An Integrated Math 1 Guide

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Everything presented here is to help you understand, learn, and how to a tackle problem. It is not intended to do you work. That is for you to do to be successful in life as you always have to learn personally, academically, and in work-related areas. Without further ado, let's learn math! 🤓

Finding the Slope

Let's say we have a problem with two points:

$$(1,0)(0,-1) \tag{1}$$

And it states to find the slope given these two points. To do this, remember the Slope Formula for a line:

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \tag{2}$$

Also remember a point has an xy-pair such as (x, y). Let's label (1) with the first point having an xy-pair as (x_1, y_1) while the second point has (x_2, y_2):

$$(\underbrace{1}_{\text{x1}}, \underbrace{0}_{\text{y1}})(\underbrace{0}_{\text{x2}}, \underbrace{-1}_{\text{y2}}) \tag{3}$$

Notice how the labels x_1, y_1, x_2, y_2 are underneath the numbers? It is recommended to do this so you could keep track and make less mistakes. With the labeling now complete, use the Slope Formula from (2) to plug in the numbers from (3) into the correct labels:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-0}{0-1} \tag{4}$$

$$= \frac{-1}{-1} \xrightarrow{\text{becomes positive}} \frac{1}{1} \tag{5}$$

$$\boxed{m = 1} \tag{6}$$

Notice how on (5) the two negatives on top and bottom give you a positive since dividing two negatives gives you a positive.

And that is how you Find the Slope of a Line! 🎯

Substitution Method

For the Substitution Method, there needs to be at least 2 equations, and they could have have two variables such as x and y. 👉 NOTE: The variables does not need to be in x or y, but they could be any type of letter!

Either of the 2 equations could be in a form where both variables are by each other: $x + y = 10$, $y - x = -1$, and so on. The variables could be opposite of each other where one is on the left side of the equal sign while the other is on the right side: $x = y$, $y - 5 = x$, $x + 1 = 2 + y$, and so on.

Lastly, the equations might only have one variable equal to a number: $x = 1$, $y = 0$, and so on.

Let's say we have a problem like so:

$$\begin{align} x+y=2 \xrightarrow{\text{This is (7)}} \tag{7}\\ 4x-y=8 \xrightarrow[\text{This is (8)}]{} \tag{8}\end{align}$$

To solve this Systems of Equations by using the Substitution Method, you have to isolate one of the variables from either (7) or (8). Let's pick (7) since that looks the easiest, and let's start isolating the variable y by subtracting x from both sides:

$$\begin{align} x-x+y=2-x \tag{9}\\ y=2-x \tag{10}\end{align}$$

The variable y is now isolated, and now let's plug (10) into (8) in order to solve for x:

$$4x-(2-x)=8 \tag{11}$$

Notice how on (11) the y gets replaced with (10) since that is what y equals to, and also noticed how on (11) the parenthesis $()$ is surrounding $2-x$? Whenever you replace a variable with an equation, you always put that equation in parenthesis in order to avoid getting the wrong answer! IMPORTANT 🚨: Always put parenthesis no what it is such as $2-x$, or a single variable like x, or a number with no variable like 2, or a single variable that is negative like $-x$, and so on.

Let's continue from (11) by distributing the minus sign in front of the left parenthesis onto 2 and $-x$:

$$\begin{align} 4x-2--x=8 \tag{12}\\ 4x-2+x=8 \end{align}$$

Notice how on (12) the x becomes a ***positive*** since the two negatives give you a minus sign. Let's continue where on the left side of the equal sign we have *like terms* of $4x$ and x, and combining them gives you $4x+x=5x$. This gives you $5x-2=8$. Now, you need x to be by itself and to do that is by first adding 2 to both sides of the equal sign, and we *add* since to get rid of -2 is by taking its *opposite* which is $+2$:

$$\begin{align} 5x-2+2=8+2 \tag{13}\\ 5x=10 \end{align}$$

Now, you have x being multiplied by 5. To get rid of 5, take the *reciprocal* of 5 on both sides of the equation. Another way to think about this is since x is being *multiplied* by 5, take the opposite of *multiplication* which is *division*:

$$\begin{align} \frac{5x}{5}=\frac{10}{5} \tag{14}\\ x=\frac{10}{5} \tag{15}\\ x=2 \tag{16}\end{align}$$

