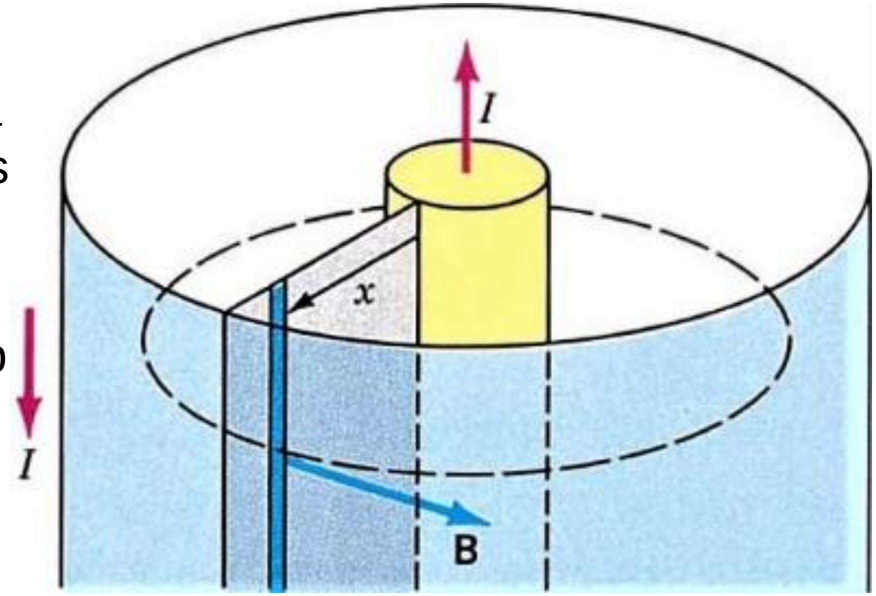
Example 32.2: A coaxial cable (同軸電纜), as the figure shows, consists of an inner wire of radius $a$ that carries a current $I$ upward, and an outer cylindrical conductor of radius $b$ that carries the same current downward. Find the self-inductance of a coaxial cable of length $\ell$. Ignore the magnetic flux within the inner wire.

Solution:




Field produced by the inner wire at a distance $x$ ($> a$) from its center is

$$B = \frac{\mu_0 I}{2\pi x}$$

The flux through an infinitesimal strip of width $dx$ and area $dA = \ell dx$:

$$d\Phi = B dA = \frac{\mu_0 I}{2\pi x}(\ell dx) = \frac{\mu_0 I \ell}{2\pi}\frac{dx}{x}$$

The total flux through the loop is

$$\Phi = \frac{\mu_0 I \ell}{2\pi}\int_a^b \frac{dx}{x} = \frac{\mu_0 I \ell}{2\pi}\left[\ln x\right]_a^b = \frac{\mu_0 I \ell}{2\pi}(\ln b - \ln a) = \frac{\mu_0 I \ell}{2\pi}\ln\frac{b}{a}$$

The self-inductance of the coaxial cable is $N\Phi = LI \quad \rightarrow L = \frac{\Phi}{I} = \frac{\mu_0 \ell}{2\pi}\ln\frac{b}{a}$

Example 32.6: Use the expression for the energy density of the magnetic field to calculate the self-inductance of a toroid (環形螺管, 螺形管) with a rectangular (長方形的, 矩形的) cross section; see the figure.

Solution:

$$\oint \mathbf{B}\cdot d\ell = B\oint d\ell = B(2\pi r) = \mu_0(NI) \quad \rightarrow \quad B = \frac{\mu_0 NI}{2\pi r}$$

