Power Consumption Optimization for Multiple Parallel Centrifugal Pumps

Kasper L. Jepsen[1 2], Leif Hansen[1], Christian Mai[1] and Zhenyu Yang[1 3]

Abstract— **Large amounts of energy is being used in a wide range of applications to transport liquid. This paper proposes a generic solution for minimizing power consumption of a generic pumping station equipped with identical variable speed pumps. The proposed solution consists of two sequential steps; firstly, the energy consumption for a pre-selected set of active pumps are formulated as a convex optimization problem and that can be solved analytically. Secondly, the problem of choosing the number of active pumps are formulated and solved using a convex solver subject to constraints. The proposed solution is compared with a conventional affinity-law based solution. The experimental analysis showed that the proposed solution and method lead to more precise pump modeling and simplicity in solving this type of pump optimization problem.**

I. INTRODUCTION

Transporting liquid by means of electrically powered pumps is widely used in process systems, such as oil and gas production plants, power plants, cooling systems, water delivery systems, etc. Compared to other groups of industrial equipment, pumps are responsible for consuming the largest amounts of energy in the European union's industry, using an extraordinary 160 TWh per annum [1]. In the USA, almost 1% of the energy usage is consumed by municipal water utilities, and a major part of the energy is consumed by pumping systems [2].

Many conventional pumping systems still deploy on/off switching control, meaning that only a fixed set of pressure and flow rate combined is achievable, as a result of on/off control strategy, potentially higher pressure and flow rates then desired is achieved [3]. Furthermore, there are several challenges in cost-effective operations and scheduling of pumping systems, where multiple factors cause undesirable high energy usage, such as, poor pump designs, suboptimal combination of pumps, and inefficient scheduling of the pumps [4], [5].

Energy efficient operation and scheduling of multiple pumps is challenging due to individual pump's hydraulic characteristics, different operating configurations, and performance requirements [6]. The Variable-Frequency Drive (VFD) technique provide additional degrees of freedom to the pumping systems, this can lead to a more energy efficient solution than the conventional pump on/off switching solution. Operating pumps far from the designed operating point will greatly reduce the efficiency [7]. For the purpose of modeling variable-speed pump characteristics, the affinity-law is commonly used to describe the static relationships among pump head, capacity, and electrical power consumption [6], [8], [9], [7], [10], [11]. The goal of this paper is to develop an energy-oriented optimal control solution for a group of generic centrifugal pumps, each equipped with a VFD.

The proposed solution is divided into two sequential steps; firstly, the minimization of total energy consumption for a set of pre-selected pumps are formulated into an optimization problem, which is convex and can be solved analytically. Secondly, the selection of the number of operating pumps are formulated and solved using a convex solver subject to speed constraints. By dividing the problem into 2 steps only the number of pumps must be optimized online and not the speed.

The proposed solution is compared with conventional affinity-law based models. The experimental analysis showed that the proposed solution results in more precise pump modeling and efficiently solve this type of pump optimization problem. The potential applications of the proposed solution can be found in many areas, such as optimizing the energy consumption of membrane filtration used for offshore produced- or injection water treatment [12], [13].

The rest of this paper is organized as follows; section II presents the experimental setup; section III identifies the pump model structure and parameters based on experimental data; section IV provides a solution for the optimal balancing problem for a group of parallel pumps, which all will be put into operation; section V formulates and solves the optimal problem in terms of number of pumps to be selected into operation subject to required head and capacity; section VI illustrates some experimental results using the proposed solution; Lastly the paper is concluded in section VII.

II. EXPERIMENTAL SETUP

An experimental setup has been used for the experiments and validating the results in this paper. The Piping and Instrumentation Diagram (P&ID) of the pumping station is shown in Fig. 1. This testing facility consists of three identical variable speed pumps, each with differential pressure and flow rate measurements. To emulate different hydrodynamic resistances downstream a control valve is deployed. The flow meters are magnetic flow meters from Emerson, and the pumps are Grundfos CRE 5-5, all equipped with VFDs. The nominal design flow rate for all the pumps are $6.9\frac{m^3}{hr}$ at a head of $32m$. For power measurements, a Zimmer power analyzer LMG670 is used.

