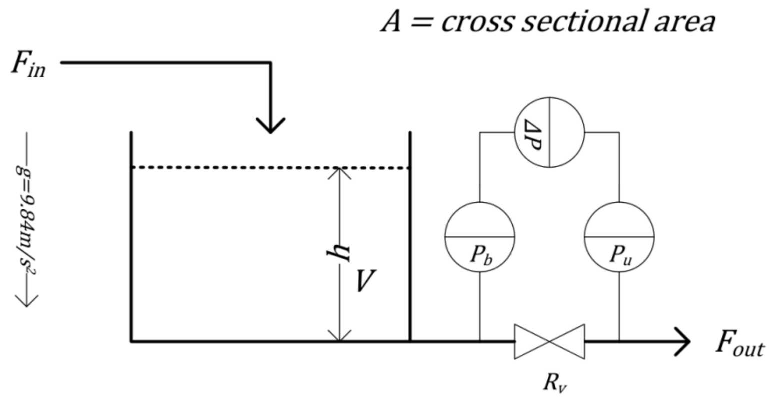
Where
V =Volume of the tank
g = gravitational acceleration (**constant**)
g_c = conversion factor (**constant**)
ρ = is the density of water
P_b & P_u =Bottom and underflow pressure

Thus the Pressure at the bottom is governed by the hydrostatic pressure

And substituting $P_b = P_u + \frac{\rho g}{g_c} h$ and $F = C_v^* \sqrt{\frac{P_b - P_u}{\rho}}$ into:

$$A \frac{dh(t)}{dt} = F_{in}(t) - F_{out}(t)$$

We get…



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Final Non-Linear Equation

We now have the model of the tank system, in this case the valve has one setting, i.e. I is a resistance depending on the C_v value:

$$A\frac{dh(t)}{dt} = F_{in}(t) - C_v \cdot \sqrt{h(t)}$$

Where

$$C_v \triangleq C_v^* \sqrt{\frac{g}{g_c}}$$

Where we have a **nonlinearity** in the square root term

Next step is to linearize this term, if we with to work with linear models…

Final Non-Linear Equation

We now have the model of the tank system, in this case the valve has one setting, i.e. I is a resistance depending on the C_v value:

$$A\frac{dh(t)}{dt} = F_{in}(t) - C_v \cdot \sqrt{h(t)}$$

This assumes that the tank is not pressurized, any pressurization pressure will be added to the hydrostatic pressure

Where

$$C_v \triangleq C_v^* \sqrt{\frac{g}{g_c}}$$

Where we have a **nonlinearity** in the square root term

Next step is to linearize this term, if we with to work with linear models…

Linearization

$$A\frac{dh(t)}{dt} = F_{in}(t) - C_v \cdot \sqrt{h(t)}$$

We first find the steady state conditions, $(h_s, F_{in,s})$, such that $h' = h - h_s$ and $F_{in}' = F_{in} - F_{in,s}$, i.e. $'$ denotes the deviations from steady state, or the reference point for linearization.

We now follow the standard Taylor linearization, for:

$$\frac{dy}{dt} = f(y, x)$$

Where $y = h$ and $x = F_{in}$

Can be linearized by Taylor series expansion, with higher order than linear truncated:

$$f(y,x) \cong f(y_s, x_s) + \left.\frac{\delta f}{\delta \mathrm{y}}\right|_{y_s, x_s} (y - x_s) + \left.\frac{\delta f}{\delta x}\right|_{y_s, x_s} (x - x_s)$$

If we have that the steady state condition$f(y_s, x_s) = 0$ and substituting for $y' = y - y_s$ and $x' = x - x_s$, we have

$$\frac{dy'}{dt} = \left.\frac{\delta f}{\delta \mathrm{y}}\right|_s y' + \left.\frac{\delta f}{\delta x}\right|_s x'$$

Linearization

$$\frac{dy'}{dt} = \frac{\delta f}{\delta y}\bigg|_s y' + \frac{\delta f}{\delta x}\bigg|_s x'$$

$$\frac{dy}{dt} = f(y,x) \rightarrow A\frac{dh(t)}{dt} = F_{in}(t) - C_v \cdot \sqrt{h(t)}$$

