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GLOBAL OPTIMIZATION OF PUMP CONFIGURATIONS USING BINARY SEPARABLE PROGRAMMING

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Abstract—In a recent paper (Westerlund et al., 1994) a pump configuration problem was presented and solved using mixed integer non-linear programming methods. The problem was found to be nonconvex. The objective function contained several local optima and global optimality of the obtained solutions could not be ensured, or even reached, in all cases with the MINLP codes. In this paper a concept of binary separable programming will be used in order to obtain the global optimum for the pump configuration problem. The pump configuration consists of centrifugal pumps of different sizes coupled in series and parallel. When the required pressure rise and flow are given the configuration giving the lowest total cost is sought. The total cost includes investment and running costs for the pumps. The capacity regulation can be performed with both speed and throttle control. An optimal flexibility study of the pump configuration problem will also be presented. Copyright © 1996 Elsevier Science Ltd

INTRODUCTION

Pumps are one of the most common process units in industry and a great deal of the electricity produced is consumed by pumping. Pumping is though a relatively efficient process unit. Nevertheless, the savings that can be achieved are considerable, primarily due to the widespread use of pumping. An optimal selection of pumps for different tasks is not a trivial task today due to the large number of different pump types and sizes available. This is especially true if the possibility to connect pumps in series and parallel is considered. Besides finding a pump that can meet the specified process requirements, a solution yielding the lowest total pumping cost possible is also naturally desired.

Much work on pumping has focused on the development of single pumps. The design of pumps is a relatively well covered topic today. However, very little work on methods for the optimal selection of pumps or configuration of pumps has been carried out.

Recent developments in the field of process synthesis have given rise to new methods for the solution of pump configuration problems. Work on the formulation of general structural optimization problems (e.g. Grossmann, 1985; Grossmann, 1989) has even made it possible to obtain an effective formulation for the pump configuration problem. One commonly used approach in process synthesis has been that of solving the problems as Mixed Integer Non-linear Programming (MINLP) problems (e.g. Geoffrion, 1972; Duran and Grossmann, 1986; Yuan *et al.*, 1989; Viswanathan and Grossmann, 1990).

In a recent paper (Westerlund *et al.*, 1994), a general superstructure for pump configurations was given. In this paper, a method for solving optimal pump configurations will be given where the problem is separated as a two level optimization problem where the lower level problem is convex under certain conditions and the upper level problem contains nonconvex separable functions. The proposed method employs the technique of separable programming when solving the upper level optimization problem. The separable nonconvex functions are discretized and the minimization task can thus be solved as an integer programming problem. Subminimization can be performed with any MINLP method that finds the global optimum for convex problems. The obtained solution is, generally, the global optimum with the selected discretization.

The purpose of this paper is, besides solving for the global optimum, to show that pump configuration problems of greater complexity can be solved effectively. An example in which 14 different pump types are considered will be given.

THE PUMP CONFIGURATION PROBLEM

The problem can be expressed as the search for the pump configuration that gives a certain pressure head and flow, while being the cheapest possible solution to the problem. The running costs as well as the investment costs of the pumps are included in the economical consideration. The pump configuration can be viewed as though it comprised various levels. On each level a number of similar centrifugal pumps are connected in series and/or parallel. The pumps on different levels are of different sizes.

The configuration problem can thus be seen as if it

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were composed of L levels with $N_{p,i}$ parallel centrifugal pumps and $N_{s,i}$ pumps in series at level i. In Fig. 1, a configuration with L levels can be seen. Each level comprises two parallel pumps and three pumps in series.

The total flow through the configuration, V_{tot}, is thereby separated into L parts. For each level the pressure rise equals the total pressure rise, Δp_{tot}.

The capacity regulation can be performed either with speed or throttle control. In speed control the pressure rise over the pump configuration is constant and the rotation speed is changed. In throttle control the rotation speed is constant and the pressure rise over the pump configuration is changed. The pump configuration problem concerning speed control is presented in Westerlund *et al.*, 1994 and concerning throttle control in Pettersson, 1994a. Therefore the derivation of the problems will not be presented here, only the resulting formulations.

