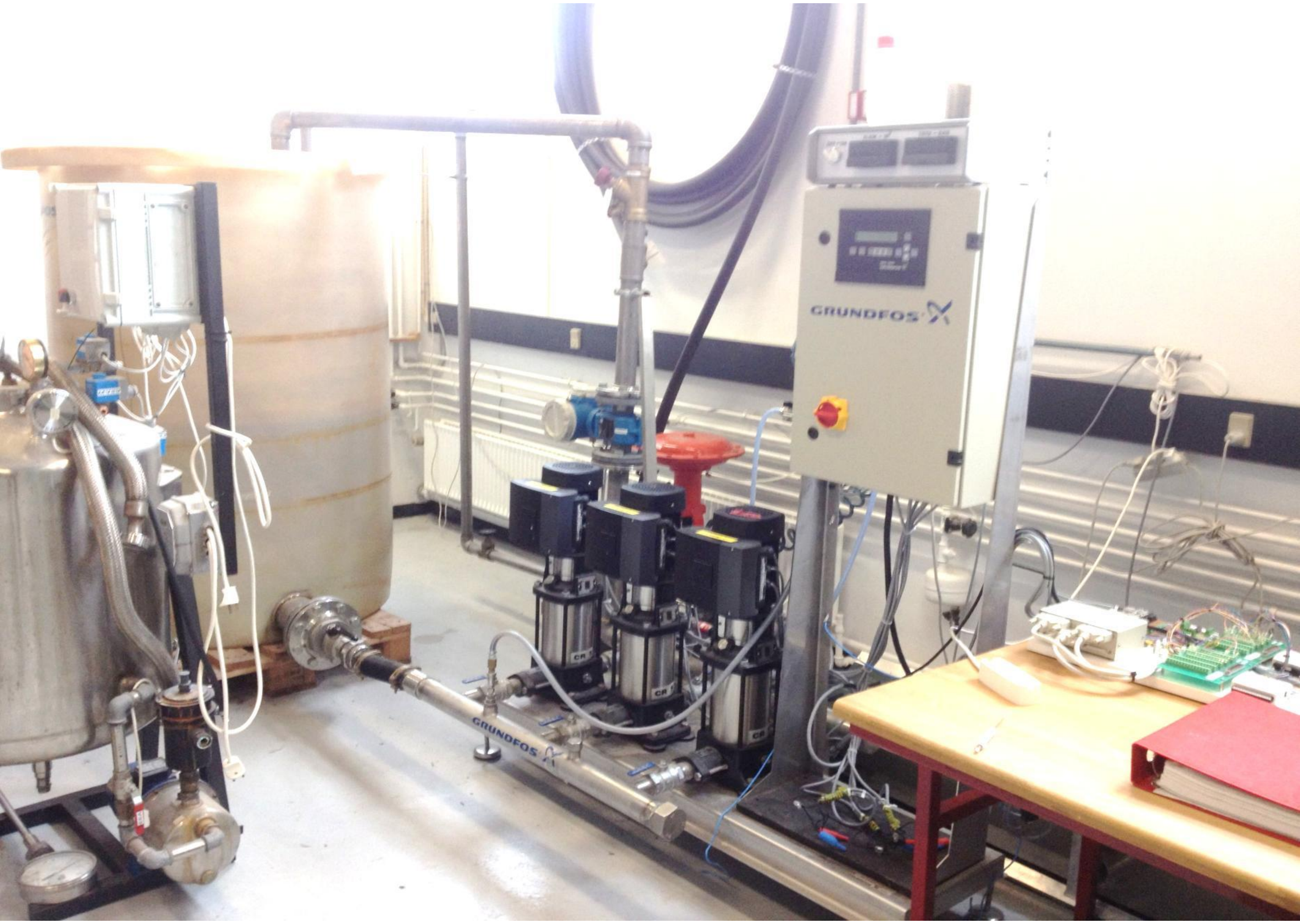
Regulation of Pump Flow



EN4-2-H228

Regulation of a pump system

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Project period: 2nd of February – 29th of May

Total prints: 9

Synopsis

The project is based on a pump system with three pumps in a view to obtain understanding and knowledge of a Proportional Integrator Derivative (PID) controller, to regulate the flow system and eventually gain a steady water flow in the pipes. Furthermore the general construction of a pump system is being studied to get a fundamental understanding of each component it takes to run this system, including an AC motor. In order to understand the flow and do calculations, basic thermodynamic equations and theory is being used as well as the affinity law.

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Indholdsfortegnelse

Preface

This report has been created at Aalborg University in Esbjerg by the group EN4-2-H228 consisting of six students. The report is the group's P4-project. The group has chosen to work with the project "Regulation of a Pump System", which is an idea from the project catalogue. The pump system is part of the facilities at Aalborg University Esbjerg.

The project is based on research, relevant lectures, and experimental work in the laboratory. The verbal use and the technical calculations are relatively easy understandable, having just a little background knowledge on regulation, pumps, and AC theory. The user interface in this project is directed towards people with interest in pump systems and regulation of pumps e.g. engineers or engineering students.

The group's goal for this project has been to get a decent understanding for regulation of a pump system, as well as a somewhat superficial overview of the AC motor and the different parts in the pump system as a whole.

We would like to thank Zhenyu Yang and Matthias Mandø for good cooperation during this project, as well as Simon Pedersen for his support on SIMULINK and the pump system.

1 Introduction

The present report deals with the issues with control of a pump system using a PID controller. The PID controller is created with SIMULINK and LabVIEW models and regulating a pump system. The project deals with some of the theoretic fluid dynamics, which is necessary for a more profound understanding of the subject. The project also gives an overview of the components in the pumping system used for the experiments relevant to the project. The focus of the components lies in the description of the pump system.

The project is useful for getting an understanding of how controlling and regulation of a pump system works. It is also useful to get an understanding of how the fluid dynamics works on a theoretical basis. Furthermore the affinity law is described.

2 Problem statement

2.1 Problem analysis

This project deals with the challenges of developing a PID. The PID controller is used in regulation of different kinds of systems, which in this project is a pump system. The purpose of the PID controller is to make sure a steady flow is obtained in a pump system in the AAU laboratory, and this controller is modeled by the use of MATLAB's SIMULINK and LabVIEW.

While the main part of this project is to make the PID controller to get a steady flow in the pump system, there will be an overview of the entire system and its components, in order to get all relevant knowledge of a system of this type. As a follow up for this, the affinity law will be reviewed in order to study the final results from the created PID controller. To make it possible to run a system like this, an AC motor is needed, which will be shortly described since it is a main component of system stability.

2.2 Issues

- What is the background of the PID controller?
- Which components are used in a pump system? – How do they work?
- How is the PID used as a controller?
- How can the pump system be analyzed?
- How does the pump operate?

2.3 Limitations

In the pump setup there are three pumps but only one is used. It is not necessary to use all the three pumps in this project, and only one is chosen to make the system as simple as possible. There is a general description of the AC motor and a short description of the three phases AC current. Based on the system some basic fluid dynamic theory is presented.

The affinity law is limited to the pumps with focus only on the change in speed and not the change of the impeller diameter, because it is impossible to change this for the setup. The project is furthermore limited to basic knowledge about PID control and the three elements P, I, and D. This knowledge is used to determine constants for P's, I's, and D's features. These constants are meant as guidelines, which will be tuned in a program in LabVIEW.

The knowledge about LabVIEW and SIMULINK is limited to the basic, because of the limited knowledge and education we have had about these programs. The knowledge about this is only superficial

3 The System

The pump system consists of three centrifugal pumps, but in this project is based on one pump only. In addition to this, the system has pressure and flow gauges. It is possible to connect extra load to the system. The system is pumping water through the pipes to the tank formed as a cylinder. By the LabVIEW program it is possible to regulate the speed of the pump. It can be regulated between 0-10 volts.

The system works by sending a voltage between 0-10 volts to an AC motor which starts the pump.

Depending on the voltage the water will float vertical up about 1.5 meters and pass through two 90 degrees bends and ends in the big cylinder tank. The tank is open at the top.

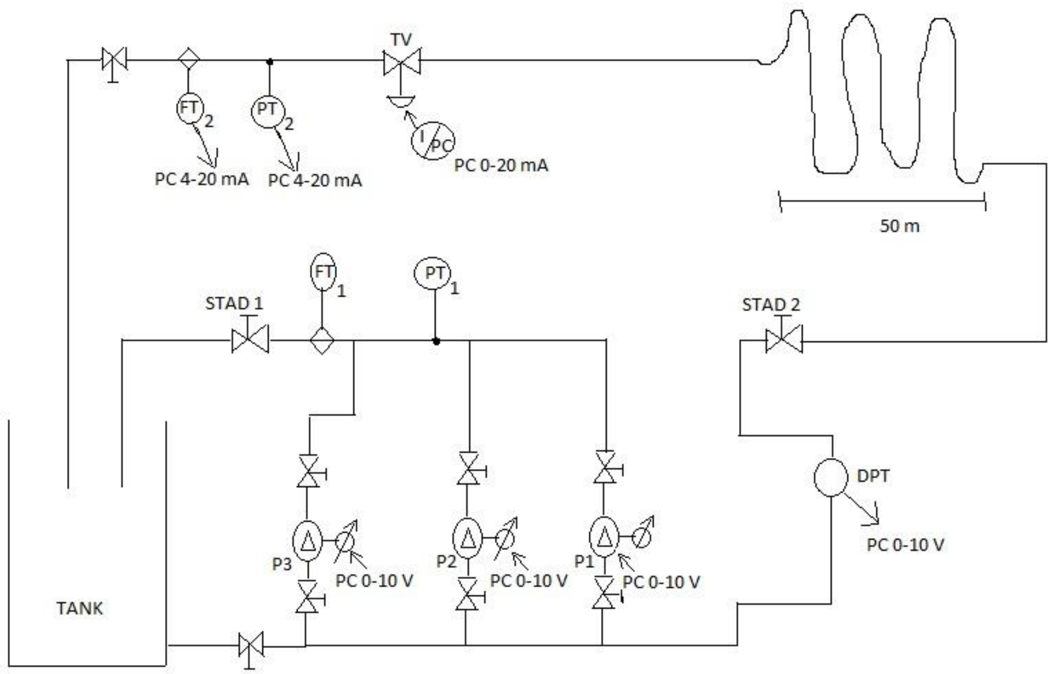


Figure 1. Own figure. An illustration of the system including all components.

Figure 2. An illustration of the pump system in the laboratory.

3.1 Centrifugal pump

In the system the pump is a centrifugal pump. A centrifugal pump is a pump which uses a rotating shovel wheel to pump the fluid through the pipes. The faster the wheel is rotating, the higher pressure it creates in the pipes. This pump is the most common for fluid pipe systems.

Like many other pumps, this pump type also converts the mechanic energy to energy of a moving fluid. Some of the energy is converted to kinetic energy as motion of the fluid and some of it is converted to potential energy, because the fluid needs extra energy to be lifted.

There are two kinds of centrifugal pumps; one is the rotation pump with a shovel wheel and another is a vortex pump (see the Figure 3 p. 5). In the rotation pump the fluid is shovelled by the rotating shovel wheel, which is placed in a closed pumping house. By rotation the mechanical work transfers to the fluid by the centrifugal effects. This happens by the fluid being pressed out of the pump house. In the middle of the pump there is a suction pipe that sucks the fluid in to the pump house by vacuum.

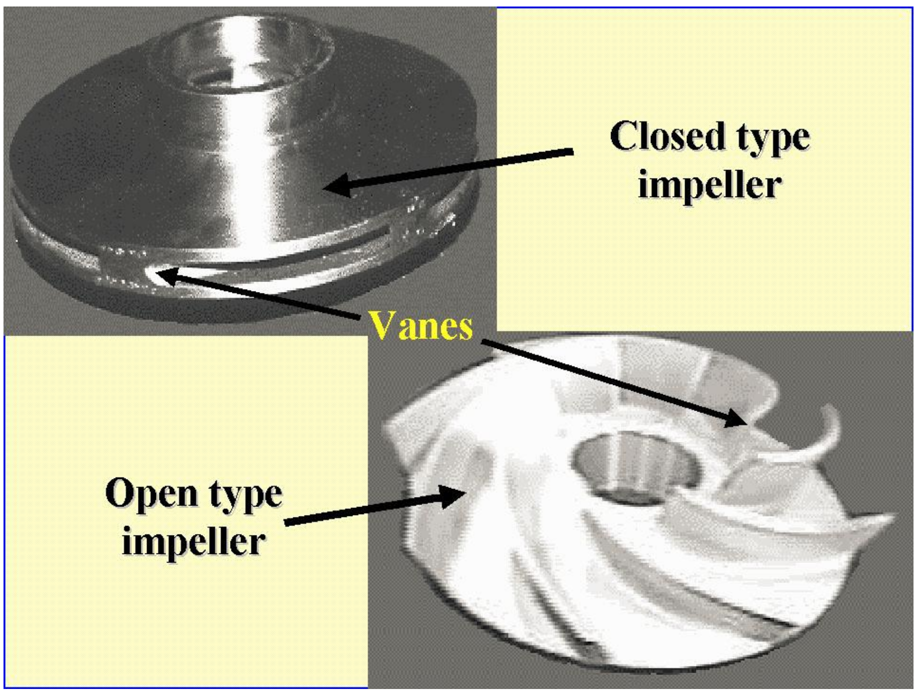


Figure 3. An illustration of a closed type impeller, which is the vortex and an open impeller, which is a rotation pump [bestcoaltrading.blogspot.com[1]] .

Then there is also the vortex pump, which is a little bit different compared to the rotation pump. This also consists of a shovel wheel, which can shovel from both sides of the pump wheel. When it is shovelling from both sides of the pump wheel, induced flow eddies[a] occurs and it gets the pressure in the pipes to increase.

[a] Eddies are rapid and disorderly fluctuations of swirling regions.

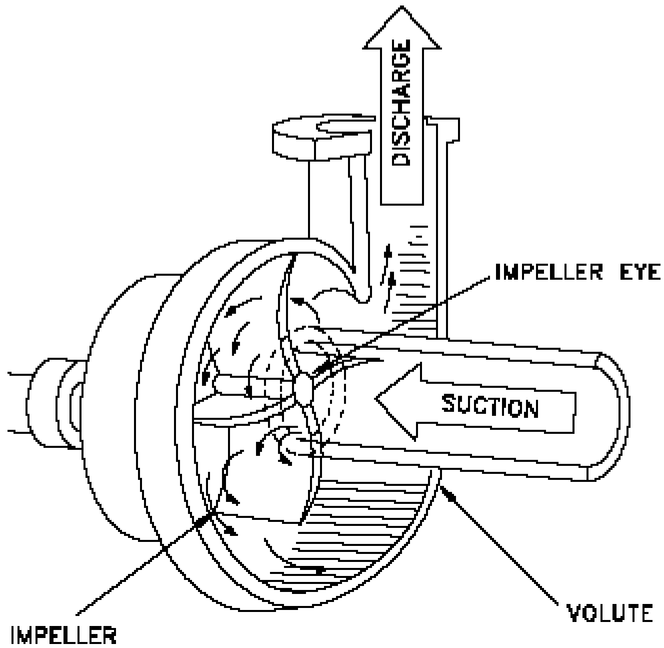


Figure 4 . A Cross section picture of centrifugal pump [netpumps.com[2]]

Figure 4 above is a picture of a centrifugal pump seen from the inside. The impeller is in the middle. The impeller is characteristic in a centrifugal pump. In the middle of the pump there is a suction side where the fluid enters. When the impeller throws the fluid it leaves the pump through the discharge side.

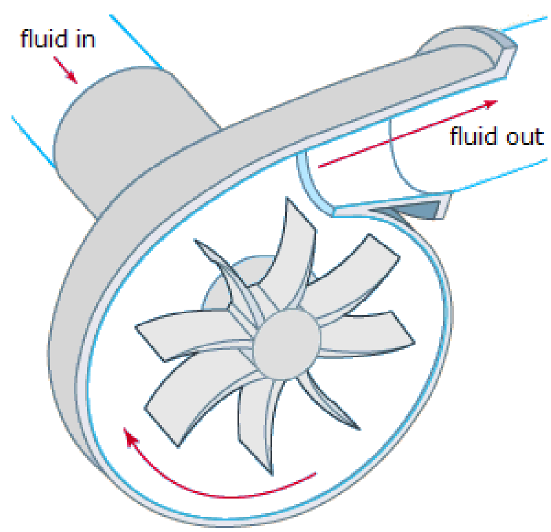


Figure 5. Impeller and its direction [wermac.org[3]]

Figure 5 shows the impeller which rotates to the right and throws the fluid out. The direction is clockwise, because if the impeller spins counter clockwise, it will shovel the fluid towards the middle and no flow will occur.

3.2 Measurement systems

There is a Promag 30 flowmeter mounted on the vertical pipe that leads the water to the top of the tank. The flowmeter works by magnetic induction, which measures the speed of the water that flows through the pipe. There are build-in electrodes in the flowmeter, which can measure the inducted voltage. This voltage is proportional to the speed of the water.

Figure 6 The Promag 30 electromagnetic flowmeter.[mywellworkvip.en.alibaba.com[4]]

The pressure gauge Rosemount 3051S, which is mounted between the inlet and exit pipes, measures the pressure difference in the pipes.

Figure 7 The Rosemount pressure gauge [emersonprocess.com[5]]

3.3 Piping and tank

The pipes in the system are made of commercial steel. The pipes where the pumps suck the water from the tank has bends of 90 degrees before it is mounted on the pump. From the point where the water is sucked from the tank until it enters the tank again, the water flows through 5x90 degrees bends on the pipe. The pipe has different diameters, and the thickest part is the part where the water is sucked from the tank. The tank is 1.5 meters tall and the diameter is 1 meter.

3.4 Partial conclusion

In this chapter the pumping system has been evaluated, and the different components have been described with focus on the centrifugal pump.

The centrifugal pump is different from other pumps. There are two kinds of this pump type, the rotation pump and the vortex pump. Both can convert mechanical energy to energy of moving fluid. The mechanical work transfers to the fluid by the centrifugal effects.

4 The Affinity Law

When the pump is operating the performance of the pump can be predicted with respect to different parameters.

The affinity laws are mathematical relations that allow an estimation of changes in pump performance, as a result of a change in one of the basic pump parameters: the flow, total head, and power. There are two ways of doing this; to set up two sets of affinity laws based on the impeller diameter and the speed. These two sets are constructed from the following origin:

Flow, diameter, and speed:

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2}\frac{D_1^3}{D_2^3}$$

Total head, diameter, and speed:

$$\frac{H_1}{H_2} = \frac{N_1^2}{N_2^2}\frac{D_1^2}{D_2^2}$$

Power, diameter, and speed:

$$\frac{P_1}{P_2} = \frac{N_1^3}{N_2^3}\frac{D_1^5}{D_2^5}$$

Where:

Q is the flow in cubic meters per second, $\frac{m^3}{s}$
N is the speed of the pump in revolutions per minute, rpm
D Is the impeller diameter in meter, m
H is the total head of the pump in meter, m
P is the power in watts, W

The parameters "1" and "2" indicates the value before and after a change done to the system. When using these laws, there is either a fixed speed or a fixed impeller diameter and because of this, the equations can be simplified in order to describe whatever is desired. In this case, it is not possible to change the impeller diameter in the pump so the affinity laws are used and rewritten for the speed as a variable:

$$\left\{\frac{Q_1}{Q_2} = \frac{N_1}{N_2} \qquad \frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2 \qquad \frac{P_1}{P_2} = \left(\frac{N_1}{N_2}\right)^3\right\}$$

In order to determine the variables after a change, a simple change is done to the equations:

$$\left\{Q_2 = Q_1\left(\frac{N_2}{N_1}\right) \qquad H_2 = H_1\left(\frac{N_2}{N_1}\right)^2 \qquad P_2 = P_1\left(\frac{N_2}{N_1}\right)^3\right\}$$

Equation 1

4.1 Partial conclusion

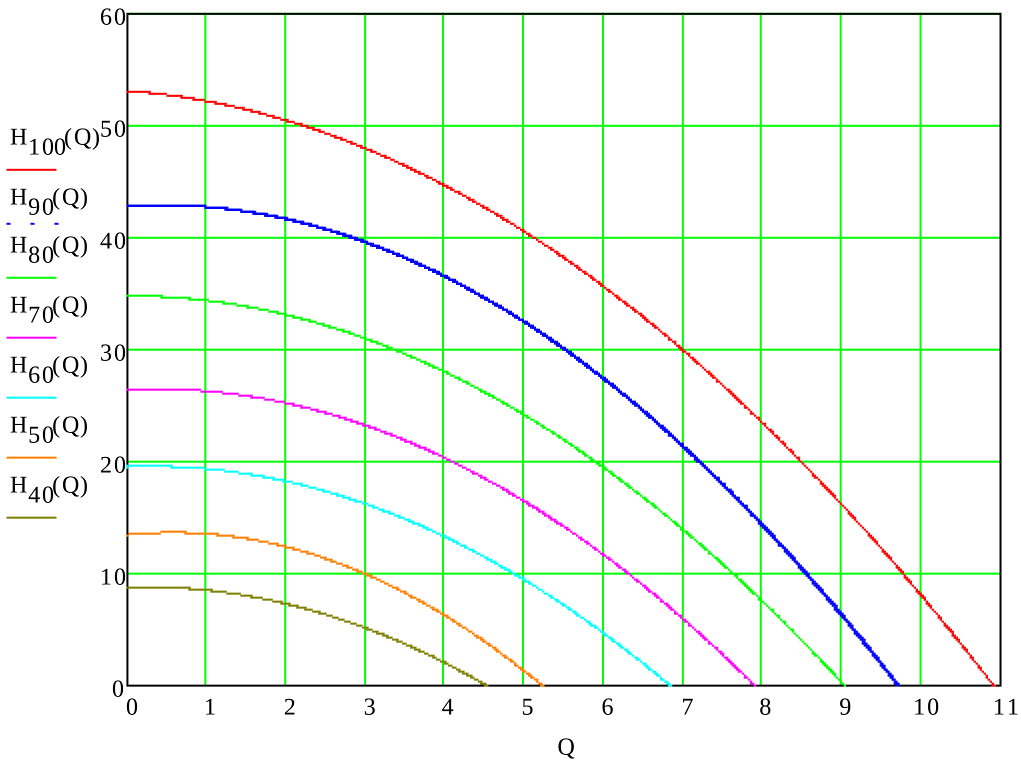
The purpose of the affinity laws is to predict another operation point on the pump curves[b] with a different speed in rpm. It determines the flow rate, total head, and power with one variable changing, the speed. Since it is an approximate theoretical assumption of the operation point with a different speed, it can only be used roughly – in this project it is used to find a system curve made by use of the affinity law and then comparing it to a system curve made by laboratory data – this is done in the following section. [pumpfundamentals.com[6]], [phdonline.org[7]].

[b] Pump curves describes the relation between head and flow rate at different pump speeds.

5 Pump and System Curves

To find the operation points the system curves are used.

When operating a pump system the most common way of describing the way the pump operates, is by the use of pump- and system curves. This project is using the CRE 5-8 pump and in order to find the pump curves, a program called WebCAPS by Grundfoss is used. The following pump curves are for the system's pump. The x-axis indicates the flow rate and the y-axis indicates the head[c]:



Graph 1: Own graph from Mathcad. This figure shows the pump curves when the pump operates at 100% down to 40% respectively. The x-axis indicates the flow rate and the y-axis indicates the head.

An example of one of the pump curves is:

$$H_{100}(Q) := -0.3987Q^2 - 0.5196Q + 53.1$$

To see the rest, look at appendix 1.

[c] The head is the height the pump can lift the fluid.

The pump curves are general for this specific pump. Afterwards the system curves can be plotted into the same graph – the intersection points are the operating points for this system. At these points the pump can operate. This system is not just the pump when referring to the system curve, but the system as a whole. This includes bends, valves etc. Each of these points has some kind of influence on the flow and therefore they all have a value, k_L, which is the minor losses of the system.

When these k_L values have been determined, see appendix 1 for further description, the system curve can be described by the equation:

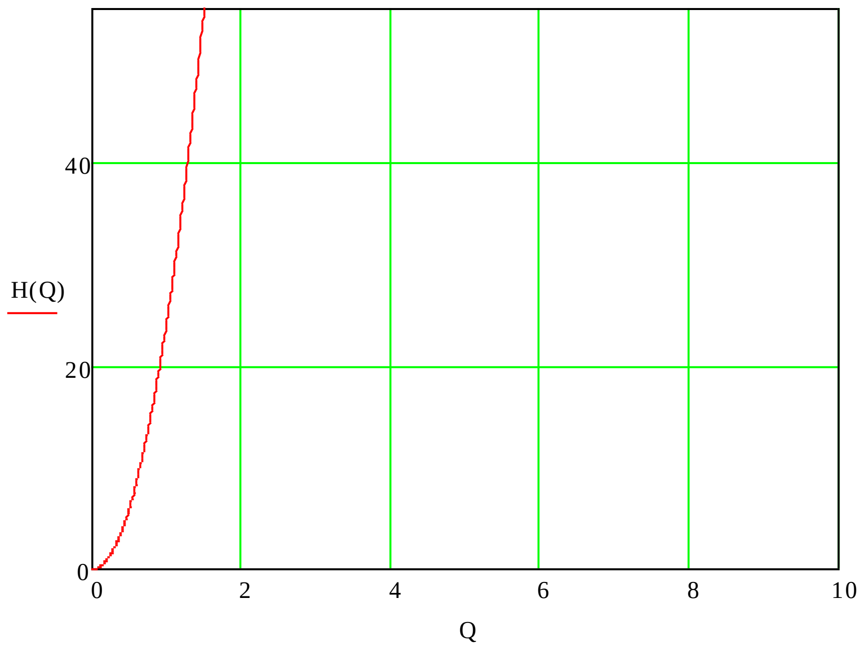
$$\Delta P = \left(\Sigma k_L + f\frac{L}{D}\right) \cdot \frac{1}{2}\rho v^2$$
$$\Updownarrow$$
$$\Delta P(v) = \left(\Sigma k_L + f\frac{L}{D}\right) \cdot \frac{1}{2}\rho v^2$$

Equation 2

This equation is derived and a rewritten to show the system curve in a H-Q diagram which turned out as following equation and figure, see appendix 1:

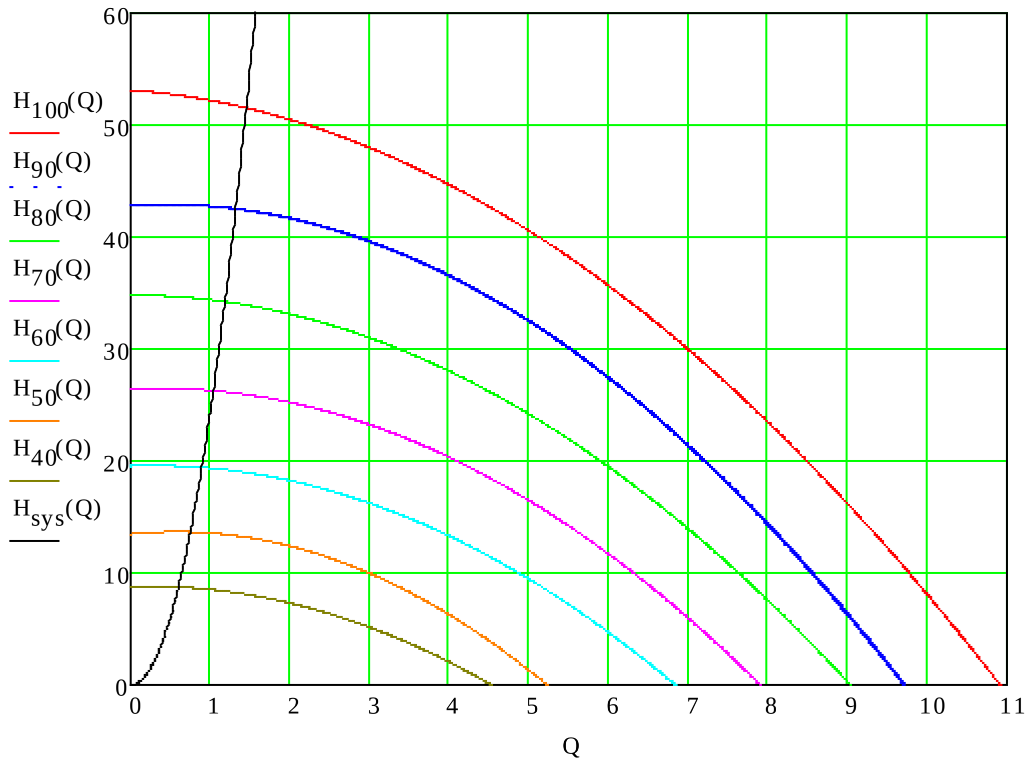
$$H(Q) = 24{,}528Q^2 - 0{,}0026Q + 0{,}002$$

Equation 3. The theoretic system curve.



Graph 2. Own figure. This figure shows the system curve for this specific system the project focuses on. The x-axis indicates the flow rate and the y-axis indicates the head.

When this system curve is created, it is ready to be combined with the pump curves which give following graph, see Graph 3 p. 13:



Graph 3. Own graph. This is a combined figure which shows the pump curves and system curve. The x-axis indicates the flow rate and the y-axis indicates the head.

The final graph has been created to describe how the pump will operate for this specific system. Each intersection point is the operating point for the pump and shows exactly how much head there will be for every flow rate. The operating points for this pump are the following:

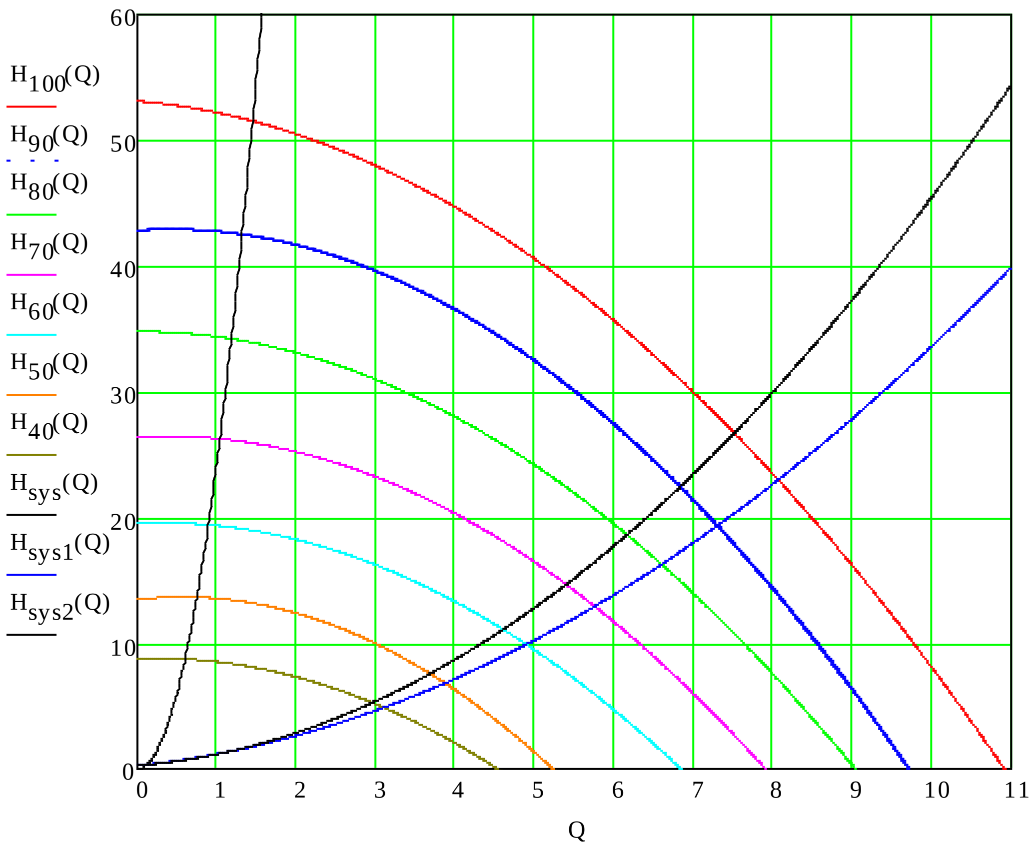
Power, %	H, m	Q, m^3/h
100	51,53	1,45
90	42,52	1,32
80	34,22	1,18
70	26,27	1,03
60	19,42	0,89
50	13,66	0,75
40	8,74	0,60

Table 1. Theoretical operating points for the pump system.

From Table 1 : the pump is running at full speed, **100** %, it will produce a head of 51.53 meters and a flow of 1.45 cubic meters per hour. This should theoretically be the operation points of this pump at those percentages however; this is not the actual operation points for this system.

During the laboratory work it turned out that the real system curves are way different and therefore the operation points are way different. The results from the tests came up with two different system curves, with the variable, making them unequal to each other, being the resistance to the flow, made by shutting a valve more and more. The two systemcurves for the real system looks like following:

$$H_{sys1}(Q) = 0{,}2676Q^2 + 0{,}6637Q + 0{,}2503$$
$$H_{sys2}(Q) = 0{,}4023Q^2 + 0{,}5014Q + 0{,}2657$$



Graph 4.This graph shows the figure with two additional system curves, determined by laboratory work. The x-axis indicates the flow rate and the y-axis indicates the head.

The two new system curves are the black and the blue ones – the blue one is without resistance while the black one is with the valve 33 % shut. These two system curves give operation points, different from the theoretical one. These new operation points have been listed in the following tables.

Intersection points without resistance

Power, %	H, m	Q, m^3/h
100	23,00	8,06
90	19,33	7,29
80	16,27	6,60
70	12,93	5,75
60	9,90	4,89
50	6,82	3,87
40	4,88	3,10

Table 2. Operating points for the pump system without resistance. Own data.

Intersection points with 33 % resistance:

Power, %	H, m	Q, m^3/h
100	26,73	7,51
90	22,43	6,83
80	18,63	6,16
70	14,68	5,39
60	11,01	4,60
50	7,57	3,68
40	5,26	2,96

Table 3 Operating points for the pump system with 33% resistance. Own data.

The reason why these are different from the theoretical values the system curve is rather unclear.

5.1 System curves by the affinity law

The affinity law can be used to make another theoretical form of system curves, looking at how the system should react on different rotor speeds measured in rounds per minute. In order to make the system curves with the affinity law, only two parts of the law has been used – the head and the flow section:

$$Q_2 = Q_1 \cdot \frac{N_2}{N_1}$$

Equation 4

$$H_2 = H_1 \cdot \left(\frac{N_2}{N_1}\right)^2$$

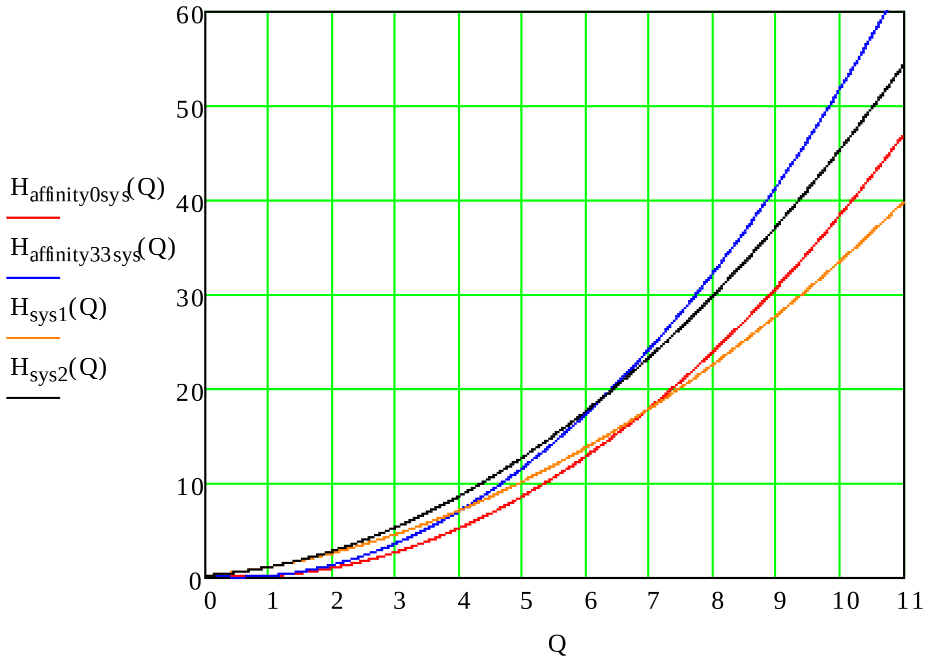
Equation 5

The data inserted in this can be found in the appendix 1 and ends up with following two system curves for respectively no resistance and 33 % resistance:

$$H_{affinity0sys}(Q) = 0{,}4277Q^2 - 0{,}4471Q + 0{,}2465$$

$$H_{affinity33sys}(Q) = 0{,}5752Q^2 - 0{,}5817Q + 0{,}2531$$

These two curves are slightly off the previously system curves determined from the tests:



Graph 5 shows the system curves determined from the affinity law as red and blue, along with the system curves made from test results. The x-axis indicates the flow rate and the y-axis indicates the head.

