Analiza numeryczna, Lista 10, 17/12/2019

Zadanie 2. $y(x) = a(2018x - 2019) + 1977$

$$E(a) = \sum_{k=0}^{n} \left[f(x_k) - y(x_k)\right]^2 = \sum_{k=0}^{n} \left[f(x_k) - a(2018x_k - 2019) - 1977\right]^2 =$$

Szukamy minimum $E(a)$, więc policzmy pochodną tej funkcji:

$$E'(a) = -2 \sum_{k=0}^{n} \left(f(x_k) - a(2018x_k - 2019) - 1977\right)(2018x_k - 2019) =$$

$$= -2\left(\sum_{k=0}^{n} (f(x_k) - 1977)(2018x_k - 2019) - a\sum_{k=0}^{n} (2018x_k - 2019)^2\right)$$

$$E'(a) = 0 \implies a = \frac{\sum_{k=0}^{n} (f(x_k) - 1977)(2018x_k - 2019)}{\sum_{k=0}^{n} (2018x_k - 2019)^2}$$

Zadanie 5. $S = aT + b$

		0	1	2	3	4	5	6	7 ← N
x_k	T	0	10	20	30	40	80	90	95
y_k	S	68.0	67.1	66.4	65.6	64.6	61.8	61.0	60.0

$$\begin{cases} a = \dfrac{(N+1)S_4 - S_1S_3}{(N+1)S_2 - S_1^2} = \dfrac{8S_4 - S_1S_3}{8S_2 - S_1^2} \\[2ex] b = \dfrac{S_2S_3 - S_1S_4}{(N+1)S_2 - S_1^2} = \dfrac{S_2S_3 - S_1S_4}{8S_2 - S_1^2} \end{cases}$$

$S_1 = \sum_k x_k$ $\quad S_2 = \sum_k x_k^2$

$S_3 = \sum_k y_k$ $\quad S_4 = \sum x_k y_k$

$S_1 = 365$

$S_2 = 26525$

$S_3 = 514.5$

$S_4 = 22685$

$a = -0{,}07993$

$b = 67.95932$

$$E(a,b) = \sum_{k=0}^{N} (y_k - aT - b)^2, \quad \begin{cases} \dfrac{\partial E(a,b)}{\partial a} = -2\sum_{k=0}^{N} (\overset{S}{y_k} - aT - b)^1 T = 0 \\[2ex] \dfrac{\partial E(a,b)}{\partial b} = -2\sum_{k=0}^{N} (y_k - aT - b)^1 = 0 \end{cases} \implies \begin{cases} aS_4 + bS_1 = S_4 \\ aS_1 + b(N+1) = S_3 \end{cases}$$

Zadanie 1. $\|f\| := \sqrt{\sum_{k=0}^{N} p(x_k) f(x_k)^2}$, $p(x) > 0$, $f(x_k)^2 \geq 0$

$$\|f\|^2 = \sum_{k=0}^{n} (\sqrt{p_k} f_k)(\sqrt{p_k} f_k) = \langle f, f \rangle$$
$$\Rightarrow \langle f, g \rangle = \sum_{k=0}^{n} (\sqrt{p_k} f_k)(\sqrt{p_k} g_k)$$

1° $\|f\| = 0 \Rightarrow f = 0$

$f(x_k)^2 \geq 0$, $\|f\| \geq 0$

2° $\|\alpha f\| = |\alpha| \cdot \|f\|$

$$\|\alpha f\| = \sqrt{\sum_{k=0}^{N} p(x_k)(\alpha f(x_k))^2} = |\alpha| \sqrt{\sum_{k=0}^{N} p(x_k) f(x_k)^2} = |\alpha| \cdot \|f\|$$

3° $\|f+g\| \leq \|f\| + \|g\|$

$$\sqrt{\sum_{k=0}^{N} p_k (f_k + g_k)^2} \leq \sqrt{\sum_{k=0}^{N} p_k f_k^2} + \sqrt{\sum_{k=0}^{N} p_k g_k^2}$$

$$\sqrt{\sum_{k=0}^{N} p_k (f_k^2 + 2 f_k g_k + g_k^2)} \leq \sqrt{\sum_{k=0}^{N} p_k f_k^2} + \sqrt{\sum_{k=0}^{N} p_k g_k^2} \quad /^2$$

$$\sum p_k f_k^2 + 2 \sum p_k f_k g_k + \sum p_k g_k^2 \leq \sum p_k f_k^2 + \sum p_k g_k^2 + 2 \cdot \sqrt{\sum p_k f_k^2} \cdot \sqrt{\sum p_k g_k^2}$$

$$\underbrace{\sum p_k f_k g_k}_{(\sqrt{p_k} \cdot f_k)(\sqrt{p_k} \cdot g_k)} \leq \sqrt{\sum \underbrace{p_k f_k^2}_{(\sqrt{p_k} f_k)^2}} \cdot \sqrt{\sum \underbrace{p_k g_k^2}_{(\sqrt{p_k} g_k)^2}} \leftarrow$$