The pump configuration problem when speed control is used can be stated as follows,

$$\min_{N_{p,i},N_{s,i},x_i,\omega_i,i=1,\ldots,L} \left\{ \sum_{i=1}^{L} (C_i+C_i'P_i)N_{p,i}N_{s,i} \right\} \tag{1}$$

subject to,

$$P_i = q_{1,i}(\dot{V}_i,\omega_i) \tag{2}$$

$$\Delta p_i = q_{2,i}(\dot{V}_i,\omega_i) \tag{3}$$

$$\omega_i - \omega_{i,\max} \le 0 \tag{4}$$

where,

$$\dot{V}_i = \frac{x_i}{N_{p,i}} \dot{V}_{tot} \tag{5}$$

$$\Delta p_i = \frac{1}{N_{s,i}} \Delta p_{tot} \tag{6}$$

$$\sum_{i=1}^{L} x_i = 1 \tag{7}$$

x_i is the fraction of the total flow pumped through level i and ω_i is the rotation speed of the pumps on the same level. (2) and (3) give the pressure rise and the required power for one pump on level i as a function of the flow through the level. C_i is the yearly installment cost for one pump at level i. C'_i is the electricity cost. $N_{p,i}$ and $N_{s,i}$ are integer variables while x_i and ω_i are non-negative real variables.If capacity controlling is performed by throttling, the rotation speed of the pumps is constant and the problem is easier to solve. The pump configuration problem for throttle control can be stated as follows,

$$\min_{N_{p,i},N_{s,i},x_i,i=1,\ldots,L} \left\{ \sum_{i=1}^{L} (C_i+C_i'P_i)N_{p,i}N_{s,i} \right\} \tag{8}$$

subject to

$$P_i = q_{1,i}(\dot{V}_i) \tag{9}$$

$$\Delta p_i = q_{2,i}(\dot{V}_i) \tag{10}$$

$$\dot{V}_i = \frac{x_i}{N_{p,i}} \dot{V}_{tot} \tag{11}$$

$$\Delta p_i = \frac{1}{N_{s,i}} \Delta p_{tot} \tag{12}$$

$$\sum_{i=1}^{L} x_i = 1 \tag{13}$$

The separation of different pump types into different levels is done because it leads to a quite useful superstructure. Configurations that do not seem to be obtainable with the formulation can, however, also be obtained by replacing single pumps in the original configuration with a corresponding pump configuration with single pumps in parallel and series. For example, if configurations where different pump types are connected in series can be accepted, the formulation can easily be modified to include these possibilities. The modified

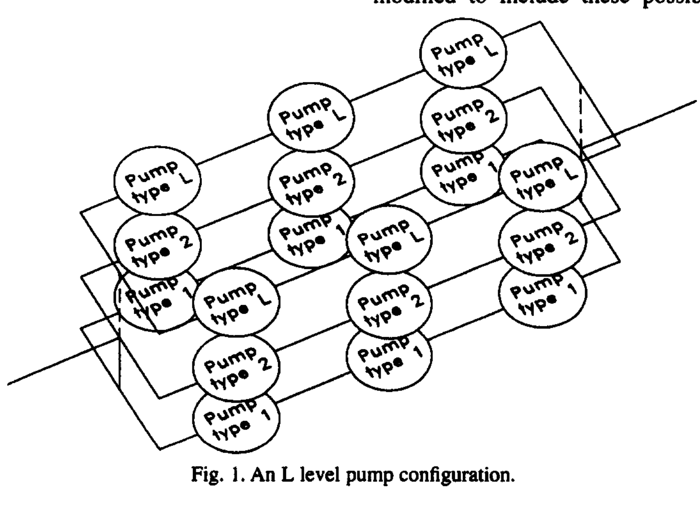

Fig. 1. An L level pump configuration.

formulation would consist of two or more pump structures, as defined above, connected in series. A problem of this kind has been shown in Pettersson, 1994a. On the other hand, if pumps of the same type, connected in parallel but with different rotation speed can also be accepted, the pump type may be defined at two or more levels.

THE PUMP PROBLEM AS A TWO LEVEL OPTIMIZATION PROBLEM USING BINARY SEPARABLE PROGRAMMING

A nonconvex problem may be solved simpler by separating the problem into a two level optimization problem. The general aim of formatting the pump problem as a two level optimization problem would in this case be to separate the convex and nonconvex parts of the problem.

Consider a problem,

$$\min_{z_1,z_2} f(z_1,z_2) \tag{14}$$

The problem can be written as a two level optimization problem according to,

$$\min_{z_1} f(z_1,z_2') \tag{15}$$

subject to

$$z_2' = arg \min_{z_2} \{f(z_1,z_2)\} \tag{16}$$

Some of the sub-problems might now be simpler to solve making also the solution of the overall problem simpler. As can be found in Westerlund *et al.* (1994) and in Appendix A the only variable that affects the variables in the different pumping levels is x_i in the multi-level pump problem. Thus the pump configuration problem, if speed control is employed, can be written as a two level optimization problem as follows,

$$\min_{x_i} \sum_{i=1}^{L} f_i(x_i,N_{p,i}',N_{s,i}',\omega_i') \tag{17}$$

subject to

$$\begin{pmatrix} N'_{p,i} \\ N'_{s,i} \\ \omega'_i \end{pmatrix}_{i=1,\dots,L} = \arg\min_{N_{p,i},N_{s,i},\omega_i} \{f_i(x_i,N_{p,i},N_{s,i},\omega_i)\} \tag{18}$$

where

$$f_i(x_i,N_{p,i},N_{s,i},\omega_i) = (C_i+C_i'P_i)N_{p,i}N_{s,i} \tag{19}$$

If the problem is defined for throttle control the rotation speed is naturally not included in (17)–(19) as variable.