The reason why the affinity law curves are slightly higher compared to the test curves is how they have been constructed. The affinity laws are only having the previous flow or head along with the rotor speed as variables, which ignores every single bend, joint, and even the length of the pipe system. Despite this, the affinity law is decent for rough estimations of flow, if there is no actual chance to test it on the system other than having a start point work at. The differences of the variables are quite clear when lining the equations up:

$$Q_2 = Q_1 \cdot \frac{N_2}{N_1} \; and \; H_2 = H_1 \cdot \left(\frac{N_2}{N_1}\right)^2$$

Compared to

$$\Delta P(v) = \left(\Sigma k_L + f\frac{L}{D}\right) \cdot \frac{1}{2}\rho v^2$$

There is simply just more factors influencing the outcome.

5.2 Partial conclusion

During this section, the pump curves and system curves have been described. These are used to give a proper look into the pump and how they operate at different speeds with different resistances. The CRE 5-8 pump was described based on its pump curves, by comparing them with the system curves – these have been found and described theoretically and practically.

Five system curves have been created for this specific system. During the research, the two most accurate have been determined; these are the ones made from actual measurements with and without resistance. The theoretical system curves for this system are all deviant from the measured curves: The first based on thermodynamics was, as shown in the section, really far off but for reasons which are not certain. The last theoretical curves by the affinity laws turned out to only be useable if not being able to do actual measurements but approximations if one operation point is known.

After all, the system curve which are most fitting for this system as it describes the pump best, turned out to be the ones based on the laboratory experiment, $H(Q) = 0{,}2676Q^2 + 0{,}6637Q + 0{,}2503$.

6 Fluid Mechanics

Fluid mechanics are important in order to understand the flow in the system.

6.1 Reynolds number

When a fluid flows through a pipe, the flow can be defined as laminar or turbulent. Here, the Reynolds number helps giving a quantitative indication of whether it is laminar or turbulent: at low Reynolds numbers laminar flow occurs and at high Reynolds numbers turbulent flow occurs. The Reynolds number is a dimensionless number.

The Reynolds number is defined by:

$$Re = \frac{Inertial\ forces}{Viscous\ forces} = \frac{V_{avg}D}{v} = \frac{\rho V_{avg}D}{\mu}$$

Equation 6

Where:

- ρ is the density of the fluid, $\frac{kg}{m^3}$
- V_{avg} is the mean velocity of the fluid, $\frac{m}{s}$
- D is the diameter of the tube. For noncircular pipes the hydraulic diameter is , m

$$D_h = \frac{4A_c}{p} = \frac{4\left(\frac{\pi D^2}{4}\right)}{\pi D}$$

Equation 7

Where:

- A_c is the cross sectional area of the pipe, m^2
- p is the wetted perimeter, m
- μ is the dynamic viscosity, $\frac{kg}{s{\cdot}m}$
- v is the kinematic viscosity, $\frac{\mu}{\rho}$, $\frac{m^2}{s}$

At the point where the flow becomes turbulent, one has the critical Reynolds number, Re_{cr}. This point is for circular pipes said to be $Re \lesssim 2300$.

The values for the different types of flow are:

$Re \lesssim 2300$	Laminar flow
$2300 \lesssim Re \lesssim 10.000$	Transitional flow
$Re \gtrsim 10.000$	Turbulent flow

These values are useable under most practical conditions. This means that the flow depends of a row of factors such as surface roughness, pipe vibrations, and fluctuations in the flow. If the pipes and the system are handled in the optimal way, Reynolds numbers as high as **100.000** can be reached while still having laminar flow. In transitional flow, the flow switches between laminar and turbulent flow in a disorderly fashion. This phenomenon is shown in Figure 8.

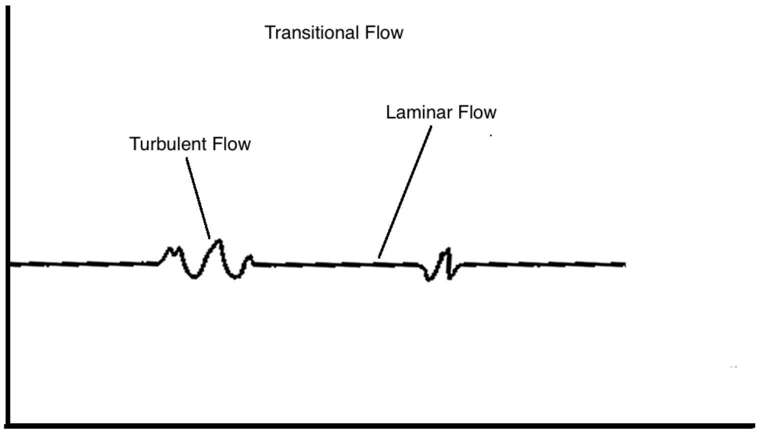


Figure 8. Shows how the flow switches between turbulent and laminar in the transitional region. [eng.cam.ac.uk[8]]

The Reynolds numbers for the pump system are at different pump speeds and flow rates:

Pump speed given in %	Flow rate	Reynolds number
100%	7,89	51496,04856
90%	7,48	48820,08152
80%	6,63	43272,34499
70%	5,75	37528,80598
60%	4,87	31785,26698
50%	3,92	25584,85556
40%	2,93	19123,37418
30%	1,94	12661,8928
20%	0,87	5678,271514

Table 4 shows the Reynolds number of the pump system at different flow rates. To see further calculations see Appendix 2.

An example of a calculation for the Reynolds number is:

- The velocity:

$$V = \frac{Q}{A} = \frac{flow\ rate}{\frac{\pi}{4} \cdot 0{,}054 m^2} = \frac{7.89\, {m^3}/{h} \cdot \frac{1h}{3600s}}{\frac{\pi}{4} \cdot 0.054 m^2} = 0.96\, {m}/{s}$$

- The Reynolds number:

$$Re = \frac{\rho V D}{\mu} = \frac{998\,\frac{kg}{m^3} \cdot V \cdot 0{,}054\,m}{1{,}002 \cdot 10^{-3} kg/m \cdot s} = \frac{998\,\frac{kg}{m^3} \cdot 0{,}96\,{}^{m}/_{s} \cdot 0{,}054\,m}{1{,}002 \cdot 10^{-3} kg/m \cdot s} = 51633{,}05$$

From Table 4 p. 19 it is possible to see that at a higher flow rate the Reynolds number is higher, which means it becomes more turbulent. The flow of the system is at the start at the flow rate, 0.87 $\frac{m^3}{h}$, a transitional flow. For every other flow rate for the system the flow is turbulent.

6.2 Entrance region

When a fluid enters a circular pipe at a uniform velocity, the fluid particles in the layer in contact with the inner surface of the pipe will come to a full stop due to the friction forces. This layer causes the adjacent layers to slow down gradually as well whilst this action causes a higher velocity for the fluid in the midsection of the pipe as shown in Figure 9 p. 20.

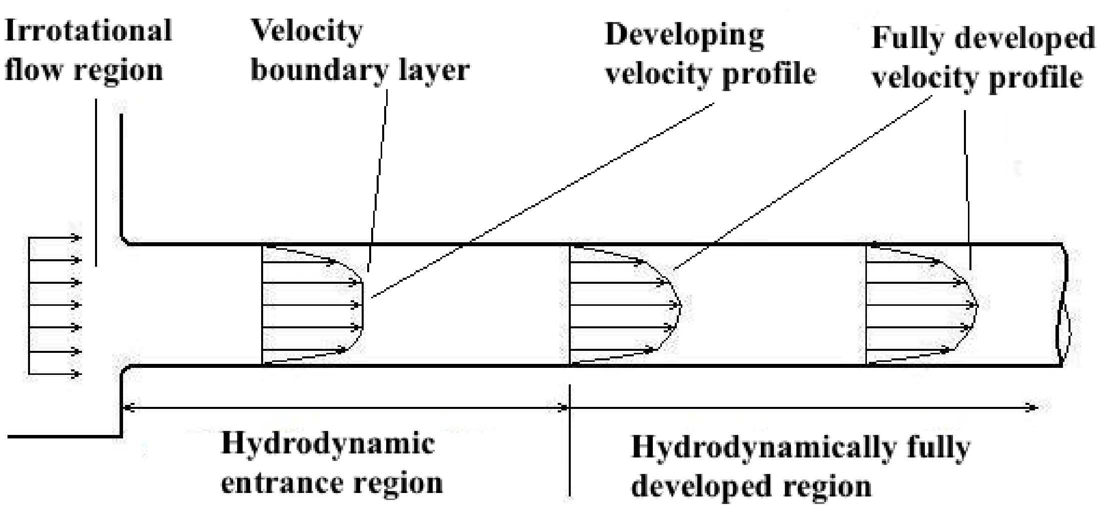


Figure 9 shows how a flow develops throughout the entry region of a pipe. The average velocity profile is parabolic in laminar flow, as shown, but somewhat flatter or fuller in turbulent flow. [brighthub.com[9]]

The hypothetical boundary surface divides the flow in a pipe into two regions:

- The velocity boundary layer
- The irrotational flow region

The velocity boundary layer is the region of the flow, where the viscous shearing forces caused by fluid viscosity are felt. Meaning the changes in velocity and viscous effect are significant. The frictional effects are negligible in the irrotational flow region. Essentially the velocity also remains constant in the radial direction.

The region from the entrance of the pipe to the point, where the boundary layer merges at the centre of the pipe is called the hydrodynamic entrance region. The length of this region is called the entry length, L_h, and the flow in the entrance region is called the hydrodynamically developing flow. After the entrance region, the flow and the velocity profile becomes fully developed and this region is called the hydrodynamically fully developed region. The hydrodynamically fully developed flow and fully developed flow is equivalent when there are no changes in the temperature of the fluid.

When the flow is fully developed, the time-averaged velocity remains unchanged, therefore the hydrodynamically fully developed flow is:

$$\frac{\delta u(r,x)}{\delta x} = 0 \rightarrow u = u(r)$$

Equation 8. If there is no change in the r-axis the flow is fully developed.

Where:

- r is the length in the y direction, m
- x is the length of the pipe in the x direction, m
- u is the velocity profile

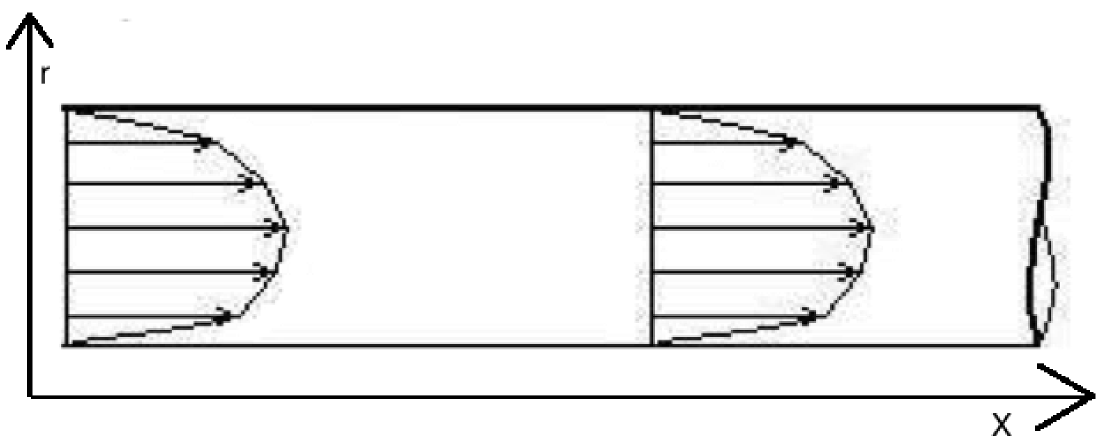


Figure 10 shows how the velocity profile can be described in a co-ordinate system. [brighthumb.com[8]]

Equation 8 p. 21 says that the velocity profile can be described in a system of coordinates as shown in Figure 10. The function says that when a flow is fully developed, the change in x is insignificant for the value of u. In other words; the velocity in a certain point on the velocity profile along the x-axis remains constant if:

$$\frac{\delta u}{\delta x} = 0$$

Meaning that the slope of the velocity curve is **0** as shown in Figure 11.

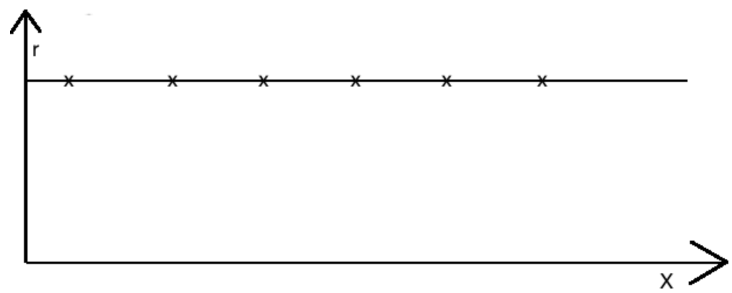


Figure 11 shows how the velocity in a certain point on a velocity profile behaves when the flow is fully developed. Own figure.

The slope of the velocity profile is in direct relation to the shear stress at the pipe wall τ_w. Because the velocity profile remains the same when in the hydrodynamically fully developed region the wall shear stress also remains constant as shown in Figure 12.

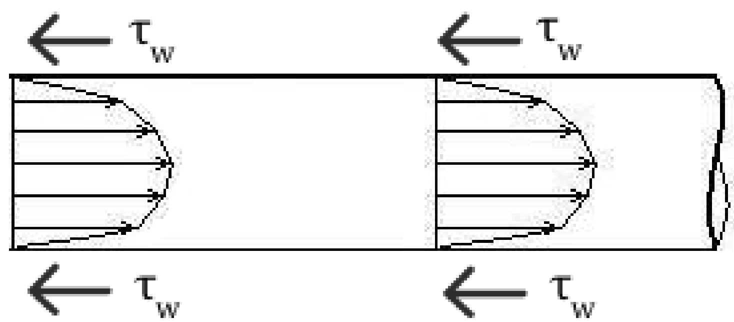


Figure 12 As well as the fully developed flow, the wall shear stress does not change downstream. [brighthumb.com[8]]

The wall shear stress is always to be highest at the inlet of a pipe. As shown in Figure 13 and Figure 14 the wall shear stress gradually drops as the flow becomes fully developed. This also means that the pressure drop is higher in the entrance region of a pipe. As a consequence this will always cause the friction factor to increase for the entire pipe. This increase may be significant for short pipes but is negligible for long ones.

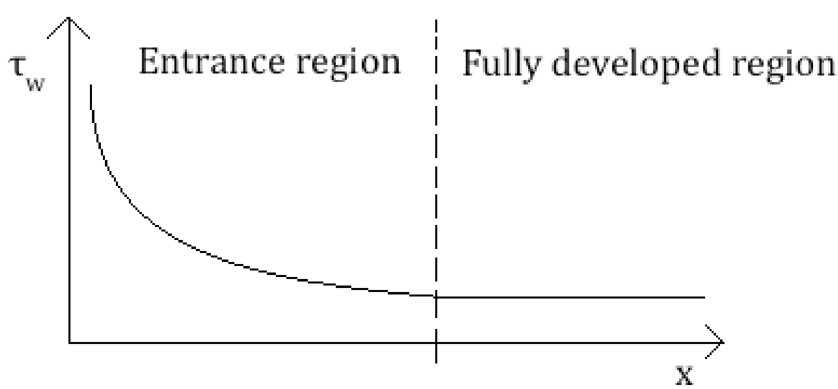


Figure 13 shows the behaviour of the wall shear stress in the entrance and fully developed region. Own figure. The x-axis indicates the length and the y-axis indicates the wall shear stress.

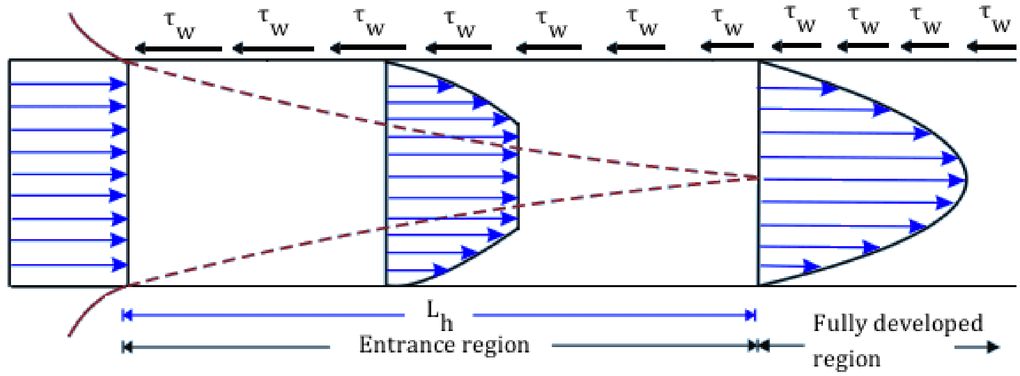


Figure 14 shows how the wall shear stress is highest at the inlet and decreases as the flow enters the fully developed region where it remains constant throughout the length of the pipe. [nptel.iitm.ac.in [10]]

6.3 Entry length

The distance from the pipe entrance to where the wall shear stress reaches within about two percent of the fully developed value, is called the hydrodynamic entry length. The hydrodynamic entry length is in laminar flow given approximately as:

$$\frac{L_{h,laminar}}{D} \cong 0{,}05Re$$

Equation 9

If $Re = 20$ the entry length is about the same as the size of the diameter, and when $Re = 2300$, the entry length is 115D.

Other values for the entry length apply when in the turbulent region. The entry length for turbulent flow is approximately:

$$\frac{L_{h,turbulent}}{D} = 1{,}359Re^{1/4}$$

Equation 10

As the equation shows, the dependence of the Reynolds number is much lower in the turbulent region. Also the entry length is shorter when in the turbulent region. In many practical cases the entrance effects become insignificant due to the overall length of the pipe. It is given that the entrance effects become insignificant when the length of the pipe exceeds 10 diameters. The entry length is therefore to be approximated as:

$$\frac{L_{h,turbulent}}{D} \approx 10$$

Equation 11

As said, in practice, the overall length of the pipes usually exceeds the length of 10 diameters, which is why assuming the flow is fully developed throughout the entire pipe can approach reasonable results. This is only applicable when dealing with longer pipes, because in shorter pipes the wall shear stress and friction factor are negligible.

The entry length for the system is based on a turbulent flow:

Pump speed given in %	Reynolds number	Entry length
100%	51496,04856	1,105494772
90%	48820,08152	1,090844488
80%	43272,34499	1,058438977
70%	37528,80598	1,021420108
60%	31785,26698	0,979872755
50%	25584,85556	0,928130317
40%	19123,37418	0,862987186
30%	12661,8928	0,778462819
20%	5678,271514	0,637040811

Table 5 shows the calculated entry length for different Reynolds numbers. For further calculations see Appendix 2.

An example for pump at 100% speed:

$$\frac{L_{h,turbulent}}{D} = 1{,}359Re^{\frac{1}{4}} \Leftrightarrow L_{h,laminar} = 1{,}359Re^{1/4}D$$

$\Updownarrow$

$$L_{h,turbulent} = 1{,}359 \cdot 51633{,}05^{1/4} \cdot 0{,}054m = 1{,}11m$$

From Table 5 p. 24 the entry length is shown. The entry length becomes longer for higher Reynolds number and pump speeds. In general the entry length is about 0,9 m.

6.4 Laminar flow

Laminar flow often appears when dealing with small pipes and low flow velocities. When a flow is laminar, the particles moves with a constant velocity parallel to the pipe. This also means that there is no change in the velocity profile, and the velocity and acceleration of any particle in the direction normal to the pipe is 0, meaning the fluid boundary layer just touching the inner side of the pipe has a velocity equal to 0.

The velocity profile of a fully developed laminar flow is parabolic; see Figure 14 p. 23:

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

Equation 12

Where:

- r is the distance from the centreline to a point on the velocity profile, m
- R is the distance from the centreline to the inner surface of the pipe, m

- μ is the dynamic viscosity $\frac{kg}{s \cdot m}$
- P is the pressure, Pa

The laminar flow has a centreline and a minimum (0) at the pipe wall, see Figure 14 p. 23. Furthermore the average geometric velocity can be determined by:

$$V_{avg} = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

Equation 13

By substituting $-\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)$ by $2V_{avg}$ the velocity profile can be written as:

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

Equation 14

This can be convenient because the average velocity can be determined through information about the flow rate. As shown in Figure 14 p. 23, the maximum velocity occurs at the centreline. This can be determined from the equation above by substituting$R = 0$, thus:

$$u_{max} = 2V_{avg}$$

Equation 15

This shows that the average velocity is one-half of the maximum velocity.

For the system the entry length is:

6.5 Pressure drop and head loss

The pressure drop ΔP is directly related to the requirements of the pump to maintain a certain flow.

It is given that for laminar flow the pressure drop is to be:

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

Equation 16

It is worth noticing, that the pressure drop is proportional to the viscosity of the fluid. Therefore the pressure drop would be 0 if there were no friction. Usually, the ΔP is used to indicate the difference between the final and the initial value. In this case though, ΔP is used to describe a pressure drop, and thus it is $P_1 - P_2$ instead of $P_2 - P_1$. A pressure drop is an irreversible loss, when it is due to viscous forces and is written as ΔP_L to show it is a loss.

The pressure loss can be described by:

$$\Delta P_L = f \frac{L}{D} \frac{\rho V_{avg}^2}{2}$$

Equation 17

Where $\frac{\rho V_{avg}^2}{2}$ is the dynamic pressure and f is the Darcy friction factor[d]. Also:

$$f = \frac{8\tau_w}{\rho V_{avg}^2}$$

Equation 18

Where

- τ_w is the wall shear stress.

The equation can be simplified for fully developed flow in a circular pipe, so that:

$$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{Re}$$

Equation 19

It is seen that the friction factor is a function of the Reynolds number. It is furthermore independent of the pipe surface roughness.

Pressure losses are often described in terms of the equivalent fluid column height when analysing piping systems. It is called the head loss and is written as h_L. The head loss comes from knowing that $\Delta P = \rho g h \Leftrightarrow h = \frac{\Delta P}{\rho g}$ combined with pressure drop, see Equation 17, gives the equation:

$$h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V_{avg}^2}{2g}$$

Equation 20

The equation is applicable for both laminar and turbulent flow and for circular and non-circular pipes. The head loss describes the additional height a pump needs to raise the fluid for it to overcome the frictional losses in the pipe, and it is caused by viscosity. It is also directly related to the shear wall stress of the pipe.

---

[d] The Darcy friction factor predicts the frictional energy loss in a pipe. It is given by the velocity of the fluid and the resistance due to friction.

6.6 Turbulent flow

In engineering practice the flows are most of the time turbulent. It is therefore important to understand how a turbulent flow can affect the shear wall stress of a pipe. Turbulent flow is characterized by rapid and disorderly fluctuations of swirling regions of fluid throughout the flow. These regions are called eddies, and they provide additional mechanisms for momentum and transfer of energy. This stands in contrast to the behaviour of laminar flow, where the fluid particles flow in an orderly manner along path lines as shown in Figure 15. Here is the momentum and energy transferred across streamlines by molecular diffusion. Unlike the molecular diffusion, the swirling eddies transfer mass, momentum, and energy to other regions of the flow much more rapidly. This significantly enhances the transfer of mass, momentum, and heat, which, as a result of the enhancement, is associated with much higher values of heat transfer, mass transfer coefficients, and friction.

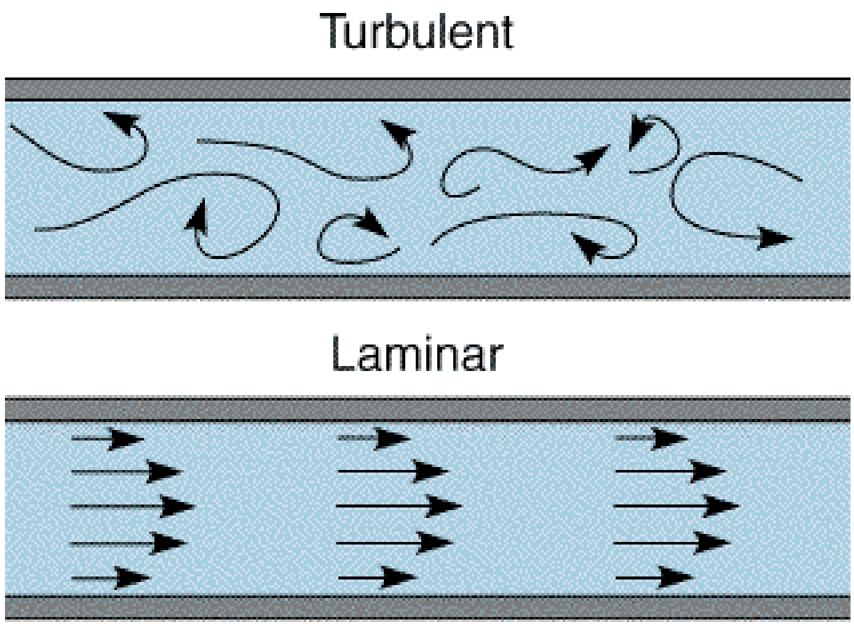


Figure 15 Flow velocity profile. Illustrates how laminar and turbulent flow behaves in a pipe. [ceb.cam.ac.uk[11]]

The velocity profile of a turbulent flow is different than the ditto for at laminar flow. As described in the former Figure 14 p. 23, the velocity profile for a laminar flow is parabolic. In turbulent flow the velocity profile is fuller than the parabolic velocity profile of the laminar flow; it also has a sharp drop near the pipe wall.

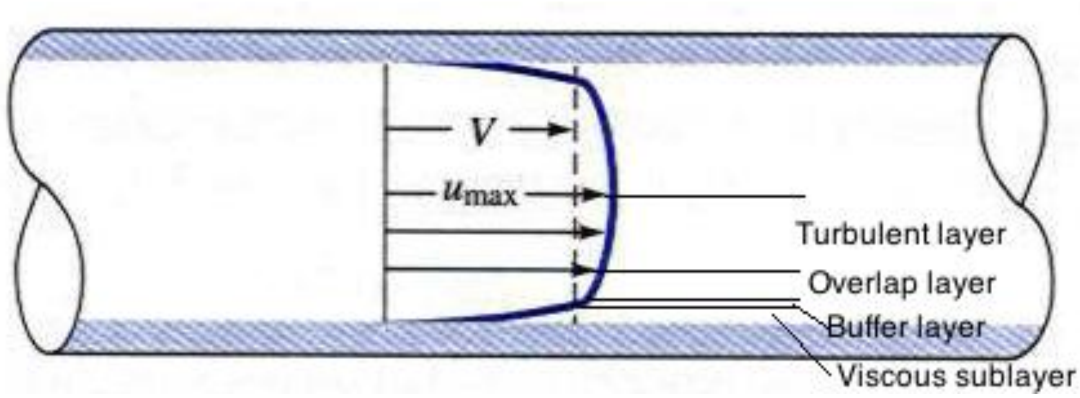


Figure 16 shows the velocity profile for a fully developed turbulent flow. The lines show the different layers of the turbulent flow. [me.queensu.ca[12]]

The velocity profile of a turbulent flow can be considered to consist of four layers characterized by the distance to the pipe wall as shown in Figure 16.

- The first layer is a very thin layer next to the wall where viscous forces are dominant. This layer is called the viscous sublayer, but can also be called the laminar, or linear, or wall sublayer. As some of the other names suggests, the flow in this region is streamlined, and the velocity profile is almost linear.
- The second layer is the buffer layer. The flow is still dominated by viscous effects, but the turbulent effects are more significant than in the viscous sublayer.
- The third layer is the overlap layer, also called the transition layer or inertial sublayer. The turbulent effects are even more significant in this layer but are still not dominating the molecular diffusion.
- The fourth layer is the turbulent or outer layer. This is the remaining part of the flow and is where the turbulent effects dominate the effects of molecular diffusion.

It is important to have the knowledge about the different regions of the turbulent flow. Because the characteristics are quite different in different regions, it is difficult to make an analytic relation for the velocity profile as it is done for the laminar flow.

6.7 Moody chart

In the turbulent flow the friction factors depends on the Reynolds number and the relative roughness ε/D. The roughness is the ratio of the mean height and roughness of the pipe and the pipe diameter. The friction factors can be determined by calculations of the measurements of flow rate and the pressure drop.

If the Reynolds number and relative roughness is given, the friction factor can be determined by the moody chart, see Figure 17 p. 29.

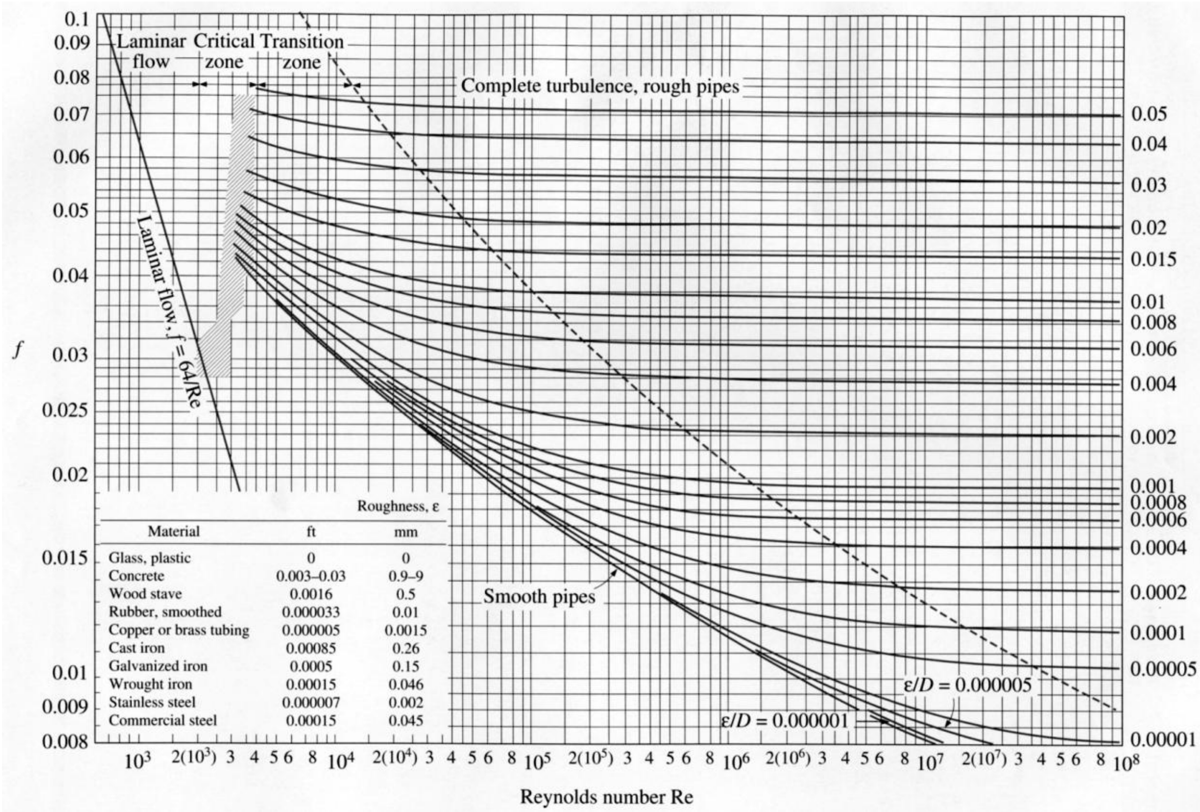


Figure 17 shows the moody chart. The x-axis indicates the Reynolds number and the left y-axis indicates the fiction factor, and the right y-axis indicates the relative roughness. To find the friction factor by the moody chart the Reynolds number and the relative roughness must be determined. [people.msoe.edu[13]]

Another way of determining the friction factor is the Colebrook equation:

$$\frac{1}{\sqrt{f}} = -2.0\ log(\frac{\frac{\varepsilon}{D}}{3.7} + \frac{2.51}{Re\sqrt{f}})$$

Equation 21

This equation is for the turbulent flow. When in laminar region, the equation for the friction factor is:

$$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{Re}$$

Equation 22

The friction factor for the pump system is:

Pump speed given in %	Reynolds number	Friction factor
100%	51496,04856	0,0235
90%	48820,08152	0,0236
80%	43272,34499	0,0241
70%	37528,80598	0,0246
60%	31785,26698	0,0253
50%	25584,85556	0,0263
40%	19123,37418	0,028
30%	12661,8928	0,0304
20%	5678,271514	0,037

Table 6. shows how the frictions factor changes depending on the pump speed. For further information see Appendix 2

An ensample for Re=5678.27:

$$\frac{1}{\sqrt{f}} = -2.0\, log\left(\frac{\frac{0.0045}{5.4}}{3.7} + \frac{2.51}{5678{,}27\sqrt{f}}\right) = -2.0\, log\left(\frac{0{,}000833}{3.7} + \frac{2.51}{51496{,}06\sqrt{f}}\right)$$

$$f = 0{,}0235$$

From Table 6 the friction factor is illustrated depending on the Reynolds number. The friction factor is not very large for this system, it is about 0.025. The friction factor becomes higher for lower Reynolds numbers, which means for this system has larger friction factor for lower pump speeds. It is also possible to determine these frictions factors from Moody Chart.

6.8 Minor losses and loss coefficients

The fluid, which flows through the pipes, will always be affected by the components of the pump system. The components are e.g. valves, instruments for measurement, pipe inlet, and exit. The bends on the pipes also has influence on the momentum of the fluid. The components cause a pressure loss because the fluid does not flow evenly throughout them. The flow mechanics of the fluid through the valves and the other components is not analyzed theoretically.

In bigger piping systems the losses are of less significance and they are called minor losses. The minor losses can be expressed by the loss coefficient K_L, which can be defined as follows:

Loss coefficient or resistance coefficient:

$$K_L = \frac{h_L}{V^2/(2g)}$$

Equation 23

h_L is the additional head loss which can be expressed as $h_L = \Delta P/\rho g$, where ΔP is the difference between P1 and P2 $(P_1 - P_2)_{valve}$. *V* is the velocity of the fluid and *g* the gravitational acceleration that is 9.8 m/s^2. This can be seen in Figure 18 a, where the valve mounted on the pipe disturbs the flow of the fluid. The majority of the head loss is located nearby the valve, while the other part of the loss happens by downstream vortices of the turbulent flow, which occurs while the fluid passes the valve.

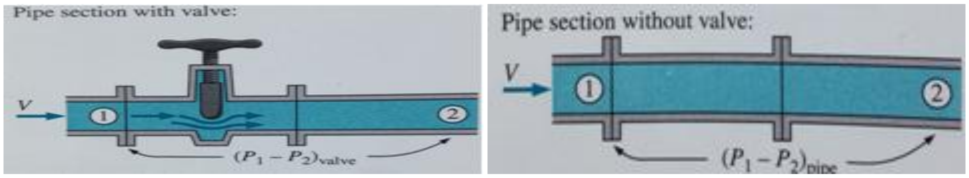


Figure 18 a) shows the pipe section with valve and b) shows the pipe section without valve.

[Thermal-Fluid Sciences p. 552[13]]

When the pipe diameters change, it is more difficult to determine the minor loss in the pipe. No matter what, there will always be a loss of mechanical energy, which would not happen if the components of the minor losses were not mounted in the pipe system. Most of the manufacturers of flow gauges recommend the flow gauges mounted a least 10-20 pipe diameter away from any bends or valves. This makes the whirling turbulent flow, which occurs by passing bends and valves, disappear and the velocity of the fluid can have time to get in a fully developed state, before it is passes the flow gauge.

When the pipes inlet diameter is equal to the outlet diameter, the loss coefficient can be determined by measuring the pressure loss through the components and divide by the dynamic pressure:

$$K_L = \frac{\Delta P}{(\frac{1}{2}\rho V^2)}.$$

Equation 24

While the loss coefficient for the component is available, the minor loss can be determined as following equation:

$$h_{L,minor} = K_L \frac{V^2}{2g}$$

Equation 25

- K_L is the loss coefficient
- v is the velocity, $\frac{m}{s}$
- g is the gravitational acceleration, $\frac{m}{s^2}$

The geometry components and the Reynolds number affect the loss coefficient, so does the friction factor. [Thermal-Fluid Sciences p. 552[14]]

Minor losses can also be linked to the equivalent length L_{equiv}:

$$h_L = K_L \frac{V^2}{2g} = f \frac{L_{equiv}}{D} \frac{V^2}{2g} \rightarrow L_{equiv} = \frac{D}{f} K_L$$

Equation 26

[Thermal-Fluid Sciences p. 553[13]]

- D is the diameter in the pipe,
- f is the friction factor and
- K_L is the loss coefficient.