[1]Department of Energy Technology, Aalborg University, Niels Bohrs Vej 8, 6700 Esbjerg, Denmark
[2]klj@et.aau.dk
[3]yang@et.aau.dk

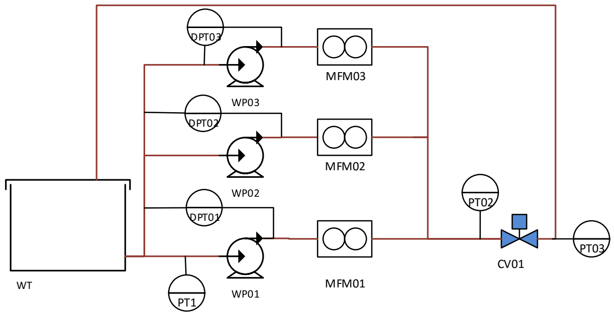


Fig. 1: P&ID of the experimental pumping station used for the experiments. For experimental proposes the water is returned and reused.

III. PUMP MODEL IDENTIFICATION

In order to find the optimal scheduling strategy, a model of the pumps will be developed in this section. The structure for the model is a function describing the power's relationship with head and flow rate, denoted as $P(H,Q)$. Previous work by Ma et al. experimentally plotted the relationship between power, flow rate, and pressure and showed that such a relationship exist. However, no model structure is suggested [9].

A. Proposed power model

The model structure in (1) is proposed by the author due to its monotonic features and fitness to the data.

$$P(H,Q) = a_1 \cdot H^{b_1} + a_2 \cdot Q^{b_2} + H \cdot Q \cdot c + d, \tag{1}$$

where P is the power consumption of the pump, H is the head, Q is the flow rate, and a_1, a_2, b_1, b_2, c, d are the coefficients. The function is restricted to the following domain; $(H,Q) \in \mathbb{R}^2_{>0}$, as neither the flow rate nor the head can be negative or zero for an actual system.

The model structure (1) does not explicitly describe the relationship with pump speed. However, it indirectly describes it, as only one possible pump speed, ω, for a given pressure and flow rate exist. This means that $\omega(H,Q)$ is well-defined and that the variables H, Q, and ω combined only have 2 degrees of freedom. For this reason the speed ω is not directly present in $P(H,Q)$.

Data for several operating points are found by manipulating the control valve CV01 and a single pump's operating speed, such that different flow rates and heads are achieved. As a result, the data points are not evenly spaced in the H-Q domain. The measured operation points that are used to find the function coefficients for pump power curve $P(H,Q)$ can be seen in Fig. 2.

The resulting fit of $P(H,Q)$ can been seen in Fig. 3. The resulting fit have an adjusted coefficient of determination ($\bar{R}^2$) of 0.992, which was concluded to be acceptable for model purpose. The resulting coefficients can be seen in Table. I.

The fitness with respect to H and Q can be seen in Fig. 4a, 4b, and 4c. The experiments were done by changing

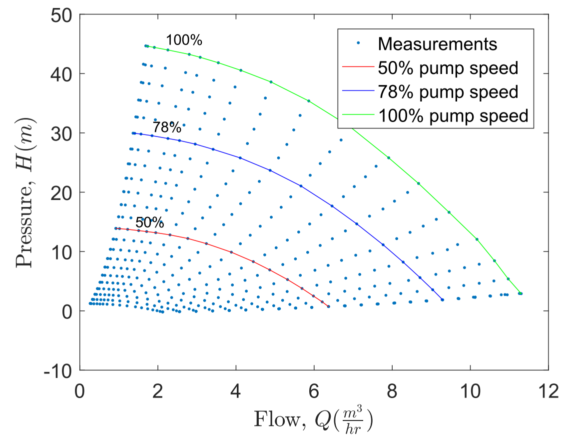


Fig. 2: The blue dots are measurements of H, Q and P, used for pump curve identification. While the red, blue, and green line are flow rates and heads at a fixed pump speed.

