Optimal Pump Scheduling in Water-Supply Networks

By Paul W. Jowitt[1] and George Germanopoulos[2]

Abstract: The electricity cost of pumping accounts for a large part of the total operating cost for water-supply networks. This study presents a method based on linear programming for determining an optimal (minimum cost) schedule of pumping on a 24-hr basis. Both unit and maximum demand electricity charges are considered. Account is taken of the relative efficiencies of the available pumps, the structure of the electricity tariff, the consumer-demand profile, and the hydraulic characteristics and operational constraints of the network. The use of extended-period simulation of the network operation in determining the parameters of the linearized network equations and constraints and in studying the optimized network operation is described. An application of the method to an existing network in the United Kingdom is presented, showing that considerable savings are possible. The method was found to be robust and with low computation-time requirements, and is therefore suitable for real-time implementation.

Introduction

The electricity cost of pumping accounts for a major part of the total operating cost of a water-supply network, estimated at £70,000,000 annually for the United Kingdom water industry ("Pump" 1985). The savings that would accrue from only a 5% reduction in the total power consumed in the United States have been estimated at $48,000,000 per year (Tarquin and Dawdy 1989). Improved operation of pumps leading to a reduction in energy cost must therefore be regarded as a priority when more efficient network operation is sought.

The electricity cost of pumping typically consists of a unit charge made for all units of energy (kWhr) consumed, and of a maximum demand charge, which is made on a monthly basis for each kVA of maximum demand at each pumping station. The full scope of possible reductions in pumping costs can only be achieved by also determining optimal operating policies for all pumping stations at a given network. The objective can be stated as

$$\min \left[\gamma \sum_n \left(\int_{t_0}^{t_f} r_n U_n \, dt \right) + \sum_n \alpha_n R_n Y_n \right] \quad \ldots\ldots\ldots\ldots (1)$$

where r_n = tariff unit charge at pumping station n; U_n = power consumption at pumping station n; Y_n = maximum half-hourly kVA demand for pumping station n over the optimization period; and R_n = charge for each kVA of monthly maximum demand at pumping station n. The constant α_n is equal

[1]Prof. of Civ. Engrg. Systems, Civ. Engrg. Systems Res. Centre, Dept. of Civ. Engrg., Heriot-Watt Univ., Edinburgh, EH14 4AS, United Kingdom.

[2]Res. Fellow, Dept. of Civ. Engrg., Section of Water Resour. and Hydr. and Marine Works, Nat. Tech. Univ., 5 Iroon Polytechneiou St., 156 73 Athens, Greece; formerly Res. Asst., Dept. of Civ. Engrg., Imperial Coll., London, SW7 2BU, United Kingdom.

Note. Discussion open until December 1, 1992. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on July 9, 1990. This paper is part of the ***Journal of Water Resources Planning and Management***, Vol. 118, No. 4, July/August, 1992. ©ASCE, ISSN 0733-9496/92/0004-0406/ $1.00 + $.15 per page. Paper No. 1468.

J. Water Resour. Plann. Manage., 1992, 118(4): 406-422

to zero if Y_n is less than the maximum kVA demand already incurred at pumping station n during the current month of account, and unity otherwise. The optimization period (t_0, t_f) is usually only a small subset of the period remaining until the end of the current monthly period of account of maximum demand charges; γ is therefore a weighting factor by which the electricity unit charges are multiplied so that the total cost is obtained on a uniform basis.

In solving the foregoing problem, account should be taken of the relative efficiencies of the available pumps, the structure of the electricity tariff, the consumer-demand pattern over the optimization period, and the reservoir storage available in the system. The interaction between the pump controls, the resulting pump power consumptions, and the pressure and flow regime in the network will have to be considered through the nonlinear network hydraulic equations and pump characteristics. Constraints relating to the quantities licenced for abstraction from borehole sources, the maximum throughput from water-treatment plants, and the maximum and minimum reservoir storage volumes and network pressures will have to be taken into account. Furthermore, a schedule of valve controls may also have to be obtained in conjunction with an optimal schedule of pumping.

It can be seen that, with the exception of very simple water-distribution systems, the problem of optimal pump scheduling is one of high complexity for which a formal approach is required. The formulation of the optimization problem must be such that sufficient representation of the network hydraulic characteristics and pumping costs is included, without the resulting solution being too complex for its computer implementation. The need for an efficient solution is emphasized by the fact that the rapidly varying nature of consumer demands requires pump schedules to be obtained in real time for the full benefits of optimal control to be achieved (Jowitt et al. 1988).

As a result, simplifying assumptions have to be made to reduce the dimensionality and complexity of the optimal pump scheduling problem. The nature of these assumptions depends on the hydraulic and operational characteristics of the particular network studied, and leads to different approaches in the formulation and solution of the problem. Such approaches have included dynamic programming (Sterling and Coulbeck 1975b; Joalland and Cohen 1980; Coulbeck and Orr 1983; Ormsbee et al. 1989); hierarchical decomposition methods (Fallside and Perry 1975; Sterling and Coulbeck 1975a; Coulbeck and Sterling 1978); and simple operating rules (Moss 1979). In the present paper a method based on linear programming is proposed. The method relies on a set of assumptions decoupling the operation of the pumps from the nonlinear hydraulic characteristics of the network. This leads to a linear set of constraints and a linear objective function involving unit charges only. The pump-network interaction is taken into account outside the optimization solution by using an extended-period network-simulation model (Germanopoulos 1988). The model gives the temporal variation of flows, pressures, and pump power consumptions throughout the network for alternative pump controls. The results are then used as input to the optimization solution. The model used can incorporate electricity tariff data, and thereby produce the cost of pumping over the period simulated. The maximum kVA demand component of the total electricity charge is taken into account in the optimization through a procedure involving repeated solutions of the linear programming problem for varying restrictions on pump usage, until the "best" answer is obtained. The proposed methodolgy has been developed for operational use within a real-time forecasting and control scheme at the Bourne End control center in the Northern Division of Thames Water Authority; this work is described by Jowitt et al. (1988). The implementation of the methodology was undertaken by Tynemarch Systems Engineering Ltd ("Real-time" 1988).