Almost done! With (16), we could plug this into either (7) or (8) to solve for y. Let's pick (7) since that looks pretty easy, and remember to put it into parenthesis:

$$\begin{align} x+y=2 \xrightarrow{\text{becomes}} (2)+y=2 \tag{17}\\ 2+y=2 \xrightarrow{\text{subtract 2 to both sides}} 2-2+y=2-2 \tag{18}\\ y=2-2 \tag{19}\\ y=0 \tag{20}\end{align}$$

🛑 THIS IS NOT THE FINAL ANSWER 🛑! Your teacher is going to want to have the final answer in *point form* such as (x, y). With $x=2$ and $y=0$, the final answer is:

$$(x, y) = \boxed{(2, 0)} \tag{21}$$

That is how you do the Substitution Method! 🚀

Graphing Systems of Equations

To graph Systems of Equations, lets use the equations (7) and (8) from the Substitution Method section. Now, to graph these two equations, one of the methods to use is finding the x and y intercepts. Once you find the x and y intercepts for one line, you can plot them and draw a line through those two points. After that, do the same procedure for the second line.

For our first line, we'll start with equation (7). The first step is to set either x or y equal to 0, and then solve for the either variable. In this case, let's set x equal to 0:

$$x + y = 2 \xrightarrow{\text{x becomes 0}} 0 + y = 2 \xrightarrow{\text{0 + y = y}} y = 2 \tag{22}$$

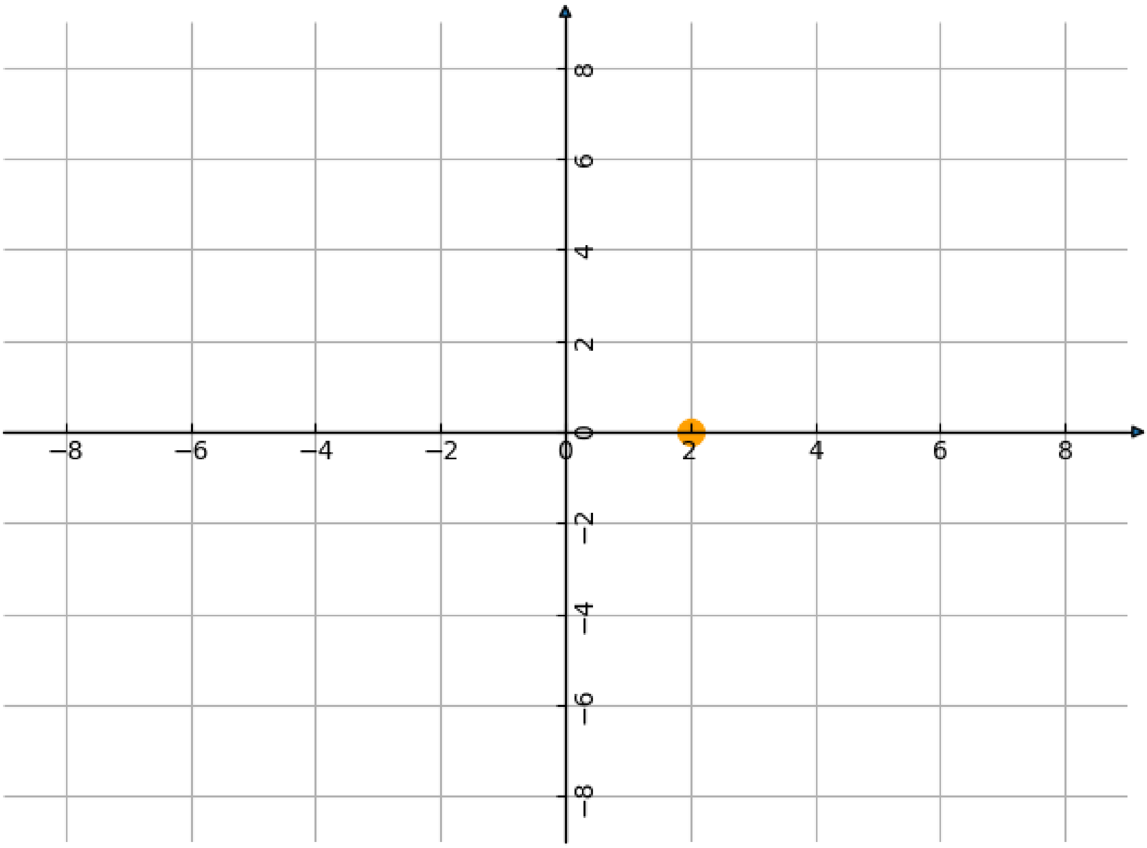
From (22), we have $y = 2$ with $x = 0$, and thus our y-intercept in point form is $(x, y) = (0, 2)$. This is for equation (7), and also remember that you need to have it in point form or your teacher is going to dock some points. Now, lets find the x-intercept on (7) by using a similar procedure, but this time setting $y = 0$:

$$x + y = 2 \xrightarrow{\text{y becomes 0}} x + 0 = 2 \xrightarrow{\text{x + 0 = x}} x = 2 \tag{23}$$

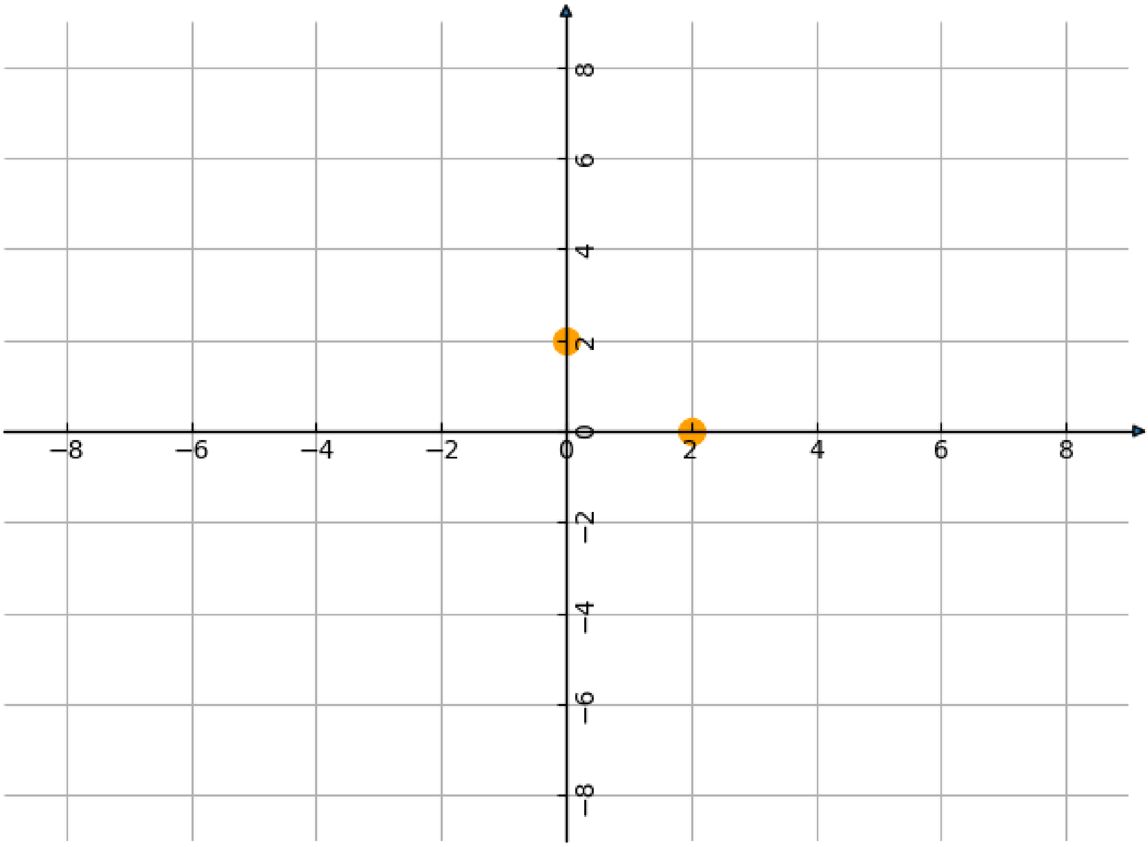
From (23), we have an x-intercept of $(x, y) = (2, 0)$ in point form where $x = 2$ and $y = 0$. With this, we could now plot these points on a graph for equation (7), and we'll do it step-by-step on plotting them. Lastly, here are the x-intercept, y-intercept, and equation (7) as reference:

$$x + y = 2, \quad \text{x-int: } (2, 0), \quad \text{y-int: } (0, 2)$$

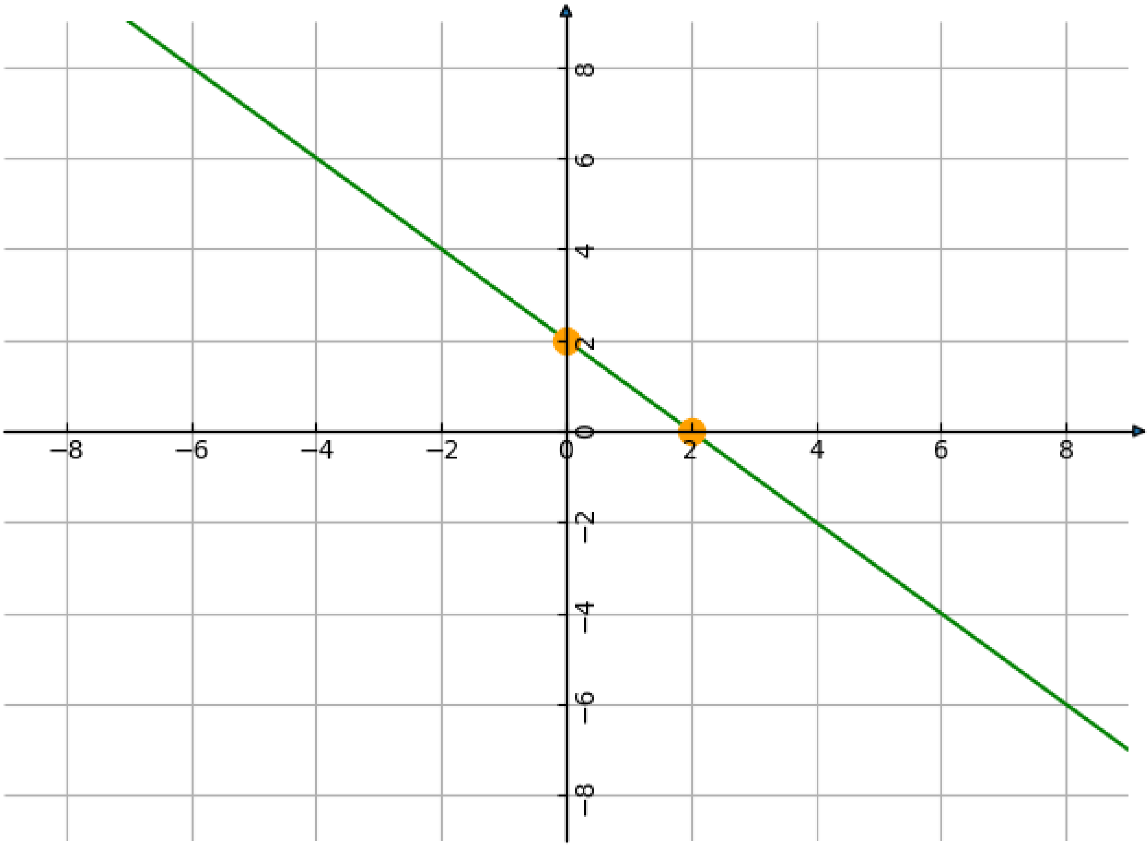
First, start from the center 0 and plot the x-intercept by moving 2 to the right for x, and since y is 0 then it stays put without going up or down:



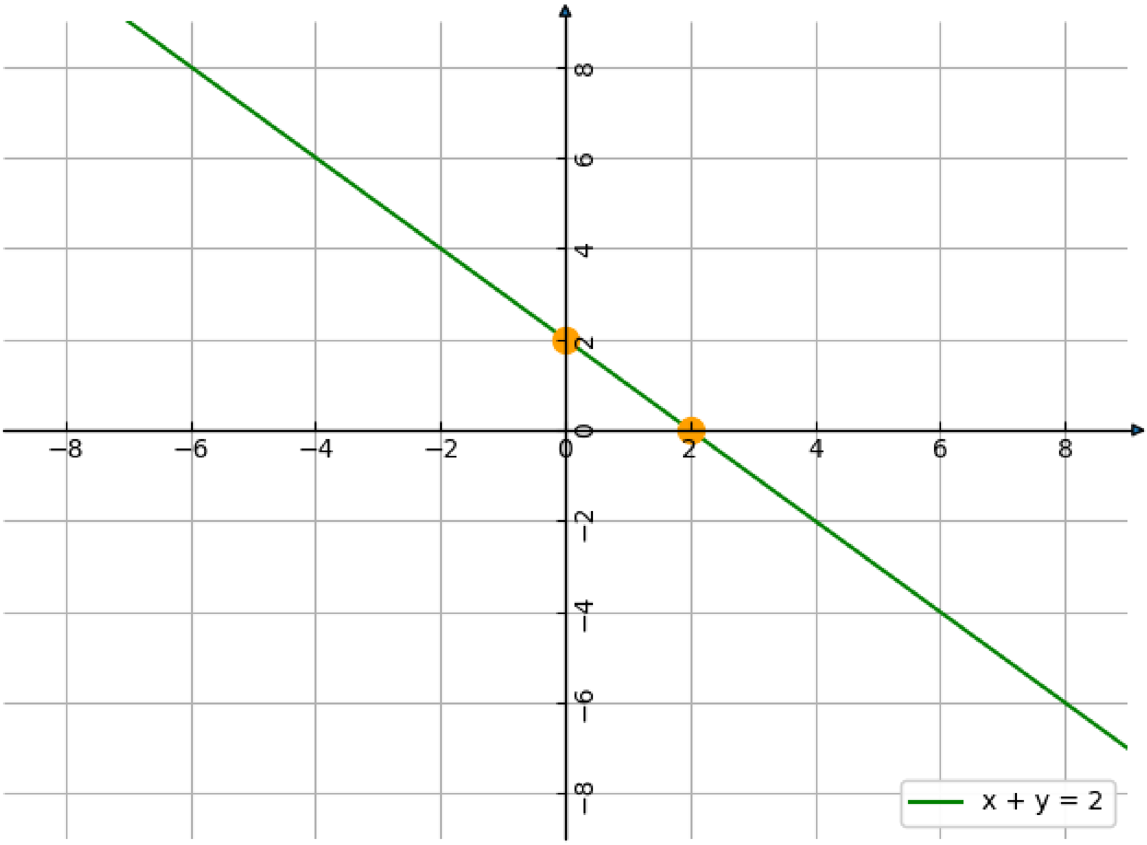
Now, plot the y-intercept by starting at the center 0 and do not move left or right since $x = 0$, but move up to the number 2 since $y = 2$:



With a straight edge like a ruler or an ID card, draw a line through these two points:



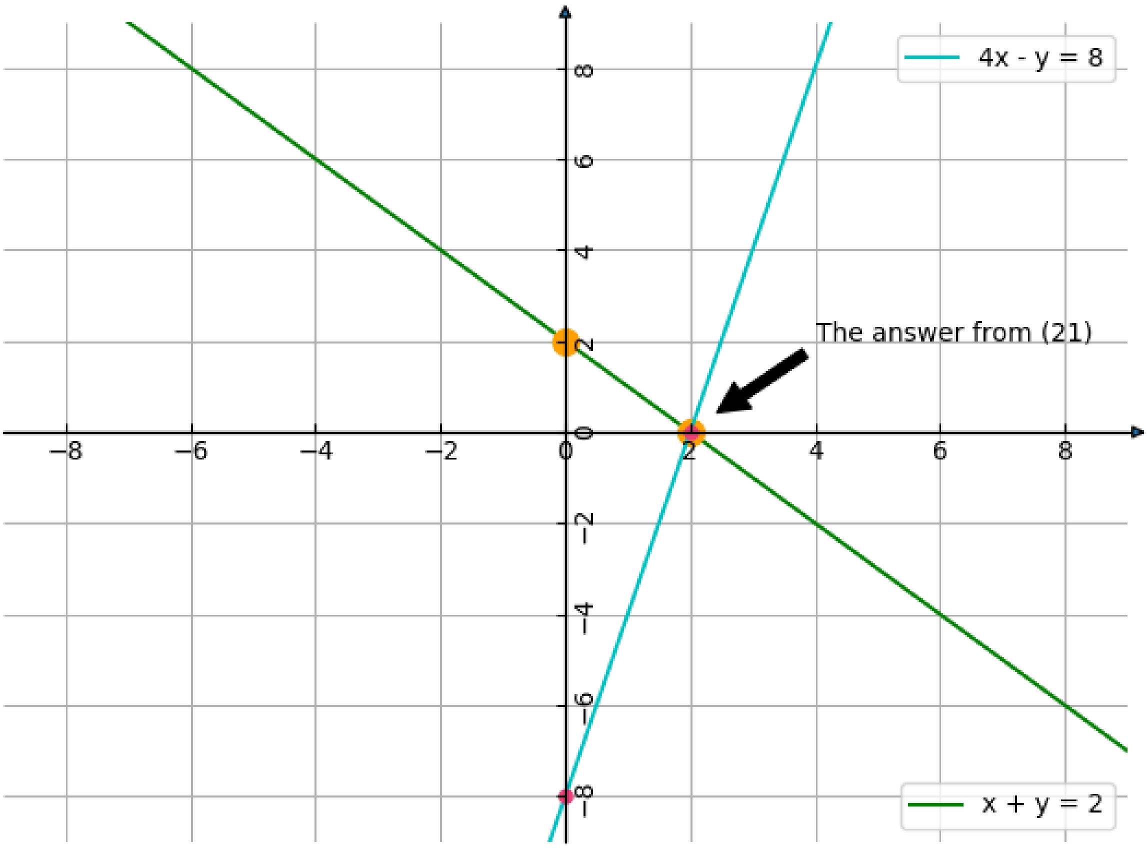
Next, use equation (7) to label the line you just drew, and label it next to the line:



Next, we need to plot equation (8) by finding the x-intercept and y-intercept. BUT, this time it is your turn to find the x-intercept and y-intercept. Here is equation (8) for your convenience:

$$4x - y = 8$$

Hopefully, you did the problem by finding the x-intercept and y-intercept. I am not going to put the answers for the x-intercept and y-intercept since I want you to do it on your own, but I am going to provide the final completed graph that includes both equations (7) and (8):



If you notice from the above graph, you see that both lines intersect (or cross) at the point $(2, 0)$. Not only is this the x-intercepts for both equations of $4x - y = 8$ and $x + y = 2$, BUT this is also what we got the answer for at (21) in the Substitution Method! So, the Substitution Method is used to find the point where two lines intersect (or cross) each other. We could also find the same answer without the Substitution Method by graphing it like we just did in the image above. YET, although doing the Substitution Method is a lot of work, it is actually easier because sometimes you might get equations that are nasty and almost impossible to graph such as this: $e^{ix} = \cos x + i \sin x$. If you want to attempt to graph this, be my guest.

There you have it! That is how you Graph Systems of Equations along with Finding the Slope and doing the Substitution Method. Keep on studying hard and never give up. Always keep pushing.

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Hopefully everything made sense in this document, but if you have any questions then please contact your Case Manager(s) or Teacher(s) for further information. Thank you and have a very good day!