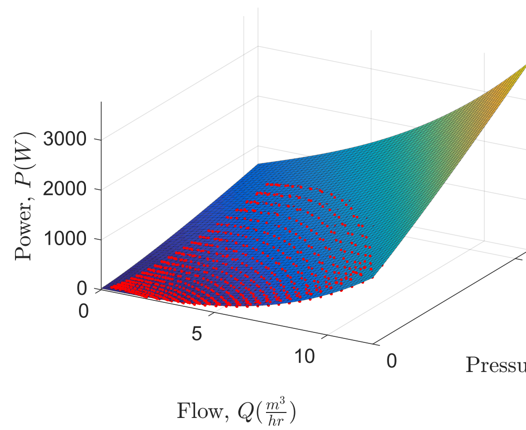


Fig. 3: Measured data from all 3 pumps (red dots) and the fitted surface.

speed and valve opening of CV01 to emulate downstream changes. The power deviation is defined in (2), where P_m is the measured power usage.

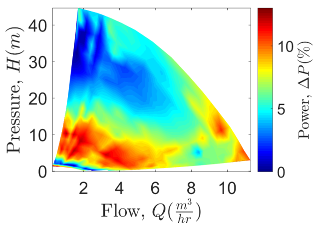
$$\Delta P = \frac{|(P(H,Q) - P_m)|}{P_m} \cdot 100\%. \tag{2}$$

The results in Fig. 4 clearly indicates that the pumps operating performances are not completely identically. This situation could be due to several reasons, such as manufacturing deviation, measurement deviations, as well as measurement noise. However, it is recognized that individual models of each pump would provide less prediction error.

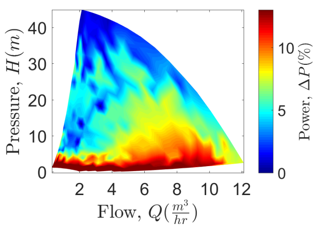
The model error is relative small while the pump is within its recommended operating range. It is only outside the recommended operating range, and at the very limit of the pumps capabilities, the error increases. Operating the pump under such condition would cause undesirable low efficiency and preferably be avoided. It is presumed that the proposed model reach a reasonable precision under the assumption of

TABLE I: $P(H,Q)$ coefficients

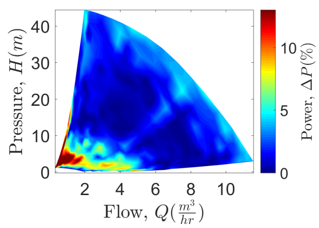
Name	a_1	a_2	b_1	b_2	c	d
Value	2.359	1.318	1.386	2.777	2.773	27.25



(a) First pump



(b) Second pump.



(c) Third pump.

Fig. 4: The Pump's deviation from $P(H,Q)$.

identical pumps, for which the following comparison of the proposed model with affinity-law-based model is committed.

B. Affinity model

The affinity-law is a commonly used rule of thumb, which is widely used to predict power consumption, flow, and pressures. However, this description will only consider the power relationship (3) [9], [7]. The affinity-law will be used as a base line to compare the proposed model structure.

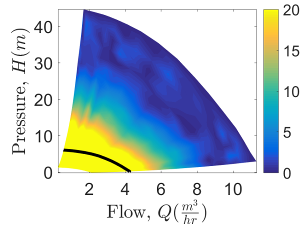
$$P_o = P_n \cdot \left(\frac{\omega_o}{\omega_n}\right)^3, \tag{3}$$

where P_n and ω_n are the nominal operating point in power and pump speed respectively. P_o and ω_o are the actual power and pump speed. The affinity model is only based on a single operating point, but is in most cases combined with a polynomial to extent estimation over the entire operating area. This is commonly done by estimating a polynomial function $H(Q)$ at rated pump speed [8], [7].

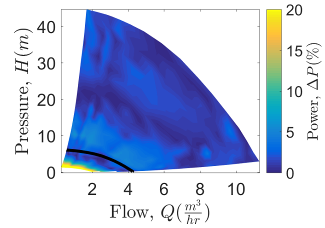
C. Model comparison

This section compares the power function $P(H,Q)$ with the affinity relation described in (3). Both models used in this section are fitted to pump number 1, to provide a common base for comparison. To avoid the error introduced by polynomial estimation in the affinity based model, measurements are used. P_n is defined as power consumption at rated pump speed (ω_n). This is repeated for different system curves, to get a two dimensional map.

