Physics 188B

Assignment 1

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In this first assignment, we are solving ODEs on python by implementing the Runge-Kutta (RK) methods and sympletic methods on a one-dimensional harmonic oscillator, and for a two-body problem.

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I. THE HARMONIC OSCILLATOR

The one-dimensional harmonic oscillator is described by the equation:

$$\ddot{x}(t) = -\omega^2 x(t), \omega > 0 \tag{1}$$

A.

For part A, the harmonic oscillator has the initial conditions of $x(0) = x_0$, $\dot{x}(0) = 0$, and with $x_0 \neq 0$. Rearranging the ODE as:

$$\ddot{x}(t) + \omega^2 x(t) = 0. \tag{2}$$

Let $x(t) = e^{rt}$, then $\ddot{x}(t) = r^2 e^{rt}$ and:

$$r^2 e^{rt} + \omega^2 e^{rt} = 0 \tag{3}$$

$$r^2 = i\omega^2 \tag{4}$$

$$r = \pm i\omega \tag{5}$$

This gives two-solutions as $x(t) = e^{i\omega}$ and $x(t) = e^{-i\omega}$, and the general solution is with the initial conditions are:

$$x(t) = Ae^{i\omega t} + Be^{-i\omega t}. \tag{6}$$

$$x(0) = A + B = x_0 \tag{7}$$

$$\dot{x}(0) = Ai\omega - Bi\omega = 0 \Longrightarrow A = B \tag{8}$$

$$x_0 = A + B = A + A \Longrightarrow A = \frac{x_0}{2}. \tag{9}$$

Thus, the general solution of the ODE with the constants solved is:

$$\boxed{x(t) = \frac{x_0}{2}(e^{i\omega t} + e^{-i\omega t})} \tag{10}$$

B.

The Lagrangian of this harmonic oscillator is such that it has both a kinetic energy and potential energy where:

$$L = T - U = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2. \tag{11}$$

The conjugate momenta is such that:

$$p = \frac{\partial L}{\partial \dot{x}} = m\dot{x}. \tag{12}$$

The equation for the Hamiltonian is:

$$H(q,p) = p\dot{q}(q,p) - L(q,\dot{q}(q,p)) \tag{13}$$

and solving gives:

$$H = p\dot{x} - L \tag{14}$$
$$H = m\dot{x}^2 - \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 \tag{15}$$
$$H = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 \tag{16}$$
$$H = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\omega x^2. \tag{17}$$

Let $p = m\dot{x} \longrightarrow \dot{x} = \frac{p}{m}$, then:

$$\boxed{H = \frac{1}{2}\frac{p^2}{m} + \frac{1}{2}m\omega x^2}. \tag{18}$$

Hamilton's equation for this system are:

$$\dot{x} = \frac{\partial H}{\partial p} = \boxed{\frac{p}{m}} \tag{19}$$
$$\dot{p} = -\frac{\partial H}{\partial x} = \boxed{-m\omega^2 x}. \tag{20}$$