As already noted, the simplifying assumptions involved in the linear programming formulation depend on the characteristics of the particular network considered. The proposed linear programming approach cannot therefore be taken to be applicable to every network; the same applies to alternative approaches that have been developed for the formulations and solution of the optimal pump-scheduling problem.

Linear Programming Formulation of Optimal Pump Scheduling Problem

Assumptions

The formulation of the optimal pump scheduling problem as a linear program relies on the following set of assumptions decoupling pumping station operation from the network hydraulic characteristics.

1. Any schedule of pumping that is feasible in terms of the reservoir storage amplitude constraints also satisfies the nodal pressure amplitude constraints.
2. For each pumping station and for given consumer demands, the flow and power consumptions can be expressed directly in terms of the pump controls at that station and are not affected by pump and valve controls elsewhere in the network.
3. For each pumping station and for given consumer demands, the proportion of the delivery flow reaching each reservoir can be expressed directly in terms of the pump controls at that station and is not affected by pump and valve controls elsewhere in the network.

As the foregoing three assumptions are central to the proposed methodology, some discussion of the conditions under which they can be expected to hold is relevant at this point. The validity of assumption (1) is a function of the correct hydraulic design of the network, i.e. in a well-designed network internal network pressures can be expected to remain within accepted bounds for allowable service reservoir storage fluctuations. Assumption (2) can be expected to be valid for networks in which the head lift across each pumping station (i.e. the difference between the suction and the delivery heads) is *large* compared to the network nodal head changes induced by pump and valve switchings elsewhere in the system. This means that whereas the flow pattern in the network may change significantly as a result of such switchings (and also as a result of fluctuations in consumer demands or service reservoir levels), the magnitude of the corresponding nodal head changes is such that the station pumps still operate practically at the same point on the pump curve. According to experience from the case study to be presented, assumption (3) can be expected to be valid in situations in which the dominant factor in determining the allocation of the delivery flow from a given pumping station between different reservoirs/zones in the system is the magnitude of the respective zonal consumer demands, and not the changes in the network head/flow pattern induced by pump or valve switchings elsewhere in the system.

If the foregoing assumptions can be confirmed, the need to incorporate an explicit nonlinear network hydraulic model in the optimization solution is eliminated and, as is seen in the case study, a very simple representation of the network is then possible. A full-network hydraulic-simulation model will still have to be used outside the formulation of the optimal pump scheduling problem in order to validate the foregoing assumptions and determine the parameters relating to pumping station flows, power consumptions, and the distribution of the pumping station flows between the network reservoirs for alternative pump controls and for different consumer demands.

Problem Variables and Constraints

In view of the case study to be presented, the formulation is confined to fixed-speed on-off controlled pumps and to borehole sources of supply. This does not involve any loss of generality, because variable-speed pumps can be included through the determination of discrete operating points. In the case of river sources, marginal water-treatment costs and constraints on the maximum throughput of water-treatment plants can be included without difficulty in a linear programming solution. An outline of the formulation is as follows.

- The optimization period is divided into a number of discrete control intervals.
- The zonal consumer demands are allocated to the corresponding network reservoirs.
- Each pumping station includes one or more combinations of parallel pumps. A parallel pump combination can be a source or booster combination. A source combination pumps water from a borehole source into one or more destination reservoirs. A booster combination pumps water out of an origin reservoir and into one or more destination reservoirs. A pumping station including a source combination is referred to as a source station. A pumping station including only booster combinations is referred to as a booster station.
- The pump controls are expressed in terms of duties at each pumping station. Each duty corresponds to a given configuration of pumps being in operation at each parallel pump combination within the station. Zero pumping from a pumping station is considered as an additional duty. The controls correspond to the length of time within each control interval for which a pumping station operates at a given duty.
- The valve controls are expressed as quantities of water transferred from an origin reservoir to a destination reservoir over each control interval.

The problem constraints can now be formulated as follows in terms of the variables as defined in Appendix II.

Constraints on the hours of pumping from each pumping station at a given duty in relation to the duration of each control interval are

$$\sum_{m=1}^{M_n} x_{nm}^k = T^k; \qquad k = 1, \ldots, K; n = 1, \ldots, N \quad \ldots\ldots\ldots\ldots\ldots (2)$$

Flow continuity constraints for each reservoir over each control interval are

$$s_i^{k-1} + \sum_{n=1}^{N} \sum_{m=1}^{M_n} \sum_{h \in Ln} x_{nm}^k a_{hmi}^k q_{hm}^k - \sum_{j \in V_i} v_{ij}^k - c_i^k = s_i^k;$$

$$i = 1, \ldots, I; k = 1, \ldots, K \quad (3)$$

Constraints on reservoir storage volume are

$$s_i^k \geq s_i^{\min}; \quad i = 1, \ldots, I; k = 1, \ldots, K - 1 \quad (4)$$

$$s_i^k \leq s_i^{\max}; \quad i = 1, \ldots, I; k = 1, \ldots, K - 1 \quad (5)$$

Constraints on initial and final reservoir storage are

$$s_i^0 = s_i^{\text{start}}; \quad i = 1, \ldots I \quad (6)$$

$$s_i^K = s_i^{\text{end}}; \quad i = 1, \ldots I \quad (7)$$

Constraints on valve flow rates are

$$v_{ij}^k \geq T^k v_{ij}^{\min}; \quad i = 1, \ldots, I; j \in V_i; \quad k = 1, \ldots, K \quad (8)$$

$$v_{ij}^k \leq T^k v_{ij}^{\max}; \quad i = 1, \ldots, I; j \in V_i; \quad k = 1, \ldots, K \quad (9)$$