The components affect the head loss. This head loss is the same as the loss, which is caused by the pipe length section. To find the total head loss it is important to know the major loss and the minor loss. These two losses together will represent the total head loss.

The total head loss can generally be determined as:

$$h_{L,total} = h_{L,major} + h_{L,minor} = \sum_i f_i \frac{L_i}{D_i} \frac{V_i^2}{2g} + \sum_j K_{L,j} \frac{V_j^2}{2g}$$

Equation 27 [Termal-Fluid Sciences p. 553[13]]

Because $\frac{V_i^2}{2g}$ occurs twice in the head loss equation it is placed outside the parenthesis.

$$h_{L,total} = (f \frac{L}{D} + \sum K_L) \frac{V_i^2}{2g}$$

Equation 28

This is valid for pipe systems with constant diameter, D.

As mentioned in section 0 p. 30, the bends on the pipes have an impact on the flow of the fluid. There are different kinds of bends and the losses depend on how sharp the edges are. This can be seen in Figure 19 where the three pumps have 90 degrees bends, also called Tee branch, see Figure 20. Behind the pumps there are the same bends on the pipes, and in the rest of the pipe system, there are three other bends.

Figure 19 illustrates the three bends before the pump. The fluid comes from the tank. After the pumps there is also three of the same bends. Own picture.

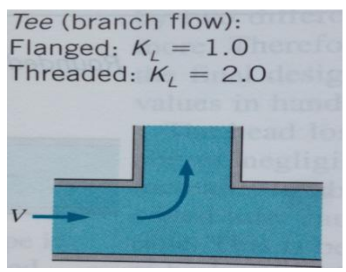


Figure 20 Illustrates the minor losses for a branch flow [Thermal-Fluid Sciences p. 556[13]]

The rest of the bends in the system are 90 degrees smooth bend threaded with a loss coefficient of 0.9. The loss coefficient can be seen in the Figure 21 p. 34.

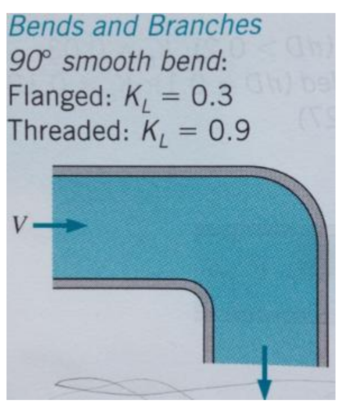


Figure 21 shows the K_L values 90 degrees smooth bend. [Thermal-Fluid Sciences p. 556[13]]

The pipe system is composed of threaded bends, as shown in Figure 22 p. 34. The two bends in figure are smooth bends threaded, and the rest are 90 degrees tee branch bends.

Figure 22 illustrates the bends down to the tank. Own picture.

The head loss becomes insignificant when the inlet pipe has a K_L value under 0.5. This can be seen at Figure 23. The sharp-edge bends causes a bigger pressure loss in the pipes because of the high K_L value.

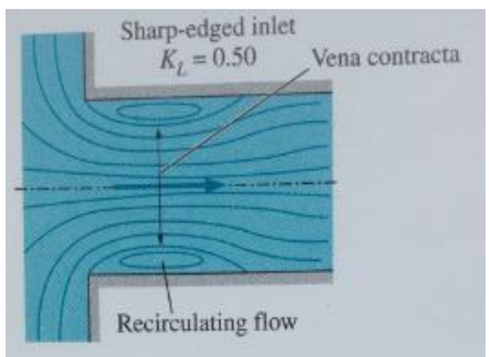


Figure 23 Picture of the sharp-edged inlet bends [Thermal-Fluid Sciences[13]]

The sharp edges results in a loss of half of the velocity. When the fluid passes the sharp edges it cannot run through the edges without getting disturbed.

In Figure 24 the well-rounded inlet does not disturb the flow and the K_L value is much lower compared with the sharp-edged inlet.

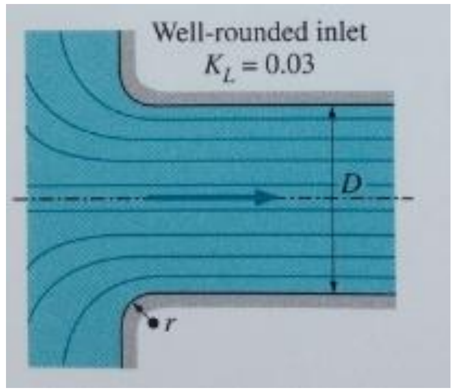


Figure 24 Picture of the well-rounded inlet bends.[Thermal Fluid Sciences p. 554 [13]]

It is assumed that for the pump system the tee branch with well-rounded inlet is used.

The head losses for the pump system is:

Reynolds number	Friction factor	Head losses
51496,04856	0,0235	1,066445023
48820,08152	0,0236	0,958937777
43272,34499	0,0241	0,755138662
37528,80598	0,0246	0,569305528
31785,26698	0,0253	0,409711153
25584,85556	0,0263	0,266685002
19123,37418	0,028	0,15015914
12661,8928	0,0304	0,06655204
5678,271514	0,037	0,013783974

Table 7. show how the Head loss depends on the friction factor. For further information see Appendix 2.

From Table 7 the Head losses are shown. The head losses becomes larger for larger Reynolds number, this means that the head losses becomes larger for smaller friction factors.

The pump system has a constant diameter, which means that following equation I valid for determining the head loss:

$$h_{L,total} = (f\frac{L}{D} + \sum K_L)\frac{V_{avg}^2}{2g}$$

Equation 29. The total head loss.

An example for friction factor f=0.0235:

$$h_{L,total} = \left(0{,}0235\frac{5{,}75m}{0{,}054m} + 20{,}299\right)\frac{(0{,}96\,{}^{m}/_{s})^2}{2 \cdot 9{,}82\,{}^{m}/_{s^2}} = 1{,}07m$$

When the head loss is known the pressure drop can also be calculated by use of the equation:

$$h_L = \frac{\Delta P_L}{\rho g} \Leftrightarrow \Delta P_L = \rho g h_L$$

The pressure drop at different head losses for the system is:

Reynolds number	Head losses	Pressure drop
51496,04856	1,066445023	10430,2589
48820,08152	0,958937777	9378,795035
43272,34499	0,755138662	7385,558168
37528,80598	0,569305528	5568,035787
31785,26698	0,409711153	4007,138961
25584,85556	0,266685002	2608,285989
19123,37418	0,15015914	1468,616455
12661,8928	0,06655204	650,905573
5678,271514	0,013783974	134,812784

Table 8 The pressure drops for different head losses. For further information see Appendix 2.

Equation 30. The pressure drop.

An example for the pressure drop at a head loss at 1.066 m.

$$\Delta P_L = 998 \, {}^{kg}/_{m^3} \cdot 9{,}80 \, {}^{m}/_{s^2} \cdot 1{,}07m = 10{,}4kPa$$

In Table 8 the pressure drops are shown at different head losses. The pressure drops become large for large Reynolds number – Higher pressure drop for turbulent flow. To avoid a large pressure drop the system has to operate at lower pump speed, and thereby lower Reynolds number and head loss.

6.9 Partial conclusion

The different measurement components are described as well as components such as the pressure gauge and flow meter. The loss coefficient is introduced as a part of the different components. This comes in handy as the fluid dynamics theory is described in detail to obtain an understanding of how a flow behaves in a pump system. The Reynolds number is introduced to give a dimensionless indication of whether the flow is laminar or turbulent. Furthermore the entrance region is described as a part of the minor losses in the system; however these losses are negligible in pipe systems longer than ten times the diameter of the pipe.

The pressure drop and head loss is described, and these are parts of the major losses. The head loss is the loss of pump force in meters caused by viscous forces. The new term friction factor is introduced in the calculation of the major losses, and it is accessible through an understanding of how the Moody chart works. The Darcy friction factor can also be determined by two different equations depending on whether the flow is laminar or turbulent. The theory gives a foundation that enhances the understanding of the behaviour of fluids through a piping system when it is affected by different viscous forces.

From the tables it is possible to see how the pump system operates and how the flow behaves at different pump speed. The Reynolds number indicates that the system is turbulent for all flow rates except when the pump is under 30% active.

The entry length, Pressure drop, and head loss becomes larger for higher velocities and Reynolds numbers.

The friction factor becomes smaller for higher velocities and Reynolds numbers.

7 Electrical parts of the pump system

For the pump system to work, a power source is needed. In this case a three-phase AC source from the grid is converted. To run the pump the AC source must go through several stages. These steps includes a frequency converter, pulse-width modulator, PWM, and an AC motor, which is described in the following chapter.

7.1 Frequency converter

The purpose of a frequency converter is to deliver an AC signal with the necessary frequency to the AC motor. This is done through three steps as shown in Figure 25. The first part of the diagram is a rectifier that changes the AC signal to a pulsating DC signal. The pulsating DC signal goes through a LC low-pass filter which converts the pulsating DC into a non-pulsating DC signal. The last part of frequency converter is an inverter that converts the DC signal to an AC signal at the desired strength and frequency. The purpose of the control unit is to supervise the three steps.

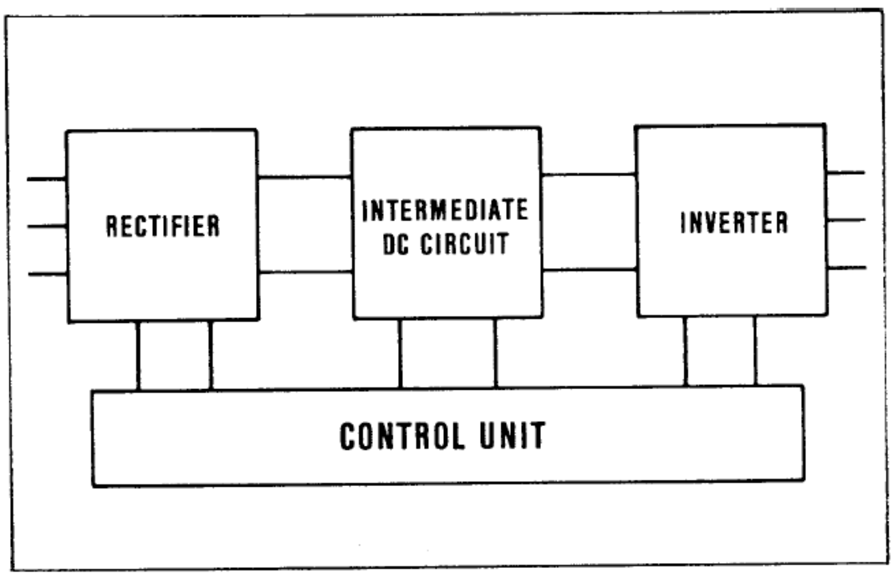


Figure 25 show a block diagram of a frequency converter. [Frequency Controlled AC Motor Drives[15]]

7.2 PWM

In short a PWM changes the width of pulses on the exit. The period is constant but the duty cycle is different. The duty cycle controls a switch that switches depending on the load of the output. So if the duty cycle is set to be 10 % the switch is "on" for 10 % of the period. Three different examples of PWM signals are shown in Figure 26 p. 39. PWM reduces the system costs and power consumption.

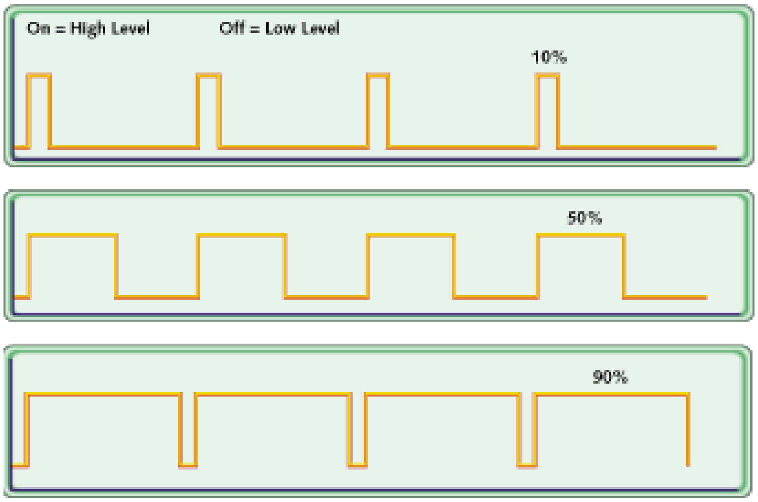


Figure 26 is an example of PWM signals at respectively 10%, 50% and 30% duty cycle [barrgroup.com[16]]

7.3 AC motor

An induction motor consists of two main parts – the rotor and the stator. The induction motor is a three phase AC motor, and is one of the most common machines of all times. [textiletechinfo.com[17]], [electrical-science.blogspot.com[18]], [sea.siemens.com[19]].

7.3.1 The stator

The stator, see Figure 27 s. 39, is constructed by a row of cores along an iron core. Since the induction motor is a three phase system, the amount of inductors must be dividable by three. The purpose of the stator is to form north- and south poles, in order to make the required magnetic fields to spin the rotor.

Figure 27 is an illustration of a stator with a total of 12 inductors and one iron core. These 12 inductors are divided by three which means each of the three poles have four inductors each. [thebackshed.com[20]]

7.3.2 The rotor

[sea.siemens.com[17]] The rotor, see Figure 28, in an AC motor is either a rotating permanent magnet or a rotating electromagnet with a north- and a south pole. Which one chosen depends on the purpose and the size of the project. Disadvantage of the permanent magnet is that it demagnetizes over time and they are very heavy. In smaller AC motors a permanent magnet can be used.

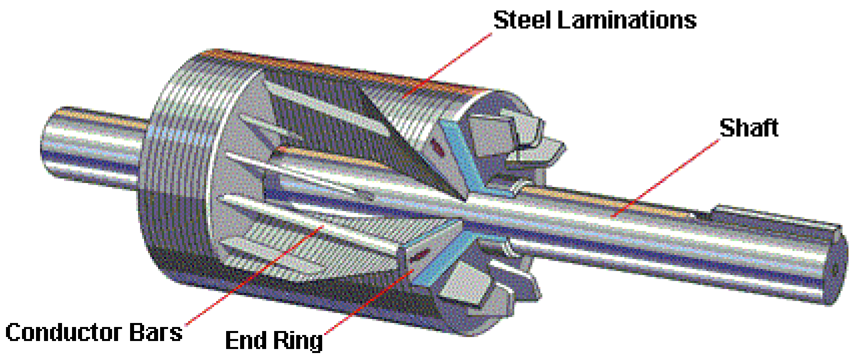


Figure 28 is an example of a rotor. [sea.siemens.com[21]]

7.3.3 How the rotor and stator works together

To make the rotor rotate, there must be a magnetic field. Since this is a three phase system, the phase windings are placed 120° apart from each other, if it is assumed to have one pole pair. In order to turn the magnet, current flows in the stator as AC which creates north and south poles in each of the three phases which switches when the current switches from positive to negative, and from negative to positive. This is rotating the rotor and drives the AC motor.

This means that each phase winding is constantly switching poles, this creates a rotating magnetic field by the positive and negative currents from the AC source.

7.4 Partial conclusion

In order to run a pump system there is need of a three-phase AC source from the grid. This AC source has to go through a few steps including a frequency converter, PWM, and the AC motor to run the pump. The frequency converter and the PWM operate on the AC source to run the motor, but are not further described. The AC motor is constructed of a stator and a rotor which are functional due to magnetism principles. In short the stator is creating north- and south poles and is constantly switching position of these which will spin the magnetic rotor.

8 Control system

To achieve the desired flow rate the motor must be supplied with the correct voltage. This is done by a controller.

For a given system there will always be an input, called the reference, and an output. The input and the output do not have to be of the same units. [facstaff.bucknell.edu[22]].For this report the pump system consists of the reference which is the adjustable voltage 0-10 V. The voltage runs a motor which creates an angular speed. The speed of the motor results in a pump speed. The pump speed creates a water flow and as an output of the system comes a certain water level. In other words; for the pump system a certain output, the water level, is desired and the reference, the voltage, must be regulated to achieve the desired output. The system is visualized in Figure 29.

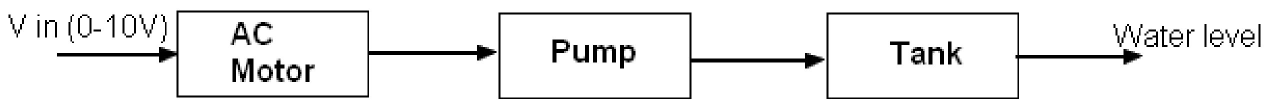


Figure 29 is an illustration of the elements of the systems, the input and the output. Own figure.

From the system description in section 3 p. 3 it is said that when the water exits the tank it returns to the system, in that case the water level will stay constant. This is not applicable from a mathematical point of view because it is non-linear. To make calculations of the system it must be considered linear – that means there is one water flow inlet and one water flow outlet. The water just exits the system and therefore the water level theoretically changes. In Figure 30 the linear version of the water tank is illustrated.

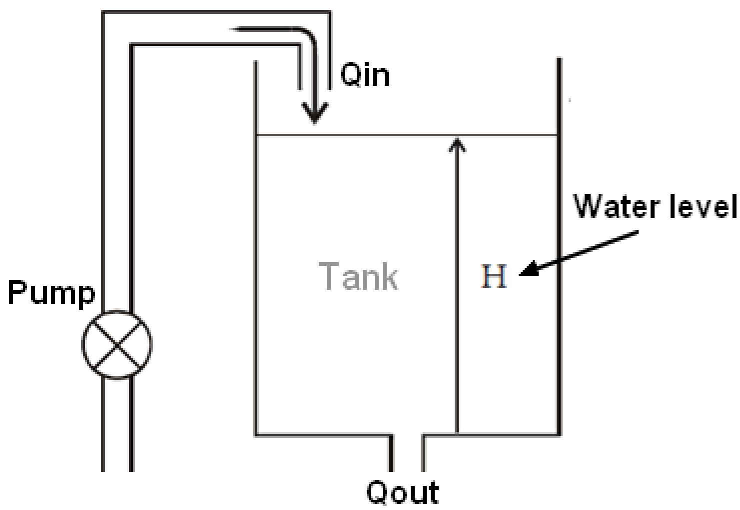


Figure 30 is an illustration of the linear version of the water tank. Own figure.

8.1 Feedback control

The process variable, PV, of the system is the water level and in feedback control this value is compared with a setpoint, SP; the setpoint is the desired water level. The difference between the PV and the SP is called the error. By manipulation of the input of the system the feedback control minimizes the error.

There are two kinds of feedback; positive and negative feedback. The positive feedback is added to the input and the negative feedback is subtracted from the input.

8.2 Open loop & closed loop systems

For an open loop control there is an input signal. The input passes elements of the entire system e.g. a motor and a pump and then an output is created. An open loop does not have any feedback and for the output to become zero the input must be zero. In Figure 31 the open loop system is shown.

Figure 31. Own figure of an open loop system

For a closed loop the system is self adjusting. The data does not go from input to output; it is sent back from the output to the start of the control system. When the data is sent back, the system will adjust itself in order to meet the requirements of the system output. Underneath the figure shows a closed loop system.

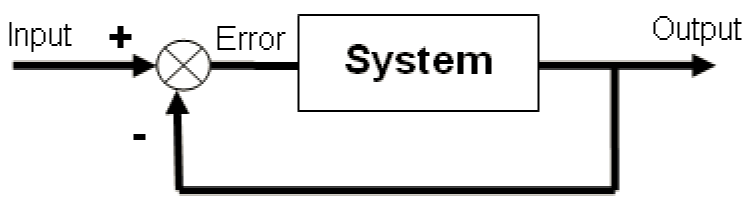


Figure 32 Own figure of a closed loop system

For the closed loop system the feedback notifies the deviation of the output from the input. The deviation is called the error and the system will adjust according to this value.

For the specific system the calculations are made based on an open loop system. In reality the system is a closed loop system but, because it is more complex, it is too hard to make calculations based on. [servocity.com[23]]

8.3 PID controller

8.3.1 Introduction to the PID

A PID controller, also known as, a Proportional-Integral-Derivative Controller, is used in feedback control systems to make sure that the system is close to the system value. The PID consists of three elements: the proportional, the integral, and the derivative, which means that there are three mathematical functions applied to eliminate the tracking error of the system. [eolss.net[24]]

The PID controller, see Figure 33 page 43, makes sure that the output variable, y, remains as close to the setpoint, r, as possible. To maintain an approximate constant curve at the setpoint the controller has to manipulate the input, u, of the system for every time there is a feedback. The feedback system is very important due to the potential disturbance, d, which influences the system performance An example of disturbances could be a heating tank with wind cooling it down – then the controller has to regulate the temperature of the tank.

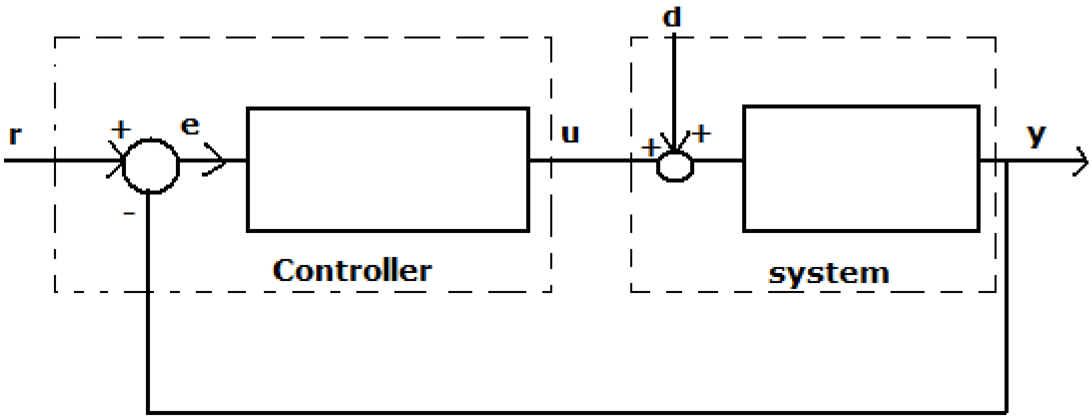


Figure 33 Own figure made from the [eolss.net p. 2.[22]] Illustrates the closed loop system.

The feedback of the system makes sure of this regulation. This happens when there is a difference in the setpoint - and output value of the system, also known as the error, e, and is defined as:

$$e = r - y$$

Equation 31

8.3.2 Introduction to the terms P, I, D

The PID controller parameters are now to be described. The behavior of the elements will be shown by graphs and explained continuously.

8.3.2.1 Introduction to P

The P part is the proportional. A P controller is the simplest controller; it is an amplifier and it does not depend on the frequency. The frequency is the amount of oscillations per second [Feedback control section 4.3.1[25]] The function of the proportional element:

$$P = K_p \cdot e(t)$$

Equation 32

Where

- K_p is the proportional constant
- $e(t)$ is the error of the system

The proportional function amplifies the error signal. In the function description of P the amplification is k_p.

The PID controller has a transfer function defined by a Laplace function and a characteristic equation. For the P controller the transfer function is:

$$\frac{N(s)}{D(s)} = \mathrm{G}(s) = k_p$$

Equation 33

Where

- $\mathrm{N}(s)$ is the numerator
- $D(s)$ is the denominator
- $\mathrm{G}(s)$ indicates the transfer function
- k_p is the proportional constant.

The transfer function describes the relationship between the input and output, where the nominator is the output and the denominator is the input. This function describes the system and its behavior. In general the transfer function can also be described as:

$$\frac{N(s)}{D(s)} = \frac{A}{s^2 + a_1 s + a_2}$$

Equation 34

Where

- s is a variable
- a_1 and a_2 are constants
- A is the value of the nominator and indicates the gain value of the controller.

From the transfer function the characteristic equation can be solved. The characteristic equation is equal to the denominator equals zero, $D(s) = 0$:

$$s^2 + a_1 s + a_2 + K_p A = 0$$

Equation 35 [Feedback control p. 205[23]]

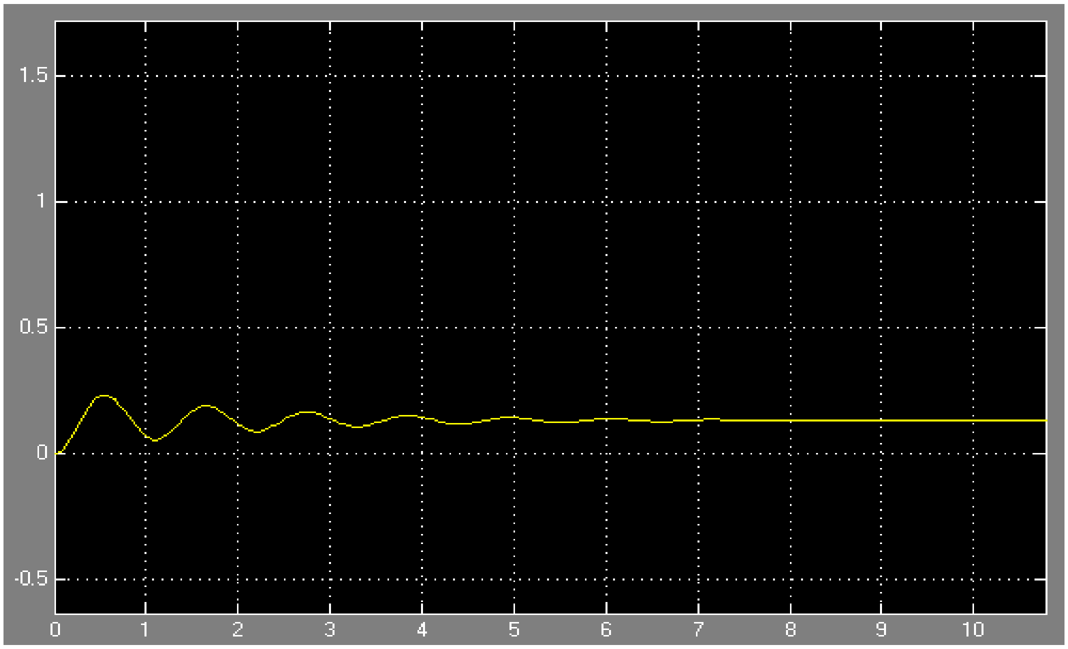
From this equation the roots and frequency can be determined. The advantage of a P controller is that the natural frequency[e] of the closed loop system can be determined and the steady state error[f] can become smaller. This is done by increasing the value, k_p. If k_p is too high, which would make the steady state error very small, there will be some problems. For a P controller there is no guarantee that the steady state system response will reach the setpoint and to make sure that it will reach it, another element is introduced; the integral part.

In Graph 6 p. 45 the P controller makes the steady state error small as time passes, but it never reaches the set-point which is 1.

---

[e] When a system is the system is exposed to an influence it will cause vibrations and the system will naturally vibrate; this action is called the natural frequency.

[f] The steady state error is the difference between the input and the output when t goes to infinity.



Graph 6. Own graph from SIMULINK. An illustration of the P element that never reaches its setpoint (1). The x-axis indicates the time and the y-axis indicates the output.

8.3.2.2 Introduction of I

The I–part of the PID controller is now introduced, and by adding this element to the controller, it becomes a PI controller. The PI controller will automatically have a reset effect, and can adjust the system in order to the setpoint. This is possible because the integral is the error over a period of time, t. [Feedback Control, Section 4.3.2[23]]

The integral has the function:

$$I = K_i \cdot \int e(t)dt$$

Equation 36

Where

- K_I is the integral constant

If the stability is not obtained by one integrator more can be added. The phase is added by -90 degrees for all omega.

The transfer function for a PI controller is:

$$\frac{N(s)}{D(s)} = \mathrm{G(s)} = k_p + \frac{k_\mathrm{i}}{s}$$

Equation 37

If it is assumed that the system is a closed-loop system of second order, the characteristic equation is:

$$s^3 + a_1 s^2 + a_2 s + Ak_i s + Ak_p = 0$$

Equation 38

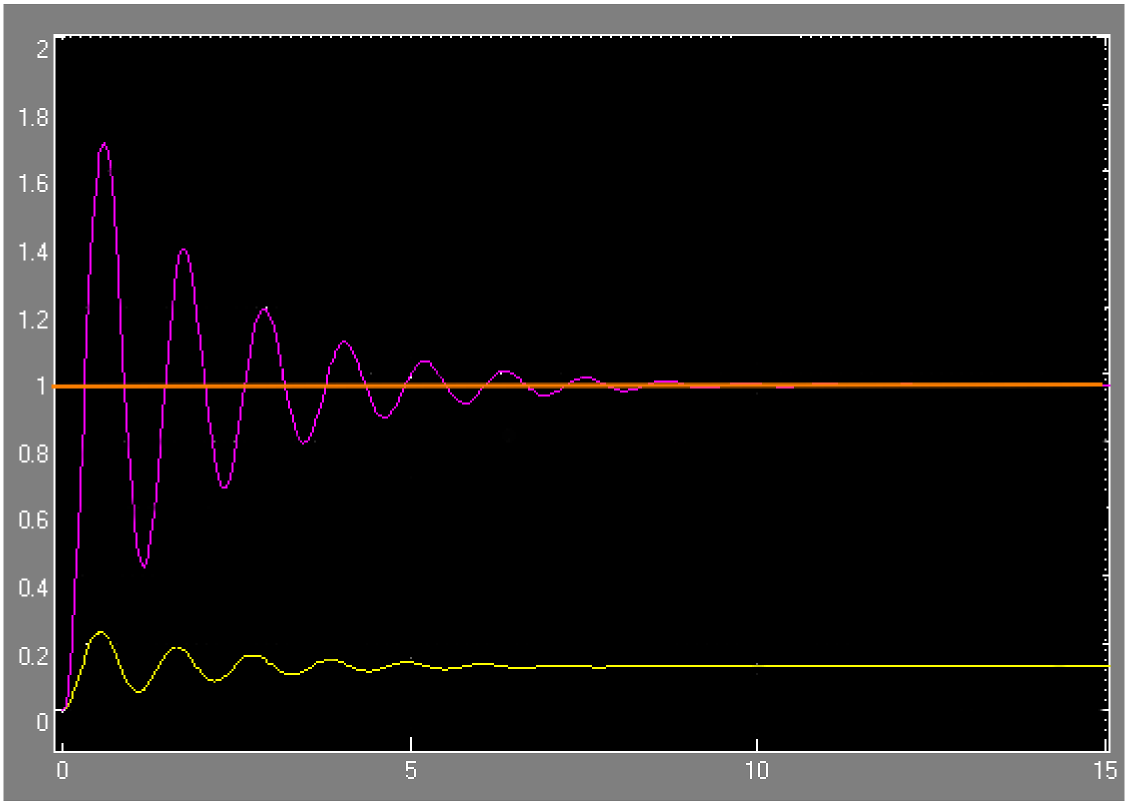
This equation is based on:

$$\frac{N(s)}{D(s)} = \frac{A}{s^2 + a_1 s + a_2}$$

Equation 39

Now there are two free constant parameters: K_i and K_p. It is now possible to determine the damping of the system output, which is not possible with only a P controller. As shown in Graph 7 it may take a lot of time for the function to settle down – to make the controller even more efficient in terms of fast response, one more element is needed.

In Graph 7 both the P and PI controller are illustrated. The PI controller almost eliminates the steady state error.



Graph 7 Own graph from SIMULINK. An illustration of the P controller (yellow) and the PI controller (purple). The x-axis indicates the time and the y-axis indicates the output. The setpoint is 1.

To make sure of less oscillation a last element is introduced to the controller, the derivative part.

8.3.2.3 Introduction of D

This element *D*, is the derivative. The derivative is the change in error per time also known as the rate of change in error. If suddenly a change in signal response appears, the derivative element will create a sharp response. The *D* part also makes sure that the signal reaches the set point very fast with a very small overshoot[g] [Feedback Control, section 4.3.3[23]].

The function for *D* is:

$$D = K_D \cdot \frac{d}{dt} e(t)$$

Equation 40

Where

- K_D is the constant of the derivivative

The transfer function of a PID is:

$$\frac{N(s)}{D(s)} = \mathrm{G}(s) = k_p + \frac{k_\mathrm{i}}{s} + s \cdot k_D$$

Equation 41

When the characteristic equation is to be determined, there is an issue to consider; the *D* element can be placed in two different ways of the system; one as feedback as shown in Figure 34 p. 48 a) or as a part of the controller shown in Figure 34 p. 48 b).

[g] When the output signal oscillates around the setpoint, the overshoot indicates the percentage difference between the peek- and the setpoint value of the output.

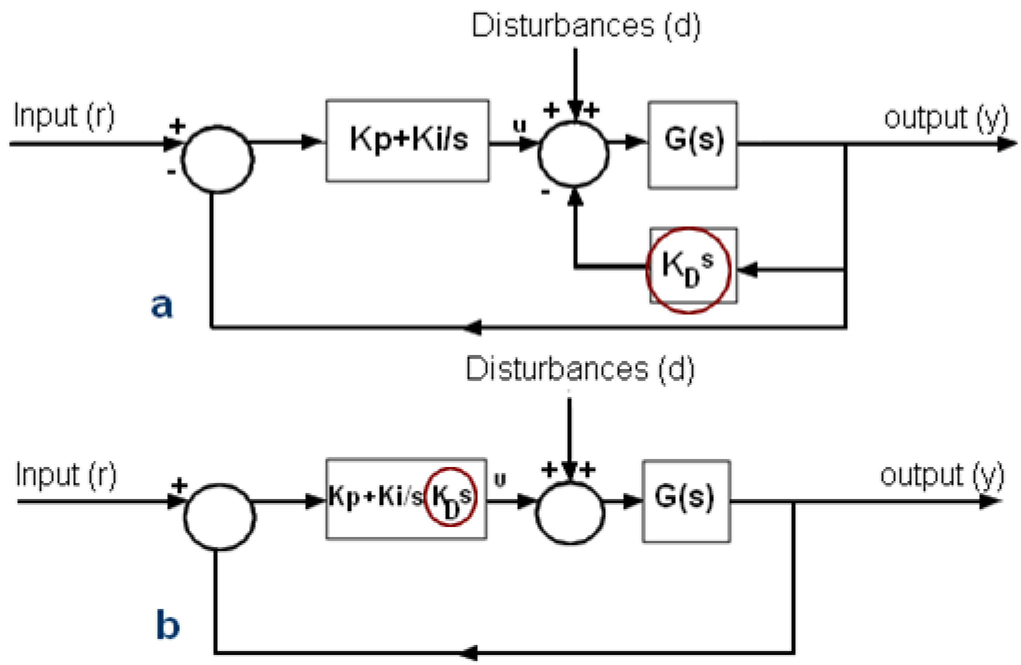


Figure 34 shows different placements of the D element. a) the D element as feedback and b) the D element is a part of the controller. Own figure.

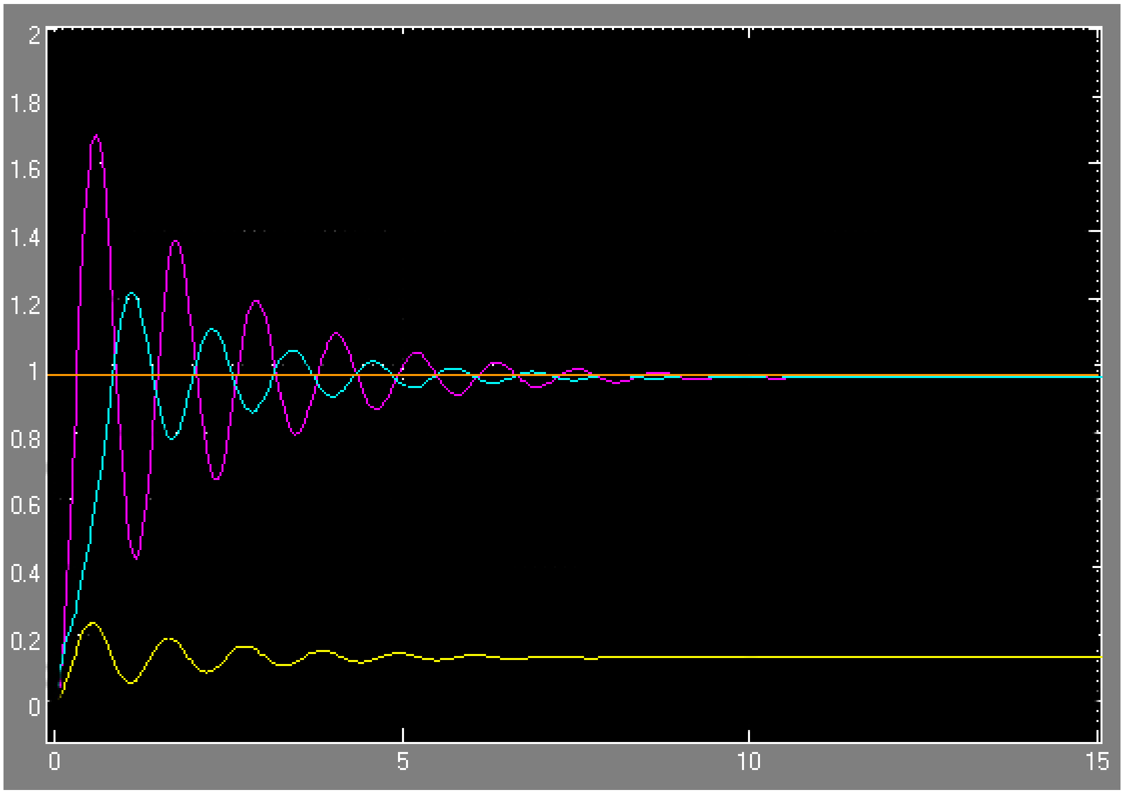
The difference between the two setups is that they have different transfer functions because of the placement of the blocks. Different transfer function will cause two different characteristic equations. For the pump system case b) Figure 34 is chosen. If case a) was chosen the effect of a quick response would not be achieved and it would act like a sensor.