Now, for $y = h$ and $x = F_{in}$, the non-linear equation becomes:

$$A\frac{dh'}{dt} = 1 \cdot F_{in}' - \frac{C_v}{2\sqrt{h_s}} \cdot h'$$

Where:

$h' = h - h_s$ and $F_{in}' = F_{in} - F_{in,s}$

Note:

$$\frac{d}{dx}\sqrt{u} = \frac{1}{2\sqrt{u}}$$

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Convert to TF

$$A\frac{dh}{dt} = F_{in} - \frac{C_v}{2\sqrt{h_s}} \cdot h$$

For simplification we set the valve resistance as $\frac{1}{R}$ and we get the following equation after Laplace transform:

$$A \cdot s \cdot H(s) = F_{in}(s) - \frac{1}{R} \cdot H(s)$$

We want to have the standard input output relationship namely, $\frac{H(s)}{F_{in}(s)}$, thus:

$$H(s) = \frac{F_{in}(s)}{As + \frac{1}{R}} \rightarrow \frac{H(s)}{F_{in}(s)} = \frac{1}{As + \frac{1}{R}} = \frac{R}{ARs + 1} = \frac{K}{\tau s + 1}$$

Exercise

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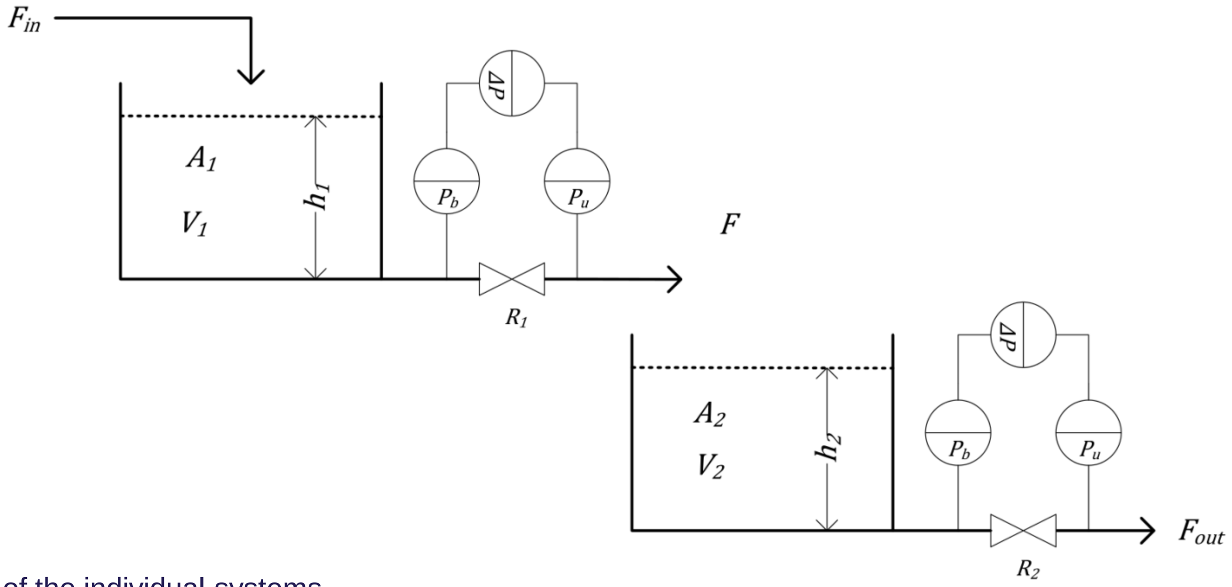
Exercise

Identify the model of the following tank system, with a fixed valve position.



1. Set up the transfer function model of the individual systems
    1. Hint: we aim at modeling the relationship between the input flow and the output flow, i.e. F_{in} and F_{out}, where the valves resistance R is a constant
2. Draw a block diagram to arrange the transfer functions
3. Construct the total transfer function
4. Implement the system into Matlab and analyze the response, try with $A_{1,2} = 0.1, 1, 10$ and simulate for 100s

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