Maximum usage constraints at each source pumping station over the optimization period are

$$\sum_{k=1}^{K} \sum_{m=1}^{M_n} \sum_{h \in L_n^s} x_{nm}^k q_{hm}^k \leq q_n^{\text{lic}}; \quad \text{for all } n \in N_s \quad (10)$$

Ignoring maximum demand charges at this stage, the problem objective function is taken to represent unit electricity charges only, and is expressed as follows:

$$\min \left(\sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{m=1}^{M_n} \sum_{h \in L_n} r_n^k x_{nm}^k U_{hm}^k \right) \quad (11)$$

It can be seen that the objective function of (11) and the constraints of (2)–(10) form a linear programming problem. The problem unknowns are the pumping times x_{nm}^k, valve transfers v_{ij}^k, and reservoir volumes s_i^k. Their determination from the linear programming solution defines the schedule of operation of the network over the optimization period.

The values of $v_{ij}^{\min}$, $v_{ij}^{\max}$, $s_i^{\min}$, $s_i^{\max}$, q_n^{lic}, are determined a priori as part of the optimization input. The same is true for the initial and final reservoir volumes s_i^{start} and s_i^{end}.

The values of the parameters q_{lm}^k, u_{lm}^k, and a_{lmi}^k represent the hydraulics of the network, and they are determined by a full extended-period simulation model independently of the optimization or directly from measured data where available. It can be seen that a necessary condition for the validity of the proposed linear programming formulation is that the parameters q_{lm}^k, u_{lm}^k, and a_{lmi}^k can be treated as constants and that their values are not dependent on the values of the decision variables x_{nm}^k, v_{ij}^k, and s_{ij}^k. This is the purpose of assumptions 1) and 2) discussed earlier.

The consumer demands c_i^k are taken to be constants, determined a priori either from typical recorded profiles in the case of off-line optimization, or by a demand-forecasting algorithm in the case of real-time control. If the values of parameters q_{lm}^k, u_{lm}^k, and a_{lmi}^k are shown by prior simulation studies of the network's hydraulic behavior to be related to changes in consumer

J. Water Resour. Plann. Manage., 1992, 118(4): 406-422

demand, then their values for each control interval k can be determined prior to the optimization in correspondence to the known values c_i^k.

Finally, with regard to (3), (8), and (9) it must be noted that the valve flow rates must be feasible in terms of the network operating heads. This can be confirmed by the use of the full extended-period network simulation model and/or by the known operational characteristics of the network.

Determination of Control Intervals

It can be seen that the number of linear programming variables and constraints increases significantly with the number of control intervals defined, leading to increased computer time and memory requirements for the solution. There is therefore an obvious incentive to limit the number of control intervals. On the other hand, the control intervals must be such that a meaningful definition of the problem is obtained. The relevant factors that must be considered for this to be the case are now discussed.

The first consideration in establishing the control intervals is the structure of the electricity tariff. Changes in the value of the energy unit charge must lead to the definition of new control intervals. The same is true for the start and end of the prescribed period for kVA charges within the day.

Another consideration must be the reservoir storage amplitude constraints. The solution produces reservoir storage values at the end of successive control intervals that will obviously satisfy the constraints on reservoir storage. However, the solution does not guarantee that the resulting schedule of pumping can be implemented in such a way that will satisfy these constraints throughout each control interval. For this reason, control interval limits must be chosen to coincide with times at which reservoir levels can be expected to reach their maximum and minimum values. For a 24-hr optimization period for example, the reservoirs can be expected to be almost full before the start of the morning peak in demand and at their lowest before the start of the nighttime period of low consumer demands.

The definition of the control intervals must also be such that pump flows, their distribution between the network reservoirs, and the pump power consumptions can be taken to be constant over each control interval with sufficient accuracy. This can be achieved within the formulation of the problem given in (2)–(11), where the parameters q_{hm}^k, u_{hm}^k, and a_{hmi}^k are defined for each interval k, by providing a more detailed schedule of operation than would be justified by the problem objective function and constraints alone.

Even if computational requirements are not critical, there is an advantage in not defining any more control intervals than is necessary; notwithstanding the issues of feasibility referred to in the preceding paragraphs, the linear programming solution may have multiple optima, and definition of an increased number of control intervals increases this possibility. The practical consequence would be an unnecessarily "strict" definition of the optimal schedule in the solution, which would reduce the flexibility of its implementation by the network operators.

Accounting for Maximum Demand Charges

The linear programming formulation minimizes the marginal electricity costs but ignores maximum kVA demand charges, which cannot be expressed as a linear combination of the problem variables. Clearly this could lead to high maximum demand charges being incurred in order to achieve a small reduction in unit charges. Because the monthly maximum demand charge could be an important part of the total electricity bill, the resulting solution could be far from optimal.

The kVA demand incurred by the use of each parallel pump combination duty can be related to the power consumption at that duty through the power factor. The power factor is the ratio of kW and kVA supplied in each month of account, and can normally be taken as a constant for each pump. It is proposed to take account of maximum demand charges by the repeated solution of the linear programming problem with varying restrictions on the use of pumping station duties.

The aim of these restrictions is to impose a ceiling on the kVA demand at each pumping station by excluding from the linear programming solution the duties whose possible use would lead to a higher kVA demand than the chosen ceiling. Obviously, a linear programming solution with restrictions on the use of pumping station duties will involve higher (or at best equal) unit charges compared with the unrestricted case. On the other hand, the maximum demand charges corresponding to a restricted solution will be lower than (or equal to) those obtained for the unrestricted solution. The aim is to determine the pumping station duty exclusions that will lead to the best trade-off between unit and maximum demand charges and hence to the minimum overall cost of pumping.