It is observed that the prediction from the affinity based model is inaccurate at lower pump speeds, especially below



(a) Pump 1's deviation from affinity law. The black line is 40% rated pump speed.



(b) Pump 1's deviation from P(H,Q). The black line is 40% rated pump speed.

Fig. 5: Deviation between pump 1, affinity law and $P(H,Q)$.

40% pump speed. Tianyi et al. reported that the affinity based model provided accurate predictions for pump speed above 40% [7]. However, conflicting results is observed. For the affinity based model, the error is over 20% even when pump speed is above 40%. The proposed power function results in a significant prediction improvement, as shown in Fig. 5. The additional prediction performance do require significantly more operation data and computation power to estimate the function coefficients. However, this is not a significant problem for the implementation.

IV. OPTIMAL OPERATIONS FOR PARALLEL PUMPS

To find the optimal scheduling strategy for model (1) an experiment is executed and analyzed with a single pump with respect to head and flow rate to determine how a given set of pumps operate most efficient with respect to $\omega(H,Q)$. Given the power function $P(H,Q)$, the total power usage $\hat{P}$ for k number of running pumps, can be described according to (4).

$$\begin{aligned}&\hat{P}(H_1,H_2\cdots H_k,Q_1,Q_2\cdots Q_k) = \\ &P(H_1,Q_1)+P(H_2,Q_2)...+P(H_k,Q_k).\end{aligned} \tag{4}$$

Since all pumps are considered in parallel, for each pump to do any work, each individual pump head must be equal to the combined pump head (H_t). Furthermore, the sum of flow rates from each pump must be equal to the total flow rate (Q_t):

$$Q_t = \sum_{j=1}^{k} Q_j. \tag{5}$$

Substituting these constraints into (4) results in (6),

$$\begin{aligned}&\hat{P}(H_t,Q_t,Q_1,Q_2\cdots Q_{k-1}) = P(H_t,Q_1)+P(H_t,Q_2)+\cdots \\ &P(H_t,Q_{k-1})+P(H_t,Q_t-Q_1-Q_2\cdots Q_{k-1}),\end{aligned} \tag{6}$$

in which $Q_1\cdots Q_{k-1}$ are considered optimization variables. Q_t and H_t are estimated constants or setpoints for the pumping station, for any given operating point. The formulated optimization problem is then given as:

$$\begin{aligned}\underset{Q_1,Q_2\cdots Q_{k-1}}{\text{minimize}} \quad & \hat{P}(H_t,Q_t,Q_1,Q_2\cdots Q_{k-1}) \\ \text{subject to:} \quad & (H_t,Q_t)\in\mathbb{R}^2_{>0}, \\ & (Q_1,Q_2\cdots Q_{k-1})\in\mathbb{R}^{(k-1)}_{\geq 0} \\ & k\in\mathbb{N}.\end{aligned} \tag{7}$$

In order to determine if the total power $\hat{P}$ have multiple minima, the Hessian matrix is used with respect to $Q_1 \cdots Q_{k-1}$. The Hessian matrix is found to be symmetric with the following elements; the expression in (8) is the i^{th} element on the diagonal, while all other elements in the matrix is expressed in (9):

$$a_2 \cdot b_2 \cdot (b_2 - 1) \cdot \left(Q_i^{b_2-2} + (Q_t - Q_1 \cdots Q_{k-1})^{b_2-2} \right), \quad (8)$$

$$a_2 \cdot b_2 \cdot (b_2 - 1) \cdot (Q_t - Q_1 \cdots Q_{k-1})^{b_2-2}. \quad (9)$$

Given the Hessian matrix it can be seen that it is positive-definite under the constrains of (7) if and only if the constrains of the function coefficients fulfill that $a_1, b_1, c, d \in \mathbb{R}$; $a_2 \in \mathbb{R}_{>0}$; and $b_2 \in \{x \in \mathbb{R} \mid x > 1\}$, meaning that when these constrains are satisfied $\hat{P}$ is a strictly convex function with respect to $Q_1 \cdots Q_{k-1}$. As it can be seen in Table. I, the presented example is within these constrains. As $\hat{P}$ is a strictly convex function on a closed domain it has a single global minimum, if this minimum is in the interior of the domain it can be found by solving the equation set:

$$\hat{P}_i' = \frac{\partial}{\partial Q_i} \hat{P}(Q_1, Q_2 \cdots Q_{k-1}) = 0 \qquad i = [1 \ldots k-1]. \quad (10)$$