$$dV = h(2\pi r dr)$$

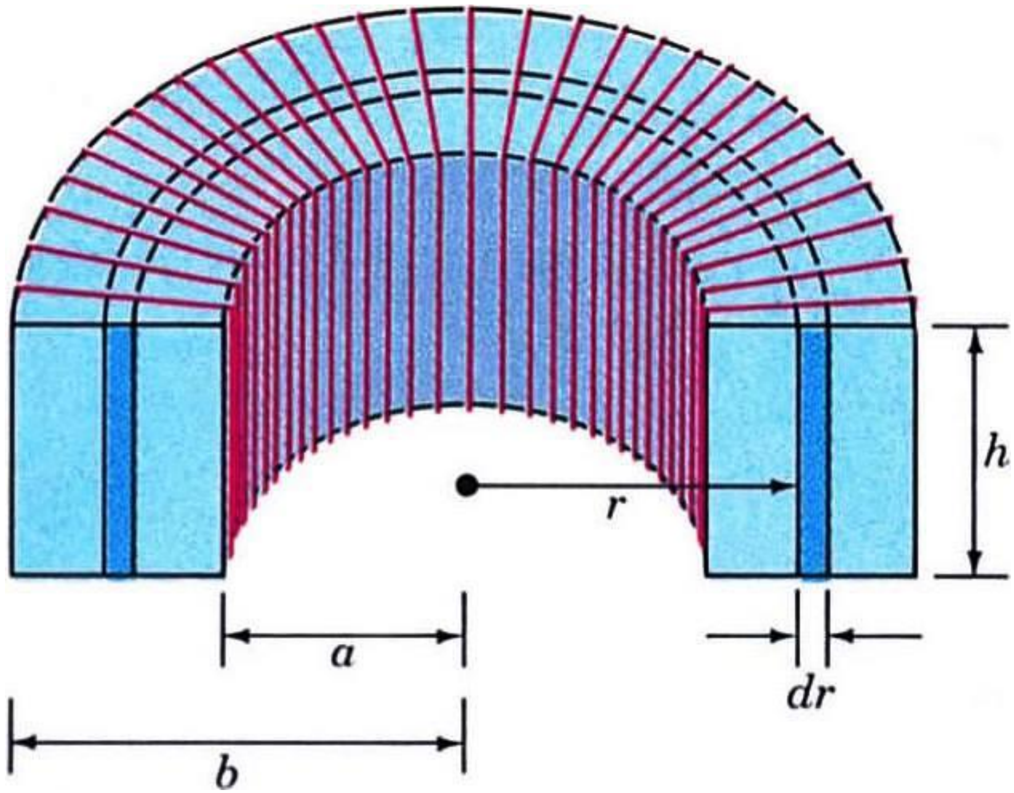
$$u_{\mathrm{B}} = \frac{B^2}{2\mu_0}$$

$$dU = u_{\mathrm{B}} dV = \frac{\mu_0 N^2 I^2 h}{4\pi}\frac{dr}{r}$$

$$U = \int dU = \frac{\mu_0 N^2 I^2 h}{4\pi}\int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 N^2 I^2 h}{4\pi}\ln\frac{b}{a} = \frac{1}{2}LI^2$$

$$L = \frac{\mu_0 N^2 h}{2\pi}\ln\frac{b}{a}$$



Example 33.3: In an $RLC$ series circuit $R = 50\ \Omega$, $C = 80\ \mu\mathrm{F}$, and $L = 30\ \mathrm{mH}$. The 60-Hz source has an rms potential difference of 120 V. Find the rms potential difference for each element.

Solution:

The rms current (均方根電流, 有效電流) $I = V/Z$ is the same for all elements.

We first need to find the impedance.

The reactances are

$$X_L = \omega L = (2\pi f)L = (2\pi)(60)(30\times10^{-3}) = 11.3\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C} = \frac{1}{(2\pi)(60)(80\times10^{-6})} = 33.2\,\Omega$$

The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50)^2 + (11.3-33.2)^2} = 54.6\,\Omega$$

Therefore, $I = \dfrac{V}{Z} = \dfrac{120\ \mathrm{V}}{54.6\,\Omega} = 2.2\ \mathrm{A}$

The rms potential difference across each element is

$$V_R = IR = 110\ \mathrm{V} \qquad V_L = IX_L = 24.9\ \mathrm{V} \qquad V_C = IX_C = 73\ \mathrm{V}$$

Example 34.2: A radio station (無線電臺) transmits a 10-kW signal (訊號) at a frequency of 100 MHz. For simplicity, assume that it radiates as a point source. At a distance of 1 km from the antenna (天線), find the amplitudes of the electric and magnetic field strengths.

Solution:

The energy of waves emitted by a point source spreads (展開, 散布) over ever-expanding spheres.