The proposed methodology is now described. The total monthly charge J_T can be written as $J_T = J_u + J_d$, where J_d is the monthly maximum demand charge; and J_u is the corresponding unit charge, projected on a monthly basis. The linear programming solution for the given problem is first obtained with no restrictions on the use of pumping station duties. This produces the minimum possible unit charge $J_u^{\min}$ for the given problem. The maximum demand charge corresponding to the unrestricted solution is defined as the reference maximum demand charge J_d^{ref}, and the resulting total charge $J_T^{\text{ref}} = J_u^{\min} + J_d^{\text{ref}}$ is defined as the reference total charge. A series of linear programming solutions for varying restrictions on the use of pumping station duties is subsequently obtained. The set of pumping restrictions yielding the minimum total charge is thus identified. It can be seen that the number of possible restrictions on the use of pumping station duties, i.e. the number of possible combinations of pumping station duty exclusions, is given by $\Pi_{n=1}^{N} M_n$. The solution of that number of linear programs would impose a heavy burden on the overall optimization solution for all but the simplest problems. Most combinations of pumping station duty exclusions, however, can be considered without a linear programming solution being needed. The basis for this is now briefly outlined.

First of all, only combinations of pumping station duty exclusions that limit the possible maximum kVA charge to a value less than J_d^{ref} are considered. Indeed, the unit charge can never be less than $J_u^{\min}$, so the maximum demand charge must be less than J_d^{ref} for there to be a possibility of the resulting total charge being less than J_T^{ref}.

Furthermore, a given set of duty exclusions must allow sufficient pumping station output flows so that the flow continuity constraints of (3) are satisfied together with the reservoir storage constraints of (4)–(7). This point can be checked prior to obtaining the linear programming solution in relation to the known consumer demands for the given problem; if the allowed pumping station output is not sufficient, the corresponding linear programming solution need not be obtained.

The number of linear programming solutions that need to be obtained for the minimization of both unit and maximum demand charges is thus smaller than $\Pi_{n=1}^{N} M_n$, leading to modest computation-time requirements. This point is demonstrated by the case study results.

Case Study

The methodology just outlined was implemented at the Bourne End telemetry and telecontrol center in the Northern Division of Thames Water Authority, at Buckinghamshire, England. The center serves the High Wycombe and Slough areas, with a total population of about 300,000.

Case Study Area

A schematic representation of the High Wycombe area network as used in the optimization is shown in Fig. 1. The simplified schematic is in direct correspondence to the proposed linear programming formulation of the pump-scheduling problem. A complete network-simulation model comprising 87 nodes and 122 interconnected elements was used for the validation of the assumptions involved in the formulation and for the determination of the parameters relating to the network's hydraulic behavior. The network includes four main pumping stations (noted as PS1, PS2, PS3, and PS4 in Fig. 1), five service reservoirs (noted as SR1, SR2, SSR3, SR4 and SR5), and three lumped consumer demands. (In Fig. 1, FL stands for floor level and TWL stands for top water level.) Pumping stations PS1 and PS2 each include a combination of submersible pumps pumping from borehole sources into a contact tank, from which a parallel combination of booster pumps

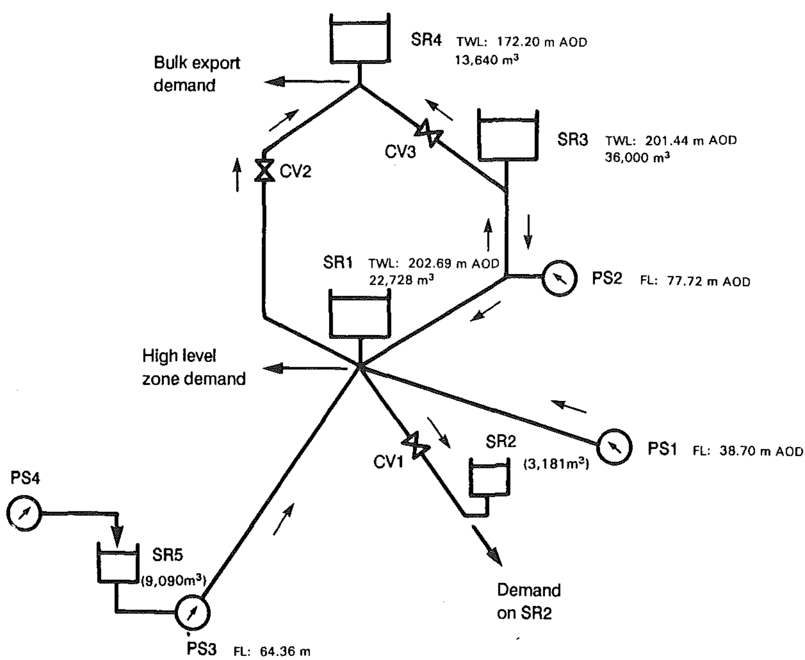


FIG. 1. Case Study Network Schematic

delivers water into supply. The booster pumps at pumping station PS1 supply a main high-level zone, whose storage is provided by service reservoir SR1. The outflow from the booster pumps at pumping station PS2 is divided between the high-level zone and service reservoir SR3. Control valve CV3 controls supplies to service reservoir SR4 with which a bulk export demand is associated. The flow between pumping station PS2 and service reservoir SR3 is bidirectional. Pumping station PS3 includes a parallel combination of booster pumps pumping water from a contact tank, shown as reservoir SR5 in Fig. 1, into the high-level zone and service reservoir SR1. Reservoir SR5 is supplied by a combination of borehole pumps at pumping station PS4. Water is exported from the high level zone through control valves CV1 and CV2. Control valve CV1 supplies service reservoir SR2, which in turn supplies a relatively small consumption zone. Control valve CV2 supplies the same bulk export demand as control valve CV3.