The characteristic equation for b) is:

$$s^3 + a_1 s^2 + A a_2 s K_D + A k_i s + A k_p = 0$$

Equation 42

The difference in P, PI and PID controllers is illustrated in Graph 8 p. 49.



Graph 8 Own graph from SIMULINK. An illustration of the P element (yellow), the I element (purple) and the D element (cyan). The x-axis indicates the time and the y-axis indicates the output. The setpoint is 1.

The graph illustrates how the output of the system is depending on the controller type. The PID and PI settle to the setpoint, which the P doesn't. The PID controller makes sure that there are fewer oscillations in the output, and the system is therefore more stable.

8.4 The meaning of the transfer functions

In section 8.3 p. 42 the transfer functions were introduced as functions of the P, I, and D. In this section the use of a transfer function is described.

The transfer function is defined as the relationship between the output and input signal as functions of *s*, which indicates that it is in the complex domain – the Laplace domain.

$$G(s) = \frac{N(s)}{D(s)}$$

Equation 33

8.4.1 Bode plot:

When the transfer function is known, all the parameters for the system can be determined. The transfer function can be used to make a diagram – a bode plot. A Bode plot is a two-subplot diagram with the amplitude, A, and phase, Δ, respect to the frequency, ω. To make the Bode plot the transfer function must be transformed into another domain –the $j\omega$ domain. This is done by the Fourier transform:

$$G(s)|_{s=j\omega} = G(j\omega)$$

Equation 43

Where

- $G(s)$ is the transfer function in the *s*-domain
- $G(j\omega)$ is the transfer function in the frequency domain

When the function is determined the amplitude and phase as function of the frequency can be found.

The complex function can be rewritten:

$$a + jb = Ae^{j\Delta}$$

Equation 44

Where

- Δ is the phase
- A is the amplitude

The amplitude A is defined as

$$A = |G(j\omega)| = \sqrt{a^2 + b^2}$$

Equation 45

$$|G(j\omega)| = \frac{|N(j\omega)|}{|D(j\omega)|}$$

Equation 46

The phase is defined as

$$\Delta = \angle G(j\omega) = \tan^{-1}\left(\frac{b}{a}\right)$$

Equation 47

$$\angle G(j\omega) = \angle N(j\omega) - \angle D(j\omega)$$

Equation 48

Normally Bode plots are sketched into a logarithmic scale, and therefore the definitions can be rewritten as:

$$|G(j\omega)|dB = 20\log\left|\frac{N(j\omega)}{D(j\omega)}\right|$$

Equation 49

$$|G(j\omega)|dB = 20\log|N(j\omega)| - 20\log|D(j\omega)|$$

Equation 50

This can be done to the PID components. From the bode plots it is possible to see how the different component affect the output signal.

The transfer functions are:

- $P: G(s) = k_p$
- $I: G(s) = \frac{1}{s}k_i$
- $D: G(s) = s \cdot k_D$

When the bode plots are to be found the values are excluded – this part will only cause a change in the amplitude curve, either it will move it upward or downward depending on the P value chosen.

8.4.1.1 The integrator

The transfer function is transformed into jω domain:

$$G(s)|_{s=j\omega} = \frac{1}{j\omega}$$

The amplitude is determined: *multiplying by j/j*

$$|G(j\omega)| = \left|\frac{1}{j\omega}\right| = \sqrt{\left(\frac{1}{\omega}\right)^2} = \frac{1}{\omega}$$

The phase is determined: *multiplying by j/j.*

$$\angle G(j\omega) = \angle j\frac{1}{-\omega} = \tan^{-1}\left(\frac{1}{-\omega}\right) = -90° \; or \; 270°$$

The degree can be determined without any frequency, because there is only a complex value which means that there will be a phase of 90 degree, see Figure 35.

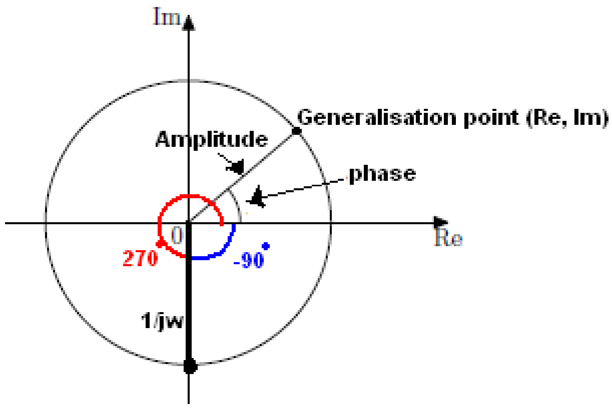
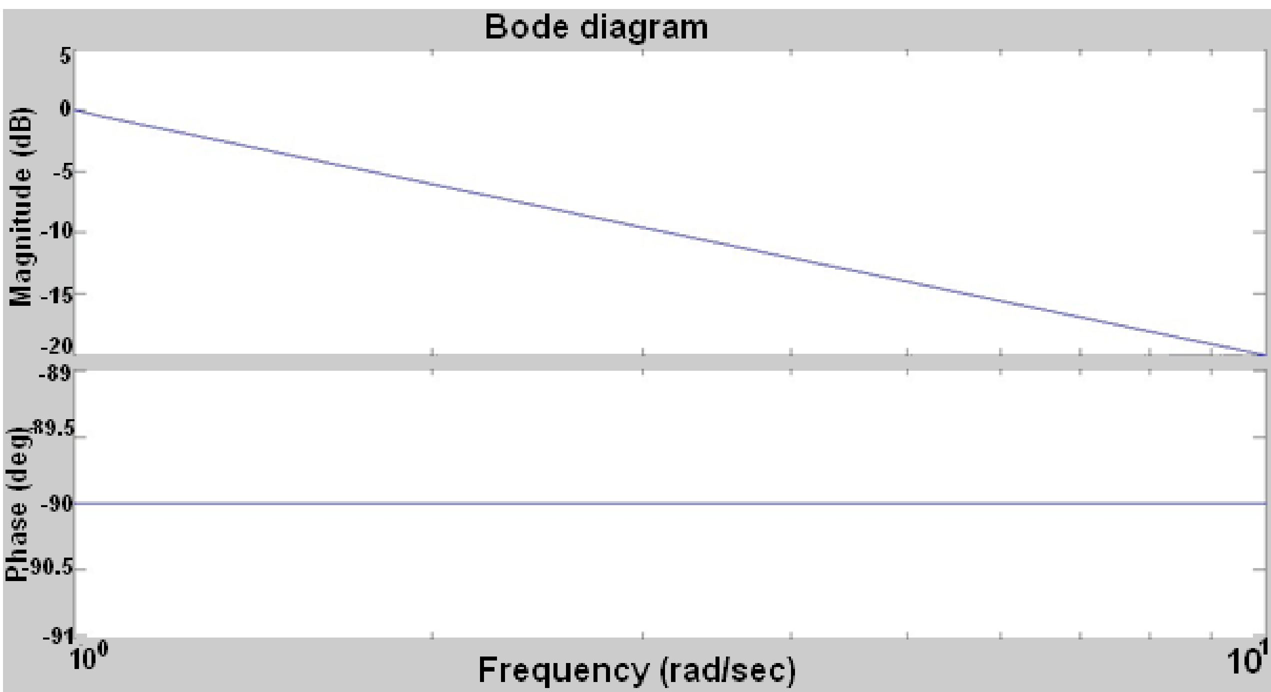


Figure 35 shows the function in the complex plane. The amplitude and phase are illustrated. Own figure.

Bode plots can be drawn by hand or made with MATLAB. The following diagram is from MATLAB.



Graph 9. Own illustration. Shows the bode plot for an integrator.

- From the Bode plot it is possible to see how the system amplitude decreases as the frequency increases, but the phase stays constant. The bode plot is a frequency illustration
- The amplitude is getting smaller as a linear function – the greater frequency the smaller amplitude

- The phase is constant at the value – 90 degree

8.4.1.2 The derivative

The transfer function of the derivative is transformed into jω domain:

$$G(s)|_{s=j\omega} = j\omega$$

The amplitude is determined: *multiplying by j/j*

$$|G(j\omega)| = |j\omega| = \sqrt{(\omega)^2} = \omega$$

The phase is determined: *multiplying by j/j.*

$$\angle G(j\omega) = \angle j\omega = \tan^{-1}(\omega) = 90°$$

The phase is plotted in Figure 36:

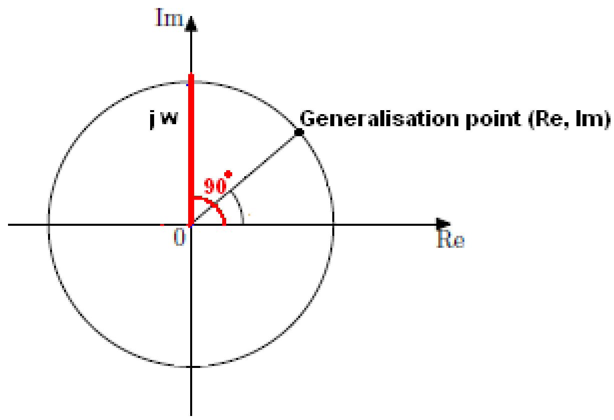
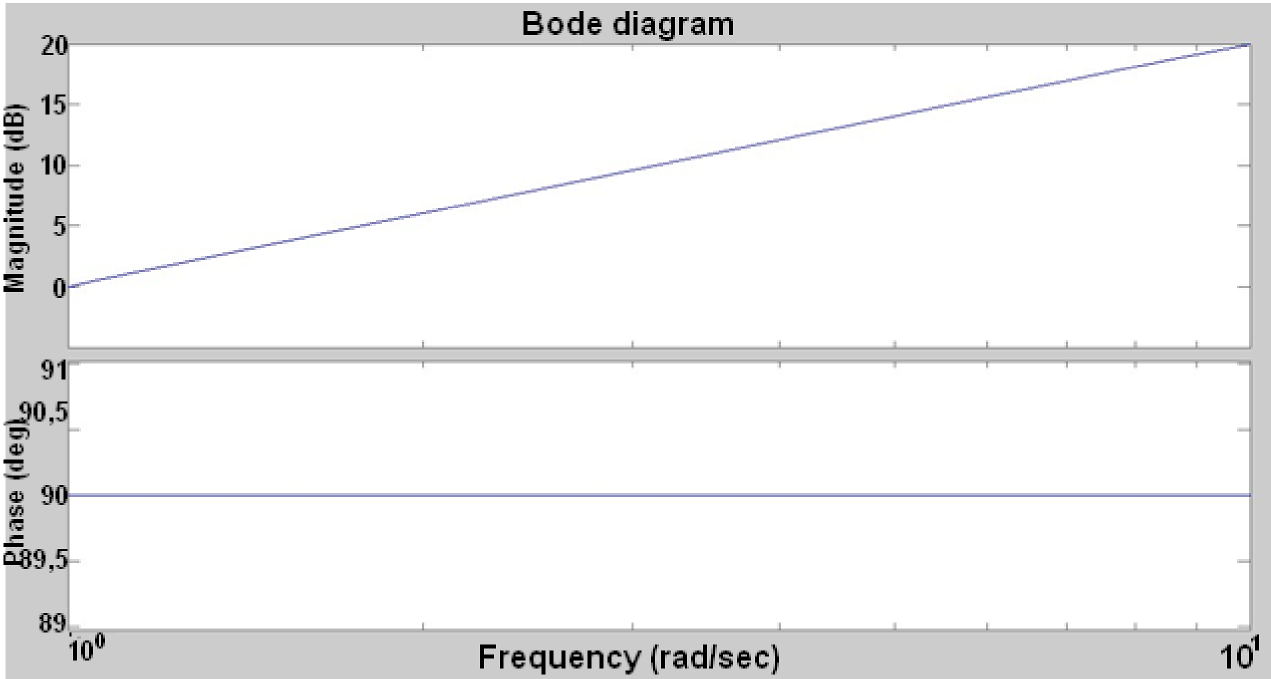


Figure 36. Own figure. Shows the phase of 90 degrees.

The bode plot in MATLAB:



Graph 10. Own figure from Matlab. Shows the Bode plot for the derivative.

- From the Bode plot it is possible to see that the system amplitude increases as the frequency increases, but the phase stays constant
- The amplitude is an increasing linear function – the greater frequency the greater amplitude
- The phase is constant at the value 90 degree

9 The transfers function of the system

In this section the mathematical tool Laplace transform is used.

The system can be described by differential equations. But instead of having a differential equation it is more convenient to have a transfer function for the system. The transfer function is easier to manipulate than the differential equation.

The system is shown below. The water enters from below and moves on through the pump where the pump changes the speed of the water, depending on the motor speed and thereby changes the volume of the tank, see Figure 37:

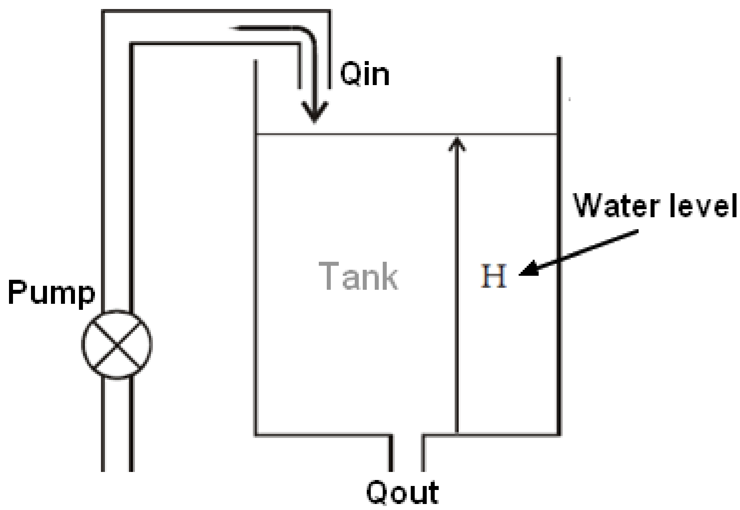


Figure 37 shows the pump system – linear version. Own figure.

When the water leaves the tank, it would normally run back to the pump, where the water flow is regulated, but to look further into the system and how it behaves, we assume the water exits at the bottom and that it does not run back to the pump. It is assumed open. This is done because the transfer function has to be determined, and to do this the system has to be assumed linear, which it is not. The system is in reality non-linear because of the pump and valves. This is necessary because transfer functions only handle linear systems.

The system is assumed linear, meaning that the valves, disturbances and friction are neglected for all the calculations.

Because of the linear assumption of the system there can be some challenges in the execution of the laboratory experiments since it really is a non-linear system. The constants of the PID might not control the system as the theory predicts. But with the constants as starting points it is possible to tune the system by Ziegler Nichols Method. For more information on the laboratory experiment and the Ziegler Nichols see appendix 3.

9.1 System function

When the function for the system is to be determined there must be a variable or variables of the system. For the specific system the volume inside the tank is the variable; it changes constantly depending on the voltage the motor is supplied. Therefore the function is based on how the volume changes.

The volume change is defined as the cross area, A, multiplied by the change in height of the water in the tank $dh(t)/dt$:

$$V(t) = A \cdot \frac{dh(t)}{dt}$$

Equation 51

Because the system is assumed linear, there are no disturbances. The volume can also be written as the change in the volume flow in the tank:

$$V(t) = \dot{q_{in}} - \dot{q}_{out}$$

⇕

$$A \cdot \frac{dh(t)}{dt} = \dot{q_{in}} - \dot{q}_{out}$$

Equation 52

The system looks like Figure 38:

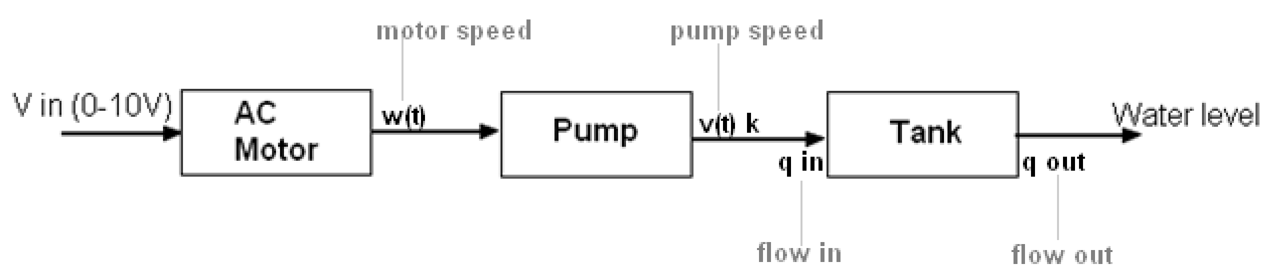


Figure 38. Own figure. A block diagram of the system.

In Control system 8 p. 41 the system was described step by step. The equation of the system is now introduced. Because the inlet flow rate must be equal to the speed of the pump the equation will be:

$$\dot{q}_{in} = k \cdot V_{in}$$

Equation 53

The outlet flow rate can be determined from a Bernoulli equation of free jets. A jet of liquid of a diameter d flows from the nozzle with a velocity v [Munson p. 121[26]].

$$\dot{q}_{out} = \sqrt{2 \cdot g \cdot h(t)}$$

The equation must be linearized in order to be Laplace transformed:

This approximation can be done by solving the Taylor function with respect to a linear function.

The Taylor function is defined by:

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)(x - x_0)^2}{2!} \ldots$$

Equation 54

$f(x_0)$ indicates the function at the equivalent point. x_0. When a function is lineralized the Taylor function becomes:

$$f(x) = f(x_0) + f'(x_0)(x - x_0)$$

Equation 55

The function which is linearized is defined by:

$$F(h) = \sqrt{2gh}$$

Determination of the function to the Taylor function, x is substituted by h.

$$f(h_0) = \sqrt{2gh_0}$$

To calculate the differentiated function, F(h) has to be process like a combined function, which is differentiated by using following statement:

$$\left(F\big(g(h)\big)\right)' = F'(g(h)) \cdot g'(h)$$

Equation 56

Where:

- $F(h) = h^{\frac{1}{2}}$
- $g(h) = 2gh$
- $F'(h) = \frac{1}{2}h^{-\frac{1}{2}}$
- $g'(h) = 2g$

This information is inserted:

$$f'(h_0) = \frac{1}{2}(2gh_0)^{-\frac{1}{2}} \cdot 2g$$

⇕

$$f'(h_0) = g \cdot (2gh_0)^{-\frac{1}{2}} = \sqrt{\frac{g}{2h_0}}$$

The Taylor function becomes:

$$f(h) = \sqrt{2gh_0} + \sqrt{\frac{g}{2h_0}}\ (h - h_0)$$

⇕

$$f(h) = \ \sqrt{2gh_0} + \sqrt{\frac{g}{2h_0}}\ h - \sqrt{\frac{g}{2h_0}} h_0$$

This function is inserted in the volume function:

$$A\frac{dh}{dt} = q_{in} - \left((\sqrt{2gh_0} + \sqrt{\frac{g}{2h_0}}\ h)\ - \sqrt{\frac{g}{2h_0}} h_0 \right)$$

⇕

$$A\frac{dh}{dt} = q_{in} - \sqrt{2gh_0} - \sqrt{\frac{g}{2h_0}}\ h + \sqrt{\frac{g}{2h_0}} h_0$$

⇕

$$A\frac{dh}{dt} = q_{in} - \left(\sqrt{2gh_0} + \sqrt{\frac{g}{2h_0}} h_0 \right) - \sqrt{\frac{g}{2h_0}}\ h$$

Equation 57

Because h_0 indicates the equivalent height of the input, and the output only will depend on the variable h, the input function can be rewritten as:

$$N(t) = q_{in} - \left(\sqrt{2gh_0} + \sqrt{\frac{g}{2h_0}} h_0 \right)$$

The function becomes:

$$A\frac{dh}{dt} = N(t) - \sqrt{\frac{g}{2h_0}}\ h$$

Equation 58

Where

- $A = \pi \cdot r^2 = \pi \cdot 0{,}5^2 = 0{,}785\ m$
- $g_{DK} = 9{,}82\frac{m}{s^2}$
- $k = 2$
- $h_o = 0{,}69m$

To simplify the function, all the parameters are inserted:

$$0{,}785\frac{dh}{dt} = N(t) - \sqrt{\frac{9{,}82}{2 \cdot 0{,}69}}\ h$$

The function for the system:

$$0{,}785\frac{dh}{dt} = N(t) - 0{,}2668 \cdot h$$

9.2 The transfer function for the system

The function for the system can be used to find the transfer function, *G(s)*. First the equation has to be Laplace transformed [Matematikbogen[27]]:

Rules:

Functions: $l\{h(t)\} = H(s)$

Derivative: $l\left\{\frac{dh(t)}{dt}\right\} = sH(s) - H(0)$

Constants: $l\{K\} = K$

The system function:

$$0{,}785\frac{dh}{dt} = N(t) - 0{,}2668 \cdot h$$

The Laplace transformed becomes:

$$0{,}785\ sH(s) - H(0) = N(s) - 0{,}2668H(s)$$

Because the Laplace function only focuses on the system and not the signal, it can be assumed that; $H(0) = 0$.

$$(0{,}785s + 0{,}2668)H(s) = N(s)$$

When the Laplace function has been found, the transfer function can be solved knowing that the transfer function is defined by the relationship between the output and input:

$$G(s) = \frac{H(s)}{N(s)} = \frac{1}{(0{,}785s + 0{,}2668)}$$

Equation 59

10 Analysis of the system

10.1 Poles and zeros

When the transfer function for the system has been determined it is possible to make visualizations of the behavior of the system, but first the poles and zeros of the function must be determined.

When the transfer function $G(s) = \frac{N(s)}{D(s)}$ and has been found, it is possible to find the s value such that $N(s) = 0$ and $D(s) = 0$ [Feedback Control p. 126[23]].

The s values satisfying, $N(s) = 0$, is called zeros, and indicates when the system response becomes zero.

The s values satisfying, $U(s) = 0$, is called poles, and indicates when the system response goes to infinity.

The poles and zeros are used in the evaluation of whether the system is stable or unstable.. To determine these values the numerator and the denominator must be solved with respect to zero.

To solve the poles and zeroes for the system, the G(s) has to be reformed:

$$G(s) = \frac{1}{(0{,}785s + 0{,}2668)}$$

$\Updownarrow$

$$\frac{H(s)}{U(s)} = \frac{\frac{1}{0{,}2668}}{\left(\frac{0{,}785}{0{,}2668}s + 1\right)}$$

$\Updownarrow$

$$\frac{H(s)}{U(s)} = \frac{3{,}7487}{(2{,}94s + 1)}$$

By solving the numerator equal to zero, it is possible to find the zeros:

$$3{,}7487 = 0 \rightarrow 3{,}7487 \neq 0$$

The equation is not valid because there are no variables in the numerator.

The poles are solved by setting the denominator equal to zero:

$$2{,}94s + 1 = 0$$

$$s = -0{,}34$$

The system is of first order and thereby it is only possible to have one pole, in this case the pole is -28.5, meaning that the system response goes to infinity when s is equal to -0,34

When the pole is found the stability can be estimated. The system is stable if the poles are located in the negative half plane of the real axis, see Figure 39. If a system has more than one pole and one of them is located at the right half plan the system is unstable.

Figure 39 illustrates where the system is stable or not dependently on the pole location.

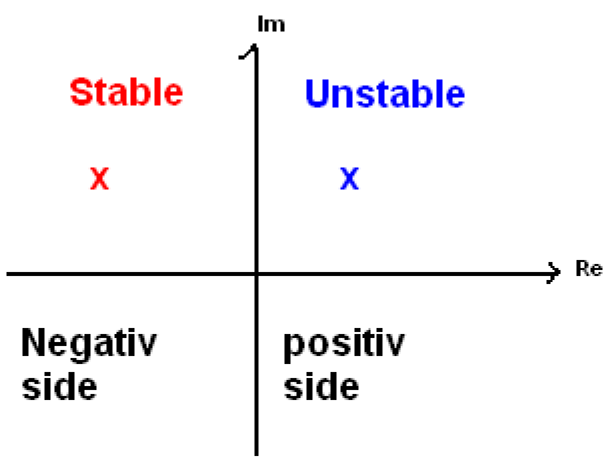


Figure 39. Own figure of the stability depending on the location of the poles.

The location of the pole of the system is illustrated below.

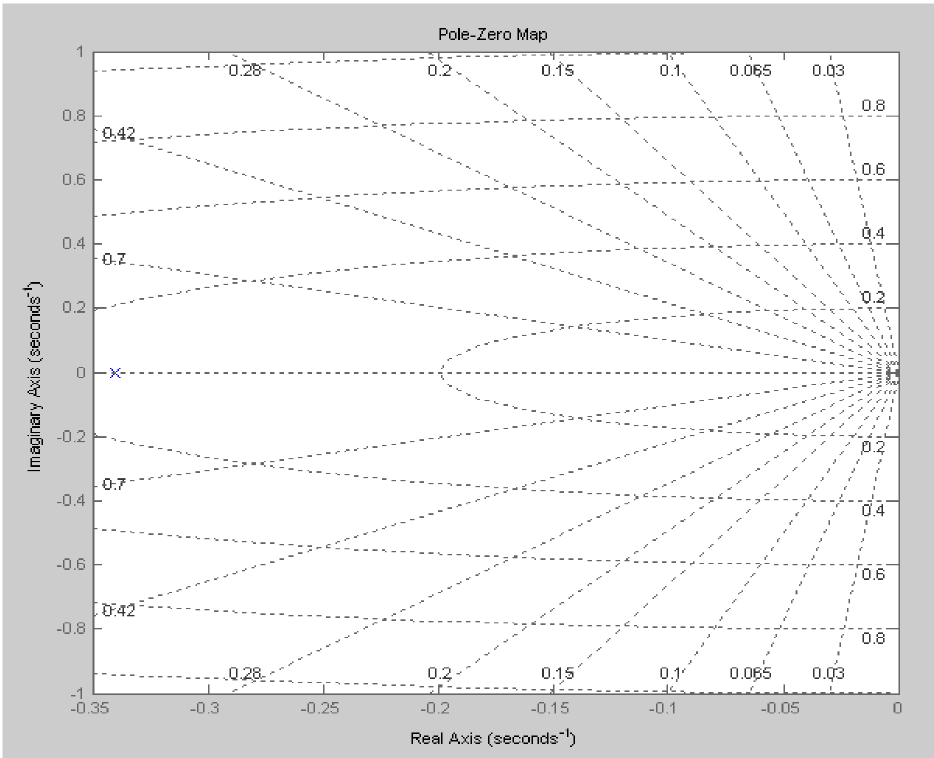


Figure 40. Own figure made with Matlab. An illustration of the location of the pole.

The diagram illustrates the pole at the point -0,34 at the real axis, because the pole is not depending on the imaginary axis. The system is stable.

Another way to establish stability and to use poles is to look at Figure 41 p. 63. In this pole diagram the corresponding impulse response is illustrated. The impulse response is the natural response of the system, and depending on where the pole is located the impulse response will have different functions.

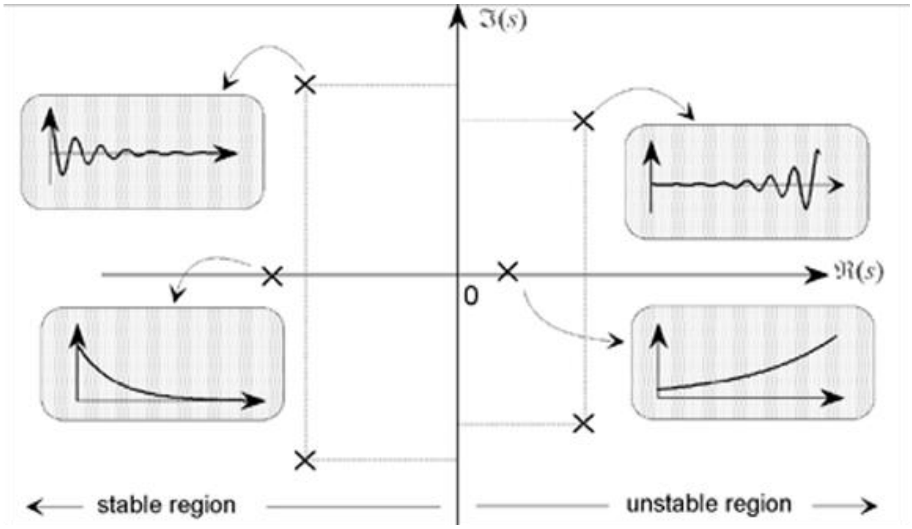


Figure 41 pole/zero curve for one-pole and higher systems [nasa.olin.edu[28]]

The pole of the system in located at the negative real axis which means that it is expected that the impulse response is exponential decreasing.

To solve the impulse response it is necessary to transform the transfer function G(s) back to the t domain g(t) [Feedback Control, p. 126[23]].

For a first order system, which is the case for the pump system, the transfer function can be written as:

$$G(s) = \frac{1}{s + \sigma}$$

Equation 60

Where

- σ is a constant

In picture Figure 41 it was possible to see that the solution for the natural response of the system has to be an exponential function. Therefore the impulse response can be expressed by following equation:

$$g(t) = e^{-\sigma t} 1(t)$$

Equation 61 [Feedback Control p. 126[23]]

For every $\sigma > 0$ the poles are at the left side of the y-axis and the response decrease and the system is stable. When $\sigma < 0$ the system response increases, and the system is unstable.

To solve the function for the impulse response, G(s) has to look like Equation 60.

The impulse response for the system is:

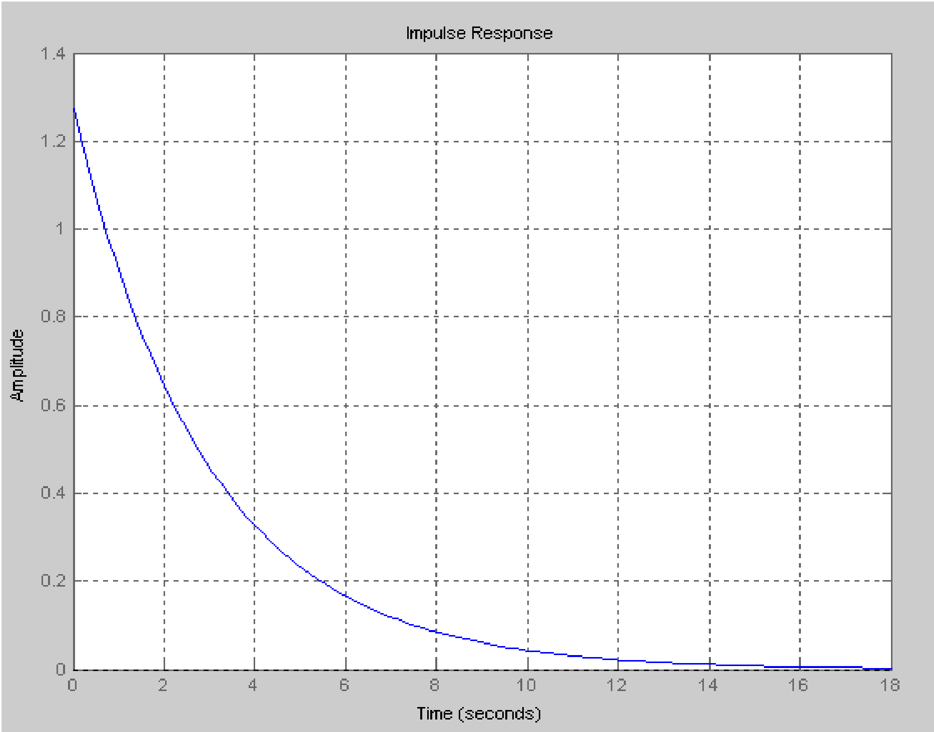
$$G(s) = \frac{1}{(0{,}785s + 0{,}2668)} = \frac{1{,}27}{s + 0{,}365}$$

Equation 62

$$g(t) = 1{,}27 \cdot e^{-0{,}365 \cdot t} \cdot \big(1(t)\big) = 1{,}27 \cdot e^{-0{,}365 \cdot t}$$

Equation 63

The impulse response is illustrated in Graph 11.



Graph 11. Own Matlab graph. Impulse response for the system.

The graph shows that the impulse response decreases as a function of time. The system is stable and $\sigma > 0$ is satisfied.

Another way to see if the system is stable, in contrast to the impulse function, is to see what happens when it goes to infinity. If the impulse response of the system becomes zero when the time goes to infinity, the system is possibly stable: a stability criterion according to the impulse response is that the impulse response of the considered system is absolutely integratable.

$$\lim_{t \to \infty} y(t) = 0$$

Equation 64

$$\lim_{t \to \infty} 1{,}27 \cdot e^{-0{,}365 \cdot t} = 0$$

Both from Graph 11and Equation 64 it is possible to conclude that the system is stable on these terms.

Beside the stability the time constant can be solved from Equation 60 p. 63:

$$\tau = \frac{1}{\sigma}$$

Equation 65

The time constant indicates how fast the responses reach 63.2% of the initial value [PDF from regulation lecture[29]]

The time constant for the system is:

$$\tau = \frac{1}{0{,}34} = 2{,}94\ sec.$$

The time constant is **0.035**sec. The time constant indicates that the system takes 2.94 sec to reach 63.2% of the initial value. If there were more poles it would be possible to estimate which pole there will course the fastest response and thereby choosing the pole after which time constant there is best for the specific system.

10.1.1 Poles for second order systems:

The system was assumed to be linear and therefore the transfer function is of first order and has only one pole. When the PID is integrated to the system, the system will not react as assumed, because there is friction, valves and the actual system equation are non-linear. Therefore a transfer function of higher orders are expected and based on this more poles are expected as well.

For a second order system the transfer function is:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Equation 66

- ξ is the damping ratio and
- ω_n is the natural frequency

The characteristic equation is:

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

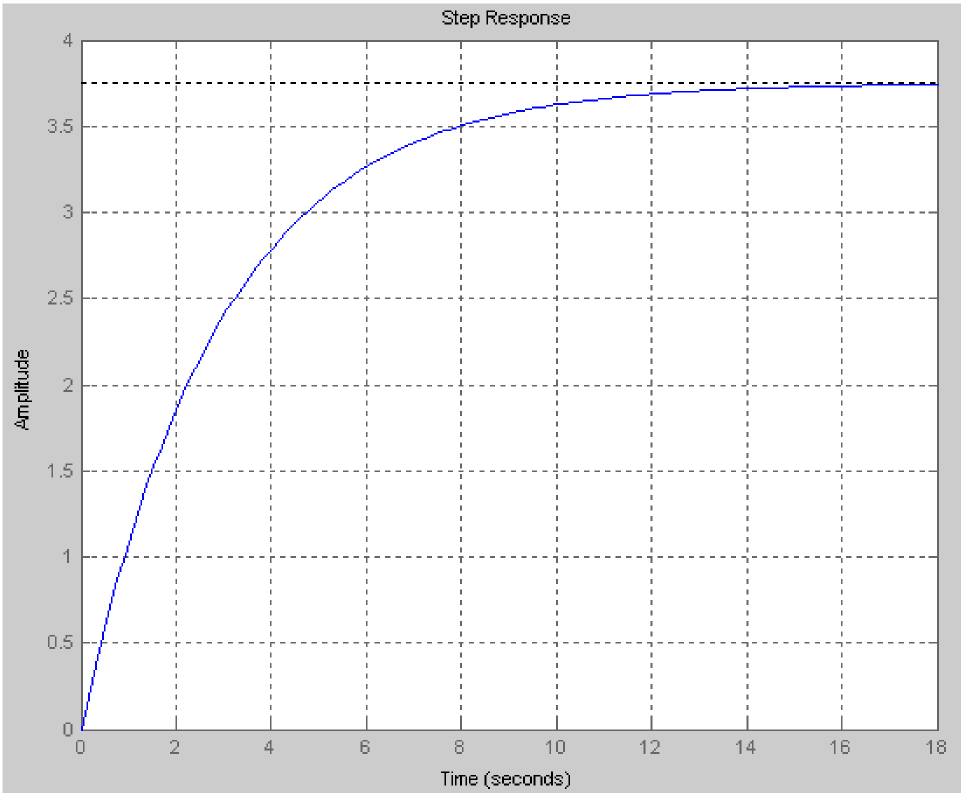
Equation 67

From this equation it is possible to determine two poles. For a second order system the poles also determines how the system response reacts; meaning that the poles determines how the step response, see the following section 10.2, and the impulse responses oscillates.