The surface area (表面積) of a sphere of radius $r$ is $4\pi r^2$, so the intensity of the waves at a distance $r$ is

(Point source) $$S_{av} = \frac{\text{Average power}}{4\pi r^2}$$

Since $E = cB$, $S_{av}$ may be written in terms of $E_0$:

$$S_{av} = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2\mu_0 c}$$

$$\rightarrow \frac{10\times 10^3\ \text{W}}{(4\pi)(10^3\ \text{m})^2} = \frac{E_0^2}{2(4\pi\times 10^{-7}\ \text{H/m})(3\times 10^8\ \text{m/s})} \quad \rightarrow E_0 = 0.775\ \text{V/m}$$

The amplitude of the magnetic field is $B_0 = \dfrac{E_0}{c} = 2.58\times 10^{-9}\ \text{T}$

Example 35.4: Kepler used total internal reflection in a glass block (玻璃磚) to deflect a beam of light as shown in the figure. For an angle of incidence $i$ at the top surface, what is the minimum refractive index needed for total internal reflection at the vertical face? The surrounding medium (周圍介質) is air, for which $n = 1$.

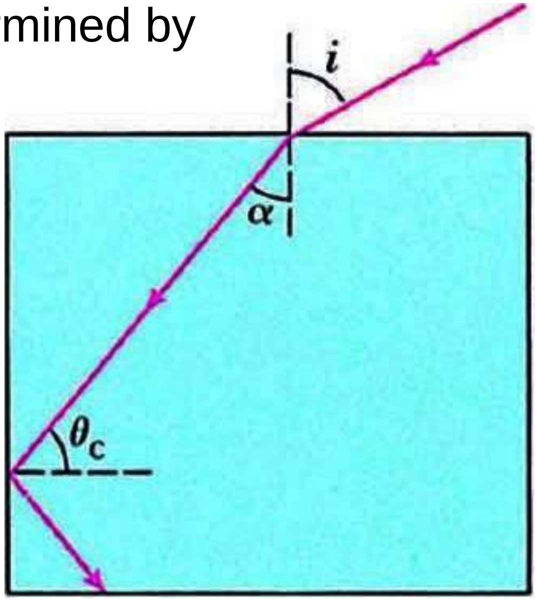
Solution:

At the top surface the angle of refraction is found from Snell's law:

$$n \sin \alpha = \sin i \qquad \text{(i)}$$

Total internal reflection will occur at the vertical face if the angle of incidence is greater than the critical angle, which is determined by

$$n \sin \theta_c = 1$$




From the diagram we see that $\theta_c = 90^\circ - \alpha$ and so $\sin \theta_c = \cos \alpha$.

Thus $n \cos \alpha = 1 \qquad$ (ii)

We square (平方) both side of equations (i) and (ii) and add them to find

$$n^2 \cos^2 \alpha + n^2 \sin^2 \alpha = n^2 = 1 + \sin^2 i$$

Therefore $n = (1 + \sin^2 i)^{1/2}$

Example 35.5: Obtain an expression for the refractive index of a prism in terms of the minimum angle of deviation (偏向角).

Solution:

The angle of deviation, $\delta$, of the beam depends on the angle $i$ at which the beam strikes the face of the prism.

It can be shown that the minimum value of the angle of deviation occurs when the ray goes through the prism symmetrically (對稱).

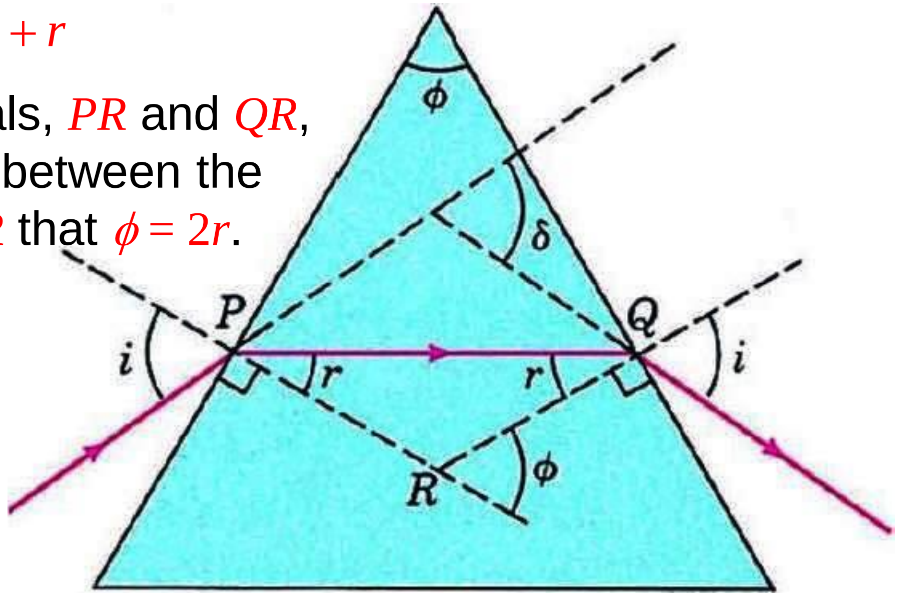
At each face the direction of the ray changes by $(i - r)$ and so the total deviation is