The operation of the submersible pumps at pumping stations PS1 and PS2 is not accounted for explicitly in the solution of the optimal pump scheduling problem. Studies of these pumping stations have shown that the output of the submersibles can be assumed to always match that of the boosters, and that contact tank storage changes can be neglected. Having therefore determined an order of switching for the submersible pumps, their operation can be deduced for each configuration of booster pumps in operation. The power consumption and kVA demand for each booster pump configuration can then be increased by the corresponding power consumption and kVA demand of the submersibles. The operating costs of the submersibles are thus passed on to the boosters.

As a result of not considering explicitly the submersible pumps at pumping stations PS1 and PS2, each of the pumping stations PS1, PS2, PS3, and PS4 includes one combination of parallel pumps. Therefore, no distinction between pumping stations and the corresponding parallel pump combination is made. Pumping stations PS1, PS2, and PS4 are source pumping stations in the formulation of the optimal pump-scheduling problem, whose operation is subject to abstraction-licence restrictions. Pumping station PS3 is a booster station pumping water from reservoir SR5 into reservoir SR3.

Pumping stations PS1, PS2, PS3 have three, five, and 10 identical fixed-speed parallel pumps, respectively, in operation with freedom of switching. The number of duties at these stations is therefore four, six, and 11, accounting for zero pumps in operation as an additional duty. Pumping station PS4 can have four different configurations of pumps in operation, giving it a total of five duties.

Electricity Tariffs

The electricity tariffs available are the M1 and M2 tariffs of the Southern Electricity Board (*Tariffs* 1987). The main difference between them is that in the M1 tariff the charges for daytime and nightime consumption are the same; in the M2 tariff a rebate for nighttime pumping is allowed but charges for daytime electricity consumption are higher than those specified by the M1 tariff. Pumping stations PS1, PS3, and PS4 operate under the M2 tariff; pumping station PS2 operates under the M1 tariff. The nighttime period for both tariffs is from 00:00 hours to 07:00 hours. The prescribed period for maximum kVA demand charges is from 07:30 hours to 19:30 hours for pumping stations operating under the M2 tariff. For pumping stations under the M1 tariff, the maximum kVA demand is metered on a 24 hr basis. Under both tariffs, maximum demand charges apply to winter months only.

Correspondingly, the 24 hr optimization period considered in this example application has control intervals defined as follows: Interval 1 between 00:00 hours and 07:00 hours; interval 2 from 07:00 hours to 19:30 hours; and interval 3 between 19:30 hours and 24:00 hours.

Results

The optimized operation of the network was obtained retrospectively for demands that have already been experienced on the network. The demands (Fig. 2) were calculated from measured reservoir level changes and pumping station outflows from midnight to midnight on a typical summer period day. Table 1 presents the pumping station flows and power consumptions. The maximum storage quantity for each reservoir is noted in Fig. 1. The minimum allowable storage quantity is taken as 40% of the corresponding maximum storage. The licenced daily quantities for abstraction at source pumping stations PS1, PS2, and PS4 are 22,730 m^3, 18,184 m^3, and 54,000 m^3, respectively. The maximum flow rates through control valves CV1, CV2, and CV3 are 56 l/s, 368 l/s, and 368 l/s, respectively. The respective minimum flow rates are zero. This information, together with the electricity tariff data, forms the basis of the input required for the optimization solution. The values of Table 1 were obtained prior to the optimization following exhaustive simulation of the network's operation for a range of possible conditions. Since no significant changes in pumping station flows and power consumptions were indicated by the simulation on a diurnal basis, the values given in Table 1 apply for the whole optimization period.

The first set of results was obtained by optimizing on unit charges only. Extended period simulation results on the observed and optimized network operation are presented in Fig. 3; the observed network operation under the consumer demands considered is included for comparison, in order to determine the performance of the optimization. The main characteristics of the optimal responses is now described. Pumping stations PS1 and PS2 are identified as the most efficient and are used to their full abstraction-licence limit. Pumping stations PS3 and PS4 are brought in to supply the remaining

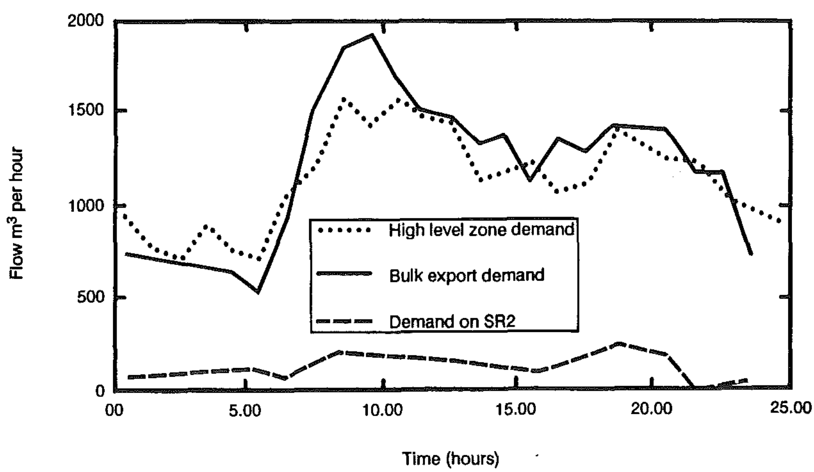


FIG. 2. Consumer Demand Profiles

J. Water Resour. Plann. Manage., 1992, 118(4): 406-422

TABLE 1. Pumping-Station Flows and Power Consumption

Number of pumps in operation (1)	Flow (l/s) (2)	Power (kW) (3)
	(*a*) Pumping Station PS1	
1	69	178
2	137	353
3	200	519
	(*b*) Pumping Station PS2	
1	65	116
2	127	229
3	186	341
4	241	438
5	290	547
	(*c*) Pumping Station PS3	
1	71	151
1	142	303
3	212	454
4	280	604
5	351	760
6	421	914
7	485	1065
8	550	1217
9	604	1363
10	638	1502
	(*d*) Pumping Station PS4	
1	160	167
2	320	334
3	480	501
4	640	668