The upper level problem now contains only the fractions x_i of the total flow to each level as variables. The objective function (17) is nonconvex but the lower level problems (18)–(19) include only $N_{p,i}$, $N_{s,i}$ and ω_i as variables. This problem was found to be convex in Westerlund *et al.* (1994) and can be seen in Appendix A. The lower-level problems in (18) can be solved with the Outer Approximation (OA) method or for example with the Extended Cutting Plane (ECP) method (Pettersson, 1994b; Westerlund and Pettersson, 1995). The solution obtained with the OA or the ECP method is known to be the global optimum when the objective function is convex.

The pump configuration problem is separable, both with speed and throttle control, when the problem is formulated as a two level minimization problem according to (17) – (19). The problem can be written according to,

$$\min_{x_i,i=1,\dots,L} \left\{ \sum_{i=1}^{L} f_i(x_i) \right\} \tag{20}$$

such that,

$$\sum_{i=1}^{L} x_i = 1 \tag{21}$$

$$0 \le x_i \le 1 \tag{22}$$

where $f_i(x_i)$ are separable functions given by the solution of the lower level optimization problem and is the cost of the optimal solution satisfying the desired pressure rise with pump type i at a certain fraction of the total volume flow, x_i. The objective function (20) contains separable functions (Reklaitis *et al.*, 1983). Since the functions are found to be nonconvex a standard separable programming approach can however not be used in this case. In this paper a method using binary variables is thus employed. In this case the separable programming problem can be formulated as a zero-one programming problem. This formulation can be called Binary Separable Programming (BSP) and it involves a binary variable in each interval. The problem can then be solved with a zero-one or an Integer Programming (IP) algorithm. The solution to the BSP problem will be the best grid point in the region defined by the linearized problem. The optimum found with the BSP method is always a feasible point, regardless of whether the problem is convex or not. The convexity or nonconvexity of the original problem does naturally not affect the performance of the search method when the problem is presented in linear form.The objective value on the optimal configuration of a given level at a certain x_i will be denoted by $f_{i,j}$ where,

$$f_{i,j} = f_i\left(x_i = \frac{j-1}{m}\right) \tag{23}$$

m denotes the number of discretization intervals and j denotes the discretization-index. When the upper minimization problem is defined according to the BSP formulation it can be rewritten as,

$$\min_{y_{i,j}, i=1,\ldots,L, j=1,\ldots,m} \left\{ \sum_{i=1}^{L} \sum_{j=1}^{m} (f_{i,j+1} - f_{i,j}) \cdot y_{i,j} \right\} \quad (24)$$

$$\sum_{i=1}^{L} \sum_{j=1}^{m} y_{i,j} = m \quad (25)$$

$$y_{i,j+1} - y_{i,j} \leq 0 \quad (26)$$

$$y_{i,j} \in \{0,1\} \quad (27)$$

(25) is obtained from (7) or (13) when the discretizations are made equidistant according to (23). The optimization is performed with respect to the binary variables. The binary variables indicate the different intervals which are included in the solution and thereby define the volume fractions, x_i, at the optimal solution.

The costs, $f_{i,j}$, corresponding to the convex sub-problems thus have to be solved before the BSP problem is solved. The CPU time for the solution of the pump configuration problem is dependent of the used discretization of the separable functions. The CPU time can be divided into two parts, the CPU time for solving the upper level BSP problem (a MILP problem) and the CPU time for solving the discretization points (the lower level convex MINLP problems). In general the CPU time for the discretization points is given by the number of separable function multiplied by the CPU time for solving each lower level convex MINLP problem. In our case the CPU time for the convex MINLP problems is between a fraction of a CPU second and a few CPU seconds when solving the MINLP problems on a VAX 6510 vp computer.

For the problem with 14 different pump types and a discretisation of the x_i:s in 30 points there will be 420 sub-problems that has to be solved. The number may seem big, but it has to be remembered that the sub-problems are convex and relatively easy to solve. The number of combinations of the volume fractions, x_i, is, however, several magnitudes greater. In addition to the subproblems, one binary programming problem with 420 binaries need to be solved. The latter problem is in our case solved in a few CPU minutes.