The resulting equations is shown in:

$$\hat{P}_i' = a_2 \cdot b_2 \cdot \left(Q_i^{(b_2-1)} - (Q_t - Q_1 \ldots Q_{k-1})^{(b_2-1)} \right) = 0. \quad (11)$$

It follows that:

$$(Q_t - Q_1 - \ldots Q_{k-1})^{(b_2-1)} = Q_i^{(b_2-1)}. \quad (12)$$

The resulting equations can then be reduced to:

$$Q_t - Q_1 - Q_2 \ldots - Q_{k-1} = Q_k = Q_i \qquad i = [1 \ldots k-1]. \quad (13)$$

The resulting (13) shows that all flow rates must be identical at the global minimum. It is clear if k pumps are chosen to be active, the highest efficiency achieved at identical speeds. However, it is under the assumption that all pumps are identical. Because the load sharing between the pumps can analytical be solved, the optimization problem can be reduced to only consider the number of active pumps.

V. OPTIMAL SCHEDULING FOR PARALLEL PUMPS

The previously estimated function $P(H,Q)$ can be used to calculate the power consumption for each possible configuration. Since the pumps are identical the same flow will be produced by each pump given the same speed. Therefore, given the result in (13), the energy cost for k identical pumps running out of a set of n is defined as:

$$P_t(H_t, Q_t, k) = P\left(H_t, \frac{Q_t}{k}\right) \cdot k \qquad k \in [1 \ldots n]. \quad (14)$$

Convexity for the cost function $P_t(H_t, Q_t, k)$ can then be checked. The optimization problem is defined with respect to the amount of operating pumps, k. If k is relaxed to $k \in \mathbb{R}_{\geq 0}$, the Hessian matrix reduces to one element shown in:

$$\frac{\partial^2}{\partial k^2} P_t(H_t, Q_t, k) = \frac{a_2 \cdot b_2 \cdot \left(\frac{Q}{k}\right)^{b_2} \cdot (b_2 - 1)}{k}. \quad (15)$$

Given the constraint for $P(H,Q)$, considering only positive flow rate and $k \in \mathbb{R}_{\geq 1}$, it is shown that (16) holds true for the defined domain.

$$\frac{\partial^2}{\partial k^2} P_t(H_t, Q_t, k) > 0. \quad (16)$$

Therefore, $P_t(H_t, Q_t, k)$ with respect to k is strictly convex given the relaxation of k. The relaxation and related problems will be addressed when formulating the optimization problem. The function $\omega(H,Q)$ is estimated and used to define constraints that keeps $P(H,Q)$ within its valid operating range. The function, $\omega(H,Q)$, is for precision purposes based on a thin-plate spline interpolation.

A. Optimal scheduling (mode 1)

Given any H_t and Q_t the proposed optimal configuration can be found by minimizing $P_t(H_t, Q_t, k)$ with respect to k. As the problem is proven to be a convex one-dimension optimization problem, any standard convex optimization algorithm can be applied. Alternatively, given a sufficient small number of combination, the problem can be solved by enumeration of $k \in [1 \ldots n]$:

$$\begin{aligned} &\underset{k}{\text{minimize}} \quad P_t(H_t, Q_t, k) \\ &\text{subject to:} \\ &\qquad (H_t, Q_t) \in \mathbb{R}^2_{>0}, \\ &k \in \left\{ i \in \mathbb{R}_{\geq 1} \;\middle|\; \omega_{min} \leq \omega\left(H_t, \frac{Q_t}{i}\right) \leq \omega_{max} \right\}, \end{aligned} \quad (17)$$

where ω_{min} and ω_{max} is the minimum and maximum pump speed respectively.

Solving the optimization problem results in a $k \in \mathbb{R}_{\geq 0}$, given the previous relaxation of k. As k must be a integer to be applied, the optimal number of active pumps can be found by using the 2 closest integers to k and analyzing these for optimality. Additional actions must be deployed to reduce extensive switching caused by signal noise, a simple hysteresis would suffice.