consumer demands in the system, with the operation of source pumping station PS4 generally following that of the booster pumps at pumping station PS3. Pumping station PS3 operates at full capacity (i.e. with all 10 pumps available) during the nighttime period, when unit charges under the M2 tariff are low. This is reflected in the profile of service reservoir SR1, which shows a wider variation than under the observed operation in order to allow full advantage of cheap nighttime pumping. Whereas the optimized solution makes full use of the storage volume available at service reservoir SR1, that is not the case with the remaining service reservoirs, whose capacities correspond to a much greater proportion of their associated demands; the corresponding constraints on storage variation are thus less likely to affect the solution. The optimal operation of control valves CV1, CV2, and CV3 (and the resulting operation of reservoirs SR2, SR3, and SR4) therefore follow from the optimal responses just outlined so that a feasible overall answer is obtained.

A second set of optimization results was produced for unit plus maximum demand charges (see Fig. 3). The main difference between the two sets of optimal responses was found to be in the operation of pumping station PS3,

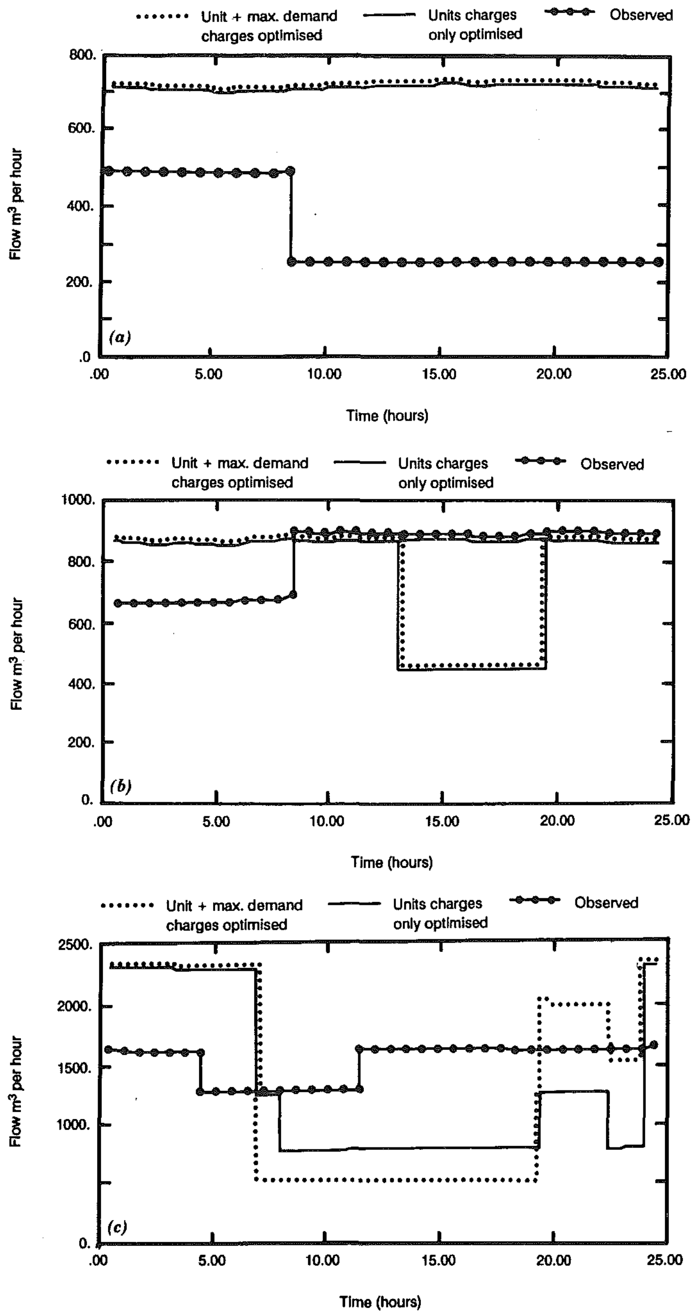
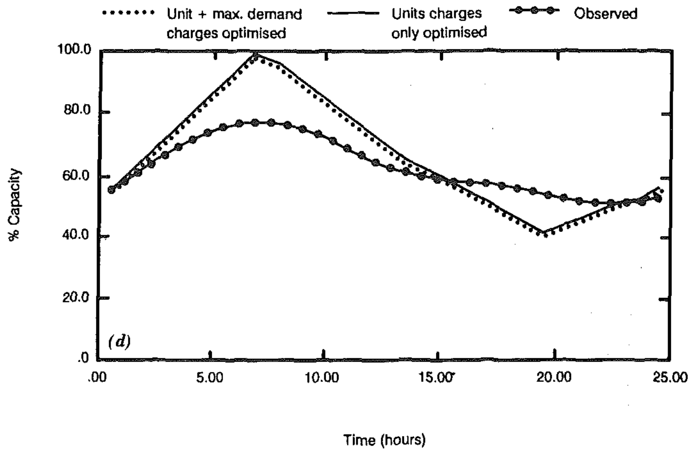


FIG. 3. Simulated Results for Observed and Optimized Network Operation: (*a*) Pumping Station PS1 Outflow; (*b*) Pumping Station PS2 Outflow; (*c*) Pumping Station PS3 Outflow; (*d*) Reservoir SR1 Storage; and (*e*) Flow through Control Valve CV2



