For the source code for the coding problems, see the attached Jupyter notebook, Matlab live script. Alternatively, the entire git repository is attached as a zip archive, and is available on GitHub. The comments in the code have been omitted here for brevity. They are present in the Jupyter notebook and Matlab live script.

(1) **Problem Statement:** Compute the interpolating polynomial $p_2(x)$ that interpolates $f = \sqrt{2}x\cos(x)$ at points $x_0 = 0$, $x_1 = \frac{\pi}{4}$, and $x_2 = \frac{\pi}{2}$ in the interval $[0, \frac{\pi}{2}]$.

$$p_2(x_0) = 0 = a(x_0)^2 + b(x_0) + c$$
$$0 = a(0)^2 + b(0) + c$$
$$0 = c$$
$$c = 0$$

We have c, let's move on to calculate b.

$$p_2(x_1) = \sqrt{2}x_1\cos(x_1) = a(x_1)^2 + b(x_1) + c$$
$$\sqrt{2}\frac{\pi}{4}\cos\left(\frac{\pi}{4}\right) = a\left(\frac{\pi}{4}\right)^2 + b\left(\frac{\pi}{4}\right) + 0$$
$$\frac{\pi\sqrt{2}}{4}\frac{\sqrt{2}}{2} = a\frac{\pi^2}{8} + b\frac{\pi}{4}$$
$$\frac{\pi}{4} = a\frac{\pi^2}{8} + b\frac{\pi}{4}$$
$$\frac{\pi}{4} - b\frac{\pi}{4} = a\frac{\pi^2}{8}$$
$$1 - b = a\frac{\pi^2}{8}\frac{4}{\pi}$$
$$1 - b = a\frac{\pi}{2}$$
$$b = 1 - a\frac{\pi}{2}$$

We have b in terms of a. Using this, we can calculate the value of a and then plug that back in here to get b.