EXAMPLE

In this section an example considering 14 different pump types will be given. The number 14 corresponds to all centrifugal pumps of a selected type (NM/NP) of a Danish pump manufacturer (Grundfos). The example has been formulated as a BSP problem and the integer optimization can be solved with any MILP code. The total head and power curves as given by the pump manufacturer can be approximated by the following equations with a good deal of accuracy,

$$\Delta p_{M,i} = a + b \cdot \dot{V}_{M,i} + c \cdot \dot{V}_{M,i}^2 \quad (28)$$

$$P_{M,i} = \alpha + \beta \cdot \dot{V}_{M,i} + \gamma \cdot \dot{V}_{M,i}^2 \quad (29)$$

The lower index M indicates that the data is provided by the manufacturer. The data is given for water at 20°C and a rotation speed of 2950 rpm. The maximum rotation speed for the pumps is 2950 rpm. The constants in (28) and (29) are given in Table 1 for the different pump types. The relation between the flow, $\dot{V}_i$, the pressure rise, Δp_i, the power, P_i, and the corresponding values $\dot{V}_{m,i}$, $\Delta p_{m,i}$ and $P_{m,i}$ given by the manufacturer can for another rotation speed, and another fluid, be obtained by proportional relations according to Coulson and Richardsson, 1985.

When the pressure head and power is approximated with (28) and (29) the one-level problem is known to be convex as long as the flow through one pump is greater than $\dot{V}_{\min}$ where,

$$\dot{V}_{\min} = \left(\frac{\omega}{\omega_{\max}} \right) \cdot \left(-\frac{2 \cdot \alpha}{\beta} + \sqrt{\left(\frac{2 \cdot \alpha}{\beta} \right)^2 - \frac{\alpha}{\gamma}} \right) \quad (30)$$

as shown in Appendix A. Thus, if the sub-minimization (i.e. the solution of the $f_{i,j}$ values) is done with the OA or the ECP method the solution obtained will be the global optimum as long as the flow through the different pumps is greater than $\dot{V}_{\min}$. $\dot{V}_{\min}$ is typically lower than the

Table 1. Data for the 14 pumps treated in this example.

	a	b	c	α	β	γ	Price
	kPa	$\frac{kPa}{m^3/h}$	$\frac{kPa}{(m^3/h)^2}$	kW	$\frac{kW}{m^3/h}$	$\frac{kW}{(m^3/h)^2}$	(FIM)
Pump1	367.4	0.3982	-0.00862	3.824	0.1041	-2.298e-4	20730
Pump2	308.7	0.4631	-0.0175	2.851	0.07942	-1.832e-4	19350
Pump3	233.3	0.2870	-0.00768	2.912	0.04634	-1.112e-4	15540
Pump4	191.0	0.2742	-0.00715	1.837	0.04806	-1.595e-4	13990
Pump5	630.1	0.5948	-0.0114	7.171	0.1736	-3.601e-4	29000
Pump6	519.4	0.6577	-0.0135	4.316	0.1713	-4.304e-4	24730
Pump7	346.2	0.9127	-0.0327	1.101	0.1293	-7.048e-4	11180
Pump8	293.7	0.4764	-0.0297	1.089	0.09029	-4.268e-4	10030
Pump9	229.6	1.119	-0.0400	1.323	0.05604	-2.786e-4	8940
Pump10	194.5	0.7625	-0.0380	0.738	0.06249	-4.286e-4	7530
Pump11	564.5	0.5932	-0.0382	2.028	0.2058	-8.537e-4	16650
Pump12	473.2	0.7863	-0.0434	1.730	0.1550	-6.179e-4	16130
Pump13	218.9	1.0311	-0.0741	0.725	0.0584	-3.786e-4	7880
Pump14	189.3	0.5331	-0.0747	0.651	0.0464	-3.050e-4	7570

lowest practical limit for pumping with a selected pump type. The bound ((30)) can thus be introduced already when solving the lower level optimization problem. In this example the required flow of water through the pump configuration is defined to be 350 m^3/h and the total pressure rise to be 400 kPa. The installment cost is obtained with an interest rate of 10% and an economical life of 10 years for the pumps. The yearly installment cost is thus obtained by multiplying of the pump price by a factor of 0.1627. The running costs are given by the electricity price, 0.30 FIM/kWh, and the yearly running time, 6000 h/year. It has to be pointed out that the prices of the speed controllers are not included in the investment costs. The numerical problem to be solved is presented explicitly in Appendix B.Using the given data the optimal pump configuration can be obtained for various problems. For example the single-level problems have been solved for both speed and throttle control and the result is reported in Table 2. The 14-level problems have also been solved and the optimal configurations are shown in Table 3.

It can be mentioned that in the case where all 14 pump types are considered the problem consists of 980 binary variables when the interval between two grid points is 5 m^3/h.

The formulation performs well and the only drawback is that the number of constraints defining $y_i \leq y_{i+1}$ grove relatively large; in this example there are 966 constraints.