10.2 Step response

Another response of the system is the step response, which indicates the behavior of the system. From the step response it is possible to see how it damps – either too slow, too fast, or something in between.

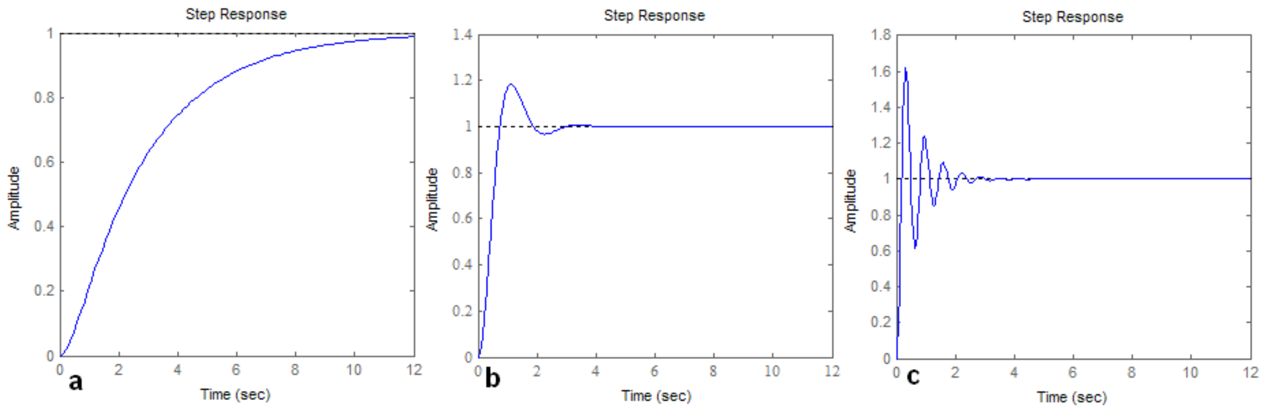
In Graph 12 the step response for the system is shown.



Graph 12 Own graph made with MATLAB.

Graph 12 illustrates the step response for the system. By looking at the step responses in Graph 13 p. 67 it either can be under-damped, critical damped or over-damped.

In the figure the 3 states are shown and the set point is one:



Graph 13 illustrates different damping ratio of step response. The figure a) is underdamped, b) is critical damped or c) overdamped. [lpsa.swarthmore.edu[30]]

(a) The system is responding too slow. (b) The system is responding very well, and this is the desired behavior of the system. In the third case (c) The system is responding too fast and it oscillates – this will make the system unstable.

The damping is depending on the damping ratio, which can only be determined for second and higher order systems.

The damping ratio is how fast the oscillations die out.

10.3 Bode plot

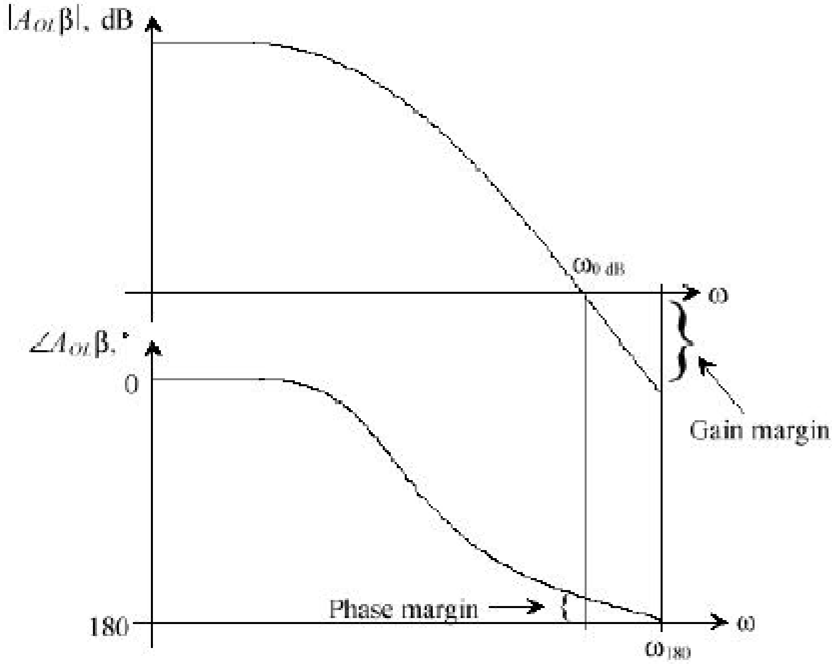
In section 8.4.1 p. 50 Bode plot was presented, but only by equation definition. In this section Bode plot is described. This description is done so that it is possible to estimate the behavior of the system.

By analyzing the Bode plot, it is possible to look further into the stability of a system with negative feedback. This can be done by finding the gain- and phase margin, see Graph 14 p. 68.

- The gain margin, see Graph 14 p. 68, is measured at the point where there is a shift of 180 degrees (-180), because exactly at this point the negative gain in the negative feedback becomes stable. It is measured between the amplitude curve and the line of 0 dB [erc.msstate.edu[31]].
  The gain margin is the amount which the gain must decrease or increase to make the loop gain reach 0 dB, when the phase angle is – 180°.
  The gain margin is the change in the open loop gain that is needed to make the system unstable. When the gain margin becomes large, the parameters for the system have to become large as well to make the system unstable.
- The phase margin is, measured at the point where the amplitude is 0 dB. At this point it is the amount of difference between the phase curve and the -180 degree.

The phase margin is the amount the phase must decrease or increase to make the phase angle -180 when the loop gain is at 0 dB. The phase margin should be 60° and should not be less than 45°.

When the gain margin becomes large, the parameters for the system have to become large as well to make the system unstable.



Graph 14 A Bode plot where the phase and gain margin are illustrated [eprints.iisc.ernet.in[32]]

In general the system is said to be stable if following criterion is satisfied:

$$\beta A_{ol} = -1$$

$$\beta A_{ol} = -180°$$

- A_{ol} is the transfer function for the for the closed loop and is indicated at the y-axis in the bode plot see Graph 1.
- β is the feedback factor
- A_{ol} is the gain for the open loop system

The closer the system comes to satisfy this, the more robust stable it becomes.

10.3.1 Stability analysis of the Bode plot for the system

The transfer function is transform into the jΩ plan and solved:

$$G(s)|_{s=j\Omega} = G(j\Omega) = \frac{1{,}27}{j\Omega + 0{,}365}$$

Equation 68

⇕

$$|G(j\Omega)| = \left|\frac{1{,}27}{j\Omega + 0{,}365}\right|$$

⇕

$$= \frac{1{,}27 \cdot (0{,}365 - j\Omega)}{(0{,}365 + j\Omega)(0{,}365 - j\Omega)}$$

⇕

$$= \frac{1{,}27 \cdot (0{,}365 - j\Omega)}{0{,}365^2 + \Omega^2}$$

⇕

$$= \frac{0{,}4635 - j1{,}27\Omega}{28{,}5^2 + \Omega^2}$$

⇕

$$Re + jIm = \frac{0{,}465}{0{,}365^2 + \Omega^2} - j\frac{1{,}27\Omega}{0{,}365^2 + \Omega^2}$$

Equation 69

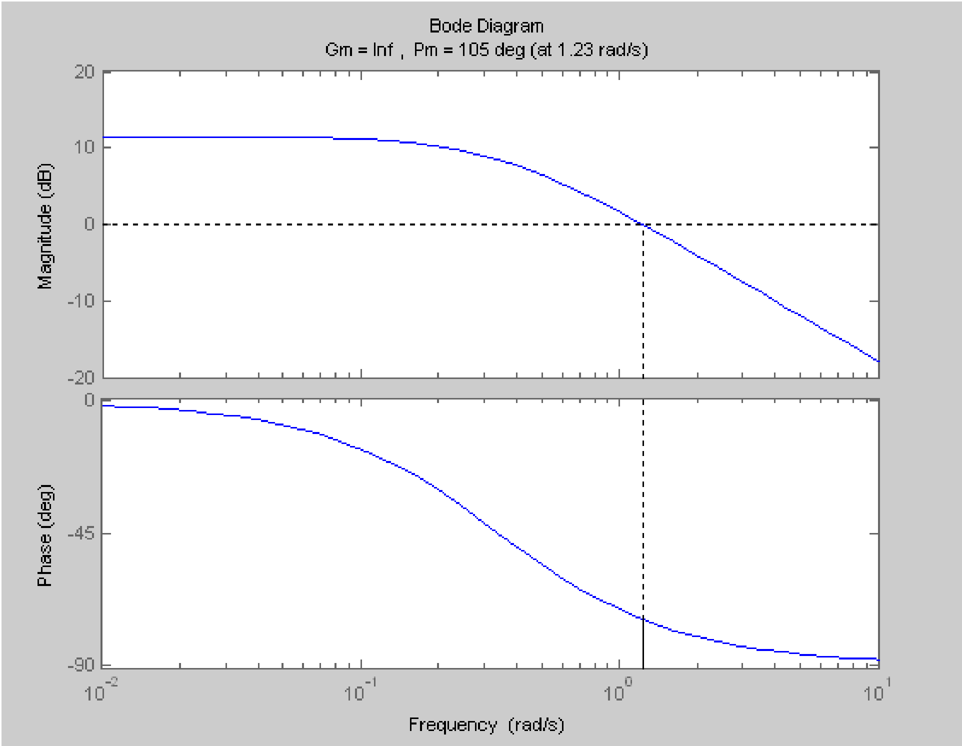
Amplitude:

$$|G(j\Omega)| = \sqrt{Re^2 + Im^2} = \sqrt{\left(\frac{0{,}465}{0{,}365^2 + \Omega^2}\right)^2 + \left(\frac{1{,}27\Omega}{0{,}365^2 + \Omega^2}\right)^2} = \frac{1{,}27\sqrt{\Omega^2 + 0{,}134}}{\Omega^2 + 0{,}133}$$

Phase:

$$\angle G(j\Omega) = \tan^{-1}\left(\frac{Im}{Re}\right) = \tan^{-1}\left(\frac{\frac{0{,}465}{0{,}365^2 + \Omega^2}}{\left(\frac{1{,}27\Omega}{0{,}365^2 + \Omega^2}\right)}\right) = -\tan^{-1}\left(\frac{0{,}366}{\Omega}\right)$$

The Bode plot of the system:



Graph 15 is a Bode plot of the system. The first diagram is for the amplitude given in dB, and the second is phase given in deg. The illustration is made with MATLAB.

The system is stable and from MATLAB the values of the Gain margin are noted as "inf", which means infinity. The Phase margin is 105 deg. The margins are very large and therefore the system is stable, and all the parameters of the characteristic equation must become very large to make the system unstable.

10.3.2 Bandwidth

By analyzing the Bode plot, it is possible to measure the Bandwidth. The bandwidth is the frequency at that point where the amplitude has decreased and reached -3 dB compared with the DC-gain – or in other words where the amplitude is constant.

The bandwidth is illustrated in Figure 42 p. 71.

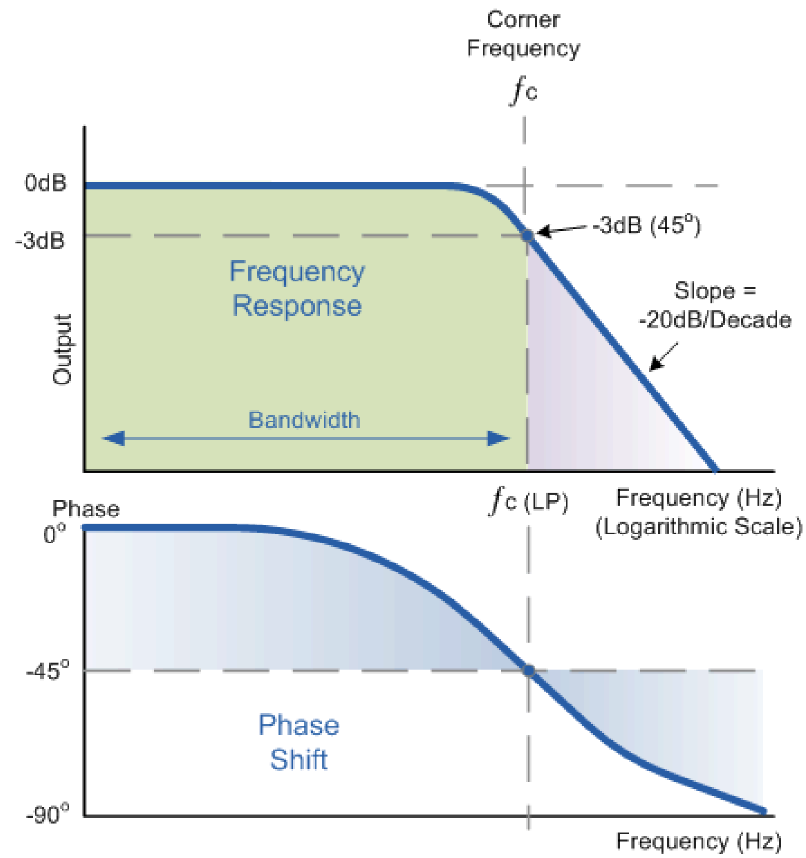


Figure 42 shows a Bode plot with illustration of the bandwidth [electronics-tutorial.ws[33]]

The bandwidth is given in Hz and is known as the pass band. It indicates the range of frequency which can be processed. It is also a measurement to see when high frequency signal can come through, without any large change in amplitude. In addition to this it is also a measurement on how fast the closed loop system is responding. [denstoredanske.dk[34]]

The bandwidth of the system is calculated with MATLAB:

$$BW = 0{,}34$$

This means that every frequency below 0.34 can be processed in the system, which is not very high.

10.4 Root locus

The root locus is a design tool that represents information about the behavior of a system in a graphical way – when the controller is working. The root locus means root location, and is a graph which illustrates possible poles depending on which gain used.

The gain is placed just before the system gain, see Figure 43.

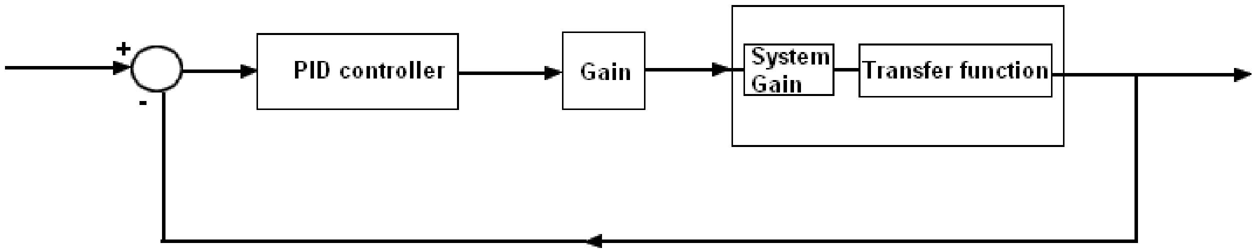


Figure 43 illustrates where the gain is placed in the system. Own figure.

Root locus can be used for closed- and open loop system, in this case the analysis is based on the open loop. The root locus for the open loop is determined by:

$$\mathrm{K} \cdot \mathrm{G(s)} = 0$$

Equation 70

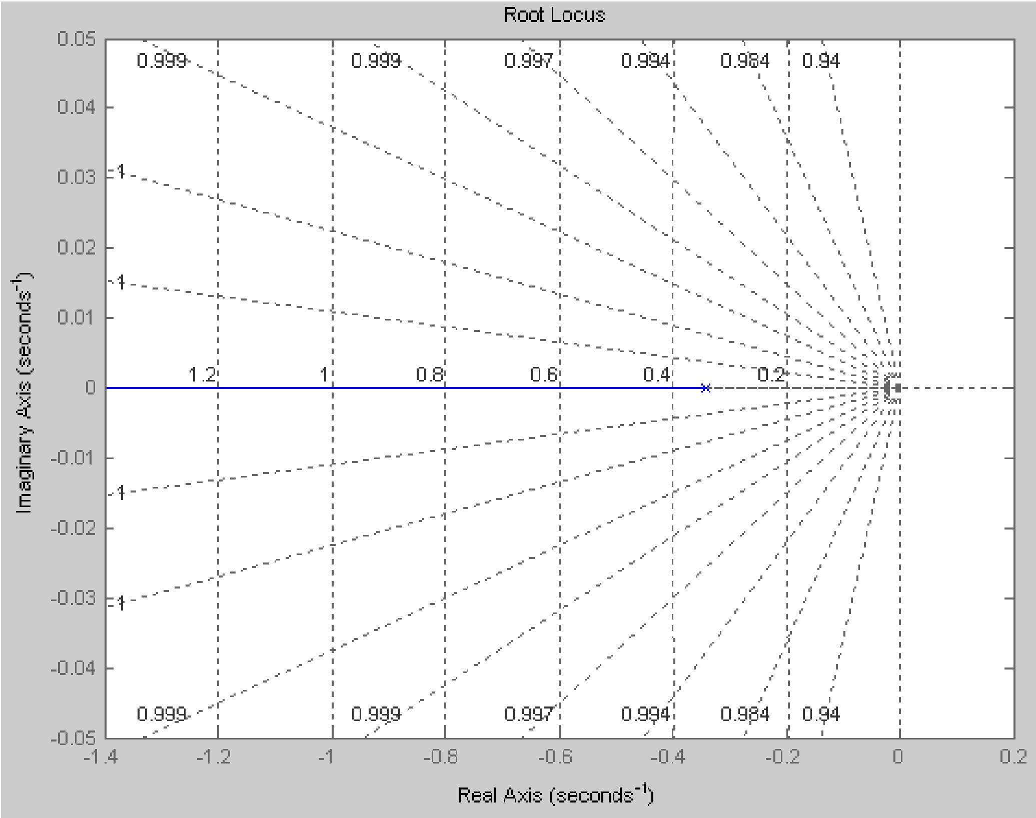
- K is the gain value and
- G(s) is the transfer function of the system

For the system the root locus equation is:

$$K \cdot \left(\frac{1{,}27}{s + 0{,}365}\right) = 0$$

Equation 71

The Root locus is plotted, see Graph 16.



Graph 16 shows the Root locus of the specific system. Own graph.

In general the gain is added to a system to make sure that the system becomes more like the critical damped step response.

The certain pump system is of first order and therefore the step response will never look like Graph 13 b) p. 67, which is critical damped, and therefore a gain is not needed. The gain is not needed because multiplying with a gain to a first order system will only cause an increase in amplitude. However it can become necessary to add a gain when tuning the PID.

If it is necessary to add a gain in the tuning of the PID one must be aware that if the gain varies the pole varies as well. When the pole varies the parameters of the system changes; meaning that a pole with a certain damping ratio or time constant can be chosen and the associated gain can be used.

Here is an example where the transfer function is of second order:

$$G(s) = \frac{N(s)}{D(s)} = \frac{K}{s^2 + 3s + K}$$

Equation 72

When the root locus graph is made the method is to make several graphs of different K-values and then analyze the graphs.

First the roots of the equation are to be calculated.

$$s^2 + 3s + K = 0$$

Equation 73

All the values are plotted – because it is complex conjugate roots, the value of K is only shown for the root with a positive imaginary part, see Figure 44 .

K	1	2	4	10	20	40	100
Roots	-2.62, -0.38	-3	-1.5 ± j1.32	-1.5 ± j2.78	-1.5 ± j4.21	-1.5 ± j6.14	-1.5 ± j9.89

Table 9 shows the roots to the according K-value.

The plotted root locus

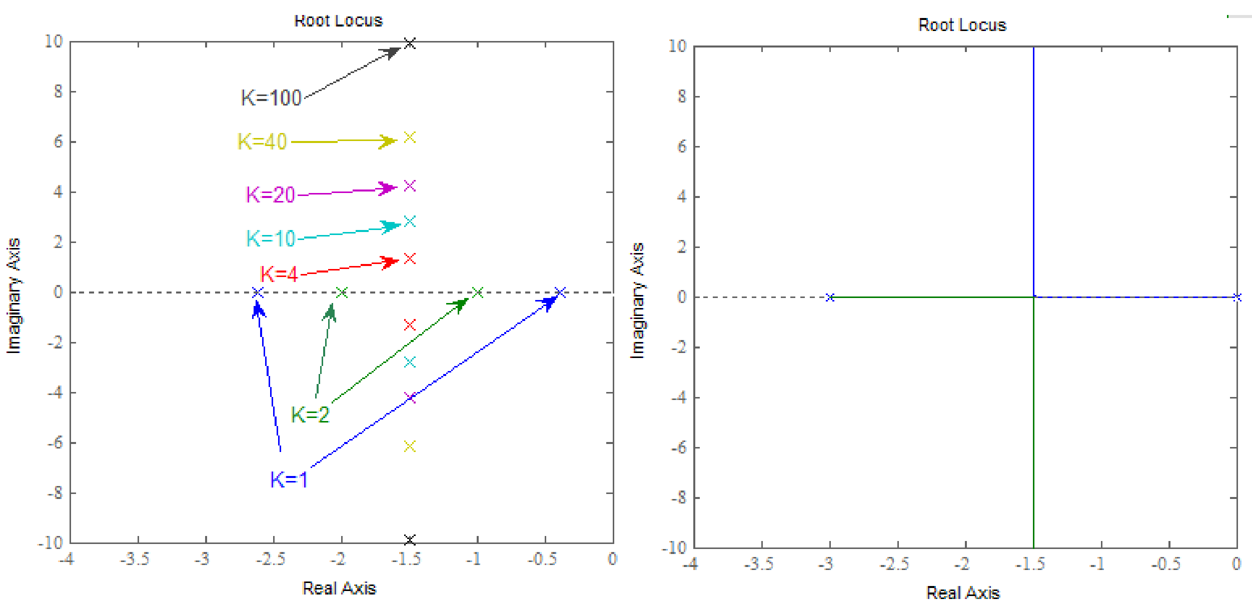
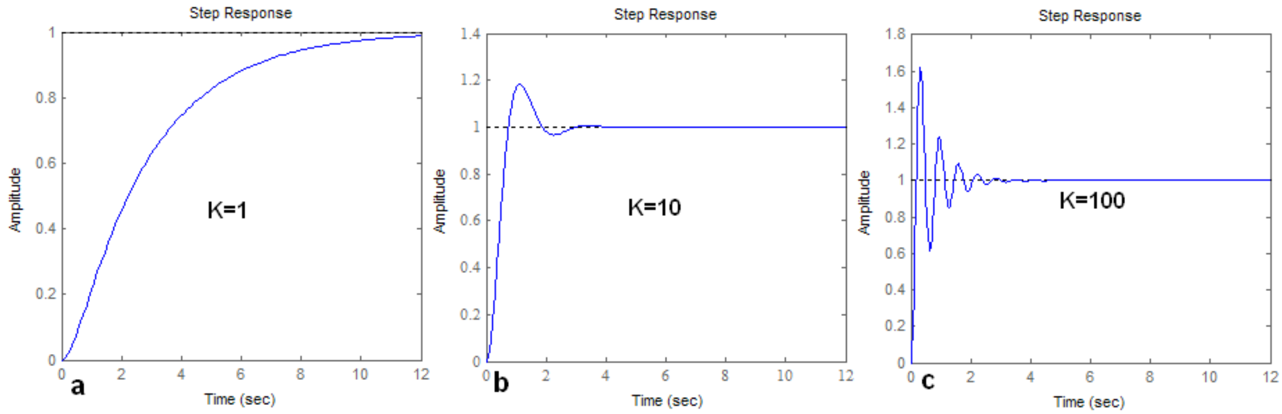


Figure 44. The roots are plotted in the Root Locus diagram according to the K-values [lpsa.swarthmore.edu[28]]

It is shown how the roots are plotted in order to the K-value.

In Graph 17 p. 75 an illustration of the step responses is shown. The tree graphs are for different$k = 1, K = 10, K = 100$.



Graph 17 illustrates different damped step responses according to the K-values. [lpsa.swarthmore.edu[28]]

The graphs a, b and c illustrate how the step responses depend on the K value. If the K value becomes too large the response begins to oscillate too much. For this case $K = 10$ is the best, unless otherwise is specified. To value is not always **10**, but is depending on the transfer function of the system.

11 Determinations of the PID constants

When the pump system is controlled by the PID the unknown constants $K_p, K_i, and\ K_D$ must be determined. The constant may not fit the system because of the linearization of the system in section 9 p. 55, but with these constants as starting points, it will be easier to adjust them to the specific system when the laboratory experiments are in progress.

The function of the PID in the time domain is:

$$PID(t) = K_P \cdot e(t) + K_i \int_0^t e(\tau)\, d\tau + K_D \frac{d}{dt} e(t)$$

Equation 74

Where

- K_i is one divided by the integration time, $\frac{1}{T_i}$, and K_D is the differential time, T_D

11.1 Constant info

An issue with the constants is that a little change in their value results in changes of the response of the system. An overview of what happens when a change appears is shown in Table 10. The table shows what happens to the different parameters, such as stability and overshoot, when the constants are increased.

Increase of the constants:

Constant	Rise time	Overshoot	Settling time	Steady state error	stability
$\boldsymbol{K_p}$	Decrease	Increase	Small change	Decrease	Degrade
$\boldsymbol{K_i}$	Decrease	Increase	Increase	Eliminate	Degrade
$\boldsymbol{K_d}$	Minor decrease	Decrease	Decrease	No effect in theory	Improve if K_d is small

Table 10 illustrates what happens to the different parameters when the constants are increased [wwwdsa.uqac.ca[35]]

The parameters are:

- Rise time
- Overshoot
- Settling time
- Steady state error
- Stability

The rise time indicates the time it takes for the response to reach 90% [wwwdsa.uqac.ca[33]]of the set-point value. The overshoot indicates the difference in the peek value and the setpoint value. The settling time indicates how fast the response takes to settle at the setpoint value. Steady state means that there is no change pr. time, so the steady state error is the difference between the final value of the response and the setpoint value. The stability indicates how the stability of the system is - if it degrades or improves.

- K_P
  This value is used to decrease the rise time, which means if the rise time is too large, a larger K_p value must be chosen. From the table it is possible to see, that when the value increases it causes a decrease in both the rise time and the steady state error, but as it increases the system will become unstable and the overshoot will increase.
- K_i
  This value is used to eliminate the steady state error, and is often chosen to be a small value. From the table it is possible to see, that when the value increases the rise time decreases, but both the overshoot and settling time will increase. These are the reason that K_i has to be of a small value.
- K_d
  This value is used to reduce the overshoot and settling time, which means the value has to be small, see Table 1076. From the table it is possible to see, that when the value increases the settle time, overshoot and the rise time decreases a little bit, but when the value is made very small it is improving the stability. A small K_D is therefore desirable, but it can be tuned to make sure that the other parameters are behaving optimally.

Before determining the constants based on this knowledge, it is very important to make some specification for the system response. Based on the specifications the PID constants can be determined.

11.2 Specifications

The system is very slow, because it takes time before the motor and pump has regulated the flow speed which affects the water level in the tank. In reality the water level will not change because of the non-linear system, but it is still assumed linear with respect to the calculations. The specifications are therefore based on a slower response, which means that it is desired for the system to have a slow rise time and settling time, but still fast enough to make the regulations happen fast.

The specifications are:

- Steady state error must be eliminated.
- The overshoot is allowed to reach maximum 15%, this is 15.225.
- The Settling time has to be 5 sec or less.
- The rise time has to be 1-2 sec or less.

11.3 Determination

To determine the exact constants SIMULINK has been used. The linear system is constructed as a block diagram in SIMULINK. The block diagram looks like this and is now considered as a closed loop, see Figure 45:

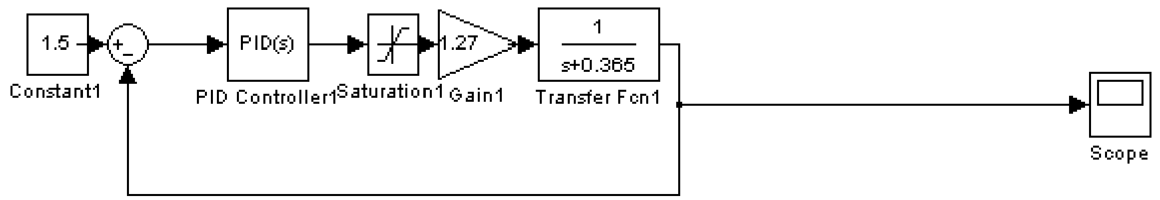


Figure 45 .The closed loop for the system. The constant input is the 1.5 m, which indicates the level height that is the desired output to make sure the flow is steady. Made with SIMULINK.

Here follows a description of the block diagram:

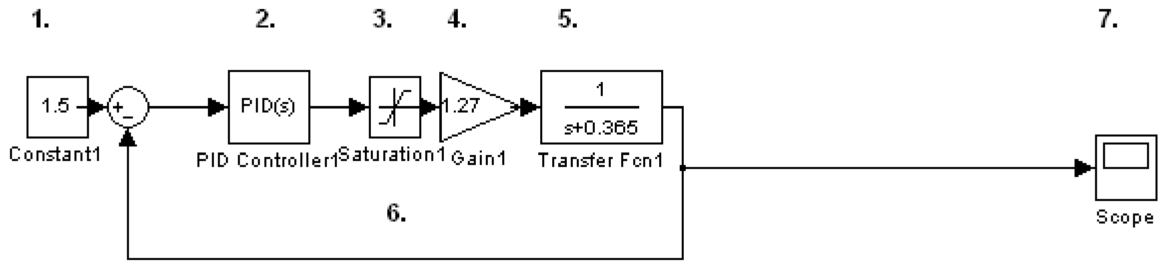


Figure 46 A block diagram that illustrates the feedback control of the system. Made with SIMULINK.

The blocks are indicated by a number:

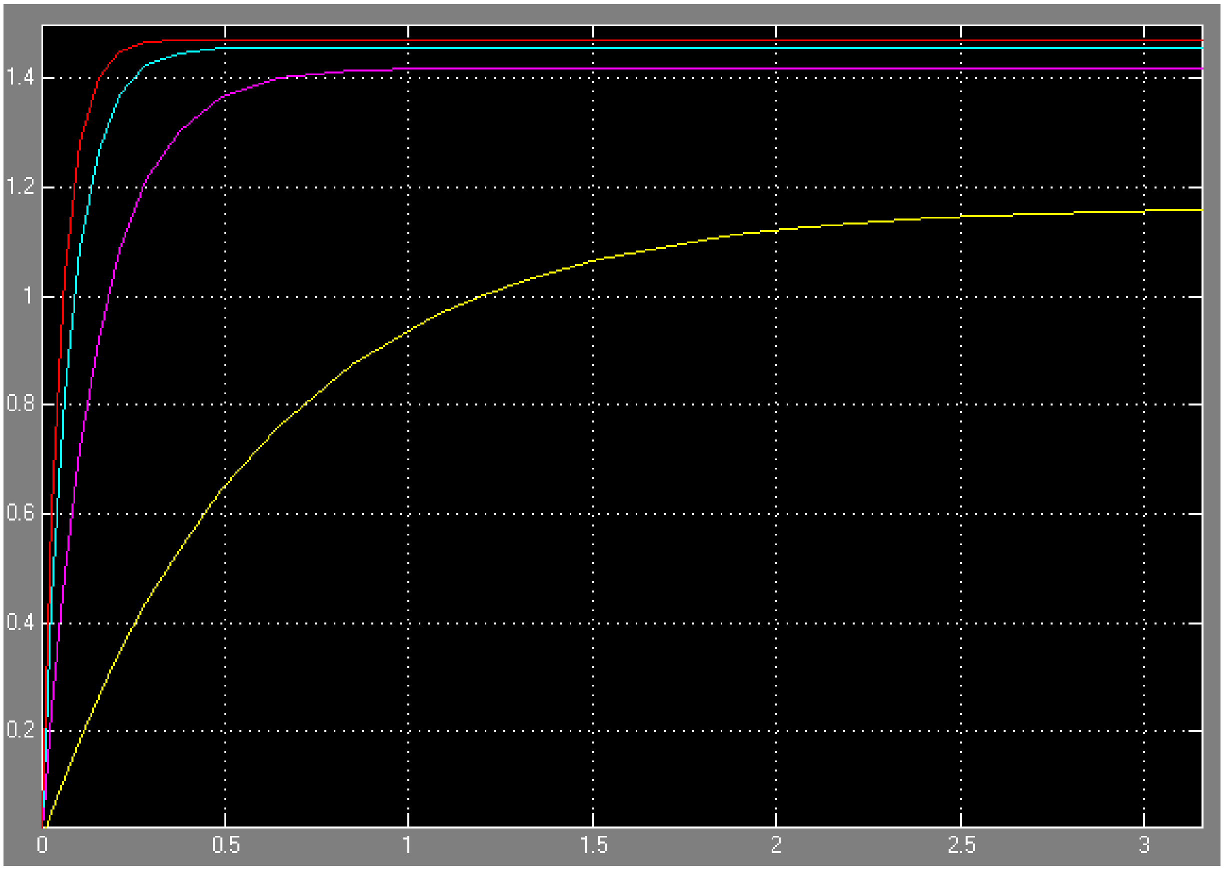
- The input, which in this case is chosen to be 1.5.
- The PID controller.
- This block is a saturation block; the saturation is necessary for a system with an integrator as controlling element. The integrator is considered to be an automatic reset element. It makes sure to reach the desired output and is considered unstable in the open loop. The saturation block makes sure to stabilize it, by "turning off" the integrator when the system reaches a limit; it saturates. This part has to be large enough to make sure that the input to the integration part is kept small under all error conditions.
- The saturation takes care of the control of the integrator and thereby reduces the overshoot.
- This block is the gain, which in this case is the numerator of the transfer function for the system. Its value is 2.9.
- This block represents the system (motor, pump and tank), and is denoted by its transfer function.
- The feedback loop. In the beginning it makes sure to calculate the error, by taking the difference between the input and output.
- The output response.

To estimate the constant values, the block diagram Figure 46 p.78 is used to illustrate the responses for different constants values. To determine the constant values, four responses are illustrated together in one response diagram and are then discussed, see Graph 18.

11.3.1 Determination of K_p

Values: $K_i = 0, K_D = 0, K_P = 1, 5, 10, 15.$

Graph 18 shows the four responses indicated by different colors; yellow is 1, purple is 5, red is 10 and cyan is 15.



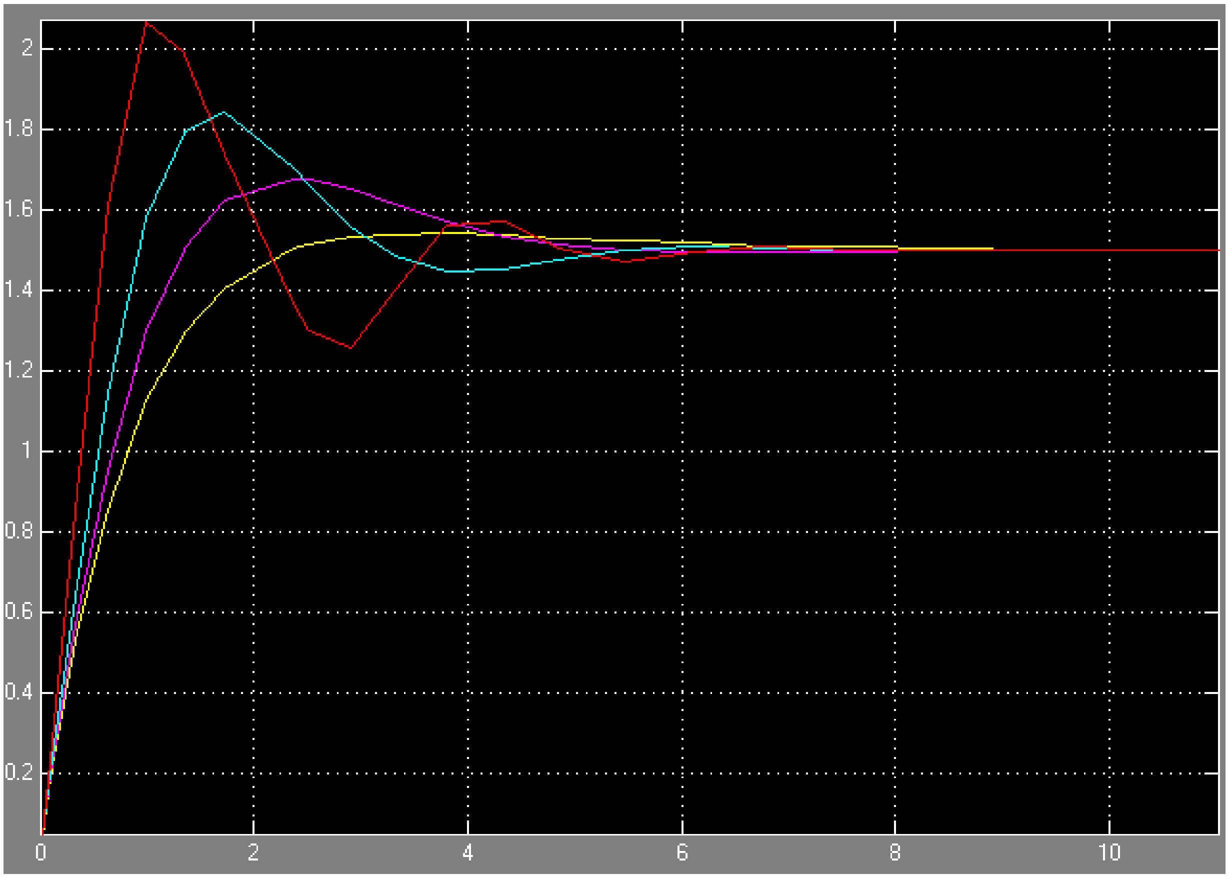
Graph 18. Illustrates the response curves, depending on the K_p value. The x-axis indicates the time, and the y-axis indicates the output value. Made with Simulink.

