Mathematica Homework #4

Email notebook to corbin@physics.ucla.edu
with a subject line: [Physics 105A]
by on/or about Friday, 19 February

- In the first cell, enter your **name**, **student ID**, **email address** and the **assignment identifier** (eg. "HW 4") as text.
- 1) You, no doubt, have done the following at least a dozen times by hand - but it's a little different when you try to set the computer up to do it. Find the gravitational potential due to a uniform sphere of mass M and radius R for all points inside and outside of the sphere.
- 2) Now, as a bit of a study in organization, find the gravitational self-energy of a spherical distribution of mass for which the mass density varies as $\rho(r) \propto r^n$. (You can probably check your answer by taking $n = 0$, if you remember doing the analogous problem in lower-division E&M)
- 3) Calculate the potential due to a thin circular ring of radius a and mass M for points lying in the plane of the ring and exterior to it. The result can be expressed as an elliptic integral. Assume that the distance from the center of the ring to the field point is large compared to the radius of the ring. Expand the expression for the potential and find the first correction term. I want you to put some thought into how you're going to use Mathematica to produce an expansion of the potential in $\frac{a}{r}$, and I want that final expansion to look nice. If you're not already familiar with the following commands, I suggest you look them up in the help browser: **Series**, **Normal**.
- 4) If you scratch around a bit on a piece of scrap paper, you should be able to convince yourself that in system of units where time is measured in years and length in astronomical units (a.u.'s), the combination $GM_{sun} = 4\pi^2$. You don't need to do that here, but it sets the stage for the problem. I would like you to (use Lagrangian Dynamics to) plot the orbital trajectory for a particle with potential energy $U = -4\pi^2 m/r^{1+\epsilon}$. Take $m = 1$ (it shouldn't matter), $\vec{r_0} = \{1, 0\}$, $\dot{\vec{r_0}} = \{-\pi, 2\pi\}$. Do this for:
  - i) $\epsilon = 0$ (gravity as we know it)
  - ii) $\epsilon = +0.1$
  - iii) $\epsilon = -0.4$

  Feel free to play around with other values (but keep in mind what you've learned about stable orbits). Let time run long enough to see some neat features. Make sure you label your axes. Set AspectRatio->Automatic, and consider bumping up MaxSteps.

- 5) Now it's time to show-off a little. Let's see if you can animate the orbits in the first problem. Put a nice, big, yellow dot at the origin for the sun, a small blue dot at the location of the planet and show the planet gliding over its superimposed trajectory. Add a line joining the sun to the planet so that you can see the area being swept-out as the planet glides along.