FLEXIBLE PUMP CONFIGURATIONS

When selecting a suitable pump type and configuration a certain degree of flexibility has to be considered. The solution of optimal flexible pump configuration problems when changes occur in the pressure head and total flow has been shown in Westerlund and Pettersson, 1993. A varying electricity price and lifetime of the pumps naturally also affect the total economy.

In Fig. 2, the pump types involved in the optimal configuration have been shown as a function of the installment and running costs. The figure is obtained for the case with speed control and it shows that in the present case the solution is relatively unaffected by small variations.

In Fig. 3, the optimal pump types for a varying pressure rise and total flow can be seen when all 14 pump types have been considered. It can again be noticed that for the present example the configuration found is best for small variations in the process definitions.

In Fig. 3, configurations consisting only of pump type 5 seem to be the best choice for most pumping conditions. It must thus be remembered that instead of only one optimal configuration of pump type 5 there exist four different ones in the figure.

Both figures in this section are obtained with a relatively coarse grid and therefore the borders between different areas are not very smooth. The small number of points is due to the relatively long time for solving one point. For example, Fig. 3 consists of 984 different solutions.

The complexity of the problem can be illustrated by the following consideration. Suppose, that instead of using the BSP formulation to solve the upper minimization problem all possible combinations would be compared and the best one of these would be selected; then there would be more than 5.2×10^{14} different possibilities to choose from for the presented example when Δp_{tot} = 400 kPa and $\dot{V}_{tot}$ = 350 m^3/h. In the same way the 984 different solutions in Fig. 3 would have to be selected from more than 7.5×10^{18} different alternatives. If the solution of one alternative would take only one second, it would take more than $2.3 \cdot 10^{11}$ years to obtain the figure.

The figures presented in this section have all been for the case employing speed control but the same figures could, of course, be produced for the case with throttle control.

Table 2. Optimal configurations for one level problems with both speed and throttle control.

	Speed control				Throttle control		
	N_p	N_s	ω_i	Yearly cost (FIM/year)	N_p	N_s	Yearly cost (FIM/year)
Pump 1	3	2	2561	116,829	3	2	158,921
Pump 2	5	2	2688	138,622	4	2	146,112
Pump 3	3	3	2775	116,417	5	2	126,280
Pump 4	4	3	2748	113,628	4	3	131,449
Pump 5	3	1	2611	103,285	3	1	135,779
Pump 6	3	1	2916	108,756	3	1	111,662
Pump 7	6	2	2580	117,003	5	2	138,763
Pump 8	6	2	2850	116,708	6	2	125,501
Pump 9	8	2	2938	115,687	8	2	116,628
Pump 10	7	3	2910	128,428	7	3	131,228
Pump 11	6	1	2769	119,188	5	1	123,803
Pump 12	7	1	2938	117,373	7	1	118,355
Pump 13	15	2	2933	138,632	15	2	140,065
Pump 14	12	3	2890	151,674	12	3	157,407

Table 3. Optimal configurations for the fourteen-level problems with both speed and throttle control.

Speed control:	Yearly cost = 103,285 FIM/year		
Pump 5	N_p =3	N_s =1	
Throttle control:	Yearly cost = 110,148 FIM/year		
Pump 4	N_p = 1	N_s =3	x =0.3143
Pump 6	N_p = 2	N_s =1	x =0.6857

DISCUSSION

In this report it has been shown that pump configuration problems can be efficiently solved with a satisfactory discretization even for problems of considerable dimensions. The method used separates the problem as a two level optimization problem. The lower level