The responses illustrate that for larger values of K_P the responses become faster – the rise time becomes smaller. There are no overshoots, because the settling point never becomes larger than the set-point value. When the K_P is to be determined it depends on the rise time. The specification for the rise time is 1-2 sec. The only value there satisfies this criterion is $K_P = 5$. this value is chosen for the P constant.

11.3.2 Determination of K_i

Values: $K_p = 1, k_D = 0, K_i = 0.5, 1, 2, 5$

Graph 19 shows the four responses indicated by different colors; yellow is 0.5, purple is 1, cyan is 2, and red is 5.



Graph 19 illustrates the response curves, depending on the K_i value. The x-axis indicates the time, and the y-axis indicates the output value. Made with SIMULINK.

In Graph 19 it is illustrated how the output responses are depending on different K_i values. In this illustration there is overshoot and oscillations.

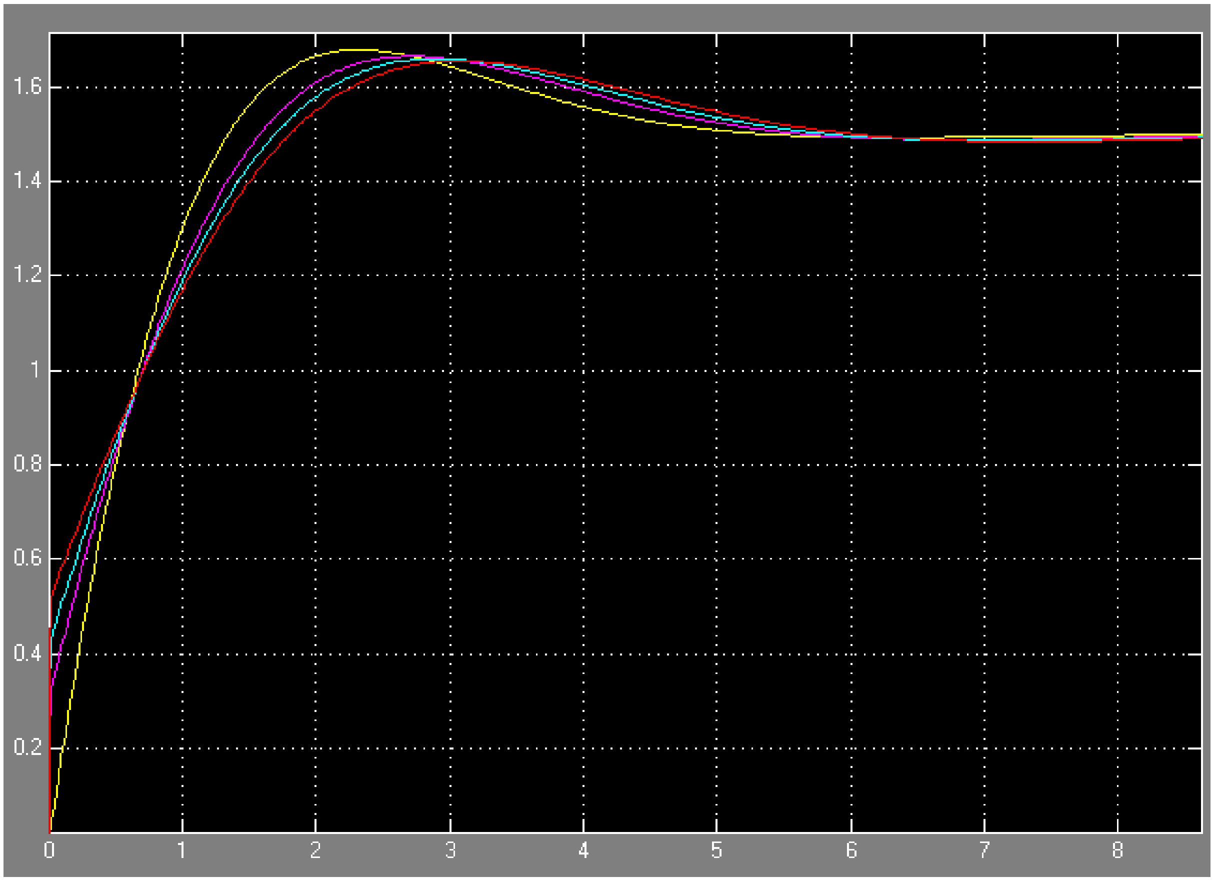
The K_i is chosen with respect to both eliminate the steady state error, which all of these values do. Because the steady state error is zero for all the responses, the specifications are the base line. Not more than 15% overshoot – this eliminates $K_i = 2$ and$K_i = 5$, because their peek value is over 1,725. These responses also oscillate too much.

The settle time has to be 4 sec, which the response with $K_i = 1$ fulfils. $K_i = 1$ is chosen for the I constant.

11.3.3 Determination of K_D

$K_P = 1, K_i = 1, K_D = 0.1, 0.2, 0.3, 0.4.$

Graph 20 Illustrates the response curves, separately, depending on the $\boldsymbol{K_D}$ value. The x-axis indicates the time, and the y-axis indicates the output value. Made with SIMULINK. $\boldsymbol{K_d = 0.1, 1, 10, 20}$illustrates responses indicated by different colors; yellow is 0.1, purple is 0.2, cyan is 0.3, and red is 0.4.



Graph 20 Illustrates the response curves, separately, depending on the K_D value. The x-axis indicates the time, and the y-axis indicates the output value. Made with SIMULINK. $K_d = \mathbf{0.1, 1, 10, 20}$

The response diagrams illustrates that a change in K_D does not cause a big change in the response, which also was indicated in Table 10 p. 76 the response with $K_D = 0.1$ is a little different than the other three responses. It is faster, and has a more overshoot, but it does not become higher than the specified overshoot.

The settling time is specified to 5 sec. which in this case it is when $K_i = 0.1$.

$K_i = 0.1$ is chosen to the D constant.

The final PID function and response is illustrated at the next page.

11.4 The system function of the PID

Here is the function of the PID of the pump system:

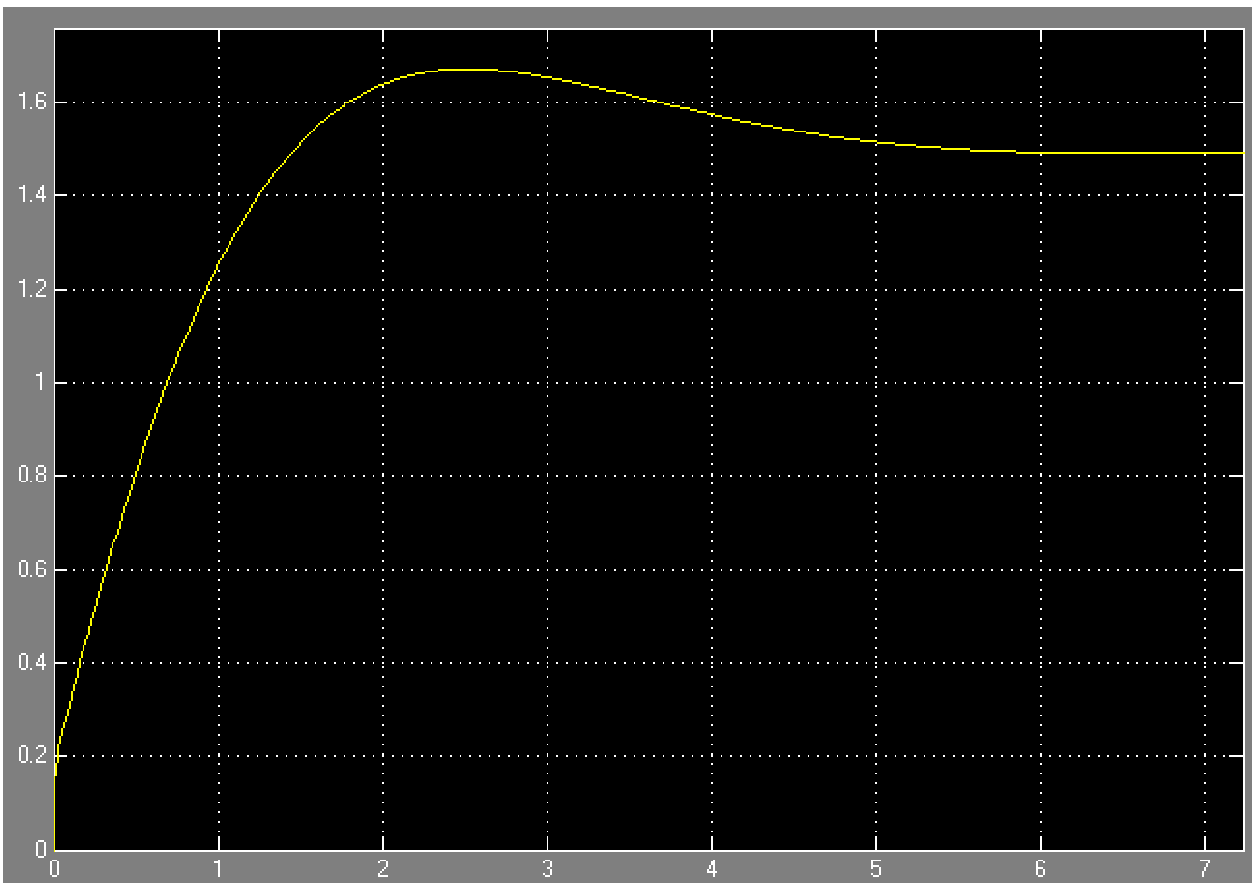
$$PID(t) = e(t) + \int_0^t e(\tau)d\tau + 0.1\frac{d}{dt}e(t)$$

Equation 75

Where:

- $K_P = 1$
- $K_i = 1$
- $K_D = 0.1$

The step response of the system with the PID, see figure Graph 21:



Graph 21 is an illustration of the response of the system with the specified PID controller. The x-axis indicates the time, and the y- axis indicates the output value. Made with Simulink.

From Graph 21 is an illustration of the response of the system with the specified PID controller. The x-axis indicates the time, and the y- axis indicates the output value. Made with Simulink. is an illustration of the response of the system with the specified PID controller

- The settling time is about 5 sec.
- The Rise time vas when it had reached 90%, which is 1.35. The rise time is about 1.1 sec.
- The peek value is 1.69 – 12.66% overshoot.
- There is no steady state error.

The stability of the system with the controller, the closed loop system, is determined in the next section.

11.4.1 Stability of the closed loop system

At last the stability of the closed loop can be determined to see if the system with the PID controller is stable. This is done by gathering all the blocks together as one – superposition principle.

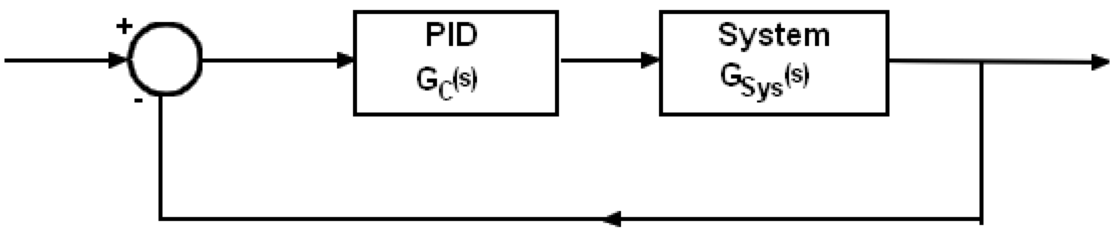


Figure 47 shows the pump system with the PID. Own figure.

When two blocks are placed like in the figure above, negative feedback, the blocks and feedback loops can be replaced by one block only. This is done by using the equation:

$$G(s) = \frac{G_C(s) \cdot G_{sys}(s)}{1 + G_C(S) \cdot G_{sys}(s)}$$

Equation 76

The transfer function for the PID is:

$$G_C(s) = 1 + \frac{1}{s} \cdot +s \cdot 0{,}1$$

The transfer function for the pump system is:

$$G_{sys}(s) = \frac{1{,}27}{s + 0{,}365}$$

$$G(s) = \frac{\left(1 + \frac{1}{s} + s \cdot 0{,}1\right)\left(\frac{1{,}27}{s + 0{,}365}\right)}{1 + \left(1 + \frac{1}{s} + s \cdot 0{,}1\right)\left(\frac{1{,}27}{s + 0{,}365}\right)}$$

⇕

$$G(s) = \frac{0{,}113(s^s + 10s + 10)}{s^2 + 1.45s + 1.123}$$

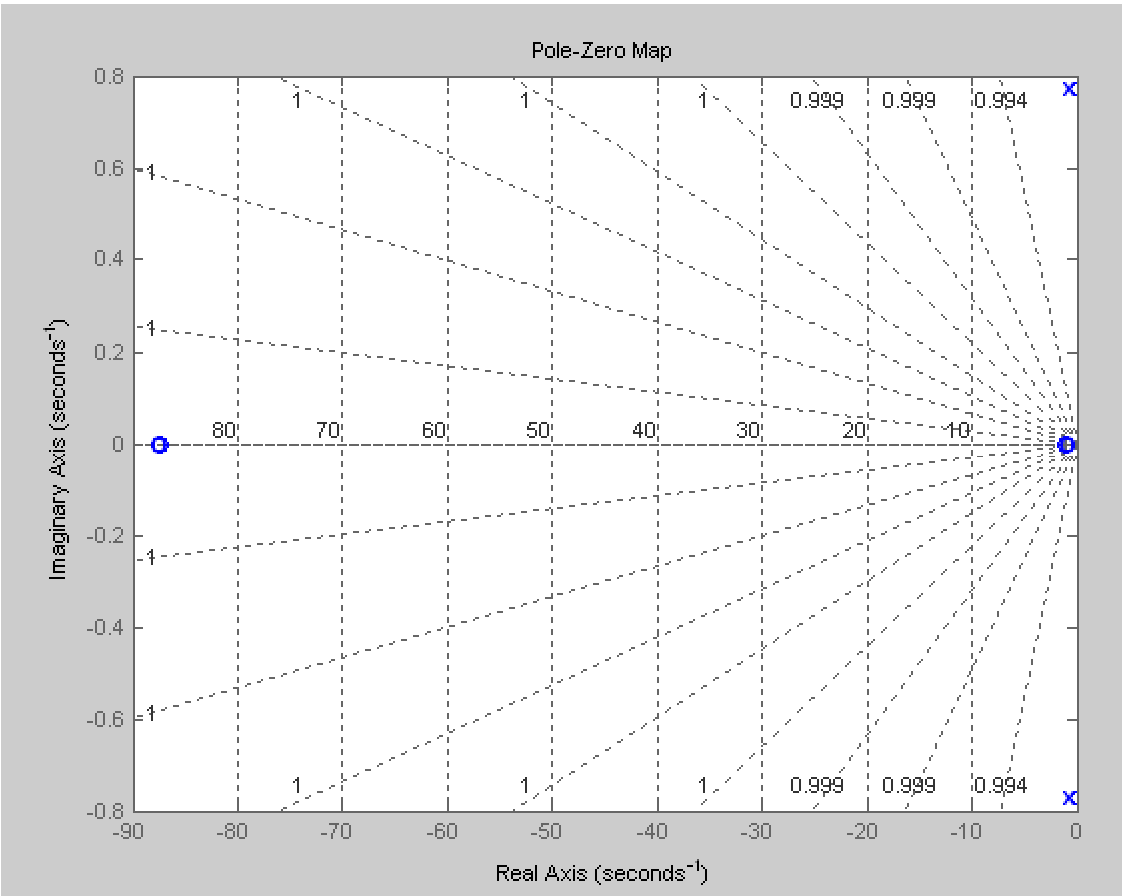
Equation 77

This equation is solved in MATLAB:

The poles are:

$$s_1 = -0{,}7250 + 0.7729i \quad and\ s_2 = -0{,}7250 - 0.7729i$$

Graph 22 illustrates the pole zero diagram:



Graph 22. own graph from MATLAB. The x-axís indicates the real axis, and the y - axis indicates the complex axis.

The poles are all located at the negative half plan of the real axis; this means that the closed loop system is stable with this controller.

It is expected that for higher order systems, which the pump system practically is, these constants for the PID have to be tuned, because the pump system will behave differently than what is estimated with the theoretically first order approximation.

When the PID controller is developed theoretically all the constants are simulated with SIMULINK, and afterwards the controller is modeled in LabVIEW. For further information on LabVIEW see appendix 4.

From the experiment it was possible to conclude that these constant which were determined for the PID controller are fitted for the system, see Figure 48.

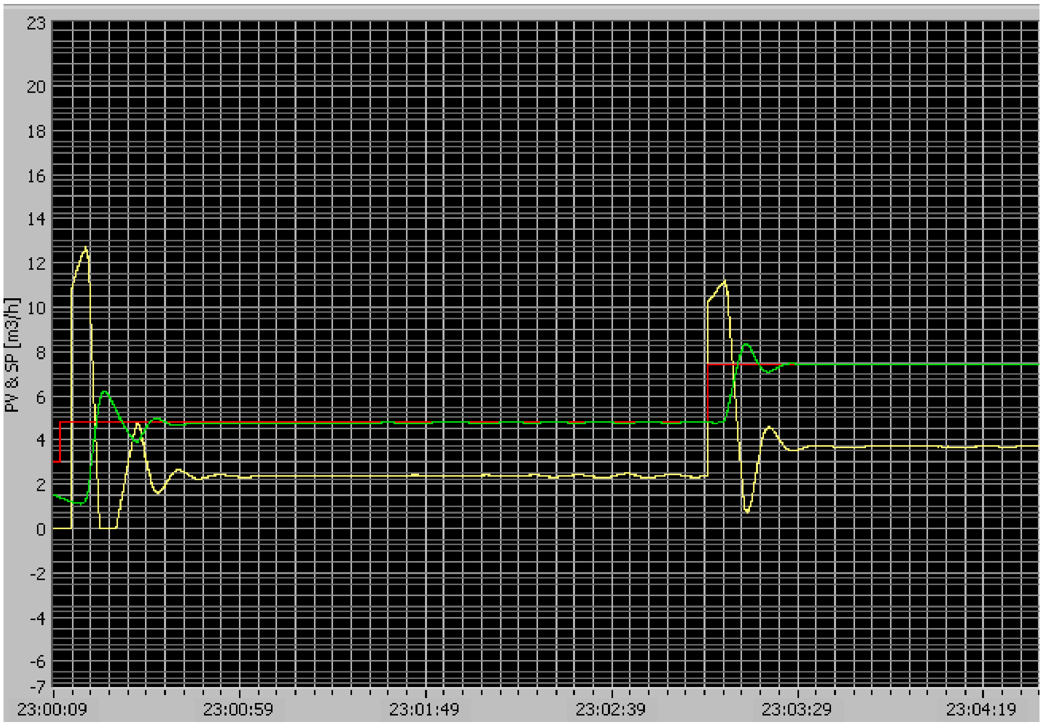


Figure 48 illustration of the pump system response. The x-axis indicates the time and the y-axis indicates the output. The PID constant are: $K_P = 1; K_i = 1; K_D = 0.1$

From the table it is possible to see that the response for the system output has a fast response and settles to the setpoint, the steady state error is 0,02-0,03 and the overshoot is about 20%. This error was not possible to eliminate with a tuning of the constants, and the overshoot is acceptable.

At the tuning process the PID made the system more unstable, so it was concluded that the constant for the PID controller which was determined to be:

$$K_P = 1; K_p = 1; K_D = 0.1$$

Is the best for this specific pump system.

Partial Conclusion

The constant for the PID control has been determined, and from this it can be concluded that the response for the system is stable and all specification are satisfied. The constants are found by both use of knowledge based on how the responses will react if the constants are increasing, and by looking at response diagrams made with SIMULINK. This PID can be used in controlling the pump system, but it might have to be tuned when the experiments take place.

12 Conclusion

The fluid dynamics has been used to get an understanding of how the fluid behaves inside the pipes, pumps, and other components. The equations described in the report have been used for a series of calculations and equations. The equations are combined with the knowledge about the different components in the system using the loss coefficients to determine the head loss, pressure drop and the entry length. In addition the different components are described including the centrifugal pump. It is also described, that it is important to consider, where different gauges is put in a pump system. This is because the most precise measurements are achieved when the flow is fully developed. Due to the equations for calculation of the entry length, this distance is possible to predict. The results are considered to be reasonable showing that the flow is very turbulent causing a significant head loss and entry length.

The Affinity law has been used to estimate the system curve of the pump system, used in this project. The system curves acquired by the Affinity law have been compared with the system curves created by the data collected from experiments. It is shown that the system curves are not equal. This is because the Affinity law does not include all the losses in the system, where the system curves made from the practical data include the losses in the entire pump system.

Several system curves have been made. During the research, the two most accurate have been determined; these are the ones made from actual measurements with and without resistance. The theoretical system curves for this system are all deviant from the measured curves: The first theoretical system curve was really far off but for reasons which are not certain. The last theoretical curves by the affinity laws turned out to be useable only for approximations to the actual measurements

The system curve which are most fitting for this system, turned out to be the ones based on the laboratory experiment, $H(Q) = 0{,}2676Q^2 + 0{,}6637Q + 0{,}2503$.

The electrical parts of the system have been shortly described and include the frequency converter that changes the AC signal to a needed AC signal at the needed frequency. The PWM is an electrical component that changes the width of pulses without changing the period it works in three stages and reduces system costs as well as power consumption. The stator and the rotor of the AC motor have been shortly described giving an understanding of how the induction AC motor operates.

The pump system has been analysed to estimate the system function and stability. A PID controller was chosen based on knowledge and graphical illustration of the PID constants, dependently on different values. These constants were used in experiments in the laboratory where they were inserted in LabVIEW. The PID controller regulated the pump system almost perfectly.

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13.1 Chronological order of literature

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[27] Matematikbogen – lav korrekt henvisning
[28] http://nasa.olin.edu/projects/2010/tec/work_introCT.htm
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[35] http://wwwdsa.uqac.ca/~rbeguena/Systemes_Asservis/PID.pdf PDF

Datasheet for the pump:
http://net.grundfos.com/Appl/WebCAPS/ProductDetailCtrl?cmd=com.grundfos.webcaps.productdetail.commands.ProductDetailCommand&productnumber=96518436&freq=50&page=0&selectedRow=2

APPENDIX 1

Experiment 1

Purpose

The purpose of this experiment is to determine different characteristics for the pump by using pump- and systemcurves.

The system setup

Picture 1: This picture shows the setup with pumps, water tank, different gauges, and valves

The system consists of several different components like the water tank, three pumps, pipes, gauges, and valves. During this experiment only one pump has been used and it has been controlled by a voltage which is regulated between 0 and 10 volts. Whenever the voltage is changed, a different flow and pressure will occur.

This experiment focuses mainly on the flow rate, but also notes the rounds per minute that the pump spins, in order to include the affinity law to the final calculations. The information gathered during this experiment, is supposed to illustrate how the pump operates by looking at the pumpcurves and systemcurves.

The experiment is split up in the following four parts:

1. To determine the theoretical system curve
2. To determine the pump curves and plot with system curve
3. To determine the system curve with the measured data
4. To determine the system curve with the Affinity law

Part one

Determination of the theoretical system curve

When determining the system curve the different components in the system must be known, along with the components loss coefficients, K_L.

Datasheet for the pump used:

Different connections in the system:

90°-bends, threaded:	3	$k_L = 1{,}5 \cdot 3 = 4{,}5$
T-bends, branch flow, flanged:	2	$k_L = 1 \cdot 2 = 2$
T-bends, line flow, threaded:	1	$k_L = 0{,}9$
Gate valves, fully open	3	$k_L = 2 \cdot 3 = 6$
Swing Check valve, forward flow:	1	$k_L = 2$
Union, threaded:	2	$k_L = 0{,}08 \cdot 2$
Reentrant, entrance:	1	$k_L = 0{,}8$
Reentrant, exit:	1	$k_L = 1$

Total k_L for connections and valves:

$k_L = 4{,}5 + 2 + 0{,}9 + 6 + 2 + 0{,}16 + 0{,}8 + 1 = 17{,}36$

Pumps and flow meter

It is assumed that the pump has a union threaded pipe:	1	$k_L = 0{,}08$
Flow meter:	1	$k_L = 2 + 0{,}861 = 2{,}861$

Total k_L for pump, flow meter, and pressure gauge:

$k_L = 0{,}08 + 2{,}861 = 2{,}869$

Determining the system curve:

$D = 5{,}4cm = 0{,}054m$, measured

$\frac{\varepsilon}{D} = \frac{0{,}0045cm}{5{,}4cm} = 8{,}33 \cdot 10^{-4}$,calculated in the project

$L = 575cm = 5{,}75m,$ length of all pipes

$k_{L,rør} = 17{,}36 + 2{,}869 = 20{,}299$, the summarized K_L values.

$\rho = 998 \frac{kg}{m^3}$, the density of water

System curve equation:

$$\Delta P = \left(\Sigma k_L + f\frac{L}{D}\right) \cdot \frac{1}{2}\rho v^2$$

Equation 1

Other equations to consider:

$$f = f\left(\frac{\varepsilon}{D}, v\right)$$

$$k_{L,rør} = f\frac{L}{D}$$

Equation 2. Friction factor

Determination of the friction factor f:

$$k_{L,rør} = f\frac{L}{D} \Leftrightarrow f = \frac{20{,}299}{\left(\frac{5{,}75m}{0{,}054m}\right)}$$

$$f = 0{,}19$$

Calculation of system curve:

$$\Delta P(v) = \left(\Sigma k_L + f\frac{L}{D}\right)\cdot ½\rho v^2 = \left(20{,}299 + 0{,}19\cdot\frac{5{,}75m}{0{,}054m}\right)\cdot ½\cdot 998\frac{kg}{m^3}\cdot v^2$$

This is the system curve for the pressure difference ΔP and velocity v:

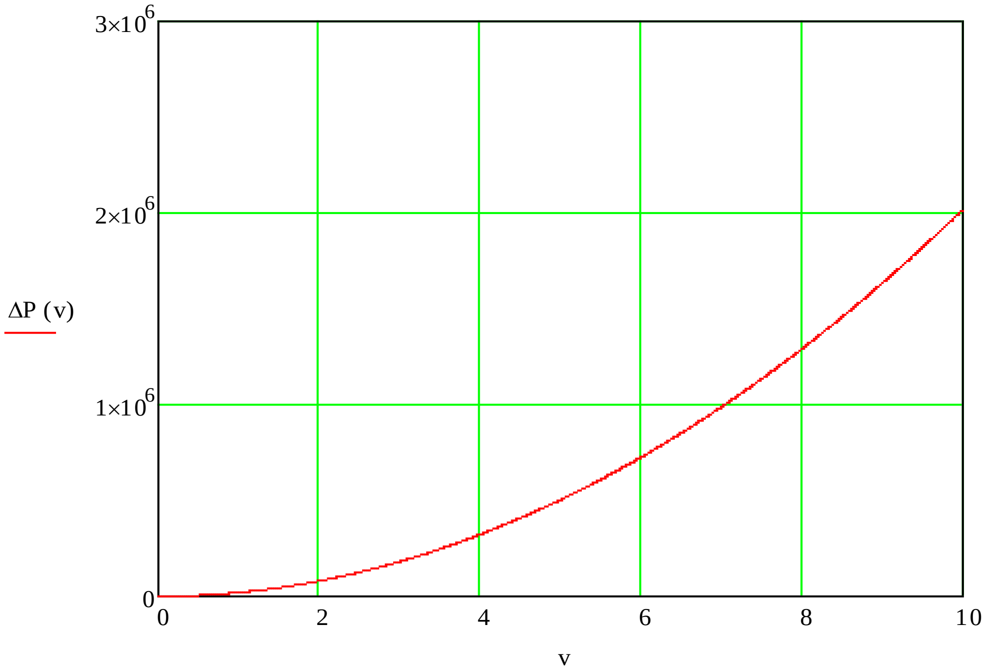


Figure 1. The theoretical system curve with velocity at the x-axis and pressure difference at the y-axis

In order to make a proper H-Q graph, a few actions have to be taken. First of all the graph will be transformed into a $H(v)$ curve by using the following relations:

$$H = \frac{\Delta P(v)}{\rho g}$$

$$\Updownarrow$$

$$H(v) = \frac{20{,}23v^2}{g}$$

This graph is turning out as:

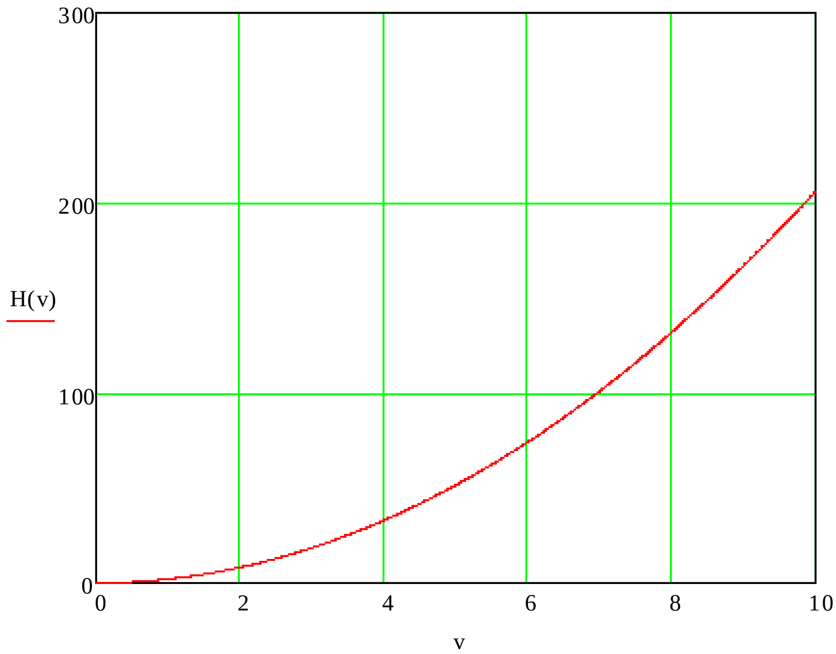


Figure 2. The theoretical system curve with velocity at the x-axis and head at the y-axis

Now this is ready to be fixed to suit an H-Q graph by tracing some speeds to find the head, and afterwards determining the flow rates at those speeds by use of:

$$Q = v_{avg} \cdot A$$

Where

- Q is the flow rate in $\frac{m^3}{h}$
- v_{avg} is the average speed in $\frac{m}{s}$
- A is the cross sectional area of the pipes

To determine the cross sectional area of the pipes, the following equation is used:

$$A = \pi \left(\frac{D}{2}\right)^2 = 2{,}29 \cdot 10^{-3} m^2$$

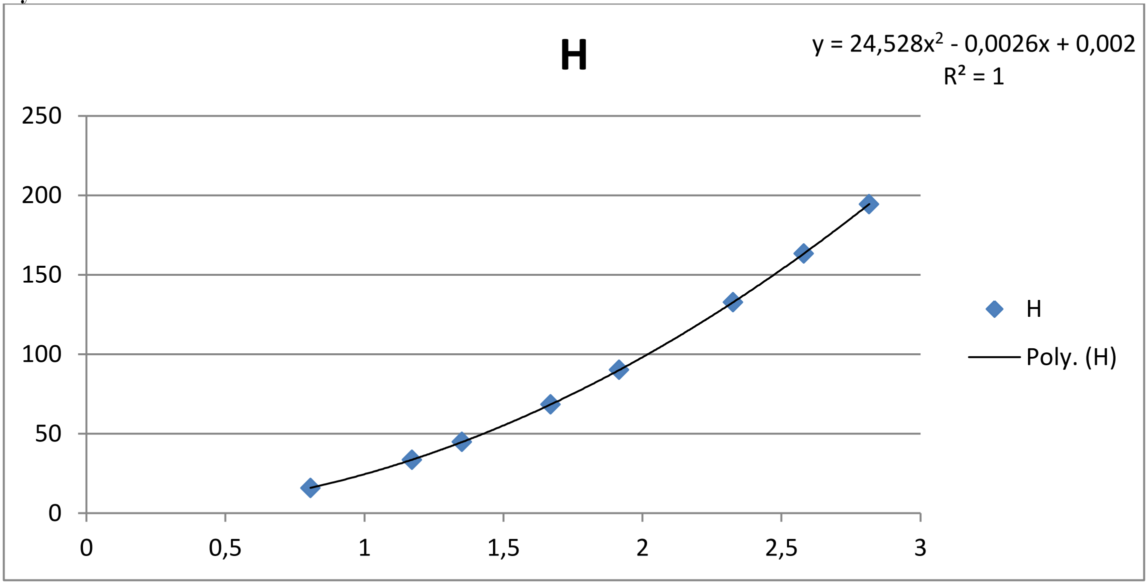
Where

- D is the diameter of the pipe

Velocity converted to flow rate				
v		Q	H	A
2,78		0,8062	15,942	0,29
4,04		1,1716	33,668	
4,66		1,3514	44,795	
5,76		1,6704	68,438	
6,61		1,9169	90,127	
8,02		2,3258	132,68	

8,9		2,581	163,39	
9,71		2,8159	194,49	

As a H-Q graph this is being showing the system curve, which should be the theoretical system curve for this entire system.



The equation for the system curve in a HQ diagram therefore is:
$H(Q) = 24{,}528Q^2 - 0{,}0026Q + 0{,}002$

The system curve in a HQ diagram:

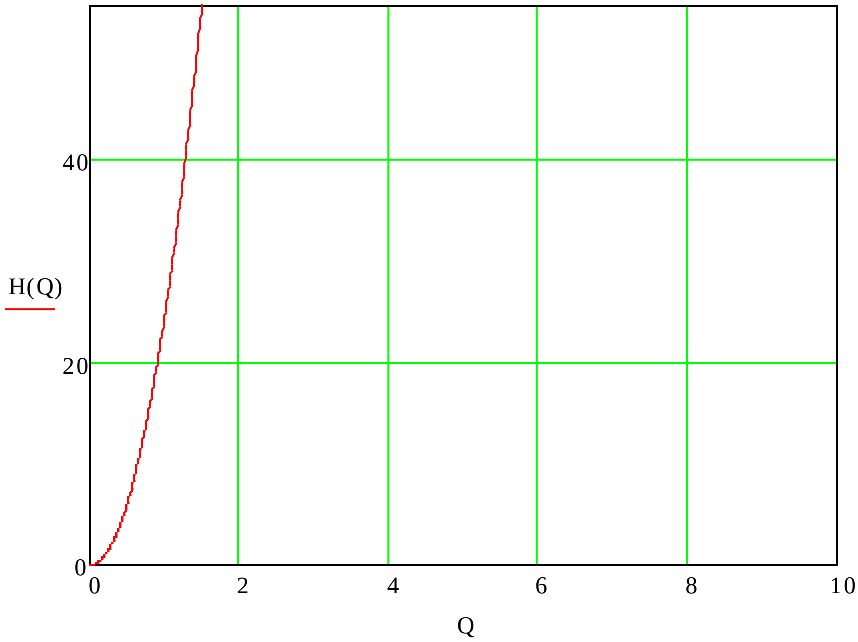


Figure 3. The theoretical system curve

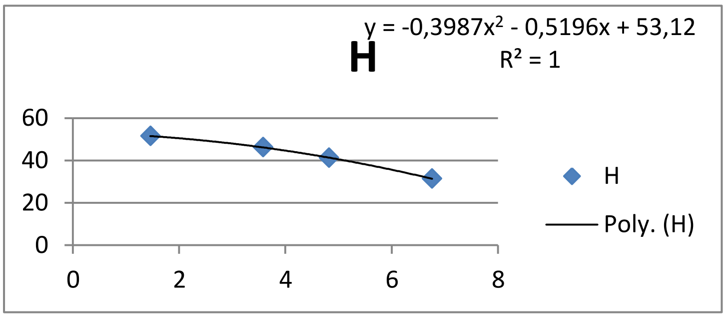
Part two

Determine the pump curves and plot the system curve

The pump curves have been determined by using data from WebCAPS at Grundfoss' webpage. The pump curves fit the particular pump that has been used. Doing this on each 10 % from 40-100 % makes it possible to draw and get seven very accurate pump curves.