COŚ W STYLU NIERÓWNOŚCI CAUCHYEGO-SCHWARZA

INACZEJ $\|f+g\| \leq \|f\| + \|g\|$

$$\langle f+g, f+g \rangle \leq \langle f, f \rangle + \langle g, g \rangle + 2\sqrt{\langle f, f \rangle}\sqrt{\langle g, g \rangle}$$

$$\langle f, f \rangle + \langle f, g \rangle + \langle g, f \rangle + \langle g, g \rangle \leq \ldots$$

$$\langle f, g \rangle \leq \langle f, f \rangle \cdot \langle g, g \rangle$$

Weźmy $g \neq 0$, $\alpha \in \mathbb{R}$, wtedy $0 \leq \langle f - \lambda g, f - \lambda g \rangle = \lambda^2 \overbrace{\langle g, g \rangle}^{>0} - 2\lambda \langle f, g \rangle + \langle f, f \rangle$

$$\langle f, g \rangle^2 \leq \langle f, f \rangle \langle g, g \rangle, \quad \langle f, g \rangle \leq |\langle f, g \rangle|$$

Zadanie 3. $\sum_{k=0}^{r} \frac{\sin(x_k)+2019}{1+e^{x_k}}\left[y_k - a\left(\ln(2x_k+2020)+x_k\right)\right]^2$

Niech $b_k = \frac{\sin(x_k)+2019}{1+e^{x_k}}$, $c_k = \ln(2x_k+2020)+x_k$,

wtedy $E(a) = \sum b_k \cdot (y_k - a\cdot c_k)^2 \Rightarrow E'(a) = -2\sum b_k(y_k - ac_k)c_k = 0$

$$\sum b_k y_k c_k - a\sum c_k^2 b_k = 0$$

$$a = \frac{\sum b_k y_k c_k}{\sum c_k^2 b_k}$$

Zadanie 5. (inaczej) $Y = ax+b$, $f_1 = 1$, $f_2 = x$

$$\begin{bmatrix} \langle f_1,f_1\rangle & \langle f_1,f_2\rangle \\ \langle f_2,f_1\rangle & \langle f_2,f_2\rangle \end{bmatrix}\begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} \langle 1,1\rangle & \langle 1,x\rangle \\ \langle x,1\rangle & \langle x,x\rangle \end{bmatrix}\begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} \langle 1,y\rangle \\ \langle x,y\rangle \end{bmatrix}$$

$\langle 1,1\rangle = \sum_0^7 1(x_i)\cdot 1(x_i) = 8$

$\langle 1,y\rangle = \sum_0^7 y_i = 514{,}5$

$\langle x,y\rangle = \sum_0^7 x_i y_i = 22685$

$\langle 1,x\rangle = \langle x,1\rangle = \sum_0^7 x_i = 365$

$\langle x,x\rangle = 26525$

$a = -0.08$

$b = 67.96$

Zadanie 7. $H(t) = h_0 + a_1 \sin\frac{2\pi t}{12} + a_2 \cos\frac{2\pi t}{12}$, $f_1 = 1$, $f_2 = \sin\frac{2\pi t}{12}$, $f_3 = \cos\frac{2\pi t}{12}$, $y = H(t)$

$a := h_0$

$b := a_1$

$c := a_2$

$$\begin{bmatrix} \langle 1,1\rangle & \langle 1, \sin\frac{2\pi t}{12}\rangle & \langle 1, \cos\frac{2\pi t}{12}\rangle \\ \langle \sin\frac{2\pi t}{12}, 1\rangle & \langle \sin\frac{2\pi t}{12}, \sin\frac{2\pi t}{12}\rangle & \langle \sin\frac{2\pi t}{12}, \cos\frac{2\pi t}{12}\rangle \\ \langle \cos\frac{2\pi t}{12}, 1\rangle & \langle \cos\frac{2\pi t}{12}, \sin\frac{2\pi t}{12}\rangle & \langle \cos\frac{2\pi t}{12}, \cos\frac{2\pi t}{12}\rangle \end{bmatrix} \begin{bmatrix} h_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \langle 1, H(t)\rangle \\ \langle \sin\frac{2\pi t}{12}, H(t)\rangle \\ \langle \cos\frac{2\pi t}{12}, H(t)\rangle \end{bmatrix}$$

$$\left\langle \sin\frac{2\pi t}{12}, \cos\frac{2\pi t}{12}\right\rangle = \sum_0^5 \sin\frac{2\pi t_i}{12}\cdot\cos\frac{2\pi t_i}{12}$$

$$\langle H, 1\rangle = \sum_0^5 H(t_i)$$

$$\langle 1,1\rangle = 6$$

Zadanie 6. $y \approx e^{ax+b}$

$y \approx e^{ax+b}$ $\quad /\ln$

$\ln y \approx ax+b$ $\longrightarrow$ $\begin{cases} f_1(x) = x \\ f_2(x) = 1 \\ y(x) = \ln y \end{cases}$

$$\begin{bmatrix} \langle x,x\rangle & \langle x,1\rangle \\ \langle 1,x\rangle & \langle 1,1\rangle \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \langle x, \ln y\rangle \\ \langle 1, \ln y\rangle \end{bmatrix}$$

Zadanie 4.

$$C(t) = \frac{t^2+3}{Ae^{2t} + B\sin(t+2)+2} \overset{x:=t}{\Rightarrow} \underbrace{Ae^{2x} + B\sin(x+2)}_{W} = \underbrace{\frac{x^2+3}{C(x)} - 2}_{f}$$

$$f_1(x) = e^{2x}$$

$$f_2(x) = \sin(x+2)$$

$$y(x) = f(x)$$