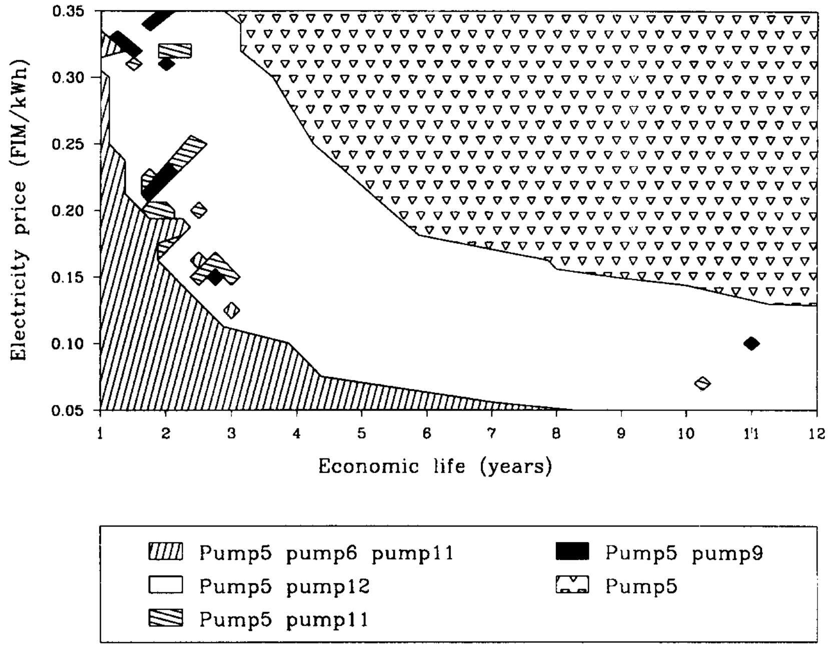

Fig. 2. The influence of installment and running costs on the pump selection. The different pump types involved in the optimal pump configuration for varying electricity price and economic life of the pumps is shown. The process demands to be achieved are 350 m³/h and 400 kPa and all of the fourteen pump types are considered.

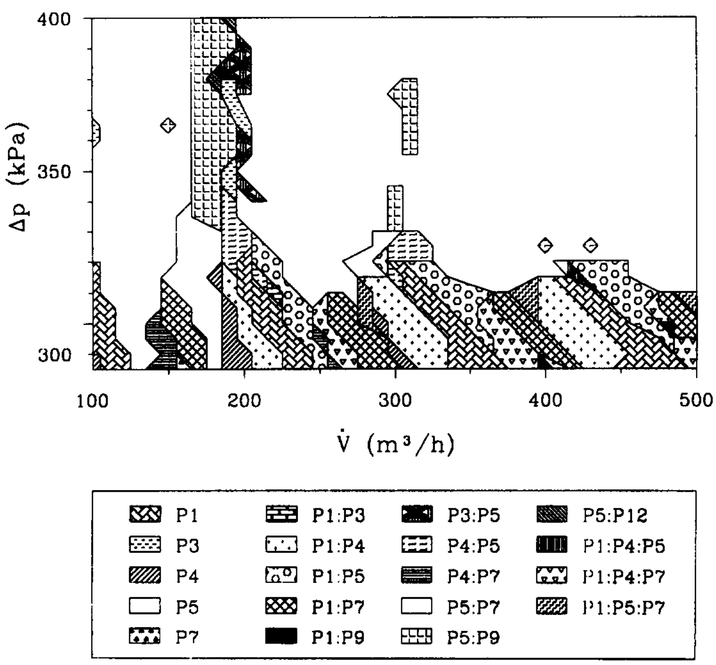

Fig. 3. Optimal pump selections for varying process demand when speed control is used. The P's stand for pump (P1:P3 = Pump 1 and Pump 3).

problems are convex MINLP problems and can globally be solved with existing MINLP codes. The upper level problem contains nonconvex functions and the minimization task has been formulated as a Binary Separable Programming problem consisting only of binary variables. The method finds the global optimal solution for a certain discretization of the separable functions.

The presented approach might also be applicable for other structural optimization problems.

APPENDIX A

The single-level problem

Consider the relaxed one-level ($L = 1$) problem (P_1)

$$\min\{(C+C'\cdot P)\cdot N_p\cdot N_s\} \tag{A.1}$$

subject to

$$\dot{V} \geq \frac{\dot{V}_{tot}}{N_p} \tag{A.2}$$

$$\Delta p \geq \frac{\Delta p_{tot}}{N_s} \tag{A.3}$$

$$\dot{V} \geq \left(\frac{\omega}{\omega_m}\right)\dot{V}_m \tag{A.4}$$

$$P \geq \left(\frac{\omega}{\omega_m}\right)^3\left(\frac{\rho}{\rho_m}\right)P_m \tag{A.5}$$

$$\Delta p \geq \left(\frac{\omega}{\omega_m}\right)^2\left(\frac{\rho}{\rho_m}\right)\Delta p_m \tag{A.6}$$

$$\omega \leq \omega_{\max} \tag{A.7}$$

$$P_m \geq g_1(\dot{V}_m) \tag{A.8}$$

$$\Delta p_m \geq g_2(\dot{V}_m) \tag{A.9}$$

where N_p and N_s are positive integers and $\dot{V}$, Δp, P, $\dot{V}_m$, Δp_m, P_m and ω are positive real variables and C, C', $\dot{V}_{tot}$, Δp_{tot}, ω_m, ρ_m, ρ, $\omega_{\max}$ given parameters and $g_1(\cdot)$ as well as $g_2(\cdot)$ given functions.The problem (P1) can be convexified by letting any variable X be replaced by $X = e^x$ (for ω we have used the symbol $w = e^{\omega}$). The problem (P1), can now be rewritten in convexified form,

$$\min\left\{\left(C+C'\cdot e^{p}\right)e^{n_p}\cdot e^{n_s}\right\} \tag{A.10}$$