$$\delta_{\min} = 2(i - r) \qquad \rightarrow \quad i = \frac{1}{2}\delta_{\min} + r$$

Since the angle between the normals, $PR$ and $QR$, to the faces is the same as that between the faces, we see from triangle $PQR$ that $\phi = 2r$.

$$\sin i = n \sin r$$

$$n = \frac{\sin i}{\sin r} = \frac{\sin\left(\dfrac{\phi + \delta_{\min}}{2}\right)}{\sin\left(\dfrac{\phi}{2}\right)}$$



Example 35.8: An object of height 1.2 cm is placed 2 cm from a spherical mirror whose radius of curvature is 8 cm. Find the position and size of the image given that the mirror is concave (凹的, 凹面的).

Solution:

We are given the focal length $f = R/2 = 4$ cm and the object distance $p = 2$ cm.

The position of the image is found by applying the mirror formula $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

On inserting the given values, we find $\frac{1}{2\,\text{cm}} + \frac{1}{q} = \frac{1}{4\,\text{cm}}$

Therefore $q = -4$ cm.

The negative sign (負號) means that the image is virtual, to the right of the mirror.

The transverse magnification is $m = \frac{y_I}{y_O} = -\frac{q}{p} = +2$

Since $m$ is positive, the image is erect.

Since its magnitude is greater than one, the image is enlarged (放大).

The size of the image is $y_I = 2(1.2\text{ cm}) = 2.4$ cm.

Example 36.4: A microscope has an objective of focal length 5 mm and an eyepiece of focal length 20 mm. The optical tube length is 15 cm and the final image is at 40 cm from the eyepiece. Find the overall magnification.

Solution:

The distance between the lenses is $d = \ell + f_O + f_E = 17.5$ cm.

Since we have the image distance for the eyepiece, the object distance may be found from $\frac{1}{p_E} + \frac{1}{q_E} = \frac{1}{f_E}$ with $f_E = 2$ cm and $q_E = -40$ cm.

This leads to $p_E = \frac{40}{21} = 1.90\,\text{cm}$

The image distance for the objective is $q_O = d - p_E = 15.6\,\text{cm}$

Finally, $\frac{1}{p_O} + \frac{1}{q_O} = \frac{1}{f_O}$ leads to $p_O = 0.517$ cm.

Note that this is slightly greater than $f_O$.

From $M = -\frac{q_O}{p_O} \cdot \frac{0.25}{p_E}$ the overall angular magnification is

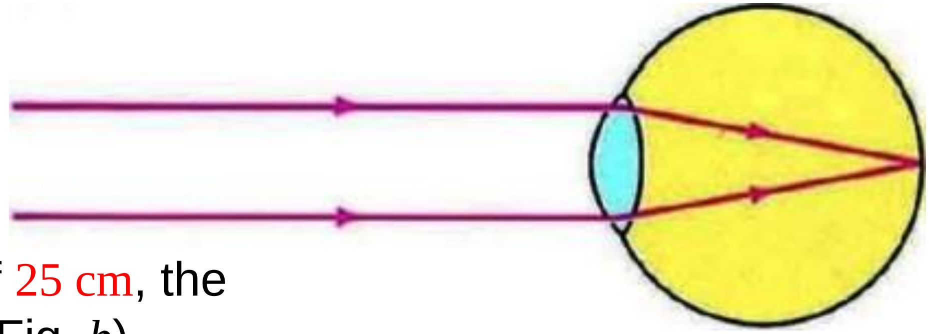
$$M = -\left(\frac{15.6}{0.517}\right)\left(\frac{25}{1.90}\right) = -397$$

Example 36.7: A normal eye has a diameter of 2 cm. What is its power of accommodation?