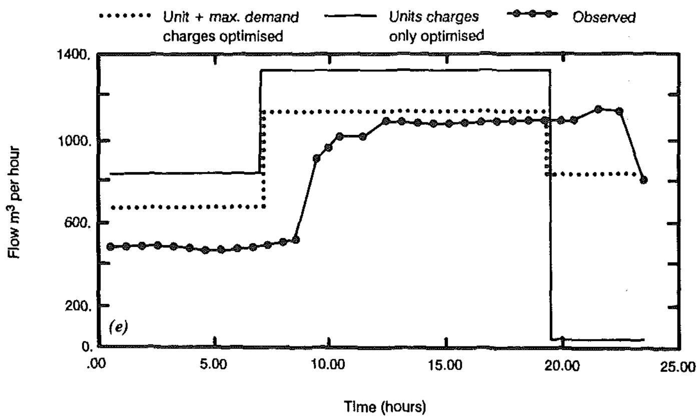


FIG. 3. (*Continued*)

followed by the source pumps at PS4. Indeed, pumping station PS3 is now restricted to three pumps in operation during the prescribed period for maximum demand charges (interval 2) in an attempt to reduce the resulting maximum demand charge. In comparison, the solution for unit charges only gave five pumps in operation during interval 2. The reverse is true for interval 3, where the solution for unit plus maximum demand charges produces a higher flow at PS3, thus compensating for the earlier restriction in pumping.

Pumping costs (as determined by the extended-period simulation of the optimized and observed network operation) show that a total saving of 7.7% can be achieved by optimizing for unit charges only. By considering unit charges and maximum kVA demand charges, the corresponding saving is 15.6%. Of course, it must be borne in mind that the foregoing energy savings correspond to a situation in which consumer demands are taken to be known exactly prior to the optimization. This will not be the case in practice; instead, a demand-forecasting algorithm would be relied upon.

The pumping station flows, power consumptions, and reservoir levels produced by the extended period simulator were found to be in good agreement with the corresponding optimization parameters and predictions. This confirms the validity of the formulation of the optimal pump scheduling problem as a linear program, and the calibrated parameter values used for the problem at hand.

With regard to the assumptions involved in the proposed method, it is also noted that extended-period simulation results have shown that the head lifts across pumping stations PS1, PS2, and PS3 are on the order of 130–170 m, and do not vary by more than 2–3 m on a diurnal basis for the consumer demands and pumping schedules shown in Figs. 2 and 3. This can be explained in terms of the large difference in elevation (see Fig. 1) between the pumping stations and the network reservoirs, whose levels in turn determine the operating heads of the main high-level zone. Indeed, this difference in elevation is much greater than normal fluctuations in network nodal heads. Each pump can therefore be taken to operate at practically the same point on its pump curve, in spite of variations in the network pressure regime.

Furthermore, the division of the outflow from pumping station PS2 between the high-level zone and reservoir SR3 was found to be determined solely by the magnitude of the high-level zone consumer demands, and can therefore be taken to be known a priori in the optimization.

Finally, it has been seen that the linear programming solution of the optimal pump scheduling problem consists of the length of the time within each control interval for which each parallel pump combination operates at a given duty, of the quantity of water transferred through each control valve over each interval, and of the resulting reservoir storage volume at the end of each interval. Control-valve flows are determined as an average flow rate corresponding to the total transferred quantity over each control interval. The pump-combination durations within each control interval do not of themselves constitute a definite schedule of operation, because they do not include on-off switching times for pump combination duties or valve controls. In the results presented here, the on-off times were determined so that pump combination duties involving higher delivery flows are used first in each control interval. The purpose of this, viewed in the context of real-time implementation, is to ensure that it will be possible in the course of the optimization period to make up for possible discrepancies between forecasted and actual consumer demands, as well as for unpredictable events in the network, such as fire-fighting, pipe breaks, and so forth. In practice, the network operators would schedule the duties heuristically to minimize switchings, provide maximum reliability, and so on.

Computational Requirements

The computer program for the formulation and solution of the optimal pump scheduling problem was developed in FORTRAN 77 on a DEC micro VAX II machine (Germanopoulos 1988). The computation time requirements for linear programming problems of different sizes are given in Table 2. An optimization for unit plus maximum demand charges was seen to involve a sequence of such linear programming solutions. Considering the case study area network, it can be deduced from the number of duties at each pumping station that there are 1,320 possible sets of limiting duties (i.e. $4 \times 6 \times 11 \times 5$). Following the procedure for the minimization of unit plus maximum demand charges described earlier, only 48 linear programming solutions had to be obtained for the optimal answer to be reached.

TABLE 2. Computation Times on (DEC Micro VAX II)

Number of control intervals (1)	Number of reservoirs (2)	Number of control valves (3)	Number of pumping stations (4)	Total number of duties (5)	Computation time (sec) (6)
2	1	0	3	21	1.7
3	2	1	3	21	5.4
3	4	3	3	21	27.4
3	5	3	4	26	41.5

This did not take into account kVA demand that may already have been incurred at each pumping station during the current month of account. Suppose, as an example, that previous pump usage includes two pumps at pumping station PS1, two pumps at pumping station PS2, one pump at pumping station PS3, and duty 1 at pumping station PS4. This reduces the number of sets of limiting duties to be considered and an optimal answer in this case was obtained after only 20 linear programming solutions. These results, together with the computation times of Table 2, indicate the suitability of the method for use in real-time environment.