subject to

$$\dot{\nu} \geq \ln\left(\dot{V}_{tot}\right)-n_p \tag{A.11}$$

$$\delta p \geq \ln\left(\Delta p_{tot}\right)-n_s \tag{A.12}$$

$$\dot{\nu} \geq w-\ln\left(\omega_m\right)+\dot{\nu}_m \tag{A.13}$$

$$p \geq 3w+\ln\left(\frac{\rho}{\omega_m^3\cdot\rho_m}\right)+p_m \tag{A.14}$$

$$\delta p \geq 2w+\ln\left(\frac{\rho}{\omega_m^2\cdot\rho_m}\right)+\delta p_m \tag{A.15}$$

$$\omega \leq \ln(\omega_{\max}) \tag{A.16}$$

$$-p_m+\ln\left(g_1\left(\dot{V}_m\right)\right) \leq 0 \tag{A.17}$$

$$-\delta p_m+\ln\left(g_2\left(\dot{V}_m\right)\right) \leq 0 \tag{A.18}$$

(A.10) is now clearly a convex function and the inequalities, (A.11)—(A.16), linear constraints. The inequalities, (A.17) and (A.18), are clearly convex constraints if the functions, $g_i(\dot{V}_M)$, are positive functions that satisfy the general condition,

$$g_i''(\dot{V}_m)\cdot\dot{V}_m+g_i'(\dot{v}_m)\cdot\left(1-\frac{g_i'(\dot{V}_m)}{g_i(\dot{V}_m)}\cdot\dot{V}_m\right) \geq 0 \tag{A.19}$$

The power curve, $P_m = g_1(\dot{V}_m)$, is generally a positive, increasing—decreasing, concave function. Thus the condition, (A.19), can be applied to the model, in order to ensure convexity. Using an approximative model of the type,

$$p_m = \alpha+\beta\cdot\dot{V}_m+\gamma\cdot\dot{V}_m^2 \tag{A.20}$$

we have the conditions $\alpha>0$, $\beta>0$ and $\gamma < 0$. Furthermore, when applying the inequality, (A.19), we obtain a lower limit for $\dot{V}_m$, in order to ensure the convexity of the problem. The lower limit is given by,

$$\dot{V}_m \geq -\frac{2\alpha}{\beta}+\sqrt{\left(\frac{2\alpha}{\beta}\right)^2-\frac{\alpha}{\gamma}} \tag{A.21}$$

and thus the lower limit of the flow through the pump (in order to ensure convexity) is given by,

$$\dot{V}_m \geq \left(\frac{\omega}{\omega_m}\right)\left(-\frac{2\alpha}{\beta}+\sqrt{\left(\frac{2\alpha}{\beta}\right)^2-\frac{\alpha}{\gamma}}\right) \tag{A.22}$$

The total head curve, $\Delta p_m = g_1(\dot{V}_m)$, is generally a positive, decreasing, concave function. Thus, condition (A.19) will generally hold. Using an approximative model of the type,

$$\Delta p_m = a + b \cdot \dot{V}_m^c \qquad (A.23)$$

we obtain the general conditions, $a>0$, $b<0$, and $c>1$. A model of the type (A.20) may also be used for approximating the total head curve.

The multi-level problem

For a multi-level problem, (P_L), each variable will obtain an index i corresponding to the level, and the objective function will be replaced with the sum of objective functions for each level, respectively. Furthermore, the inequality, (A.2), will be replaced by,

$$\dot{V}_i \geq \frac{\dot{V}_{tot}}{N_p} X_i \qquad (A.24)$$

where X_i is the fraction of the total flow through the corresponding level. Furthermore, the sum of all fractions of the total flow, X_i, must be equal to one. The corresponding relaxed inequality is thus given by,

$$\sum_{i=1}^{L} X_i \geq 1 \qquad (A.25)$$

Inequality (A.24) can in the convexified form be replaced by the linear inequality,

$$\nu \geq \ln\left(\dot{V}_{tot} \right) - n_p + x_i \qquad (A.26)$$

while inequality (A.25) can in the convexified form be replaced by

$$-\left(\sum_{i=1}^{L} e^{x_i} \right) + 1 \leq 0 \qquad (A.27)$$

The only significant difference between the single-level and the multi-level problem is inequality (A.27). Since exponential terms are nonconvex due to negative sign, constraint (A.27) is nonconvex and so is the multi-level MINLP problem.