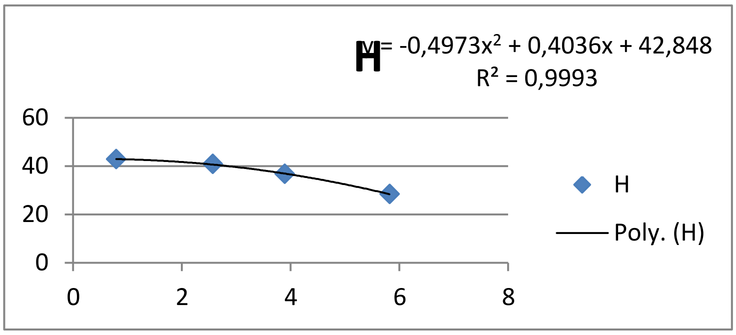
See next page.

Effectivity		Q	H
100		1,46	51,5
		3,58	46,2
		4,82	41,3
		6,76	31,4



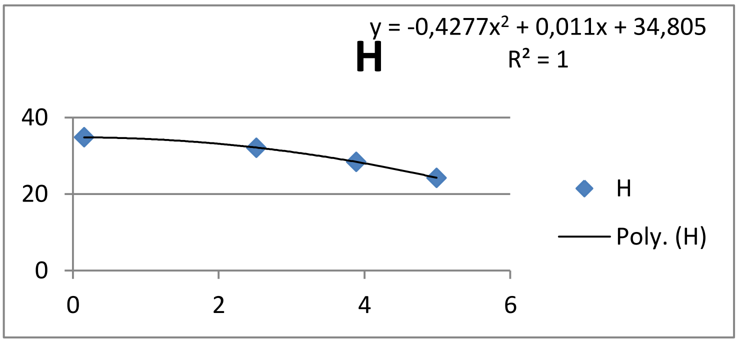
Graph 1

Effectivity		Q	H
90		0,794	42,8
		2,57	40,8
		3,89	36,7
		5,82	28,4



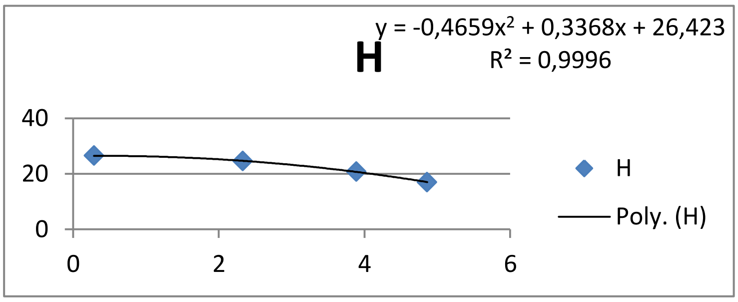
Graph 2

Effectivity		Q	H
80		0,154	34,8
		2,52	32,1
		3,89	28,4
		4,99	24,2



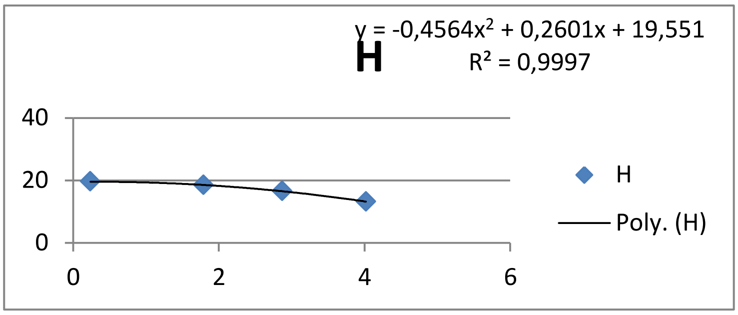
Graph 3

Effectivity		Q	H
70		0,286	26,5
		2,33	24,6
		3,89	20,8
		4,86	17



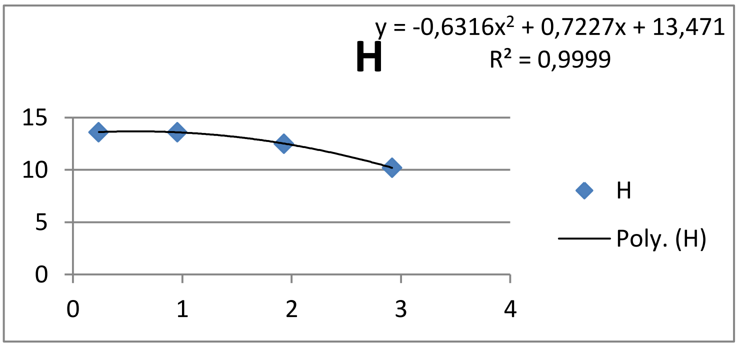
Graph 4

Effectivity		Q	H
60		0,236	19,6
		1,79	18,5
		2,87	16,6
		4,02	13,2



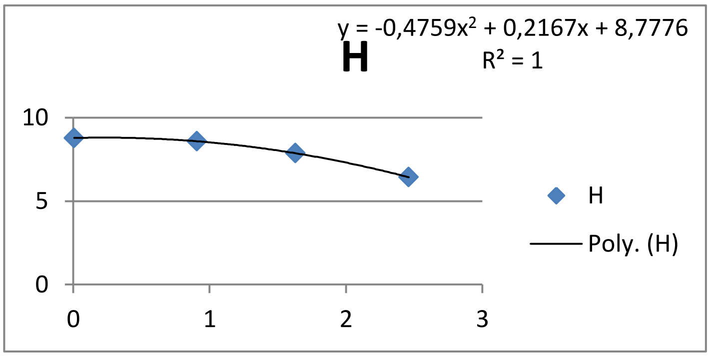
Graph 5

Effectivity		Q	H
50		0,236	13,6
		0,957	13,6
		1,93	12,5
		2,92	10,2



Graph 6

Effectivity		Q	H
40		0,007	8,78
		0,906	8,58
		1,63	7,87
		2,46	6,43



Graph 7

These seven pump curves need to be plotted in one graph containing all, in order to plot the system curve and get the operation points.

Pump curves:

$$H_{100}(Q) := -0.3987Q^2 - 0.5196Q + 53.1$$

$$H_{90}(Q) := -0.4973Q^2 + 0.4036Q + 42.84$$

$$H_{80}(Q) := -0.4277Q^2 + 0.011Q + 34.80$$

$$H_{70}(Q) := -0.4659Q^2 + 0.3368Q + 26.42$$

$$H_{60}(Q) := -0.4564Q^2 + 0.2601Q + 19.55$$

$$H_{50}(Q) := -0.6316Q^2 + 0.7227Q + 13.47$$

$$H_{40}(Q) := -0.4759Q^2 + 0.2167Q + 8.777$$

System curve:

$$H_{sys}(Q) := 24.528Q^2 - 0.0026Q + 0.00$$

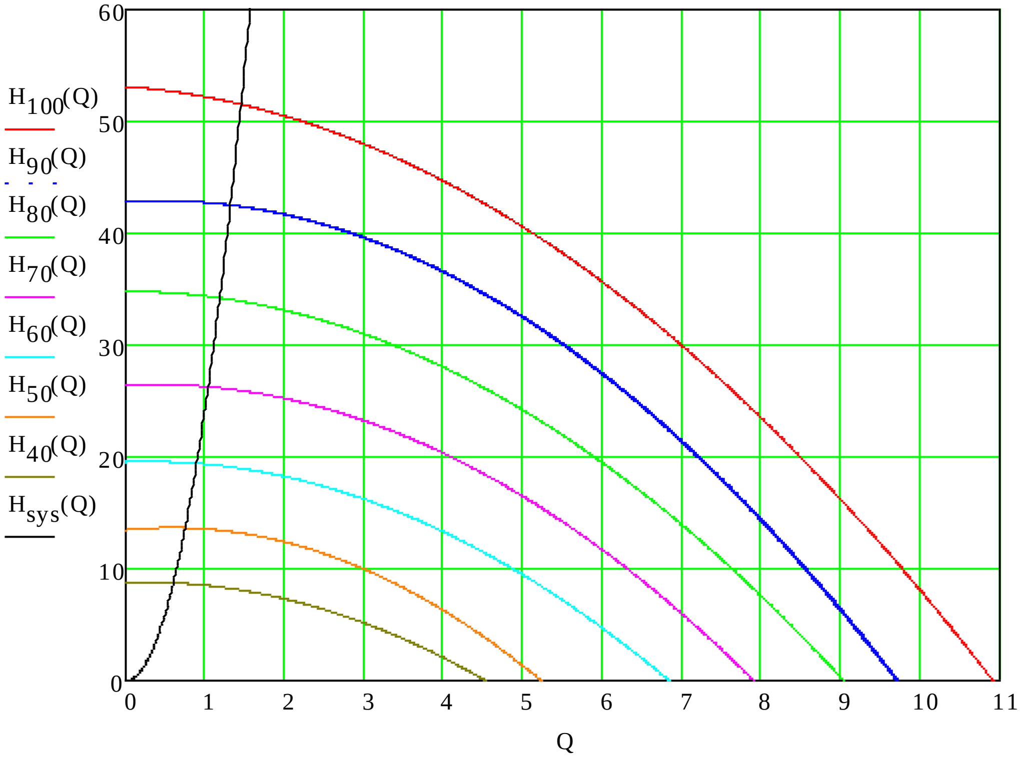


Figure 4. H-Q diagram with the theoretical system curve and the pump curves are plotted

When all the curves are been plotted in the same graph, it shows how the pump operates at different speeds. The intersection points between the system curve and all pump curves are the operation points and show how the pump should act at each speed.

Operation points for the theoretical calculated system curve:

$$H_{sys}(Q) = H_{100}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} 1.4494499745273026451 \\ -1.470190786588265387 \end{pmatrix}$$

$$H_{sys}(1.4494499745273026451) = 51.529$$

$$H_{sys}(Q) = H_{90}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} -1.300384581292467357 \\ 1.3166161549399361195 \end{pmatrix}$$

$$H_{sys}(1.3166161549399361195) = 42.517$$

$$H_{sys}(Q) = H_{80}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} -1.180655664797258122 \\ 1.1812006304764416352 \end{pmatrix}$$

$$H_{sys}(1.1812006304764416352) = 34.221$$

$$H_{sys}(Q) = H_{70}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} -1.021385440927852993 \\ 1.0349647542803109936 \end{pmatrix}$$

$$H_{sys}(1.0349647542803109936) = 26.273$$

$$H_{sys}(Q) = H_{60}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} -0.8793193903596262673 \\ 0.88983395144574400485 \end{pmatrix}$$

$$H_{sys}(0.88983395144574400485) = 19.421$$

$$H_{sys}(Q) = H_{50}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} -0.7173989301485396667 \\ 0.74622689243728829551 \end{pmatrix}$$

$$H_{sys}(0.74622689243728829551) = 13.659$$

$$H_{sys}(Q) = H_{40}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} -0.5880574913005293124 \\ 0.59682812308197140751 \end{pmatrix}$$

$$H_{sys}(0.59682812308197140751) = 8.737$$

Power, %	H, m	Q, m^3/h
100	51,53	1,45
90	42,52	1,32
80	34,22	1,18
70	26,27	1,03
60	19,42	0,89
50	13,66	0,75
40	8,74	0,60

Table 1 Operation points for the theoretical system curve.

Part three

Determine the system curve with the measured data

In the laboratory the following data was measured:

Belastning: 0						
4,3	Procent		dPT,bar	RPM	Flow LW,m3/h	Flow MÅL, m3/h
	33,3	100	0,36	2726	15,26	7,89
	30	90	0,34	2607	14,5	7,48
	26,7	80	0,32	2366	12,9	6,63
	23,4	70	0,32	2121	11,25	5,75
	20,1	60	0,3	1882	9,59	4,87
	16,8	50	0,28	1627	7,83	3,92
	13,5	40	0,28	1378	5,98	2,93
	10,2	30	0,28	1126	4,13	1,94
	6,9	20	0,28	866	2,11	0,87
	3,6	10		614		
	0,3	0		348		

Table 2. Laboratory data with 0% resistance

Belastning: 33%						
2,9	Procent		dPT,bar	RPM	Flow LW,m3/h	Flow MÅL,m3/h
	33,3	100	0,45	2726	14,26	7,34
	30	90	0,43	2607	13,6	7
	26,7	80	0,39	2366	12,11	6,2
	23,4	70	0,36	2121	10,61	5,4
	20,1	60	0,34	1882	9,08	4,59
	16,8	50	0,3	1627	7,47	3,72
	13,5	40	0,28	1378	5,73	2,79
	10,2	30	0,28	1126	4,01	1,87
	6,9	20	0,28	866	2,09	0,86
	3,6	10		614		
	0,3	0		348		

Table 3. Laboratory data with 33% resistance

With these data it is possible to construct two new system curves at 0% resistance and 33%:

$\text{Resistance1} := 0\%$

$H_{100}(7.89) = 24.2$

$H_{90}(7.48) = 18.043$

$H_{80}(6.63) = 16.078$

$H_{70}(5.75) = 12.956$

$H_{60}(4.87) = 9.993$

$H_{50}(3.92) = 6.599$

$H_{40}(2.93) = 5.327$

$\text{Resistance2} := 33\%$

$H_{100}(7.34) = 27.826$

$H_{90}(7) = 21.305$

$H_{80}(6.2) = 18.432$

$H_{70}(5.4) = 14.656$

$H_{60}(4.59) = 11.129$

$H_{50}(3.72) = 7.419$

$H_{40}(2.79) = 5.678$

Using Excel; two system curves are generated:

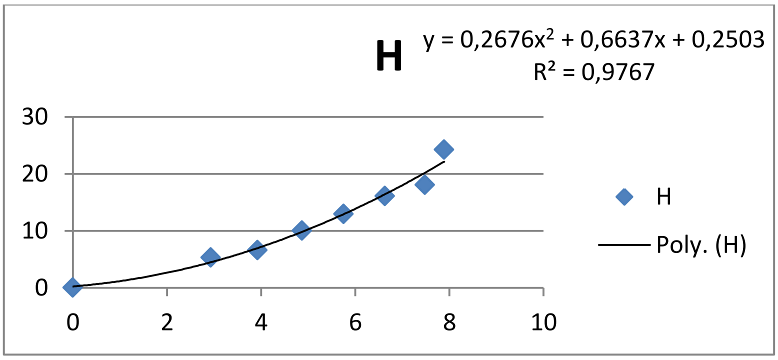
Resistance	flow	head
0%	Q	H
	7,89	24,2
	7,48	18,043
	6,63	16,078
	5,75	12,956
	4,87	9,993
	3,92	6,599
	2,93	5,327
	0	0

Table 4. Data from laboratory with 0% resistance

Resistance	flow	head
33%	Q	H
	7,34	27,826
	7	21,305
	6,2	18,432
	5,4	14,656
	4,59	11,129
	3,72	7,419
	2,79	5,678
	0	0

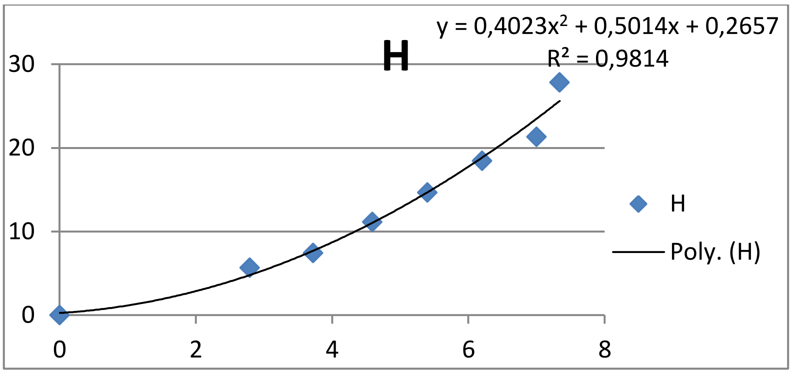
Table 5. Data from laboratory with 33 % resistance

For 0% resistance the system curve looks like this:



Graph 8 0% resitance.

For 33 % resistance the systemcurve looks like this:



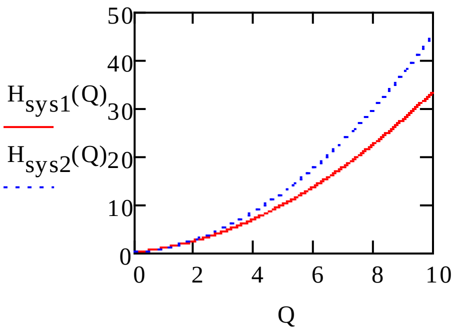
Graph 9 33% resistance.

Combined in one graph:

Practical system curves:

$$H_{sys1}(Q) := 0.2676Q^2 + 0.6637Q + 0.250$$

$$H_{sys2}(Q) := 0.4023Q^2 + 0.5014Q + 0.265$$



These practical system curves are plotted in the same graph as the theoretical system curve and the pump curves:

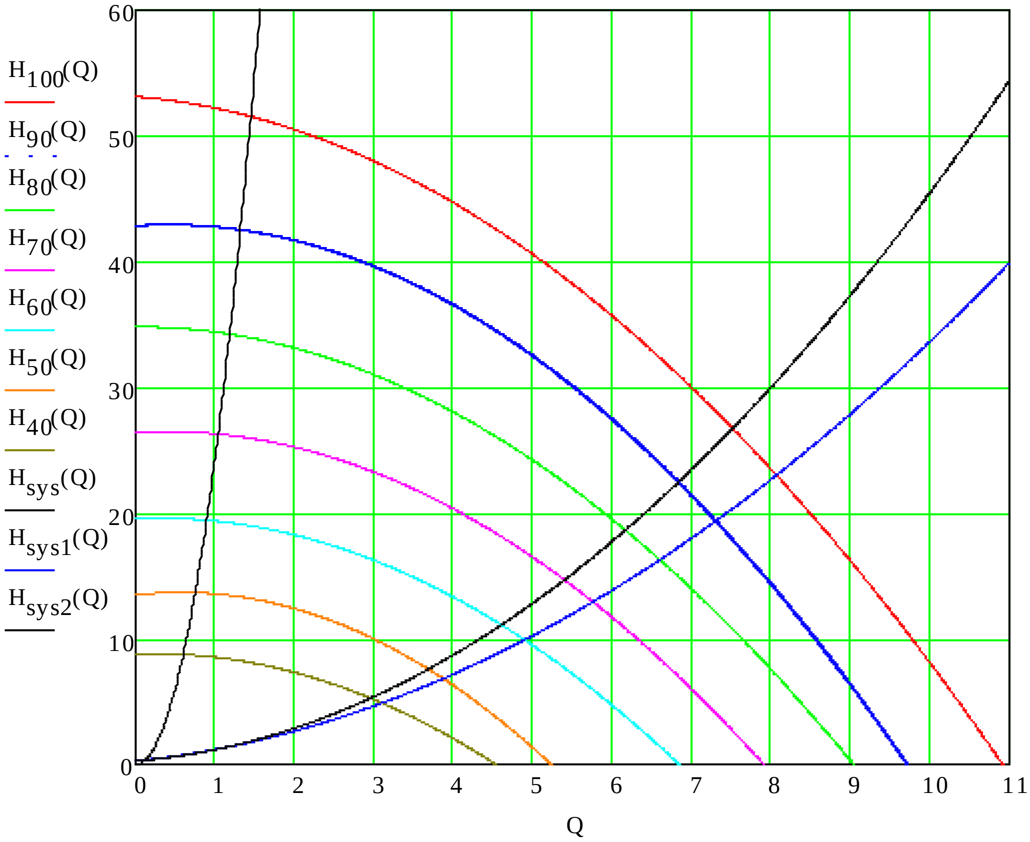


Table 6. H-Q diagram with both theoretical and practical system curve as well as the pump curves

The two new curves in this graph are the new real system curves the pump actually works from and it is shown that the blue line is the system when no resistance is attached and the black is with 33 % resistance.

Operation points with 0% resistance:

$$H_{sys1}(Q) = H_{100}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} 8.063945381366161218 \\ -9.839872141084006033 \end{pmatrix}$$

$$H_{sys1}(8.0639453813661612188) = 23.004$$

$$H_{sys1}(Q) = H_{90}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} 7.294524334514407263 \\ -7.634568784769342548 \end{pmatrix}$$

$$H_{sys1}(7.2945243345144072631) = 19.331$$

$$H_{sys1}(Q) = H_{80}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} 6.595890434355907111 \\ -7.534621917169081280 \end{pmatrix}$$

$$H_{sys1}(6.5958904343559071115) = 16.27$$

$$H_{sys1}(Q) = H_{70}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} 5.754755073245234355 \\ -6.200426511554709474 \end{pmatrix}$$

$$H_{sys1}(5.7547550732452343554) = 12.932$$

$$H_{sys1}(Q) = H_{60}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} 4.891968488057652695 \\ -5.449427051593564297 \end{pmatrix}$$

$$H_{sys1}(4.8919684880576526953) = 9.901$$

$$H_{sys1}(Q) = H_{50}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} -3.801748062112362120 \\ 3.867361941115920839 \end{pmatrix}$$

$$H_{sys1}(3.8673619411159208398) = 6.819$$

$$H_{sys1}(Q) = H_{40}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} 3.099320635364517381 \\ -3.700531126285835471 \end{pmatrix}$$

$$H_{sys1}(3.0993206353645173816) = 4.878$$

Operation points with 33% resistance:

$$H_{sys2}(Q) = H_{100}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} -8.785431227417158051 \\ 7.510774548266096877 \end{pmatrix}$$

$$H_{sys2}(7.5107745482660968779) = 26.726$$

$$H_{sys2}(Q) = H_{90}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} 6.825879353267612631 \\ -6.934594337705140421 \end{pmatrix}$$

$$H_{sys2}(6.8258793532676126319) = 22.432$$

$$H_{sys2}(Q) = H_{80}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} 6.162200819317590052 \\ -6.753044192811565955 \end{pmatrix}$$

$$H_{sys2}(6.162200819317590052) = 18.632$$

$$H_{sys2}(Q) = H_{70}(Q) \text{ solve}, Q \rightarrow \begin{pmatrix} 5.394940423877032311 \\ -5.584528076491637241 \end{pmatrix}$$

$H_{sys2}(5.394940423877032311)4 = 14.68$

$H_{sys2}(Q) = H_{60}(Q)$ solve, $Q \rightarrow \begin{pmatrix} 4.600642111243616973 \\ -4.881648283364264463 \end{pmatrix}$

$H_{sys2}(4.600642111243616973)3 = 11.088$

$H_{sys2}(Q) = H_{50}(Q)$ solve, $Q \rightarrow \begin{pmatrix} -3.468418171756391497 \\ 3.682462083159815427 \end{pmatrix}$

$H_{sys2}(3.682462083159815427)8 = 7.567$

$H_{sys2}(Q) = H_{40}(Q)$ solve, $Q \rightarrow \begin{pmatrix} 2.955392157819428738 \\ -3.279577992481236982 \end{pmatrix}$

$H_{sys2}(2.955392157819428738)3 = 5.261$

Power, %	H, m	Q, m^3/h
100	23,00	8,06
90	19,33	7,29
80	16,27	6,60
70	12,93	5,75
60	9,90	4,89
50	6,82	3,87
40	4,88	3,10

Table 7. Practical operation points for the pump with 0% resistance

Power, %	H, m	Q, m^3/h
100	26,73	7,51
90	22,43	6,83
80	18,63	6,16
70	14,68	5,39
60	11,01	4,60
50	7,57	3,68
40	5,26	2,96

Table 8. Practical operation points for the pump with 33% resistance

Part four

Determine the system curve with the Affinity law

The affinity law is used to make two extra systemcurves as in Part three, just by using the law to predict the flow rate and then use the pump curves to determine the head.

System curve with no resistance according to the affinity law:

$Q_1 := 7.89$

$$Q_2 := 7.89 \cdot \left(\frac{2607}{2726}\right) = 7.546$$

$$Q_3 := 7.48 \cdot \left(\frac{2366}{2607}\right) = 6.789$$

$$Q_4 := 6.63 \cdot \left(\frac{2121}{2366}\right) = 5.943$$

$$Q_5 := 5.75 \cdot \left(\frac{1882}{2121}\right) = 5.102$$

$$Q_6 := 4.87 \cdot \left(\frac{1627}{1882}\right) = 4.21$$

$$Q_7 := 3.92 \cdot \left(\frac{1378}{1627}\right) = 3.32$$

$$Q_8 := 2.93 \cdot \left(\frac{1126}{1378}\right) = 2.394$$

$$Q_9 := 1.94 \cdot \left(\frac{866}{1126}\right) = 1.492$$

$$Q_{10} := 0.87 \cdot \left(\frac{614}{866}\right) = 0.617$$

Pumpcurves:

$$H_{100}(Q) := -0.3987Q^2 - 0.5196Q + 53.12$$

$$H_{90}(Q) := -0.4973Q^2 + 0.4036Q + 42.848$$

$$H_{80}(Q) := -0.4277Q^2 + 0.011Q + 34.80$$

$$H_{70}(Q) := -0.4659Q^2 + 0.3368Q + 26.423$$

$$H_{60}(Q) := -0.4564Q^2 + 0.2601Q + 19.55$$

$$H_{50}(Q) := -0.6316Q^2 + 0.7227Q + 13.47$$

$$H_{40}(Q) := -0.4759Q^2 + 0.2167Q + 8.7776$$

$H_{100}(Q_1) = 24.2$

$H_{90}(Q_2) = 17.579$

$H_{80}(Q_3) = 15.17$

$H_{70}(Q_4) = 11.967$

$H_{60}(Q_5) = 8.997$

$H_{50}(Q_6) = 5.318$

$H_{40}(Q_7) = 4.251$

$H_1 := 24.2$

$$H_2 := 24.2 \cdot \left(\frac{2607}{2726}\right)^2 = 22.133$$

$$H_3 := 17.579 \cdot \left(\frac{2366}{2607}\right)^2 = 14.479$$

$$H_4 := 15.17 \cdot \left(\frac{2121}{2366}\right)^2 = 12.191$$

$$H_5 := 11.967 \cdot \left(\frac{1882}{2121}\right)^2 = 9.422$$

$$H_6 := 8.997 \cdot \left(\frac{1627}{1882}\right)^2 = 6.724$$

$$H_7 := 5.318 \cdot \left(\frac{1378}{1627}\right)^2 = 3.815$$

Systemcurve with 33% resistance according to the affinity law:

$Q_{11} := 7.34$

$Q_{12} := 7.34 \cdot \left(\frac{2607}{2726}\right) = 7.02$

$Q_{13} := 7 \cdot \left(\frac{2366}{2607}\right) = 6.353$

$Q_{14} := 6.2 \cdot \left(\frac{2121}{2366}\right) = 5.558$

$Q_{15} := 5.4 \cdot \left(\frac{1882}{2121}\right) = 4.792$

$Q_{16} := 4.59 \cdot \left(\frac{1627}{1882}\right) = 3.968$

$Q_{17} := 3.72 \cdot \left(\frac{1378}{1627}\right) = 3.151$

$Q_{18} := 2.79 \cdot \left(\frac{1126}{1378}\right) = 2.28$

$Q_{19} := 1.87 \cdot \left(\frac{866}{1126}\right) = 1.438$

$Q_{20} := 0.86 \cdot \left(\frac{614}{866}\right) = 0.61$

$H_{100}(Q_{11}) = 27.826$

$H_{90}(Q_{12}) = 21.177$

$H_{80}(Q_{13}) = 17.613$

$H_{70}(Q_{14}) = 13.903$

$H_{60}(Q_{15}) = 10.319$

$H_{50}(Q_{16}) = 6.394$

$H_{40}(Q_{17}) = 4.736$

$H_{11} := 27.826$

$H_{12} := 27.826 \cdot \left(\frac{2607}{2726}\right)^2 = 25.45$

$H_{13} := 21.177 \cdot \left(\frac{2366}{2607}\right)^2 = 17.443$

$H_{14} := 17.613 \cdot \left(\frac{2121}{2366}\right)^2 = 14.154$

$H_{15} := 13.903 \cdot \left(\frac{1882}{2121}\right)^2 = 10.946$

$H_{16} := 10.319 \cdot \left(\frac{1627}{1882}\right)^2 = 7.712$

$H_{17} := 6.394 \cdot \left(\frac{1378}{1627}\right)^2 = 4.587$

Q	H
7,89	24,2
7,546	22,133
6,789	14,479
5,943	12,191
5,102	9,422
4,21	6,724
3,32	3,815
0	0

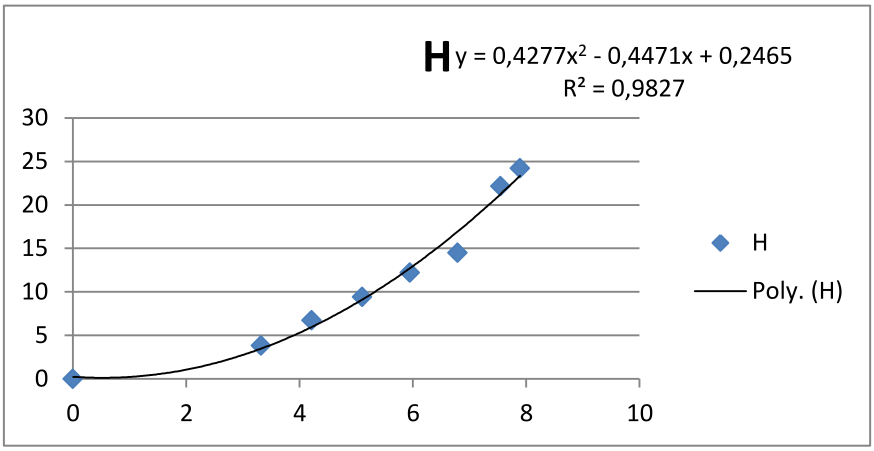


Figure 5. The acquired data from the affinity law and the system curve with 0% resistance

Q	H
7,34	27,826
7,02	25,45
6,353	17,442
5,558	14,154
4,792	10,946
3,968	7,712
3,151	4,587
0	0

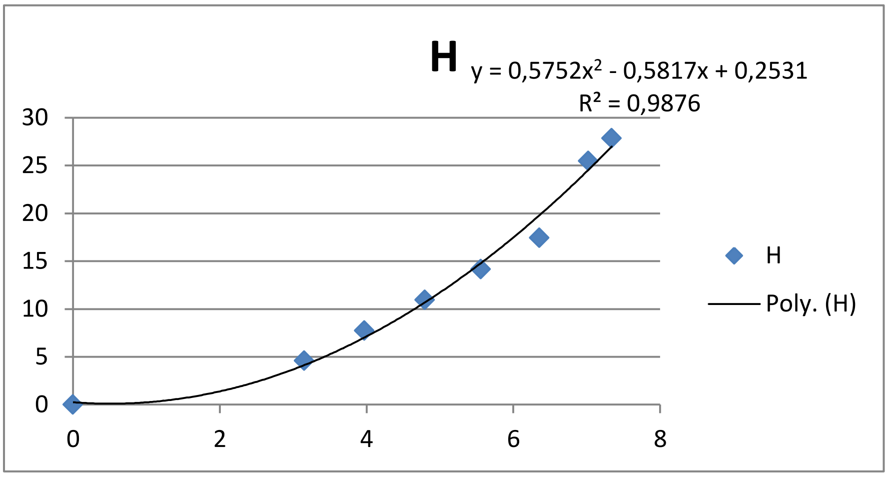


Figure 6. The acquired data from the affinity law and the system curve with 33% resistance

System curve by the Affinity law with 0% resistance:

$$H_{affinity0sys}(Q) := 0.4277Q^2 - 0.4471Q + 0.246$$

System curve by the Affinity law with 33% resistance:

$$H_{affinity33sys}(Q) := 0.5752Q^2 - 0.5817Q + 0.253$$

The practical system curve with 0% resistance and 33% resistance:

$$H_{sys1}(Q) := 0.2676Q^2 + 0.6637Q + 0.250$$

$$H_{sys2}(Q) := 0.4023Q^2 + 0.5014Q + 0.265$$

Plotting both system curves by the Affinity law and both system curves acquired from data in the laboratory in the same graph makes it possible to compare the theoretical system curve with the practical.

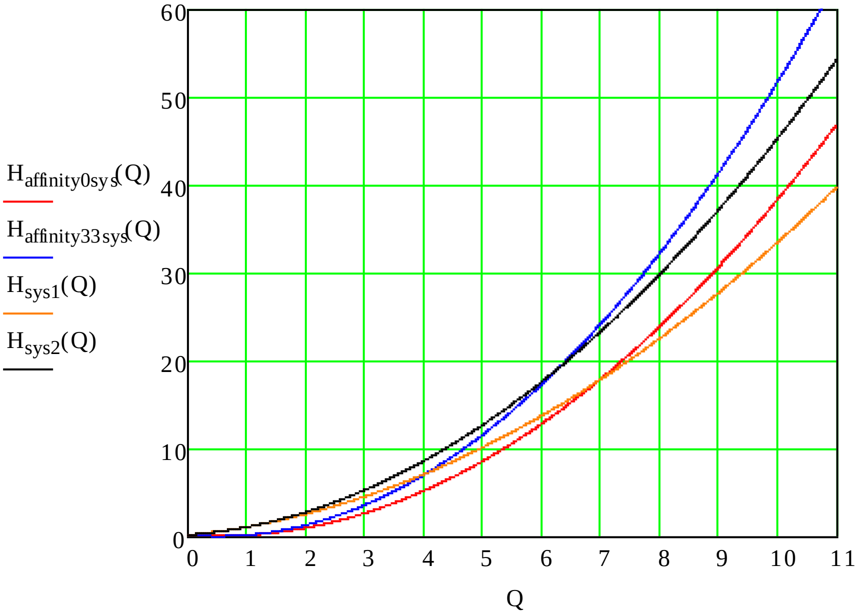


Figure 7. H-Q diagram with both system curve from the Affinity law and laboratory data

This shows a slight deviation between the theoretical and practical. This is caused by the way the affinity law because the Affinity law does not include all the systems minor losses.

Conclusion

In this experiment the pump curves and system curves have been described. These are used to give a proper look into the pump and how they operate at different speeds with different resistances. The CRE 5-8 pump was described based on its pump curves, by comparing them with the system curves – these have been found and described theoretically and practically. Five system curves ware found: one by using regular thermodynamics, two with use of affinity laws, and two practical by using recorded data from experiments.

The first system curve, determined by basic thermodynamics, turned out to be rather steep and the pump seemed to only be able to operate with a flow between 0.5 and 1.5 m^3/s. This turned out to be false, because the pump was able to operate with speeds of 9 m^3/s. Why this error was present, has not been possible to decide.

The second and third system curve was constructed from the live laboratory measurements. These curves was made by use of a resistance and no resistance; these curves can be said to be the actual true ones that describes the actual system, and since the difference from the theoretical and the practical system curve is this huge, there is something wrong with the theoretical.

Finally the system curve has been done in a different theoretical way. The affinity laws are used to find an approximate value for other speeds of a pump by using the RPM as base for changes. This law is fine to use if there is no way to do actual measurements on the pump, but only having data from one operation point and nothing about how the entire system is built. Therefore this is only acceptable for quick reviews of pumps and not for a final determination of which pump is needed.

APPENDIX 2

In the following section the head loss and pressure drop will be calculated.

Description

The water in the booster system is about 20 degree Celsius and the system has a total loss coefficient, KL, of 20.299.

The pipes are made by commercial steel and the roughness ε is 0.0045. The total lengths of the pipes in the vertical direction are 286 cm and the total lengths in horizontal direction are 289 cm. The pipe diameter is 5.4 cm. Therefore we have the following information to work with:

$T = 20\,°C$

$H = 2.86m$

$L = 5.75m$

$D = 0.054mg = 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$

$\rho = 998\,\frac{kg}{m^3}$

$\mu = 1.002 \cdot 10^{-3}\,\frac{kg}{m \cdot s}$

$\varepsilon = 0.045mm$

$\frac{\varepsilon}{D} = \frac{0.0045cm}{5.4cm} = 0.000833$ is the relative roughness

$K_L = 20.299$

The measured flow rates are:

Pump	Flow m3/h
100%	7,89
90%	7,48
80%	6,63
70%	5,75
60%	4,87
50%	3,92
40%	2,93
30%	1,94
20%	0,87

Table 9 indicates the measured flow rates at different pump speeds.

To determine the Reynolds number, the velocity in the pipe must be determined. Here is an example for pump at 100%:

$$V = \frac{Q}{A} = \frac{flow\ rate}{\frac{\pi}{4} \cdot 0{,}054m^2} = \frac{7.89\, {}^{m^3}/_{h} \cdot \frac{1h}{3600s}}{\frac{\pi}{4} \cdot 0.054m^2} = 0.96\, {}^{m}/_{s}$$

Equation 3 is used to determine the average velocity in the pipe.

