$\boxed{\text{Mathematica Homework \#2}}$

Email notebook to corbin@physics.ucla.edu
with a subject line: [Physics 105A]
by on or about Friday, 29 January

- In the first cell, enter your **name**, **student ID**, **email address** and the **assignment identifier** (eg. "HW 2") as text.
- 1) Solve the following for $x(t)$:

$$\ddot{x} = a \qquad \dot{x}(0) = v_0 \qquad x(0) = x_0$$

- 2) Solve the following for $x_1(t)$ and $x_2(t)$:

$$m\ddot{x}_1 + 2kx_1 - kx_2 = 0 \qquad m\ddot{x}_2 + 2kx_2 - kx_1 = 0$$

$$\dot{x_1}(0) = 0 \qquad \dot{x_2}(0) = 0 \qquad x_1(0) = 0 \qquad x_2(0) = a$$

- 3) An under-damped mass-spring system, initially at rest in equilibrium on a frictionless horizontal surface, is subjected to a constant driving force ($\frac{F}{m} = a$) for a length of time equal to some fraction (f) of the natural period of the system.
  - *i*) Solve for the equations of motion for the system during both the driven and subsequent undriven intervals.
  - *ii*) Take $\tau_0 = 1$, $\beta = \frac{\omega_0}{3}$, $a = 10$, $f = 0.5$ and plot the motion of the system from $0 < t < 3$ (assuming the force is first applied at $t = 0$). It may help to color the two two intervals differently, say red and blue (PlotStyle).
- 4) A pair of masses, m_1 and m_2 sit on a frictionless table. They are joined by a spring of constant k and natural (unstretched) length L. The positions of the masses at any time t (measured with respect to some arbitrary point on the line joining the masses) are given by $x_1(t)$ and $x_2(t)$, respectively.

  In a single cell...
  - *i*) Obtain the Lagrangian for the system in an inertial frame of reference.
  - *ii*) For mass/spring systems, the best generalized coordinates are those that measure the location of each body relative to its equilibrium position. Use Mathematica to re-write the Lagrangian in terms of these generalized coordinates.

- *iii*) Write a Mathematica function that will use the Euler-Lagrange equation to obtain the equation of motion for any generalized coordinate you give it as an input. Make sure you take partials and full-derivatives in the right places. You will probably need to tell Mathematica which labels correspond to quantities that are constant - you may use the **Constants** option to the full derivative command, or - probably better, use the **SetAttributes** command.
- *iv*) At this point, you should have a system of coupled differential equations. Invoke reasonable initial conditions and use DSolve[...] to solve for the motion (in generalized coordinates).
- *v*) Not required - but it might be fun to animate the result.

- 5) Consider a double-pendulum system. Let's give it a whirl in Mathematica and see what happens. Take a pair of identical physical pendula (mass m, length l), Tie the first pendulum to a fixed pivot and the second to the mass at the end of the first.
  - *i*) Write expressions for the kinetic and potential energies of the system, and the corresponding Lagrangian, in a convenient inertial frame.
  - *ii*) Convert the coordinates in the inertial frame to a more convenient set of generalized coordinates. Evaluate the expression for the Lagrangian (use **Simplify**) and see if it looks reasonable and familiar.
  - *iii*) Write a function that will use the Euler-Lagrange equation to obtain the equation of motion for any generalized coordinate you give it as an input. Make sure you take partials and full-derivatives in the right places.
  - *iv*) Use that function to obtain differential equations of motion for the pendula.

We'll have to use NDSolve[...] to go any further... Stay tuned