APPENDIX B

The explicit formulation of the 14-level problem when the capacity regulation is performed with speed control:

$$\min_{N_{p,i}, N_{s,i}, x_i, \omega_i, i=1,\ldots,L} \left\{ \sum_{i=1}^{14} \left(C_i + C_i' \cdot P_i \right) N_{p,i} \cdot N_{s,i} \right\} \qquad (B.1)$$

subject to,

$$\Delta p_1 = 367.4 \left(\frac{\omega_1}{2950} \right)^2 + 0.3982 \left(\frac{\omega_1}{2950} \right) \dot{V}_1 - 0.00862 \dot{V}_1^2$$

$$\Delta p_2 = 308.7 \left(\frac{\omega_2}{2950} \right)^2 + 0.4631 \left(\frac{\omega_2}{2950} \right) \dot{V}_2 - 0.0175 \dot{V}_2^2$$

$$\ldots$$

$$\Delta p_{14} = 189.3 \left(\frac{\omega_{14}}{2950} \right)^2 + 0.5331 \left(\frac{\omega_{14}}{2950} \right) \dot{V}_{14} - 0.0747 \dot{V}_{14}^2 \qquad (B.2)$$

$$\sum_{i=1}^{14} x_i = 1 \qquad (B.3)$$

$$\omega_i - 2950 rpm \leq 0 \qquad (B.4)$$

where,

$$P_1 = \left(\frac{\omega_1}{2950} \right) \left(3.824 \left(\frac{\omega_1}{2950} \right)^2 + 0.1041 \left(\frac{\omega_1}{2950} \right) \dot{V}_1 - 0.0002298 \dot{V}_1^2 \right)$$

$$P_2 = \left(\frac{\omega_2}{2950} \right) \left(2.851 \left(\frac{\omega_2}{2950} \right)^2 + 0.07942 \left(\frac{\omega_2}{2950} \right) \dot{V}_2 - 0.0001832 \dot{V}_2^2 \right)$$

$$\ldots$$

$$P_{14} = \left(\frac{\omega_{14}}{2950} \right) \left(0.651 \left(\frac{\omega_{14}}{2950} \right)^2 + 0.0464 \left(\frac{\omega_{14}}{2950} \right) \dot{V}_{14} - 0.0003050 \dot{V}_{14}^2 \right) \qquad (B.5)$$

$$\dot{V}_i = \frac{x_i}{N_{p,i}} \dot{V}_{tot} \qquad (B.6)$$

$$\Delta p_i = \frac{1}{N_{s,i}} \Delta p_{tot} \qquad (B.7)$$

The rest of the numerical constants for expressions (B.2) and (B.5) have to be taken from Table 1. The values of the following constants are:

$$\dot{V}_{tot} = 350\frac{m^3}{h}, \Delta p_{tot}=400kPa, \tag{B.7}$$

C_i' =0.30.6000 FIM/kW/year and the investment costs, C_i, is the pump price, from Table 1, multiplied by the factor 0.1627. For example C_1 = 0.1627·20730 FIM/year.

REFERENCES

Coulson J. M. and J. F. Richardsson, *Chemical Engineering*, Pergamon Press, **1**, (1985).

Duran M.A. and Grossmann I.E., An Outer-Approximation Algorithm for a Class of Mixed-Integer Nonlinear Programs, Math. Prog., **36**, (1986) 307–339

Geoffrion A. M., Generalized Benders Decomposition, Journal of Optimization Theory and Applications, **10**, 4 (1972) 237–260

Grossmann I.E., Mixed-Integer Programming Approach for the Synthesis of Integrated Process Flowsheets, Computers chem. Engng., **9**, 5 (1985) 463–482

Grossmann I. E., MINLP Optimization Strategies and Algorithms for Process Operations, FOCAPD, Snowmass, CO, 1989.

Pettersson F., Mixed Integer Non-Linear Programming Applied on Pump Configurations, Academic Dissertation, Process Design Laboratory, Åbo Akademi University, 1994a.

Pettersson F., ECP-An Algorithm for Non-Convex MINLP Problems, Report 94-137-A, Process Design Laboratory, Åbo Akademi University, Submitted to *Simulation*, (1994b).

Reklaitis G. V., A. Ravindran and K. M. Ragsdell, Engineering Optimization, Wiley, 1983.

Westerlund T., Pettersson F. and Grossmann I.E., Optimization of Pump Configurations as a MINLP Problem, Computers Chem. Engng., **18**, 9 (1994) 845–858

Westerlund T. and F. Pettersson, Solving Optimal Flexible Pump Configuration Design Problems by MINLP, Proceedings of the Twelfth IAESTED MIC Conference, Acta Press, 1993, 172—174.

Westerlund T. and Pettersson F., An Extended Cutting Plane Method for Solving Convex MINLP Problems, Computers Chem. Engng., **19**, (1995) S131–S136

Viswanathan J. and Grossmann I.E., A Combined Penalty Function and Outer-Approximation Method for MINLP Optimization, Computers Chem. Engng., **14**, 7 (1990) 769–782

Pipouleanu Y.X. and Domenech S., Experiments in Process Synthesis via Mixed-Integer Programming, Chem. Eng. Process, **25**, (1989) 99–116