Combined with the different flow rates, following velocities are obtained:

% active pump	Pump speed (m/s)
V100	0,957452695
V90	0,907699133
V80	0,804551504
V70	0,697763371
V60	0,590975238
V50	0,475692594
V40	0,355555944
V30	0,235419294
V20	0,105574632

Table 10 indicates the flow rates at different velocities in the pipes. From Equation 3.

Then the Reynolds number can be determined. Example:

$$Re = \frac{\rho V D}{\mu} = \frac{998\frac{kg}{m^3} \cdot V \cdot 0{,}054\, m}{1{,}002 \cdot 10^{-3} kg/m \cdot s} = \frac{998\frac{kg}{m^3} \cdot 0{,}96\, {}^{m}/_{s} \cdot 0{,}054\, m}{1{,}002 \cdot 10^{-3} kg/m \cdot s} = 51633{,}05$$

Equation 4 shows the determination of the Reynolds number.

The same step is followed for all velocities so following is obtained:

	Reynolds number
Re100	51496,04856
Re90	48820,08152
Re80	43272,34499
Re70	37528,80598
Re60	31785,26698
Re50	25584,85556
Re40	19123,37418
Re30	12661,8928
Re20	5678,271514

Table 11 are the velocities obtained by different Reynolds numbers. From Equation 4.

The Reynold number is over 10.000 for all velocities except when the pump is 20% active. This means that the flow at all flow rates, beside at the 20%, the flow is turbulent.

To determine the Darcy friction factor f, the Moody chart can be used. Another option is to use the Colebrook equation for determining the Darcy friction factor in turbulent flow. Example for pump at 100%:

$$\frac{1}{\sqrt{f}} = -2.0\ log\left(\frac{\frac{\varepsilon}{D}}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) = -2.0\ log\left(\frac{0,000833}{3.7} + \frac{2.51}{51496,06\sqrt{f}}\right)$$

Equation 5. The determination of the Darcy friction factor for a pump

Solved for f at 100% pump speed:

$$f = 0,0235$$

MathCad is used for solving f, and when this is done for all Reynolds numbers, following friction factors are found, see Table 12 at next page.

Friction factors for all Reynolds numbers:

	Friction factor
f100	0,0235
f90	0,0236
f80	0,0241
f70	0,0246
f60	0,0253
f50	0,0263
f40	0,028
f30	0,0304
f20	0,037

Table 12. The solved f values for all Reynolds numbers.

For the system the friction factor increases when the Reynolds number becomes larger.

In the pump system the diameter is constant so the following equation is valid for determining the head loss:

$$h_{L,total} = (f\frac{L}{D} + \sum K_L)\ \frac{V_{avg}^2}{2g}$$

Equation 6. The total head loss.

An example of this calculation for the pump at 100%:

$$h_{L,total} = \left(0{,}0235\frac{5{,}75m}{0{,}054m} + 20{,}299\right)\frac{(0{,}96\,{}^{m}/_{s})^2}{2\cdot 9{,}82\,{}^{m}/_{s^2}} = 1{,}07m$$

The head loss for all friction factors and velocities is:

	Headloss
HL100	1,066445023
HL90	0,958937777
HL80	0,755138662
HL70	0,569305528
HL60	0,409711153
HL50	0,266685002
HL40	0,15015914
HL30	0,06655204
HL20	0,013783974

Table 13 shows the result of the head loss for all friction factors and velocities.

The head losses become larger for higher Reynolds numbers.

The pressure drop can also be calculated by using the equation:

$$h_L = \frac{\Delta P_L}{\rho g} \Leftrightarrow \Delta P_L = \rho g h_L$$

Equation 7. The pressure drop.

An example for a pump at 100%:

$$\Delta P_L = 998\, {}^{kg}/_{m^3} \cdot 9{,}80\, {}^{m}/_{s^2} \cdot 1{,}07m = 10{,}4kPa$$

All the pressure drops are calculated:

	Pressure drop
PL100	10430,2589
PL90	9378,795035
PL80	7385,558168
PL70	5568,035787
PL60	4007,138961
PL50	2608,285989
PL40	1468,616455
PL30	650,905573
PL20	134,812784

Table 14. All the head losses for different pressure drops.

The pressure drop becomes larger for higher Reynolds numbers.

As an addition to the Reynolds number, the entry length can be determined through a simple equation:

$$\frac{L_{h,turbulent}}{D} = 1{,}359Re^{1/4}$$

Equation 8. The entry length

An example for pump at 100%:

$$\frac{L_{h,turbulent}}{D} = 1{,}359Re^{\frac{1}{4}} \Leftrightarrow L_{h,laminar} = 1{,}359Re^{1/4}D$$

$$L_{h,turbulent} = 1{,}359 \cdot 51633{,}05^{1/4} \cdot 0{,}054m = 1{,}11m$$

And for all Reynolds numbers:

	Entry length
Lh100	1,105494772
Lh90	1,090844488
Lh80	1,058438977
Lh70	1,021420108
Lh60	0,979872755
Lh50	0,928130317
Lh40	0,862987186
Lh30	0,778462819
Lh20	0,637040811

Table 15 shows the calculated entry length for different Reynolds numbers.

The entry length is longer for higher Reynolds numbers. Which means for higher Reynolds number it takes longer time, for the flow, to achieve a fully developed flow.

APPENDIX 3

The purpose of this experiment is to see if the pre-determined constants for the PID controller are valid. If they are invalid, the new information will be used for tuning the PID controller.

Description

The following constant is determined.

$$K_P = 1; K_i = 1 \rightarrow t_i = \frac{1}{1} = 1, K_D = 0{,}1$$

These constants are plotted into the front panel of LabVIEW, where the controller is implemented and running in "AUTO" mode. Then after changing the setpoint the program will regulate the flow. The controller in LabVIEW is a consisting program, where unnecessary parts are deleted.

The control platform of LabVIEW:

Figure 8 shows the control platform in LabVIEW.

- (1) The percentage speed of the pump
- (2) The switch between manual and automatic mode. This function has to be in automatic mode to run, but in manual mode to change the constant values.
- (3) Indicates the flow rate.
- (4) Indicates the setpoint for the flow rate.
- (5) Indicates the actual percentage speed of the pump
- (6) The PID panel. Here the constants values for the controller can be changed.
- (7) The display – illustrates all the parameters; flow rate (green), setpoint (red) and percentage speed (yellow)

The progress

The experiment is divided into section:

1) The constants were inserted to the controller in LabVIEW. The response is illustrated below. The red curve is the set point, the green is the flow rate, and the yellow curve is the motor speed given in %.

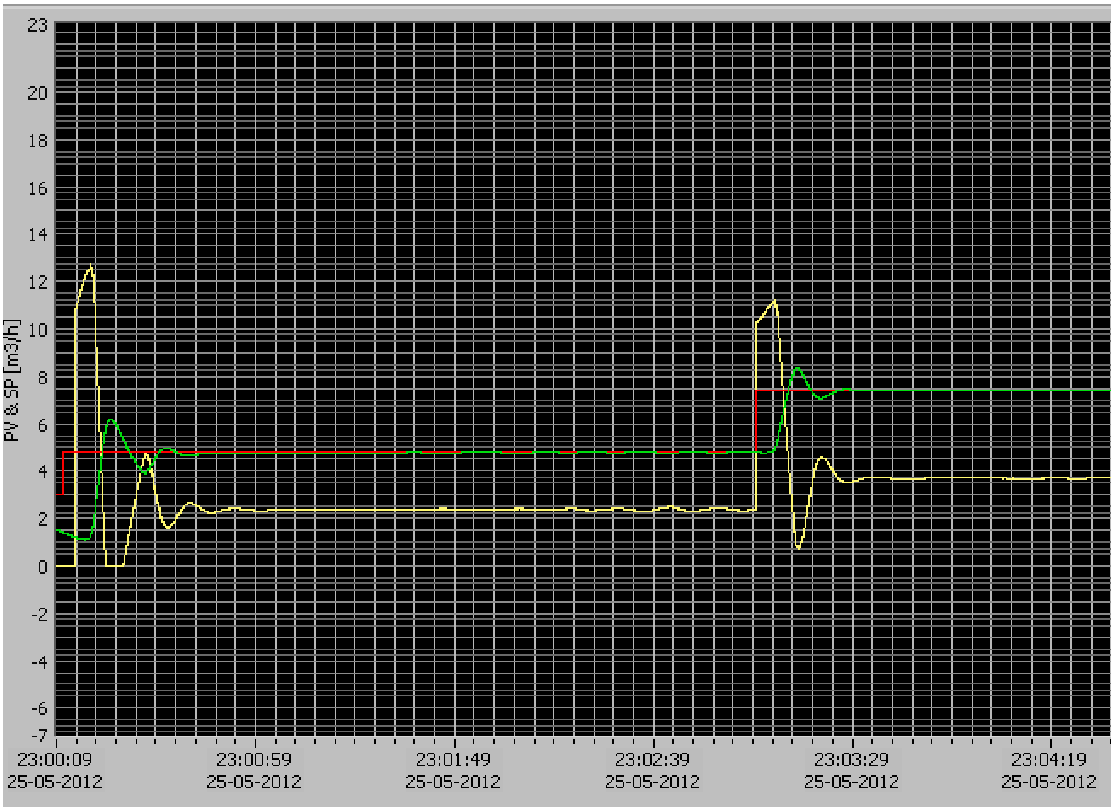


Figure 9 shows the response of the pump system. The x-axis indicates the time and the y-axis indicates the flow rate.

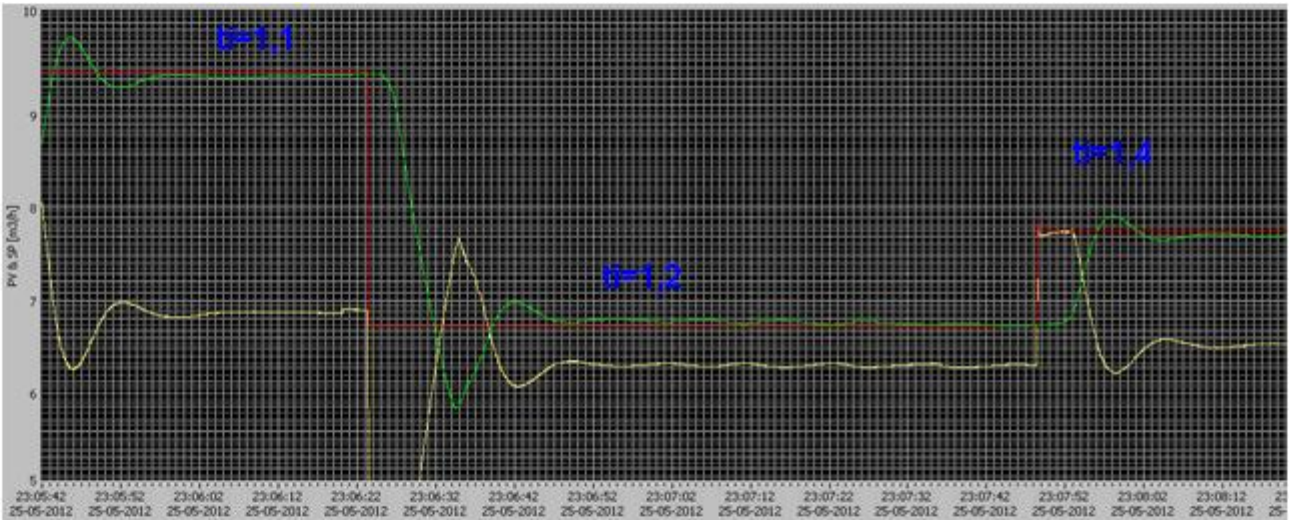
From the graph it is possible to see the response for the pump system is very stable – it doesn't oscillates too much.

At the beginning of the graph, the setpoint is given to be $5{,}2\frac{m^3}{h}$. The pump system has a fast settling- and rise time. The response settles to the setpoint in some few seconds. The overshoot is about 20 %.

Further into the graph there is another response, this is for a new setpoint, $7{,}5\frac{m^3}{h}$ - this is done to check if the pump system would settle like before, when the constants first time were inserted.

An improve could be the steady state error could be totally eliminated, it is about 0,02-0,03 for these constants.

2) To eliminate the steady state error, a larger and a smaller T_i value is chosen, $T_i = [1; 1{,}4]$. This change in this constant doesn't affect the response positively. It results in a little change in the steady state error – it becomes larger. To see this look at the graph below.



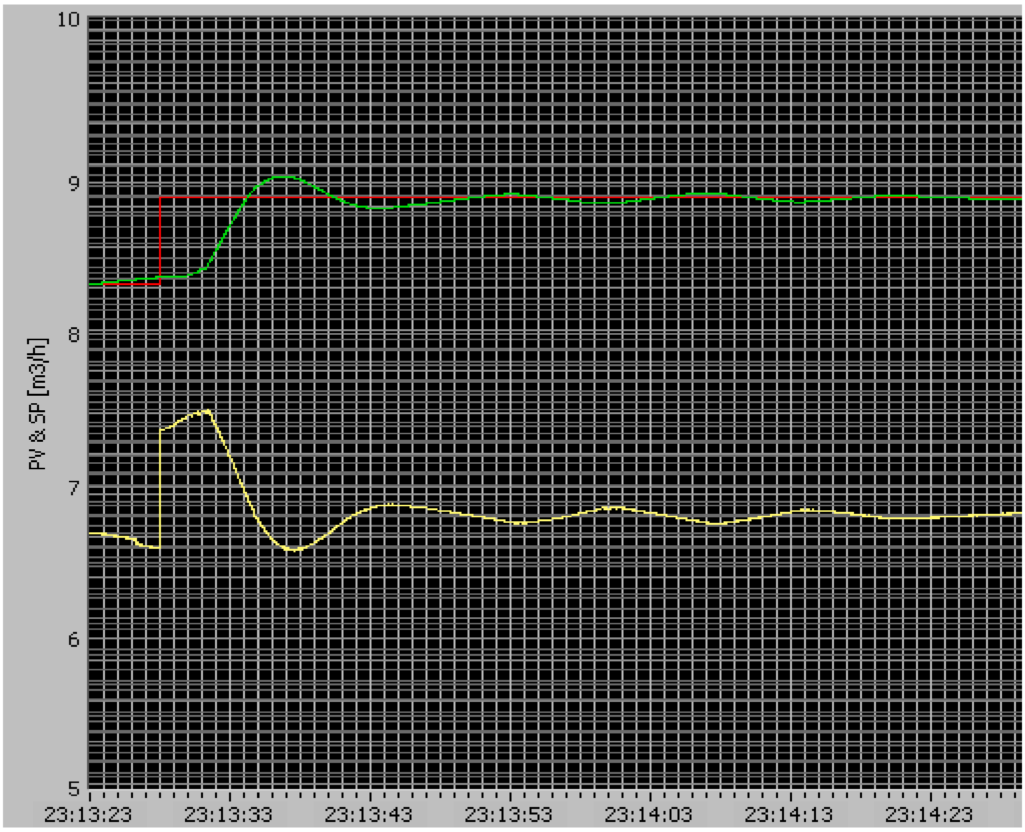
Figur 1 the graph illustrates was happens when the integral constant is changed- The graph iluustrates the response for three $K_i = 1,1, 1,2$ *and* $1,4$. The x axis indicates the time amd the y axis indicates the output.

From this graph it is possible to see that the response still settles about the setpoint but as the T_i becomes larger. And thereby $K_i$becomes smaller; the steady state error becomes larger.

A smaller constant value was also inserted but had no positive change; it coursed the same steady state error.

Based on this $T_i = 1 \rightarrow K_i = 1$ was the best constant value for this system.

4) Another way to improve the steady state error could be to change, K_D. It was changed to $\mathbf{0.2}$, but this change made the system more unstable, by that meaning it began to oscillate about the setpoint. See the graph below.



From this tuning process it can be concluded that the determined constant are the best fitted values for the controller:

$$k_P = 1; K_i = 1; K_D = 0,1$$

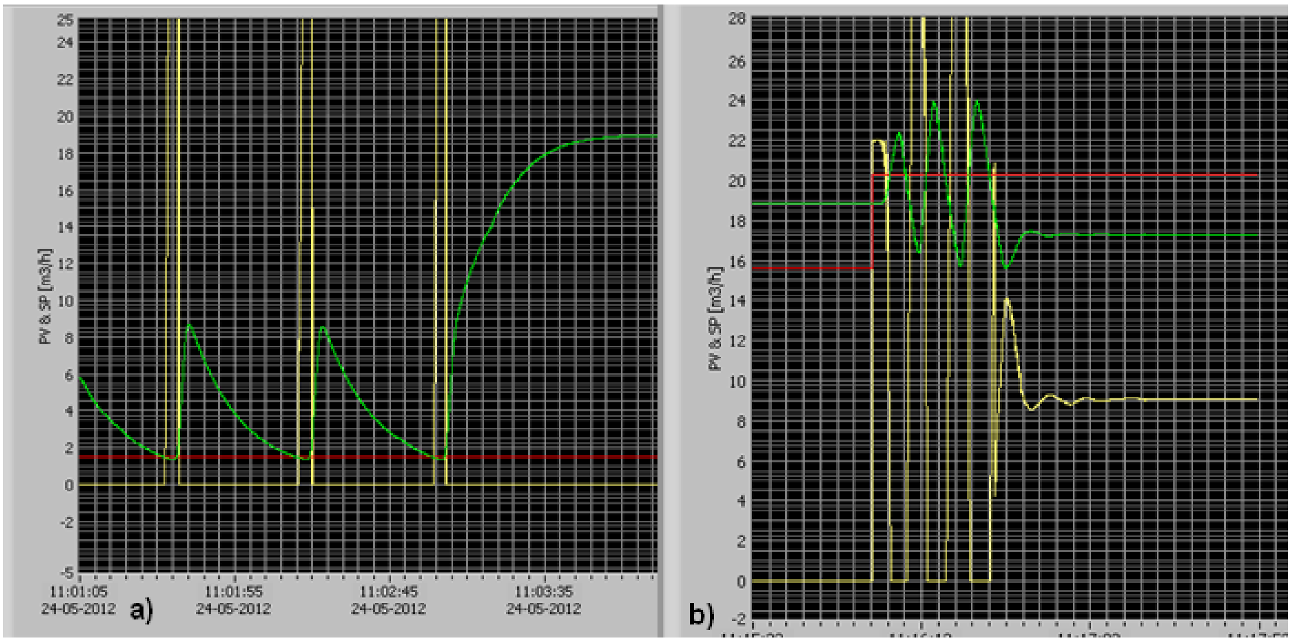
These constants have a very good response with an overshoot at 20% and a fast rise time and settle time. The overshoot is a little higher then estimated, but it still is acceptable. The rise time is about 1 second, and the settle time about some few seconds.

Discussion and conclusion:

Before in this project a different transfer function was found for the pump system, which not was correct, there were determined constants for the PID to fit this, the following properties were determined:

$$K_P = 100; K_i = 100; K_D = 0{,}1$$

There is a big difference between these constant values. Especial the proportional-and the integral constants. The response for the pump system with these constants looked like:



Figur 2 illustartions of the response. a) First time the constant were inserted b) tuning process

From the graph is it possible to see; the response in a) had a very fast rise time – too fast, and took long time to settle. In the response in b) the constants were tuned to make the steady state error smaller and the response curve better. After a long turning process the response had a smaller steady state error, but oscillated continuously around the setpoint.

The new transfer function is based on a proper linearization of the system. This transfer function has resulted in a much better response, which was concluded too be the best constant for the PID controller for this specific pump system.

APPENDIX 4

For the experiments with the pump system a premade LabVIEW setup was used. The setup included some extra pressure and flow controllers, which has been modified for the system used in this project. The modified setup consists of a differential pressure control, see Figure 10, this controller regulates the speed of the pump to give the desired pressure difference between the inlet and the outlet of the pump. The controller can be in manual mode, "MAN", or automatic mode, "AUTO". When the pump is operating in "AUTO" a setpoint is chosen and the controller will then regulate the pump speed, to get meet the desired setpoint. The program also has a box where the PID constants can be changed and a switch to turn the pump on and off. This switch is only placed at the "Differential pressure control" page but affects the whole program.

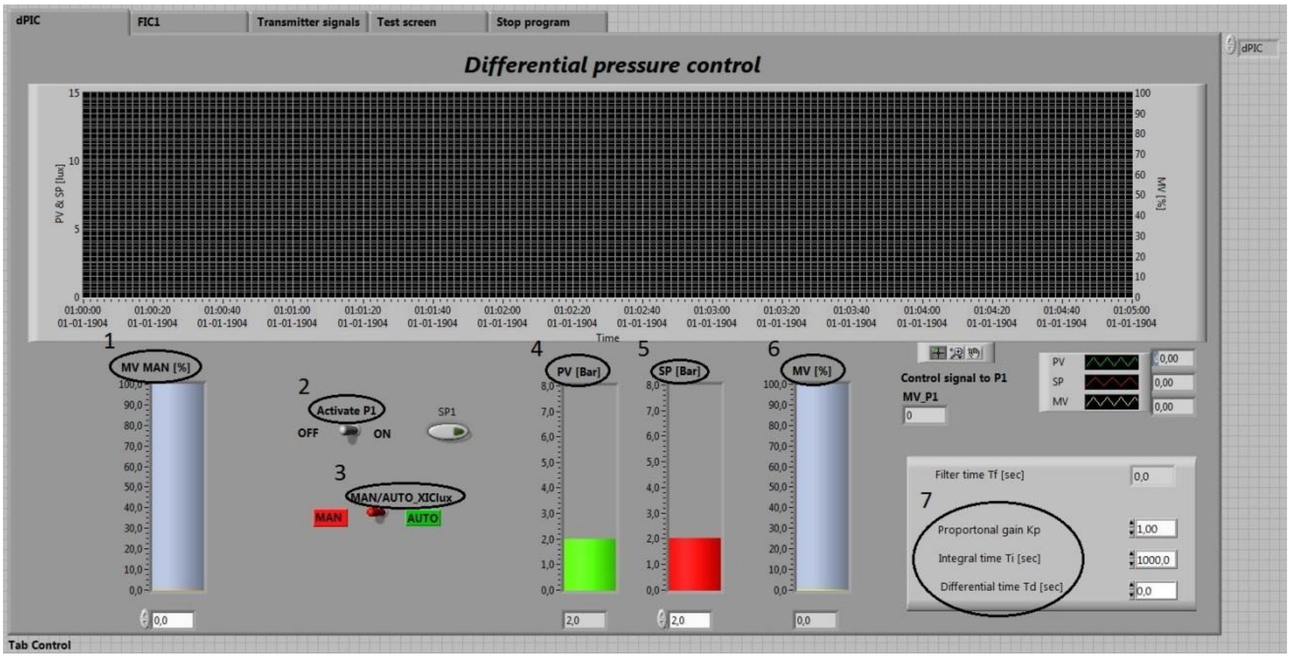


Figure 10 shows the differential pressure control in LabVIEW. Own figure.

1. MV MAN slider that controls the pump speed in "MAN" mode
2. On/off switch for the pump
3. MAN/AUTO switch for the controller
4. PV indicator that show the current pressure in Bar
5. SP slider where the desired pressure setpoint can be change while the controller is in "AUTO" mode
6. MV indicator that indicates the current pump speed
7. PID input box

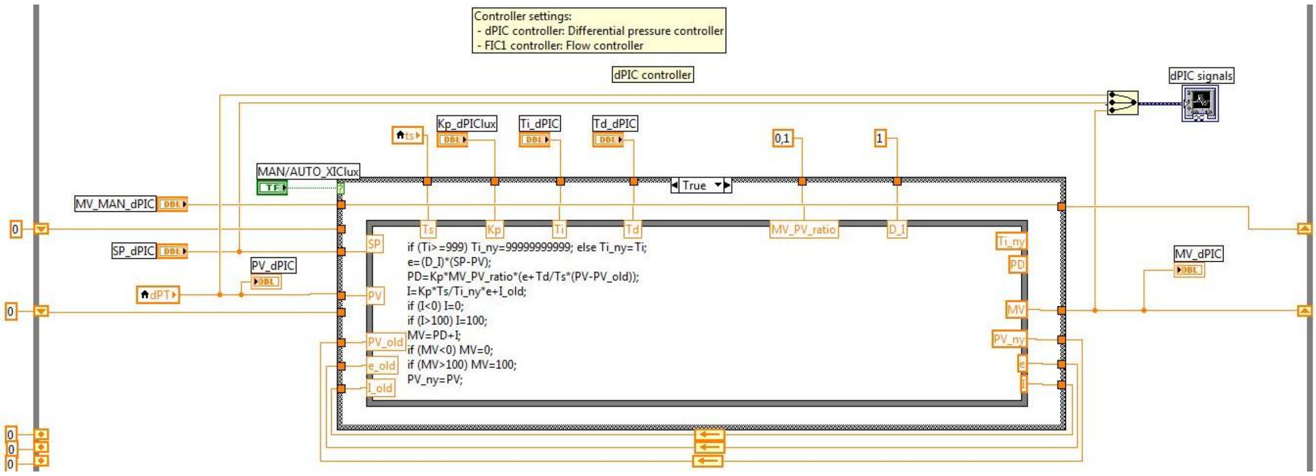


Figure 11. Own figure. The block diagram of the differential pressure control page

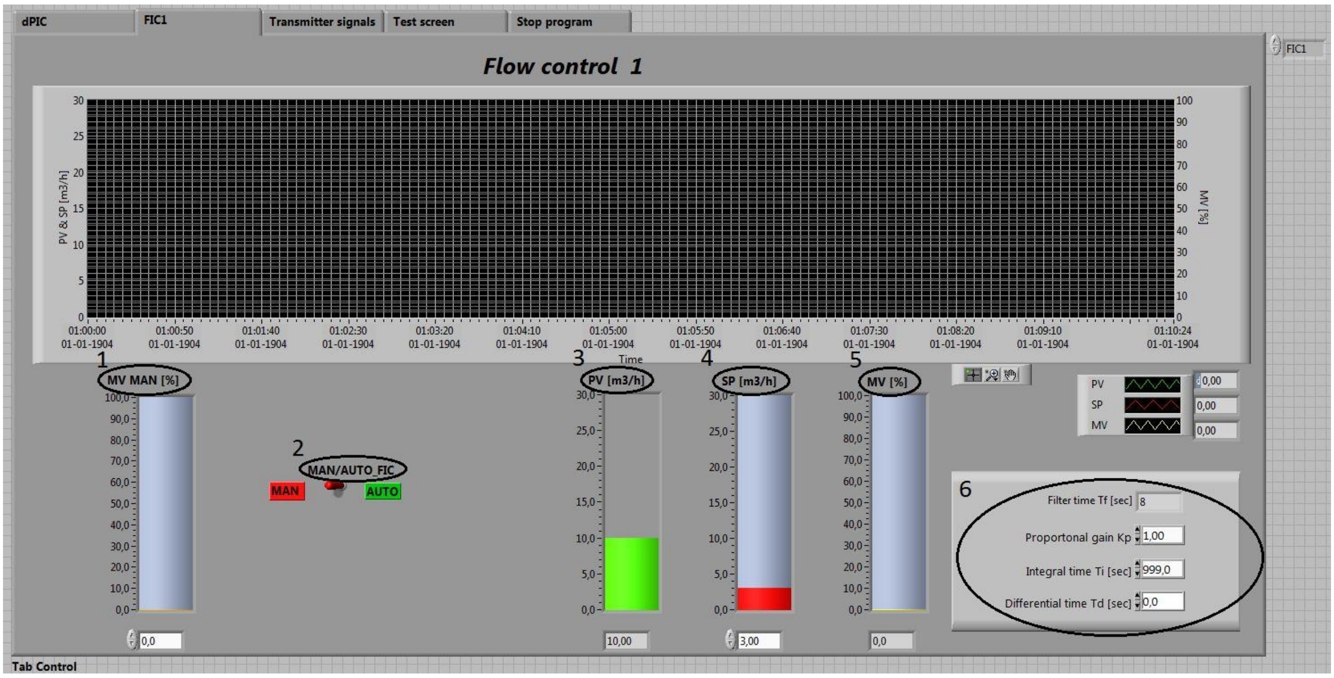


Figure 12. Own figure. The flow controller page

1. MV MAN slider that controls the pump speed in "MAN" mode
2. MAN/AUTO switch for the controller
3. PV indicator that show the current flow rate
4. SP slider where the desired flow rate setpoint can be change while the controller is in "AUTO" mode
5. MV indicator that indicates the current pump speed
6. PID input box

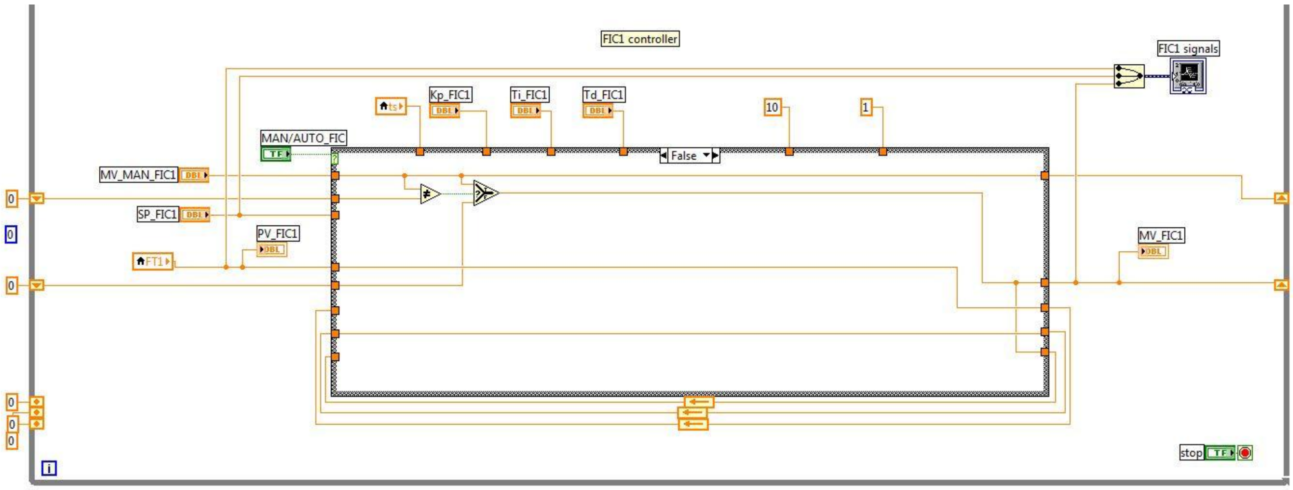


Figure 13. Own figure. The block diagram of the flow controller page

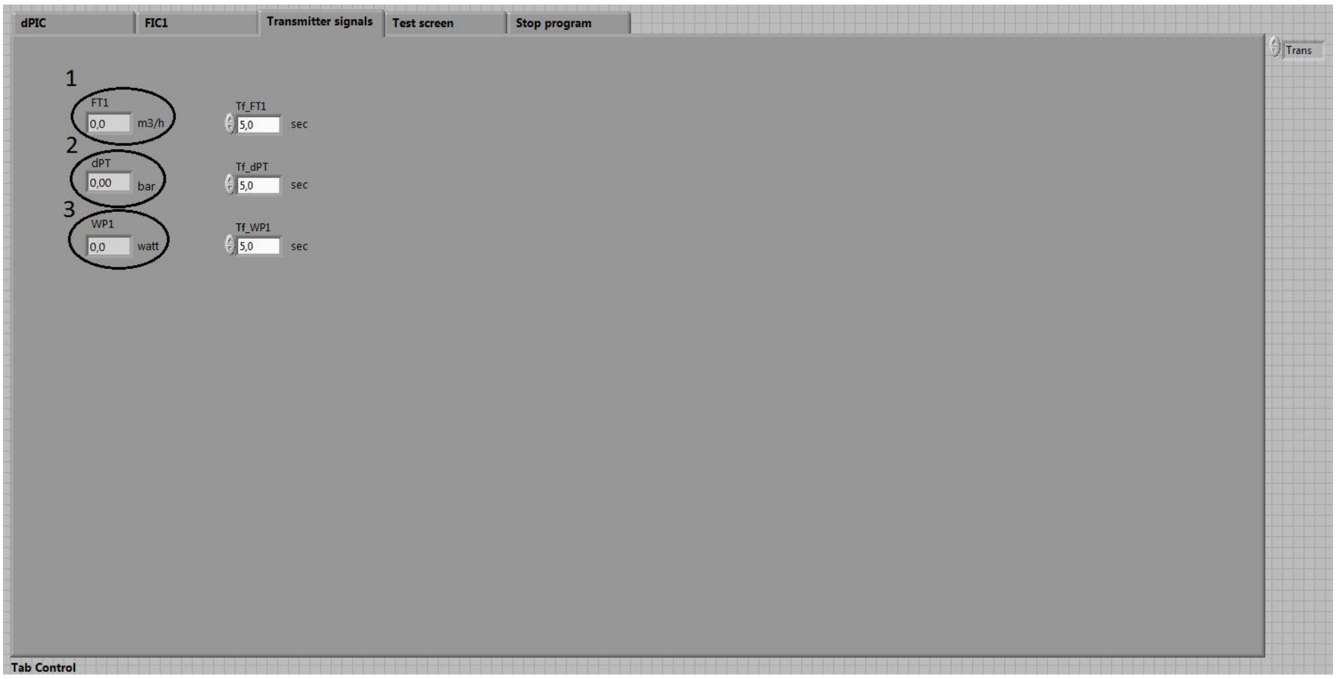


Figure 14. Own figure. The transmitter page.

1. Indicator that show the current flow rate of the system
2. Indicator that show the current pressure difference for the inlet and the outlet of the pump
3. Indicator that show the power of the pump

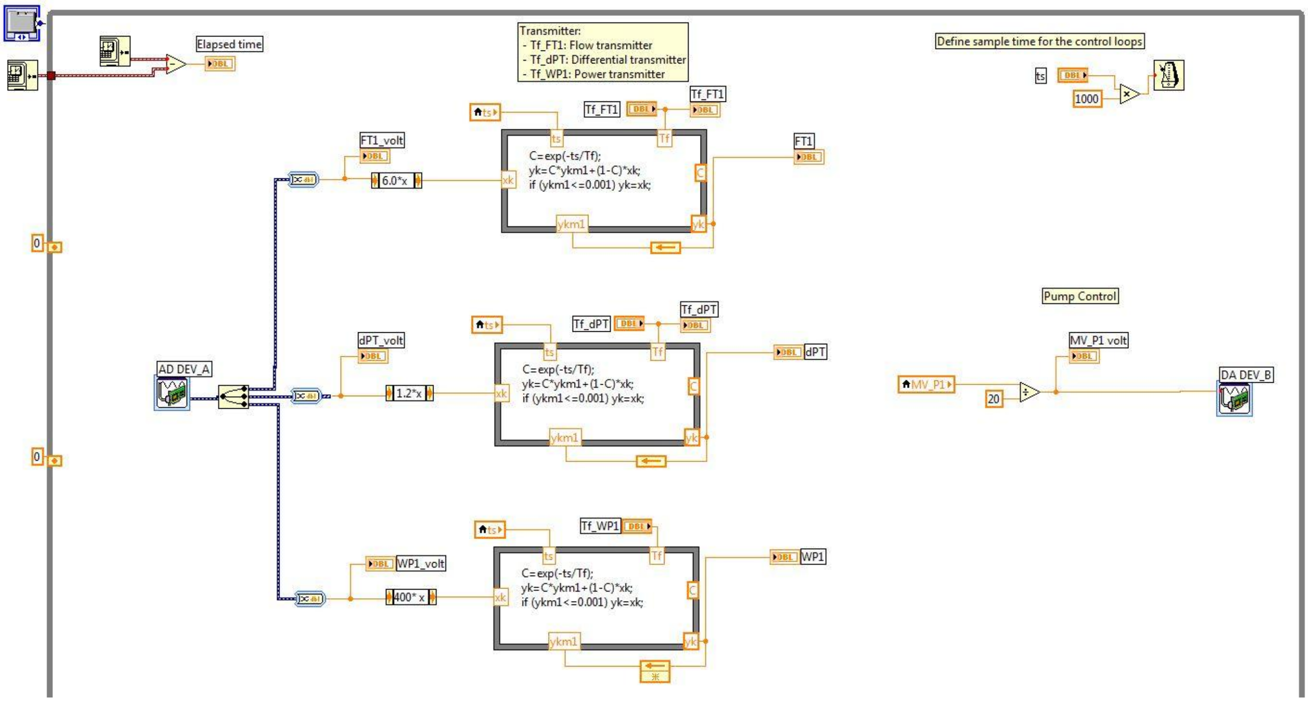


Figure 15. Own figure. The block diagram of the transmitter page