For the source code for the coding problems, see the attached Jupyter notebook, Matlab live script. Alternatively, the entire git repository is attached as a zip archive, and is available on GitHub. The comments in the code have been omitted here for brevity. They are present in the Jupyter notebook and Matlab live script.

(1) **Problem Statement:** Compute the interpolating polynomial $p_2(x)$ that interpolates $f = \sqrt{2}xcos(x)$ at points $x_0 = 0$, $x_1 = \frac{\pi}{4}$, and $x_2 = \frac{\pi}{2}$ in the interval $[0, \frac{\pi}{2}]$.

$$\begin{aligned} p_2(x_0) = 0 &= a(x_0)^2 + b(x_0) + c \\ 0 &= a(0)^2 + b(0) + c \\ 0 &= c \\ c &= 0 \end{aligned}$$

We have c, let's move on to calculate b.

$$\begin{aligned} p_2(x_1) = \sqrt{2}x_1cos(x_1) &= a(x_1)^2 + b(x_1) + c \\ \sqrt{2}\frac{\pi}{4}cos\left(\frac{\pi}{4}\right) &= a\left(\frac{\pi}{4}\right)^2 + b\left(\frac{\pi}{4}\right) + 0 \\ \frac{\pi\sqrt{2}}{4}\frac{\sqrt{2}}{2} &= a\frac{\pi^2}{8} + b\frac{\pi}{4} \\ \frac{\pi}{4} &= a\frac{\pi^2}{8} + b\frac{\pi}{4} \\ \frac{\pi}{4} - b\frac{\pi}{4} &= a\frac{\pi^2}{8} \\ 1 - b &= a\frac{\pi^2}{8}\frac{4}{\pi} \\ 1 - b &= a\frac{\pi}{2} \\ b &= 1 - a\frac{\pi}{2} \end{aligned}$$

We have b in terms of a. Using this, we can calculate the value of a and then plug that back in here to get b.

$$\begin{aligned} p_2(x_2) = \sqrt{2}x_2cos(x_2) &= a(x_2)^2 + (1 - a\frac{\pi}{2})x_2 + c \\ &= a(\frac{\pi}{2})^2 + \frac{\pi}{2} - a\frac{\pi^2}{4} + 0 \end{aligned}$$