B. Alternative scheduling (mode 2)

This mode is suggested for comparison and evaluation of mode 1 and is by no means an optimal solution, but rather a heuristic mode. The second mode utilizes a single VFD, as only a single pump is running at variable speed, while the rest utilizes on/off control. From an initial cost point of view, this is an effective configuration, as only one VFD is required. The only scheduling where all flow can be achieved is defined as:

$$\omega_1(u) = mod(u, 1), \quad (18)$$

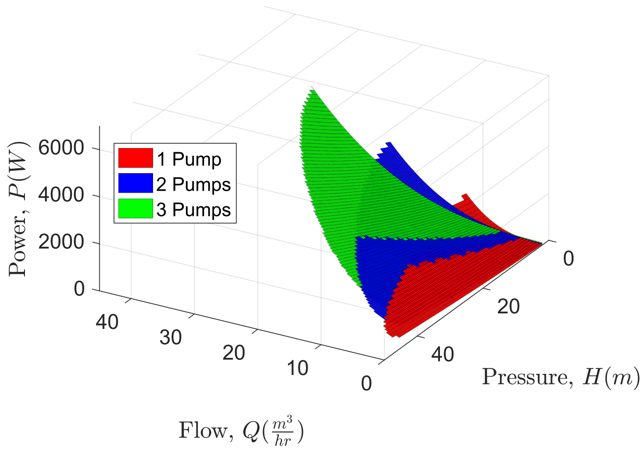


Fig. 6: Energy consumption for mode 1, with 1,2, and 3 pumps (red, blue, and green) respectively.

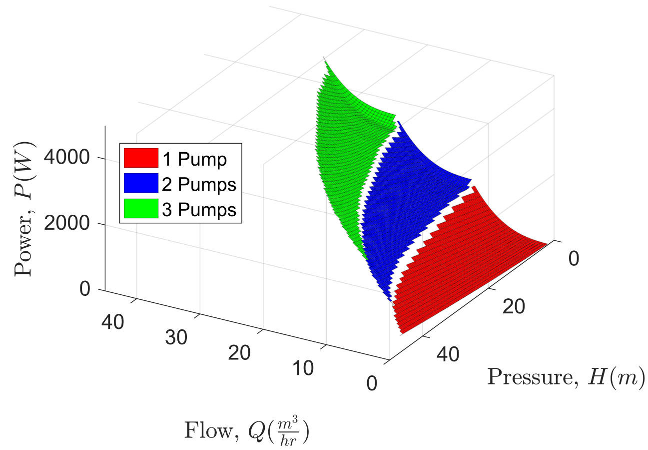


Fig. 7: Energy consumption for mode 2, with 1,2, and 3 pumps (red, blue, and green) respectively.

$$\omega_i(u) = \begin{cases} 1, & \text{if } u-(i-1) \geq 1 \\ 0, & \text{if } u-(i-1) \leq 0 \end{cases}$$
$$i = [2 \ldots n]$$
$$u \in [0,n], \tag{19}$$

where n is the number of pumps, *mod* is the remainder after division(modulo operation), ω_i is the nominal speed of the i^{th} pump, and u is the speed set point to the set of pumps.

C. Implementation procedure

Given the knowledge from the previously sections the suggested implementation steps are as follows:

1) Find $P(H,Q)$ using experimental data distributed over the valid pump range.
2) Let H_t^* be the setpoint to a controller.
3) Let the current flow be known or measurable and let the coefficient of the system curve be constant or slowly changing with no abrupt changes. The system curve can then be estimated and used to predict the flow (Q_t^*) at any given H_t^*.
4) Solve the minimization problem from (17), with $H_t = H_t^*$ and $Q_t = Q_t^*$.
5) Given that H_t^* and Q_t^* are estimated, $\omega(H_t^*, \frac{Q_t^*}{k})$ can be used to give a rough estimation of ω to maintain head and flow. This should be combined with a feedback controller to compensate for any modeling errors [8].