Conclusions

This study showed that linear programming, subject to a set of necessary assumptions, can be used for the solution of the optimal pump scheduling problem in a water-supply network, taking account of both unit and maximum kVA demand charges. The method was applied successfully to an existing network, for which it was found to produce significant reductions in pumping costs. Computation-time requirements were found to be modest, thereby indicating the suitability of the method for use in real-time control. The availability of robust and efficient methods for the solution of linear programming problems (such as the revised simplex method) is also an advantage. The use of an extended-period network-simulation model was shown to be an important part of the overall optimization procedure, in order to provide parameter estimates of the linearized network equations and constraints prior to optimization, and, subsequently, validation of these parameter values and the detailed response of the network to the optimized pump schedules. Of course, once the overall validity of the approach has been confirmed, there is no further need for the extended-period simulation model to be involved in the real-time application.

Acknowledgments

The research described in this paper relates to a program of work undertaken by Tynemarch Systems Engineering Ltd for the Thames Water Authority (now Thames Water plc). The second writer is grateful to Tynemarch Systems Engineering Ltd for funding a Research Assistantship at Imperial College, London, and to Thames Water for their assistance with data, technical information, and so forth.

APPENDIX I. REFERENCES

Coulbeck, B., and Orr, C. H. (1983). "Computer control of water supply: Optimised pumping in water supply systems—2." *Res. Resport 33*, Leicester Polytechnic, Leicester, England.

Coulbeck, B., and Sterling, M.J.H. (1978). "Optimised control of water distribution systems." *Proc., IEE*, London, England, 125(9), 1039–1044.

Fallside, F., and Perry, P. F. (1975). "Hierarchical optimisation of a water supply network." *Proc. IEE*, London, England, 122(2), 202–208.

Germanopoulos, G. (1988). "Modelling and operational control of water supply networks." PhD thesis, Imperial College of Science and Technology, London, England.

Joalland, G., and Cohen, G. (1980). "Optimal control of a water distribution network by two multilevel methods." *Automatica*, Oxford, England, 16(1), 83–88.

Jowitt, P. W., Garrett, R. T., Cook, S. C., and Germanopoulos, G. (1988). "Real-time forecasting and control for water distribution." *Computer applications in water supply. Vol. 2: Systems Optimisation and Control*, B. Coulbeck and C. H. Orr, eds., Research Studies Press Ltd, Letchworth, England, 329–355.

Moss, S. M. (1979). "On-line optimal control of a water supply network," PhD thesis, University of Cambridge, Cambridge, England.

Ormsbee, L. E., Walski, T. M., Chase, D. V., and Sharp, W. W. (1989). "Methodology for improving pump operation efficiency." *J. Water Resour. Planning and Mgmt.*, ASCE, 115(2), 148–164.

"Pump scheduling in water supply." (1985). *Tech. Report TR 232*, Water Research Centre, Swindon, England.

"Real-time forecasting and control: Water supply management." (1988). *Final Report*, Tynemarch Systems Engineering Ltd, London, England.

Sterling, M.J.H., and Coulbeck, B. (1975a). "Optimisation of water pumping costs by hierarchial methods." *Proc., Instn. Civ. Engrs.*, Pt 2, 59, 789–797.

Sterling, M.J.H., and Coulbeck, B. (1975b). "A dynamic programming solution to the optimisation of pumping costs." *Proc. Instn. Civ. Engrs.*, Pt 2, 59, 813–818.

Tariffs for industrial and commercial users. (1987). Southern Electricity, Maidenhead, England, 5–7.

Tarquin, A. J., and Dawdy, J. (1989). "Optimal pump operation in water distribution." *J. Hydr. Engrg.*, ASCE, 115(2), 158–168.

APPENDIX II. NOTATION

The following symbols are used in this paper:

a^k_{hmi} = proportion of delivery flow from pump combination h for corresponding pumping station operating at duty m that reaches reservoir i during control interval k;

c^k_i = quantity of consumer demand at reservoir i during control interval k;

I = number of reservoirs in network;

J^{ref}_d = reference maximum kVA demand charge;

J^{ref}_T = reference total electricity charge;

J^{min}_u = minimum possible unit electricity charge;

K = number of control intervals into which optimizatin period is divided;

K' = number of control intervals within the prescribed period for maximum demand charges;

L = number of parallel pump combinations in network;

L_n = all parallel pump combinations within pumping station n;

L^s_n = all source parallel pump combinations within pumping station n;

J. Water Resour. Plann. Manage., 1992, 118(4): 406-422

M_n = number of duties at pumping station n;
N = number of pumping station in network;
N^s = all source pumping stations in network;
NV = number of controlled valve transfers in network;
q_{hm}^k = flow rate delivered from pump combination h for corresponding pumping station operating at duty m during control interval k;
q_n^{lic} = quantity licenced for abstraction at source pumping station n over the optimization period;
R_n = charge for each kVA of monthly maximum demand at pumping station n;
r_n^k = unit cost of electrical energy consumed over control interval k at pumping station n;
s_i^k = storage quantity at reservoir i at end of control interval k;
s_i^{end} = storage at end of optimization period at reservoir i;
$s_i^{\max}$ = maximum allowable storage quantity at reservoir i;
$s_i^{\min}$ = minimum allowable storage quantity at reservoir i;
s_i^{start} = storage at start of optimization period at reservoir i;
T^k = duration of control interval k;
t_f = time at end of optimization period;
t_0 = time at start of optimization period;
U_n = power consumption at pumping station n;
u_{hm}^k = power consumption at pump combination h for corresponding pumping station operating at duty m during control interval k;
V_i = all reservoirs linked to reservoir i through control-valve connection;
v_{ij}^k = valve controlled quantity of water transferred from reservoir i to reservoir j during control interval k;
$v_{ij}^{\max}$ = maximum flow rate through valve connecting reservoirs i and j;

$v_{ij}^{\min}$ = minimum flow rate through valve connecting reservoirs i and j;
x_{nm}^k = length of time within control interval k for which pumping station n operates at duty m;
Y_n = maximum kVA demand at pumping station n over optimization period;
α_n = constant [see (1)]; and
γ = weighting factor [see (1)].