Solution: A normal eye can focus from 25 cm to infinity.

The diameter of the eyeball (眼球) is equal to the focal length for objects at infinity (Fig. *a*).

$$\frac{1}{f_1} = P_1 = \frac{1}{0.02\ \mathrm{m}} = 50\ \mathrm{D}$$



(*a*)

For an object at the near point of 25 cm, the image distance is still 2 cm (Fig. *b*).

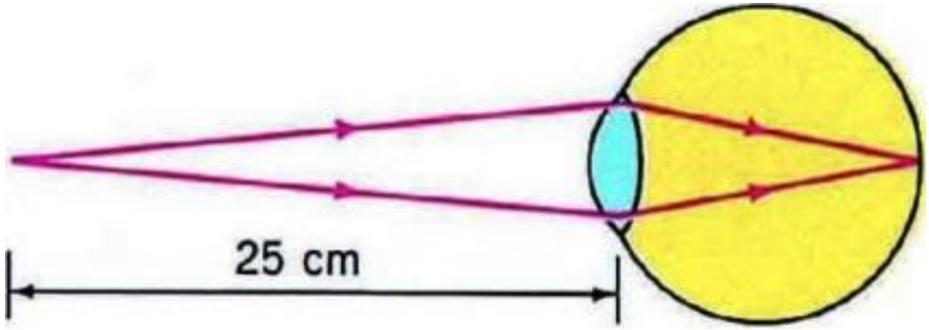
The focal length is given by

$$\frac{1}{0.25\ \mathrm{m}} + \frac{1}{0.02\ \mathrm{m}} = \frac{1}{f_2} = P_2$$

Thus $P_2 = 54\ \mathrm{D}$.

The power of accommodation is

$$P_2 - P_1 = 4\ \mathrm{D}$$




(*b*)

Example 37.5: In an experiment on Newton's rings the light has a wavelength of 600 nm. The lens has a refractive index of 1.5 and a radius of curvature of 2.5 m. Find the radius of the 5th bright fringe.

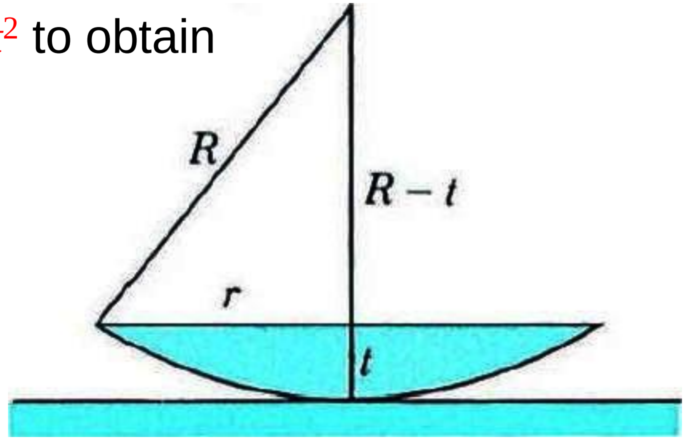
Solution: If $R$ is the radius of curvature of the lens, then from the figure we see that

$$r^2 = R^2 - (R - t)^2$$

where $r$ is the radius of a fringe and $t$ is the thickness of the film.

Since $t$ is very small, we may drop terms in $t^2$ to obtain

$$r^2 \approx 2Rt$$




In order to find $r$, we must first find $t$.

The condition for a bright fringe is

$$2t = \left(m + \frac{1}{2}\right)\lambda_F$$

We note that $n = 1$ for the air film (the index for the glass is irrelevant) and that $m = 4$ for the fifth bright fringe.

$$t = \frac{(4.5)(6\times10^{-7})}{2} = 1.35\times10^{-6}\ \text{m} \qquad \rightarrow\ r = \sqrt{2Rt} = 2.6\times10^{-3}\ \text{m}$$