VI. EXPERIMENTAL VALIDATION AND RESULTS

Fig. 6 can be generated based on the model (1). The surface plot shows the energy consumption of 1, 2, and 3 pumps running in mode 1 from minimum to maximum speed. Fig. 7 shows mode 2 defined by (19). A comparison between mode 1 and 2 shows a clear difference. However, the exact difference in energy usage is unclear. Fig. 8 provide better visualization of the reduction in energy consumption. The

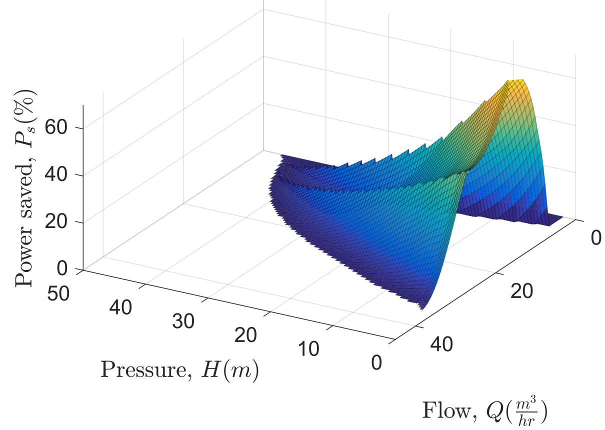


Fig. 8: Energy saved, based on (20)

energy saved (P_s) is defined in (20), where P_{m1} and P_{m2} is the total power for mode 1 and 2, respectively.

$$P_s = \frac{P_{m2} - P_{m1}}{P_{m2}} \cdot 100\%. \tag{20}$$

Fig. 8 clearly shows mode 1 to be the most effective mode with up to 70% power saved. However, it is only at unrealistic low head where the power saved is extremely high. Realistic flow rates and pressures can result in 30% power reduction. However, as pressure increase the difference is reduced to zero.

A. Validation

A comparison between simulated and measured pump configuration selections is based on simulated and measured response. The simulated response is based on the estimated function $P(H,Q)$, while the measured response is measurements from running all configurations and choosing the configuration with the least energy usage. From Fig. 9 it is clear that only in a very limited region the wrong

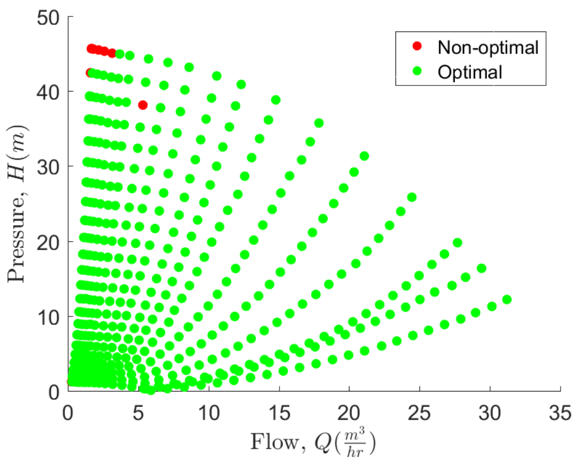


Fig. 9: Difference in simulated and measured optimal scheduling. Green is where the algorithm chose the optimal solution, while red is where a less optimal solution configuration is selected, because of modeling errors.

configuration was chosen by the algorithm. The region where selection error occur is on the boundary of valid pump operating condition and outside the efficient operating area. Therefore, any well designed pumping system should not encounter these selection errors.

VII. CONCLUSION AND FUTURE WORK

This paper investigated optimal scheduling of a group of identical variable speed pumps in terms of minimizing energy consumption subject to a satisfactory pump performance. The proposed model structure was compared to affinity-based models and the proposed model provided a significant improvement in prediction accuracy, especially at lower speeds.

The proposed solution consists of two sequential parts; firstly, the optimization problem with respect to pump speed, given a set of operating pumps and expected operation point in head and flow rate, was formulated. It was concluded that under these conditions, the optimal solution was running the pumps at identical speed. Secondly, the optimal number of active pumps was formulated as a convex problem with constraints, and solved using a standard solver.

The proposed solution presumes the performance curves remain constant over time; the scheduling can therefore be solved offline. However, in practice is not the case. The model could be extended to cover non-identical pumps in combination with performance map adaptation for taken changing behavior into considerations, such as cavitation and wear [14]. If performance map adaptation is deployed, the optimization must be done online.

The proposed scheduling method was tested and compared to an alternative method. The resulting energy reduction from the proposed method are as much as 30%.

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