

The Dirac – Kerr-Newman electron.

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We discuss the relation of the Kerr-Newman spinning particle to the Dirac electron and show that the Dirac equation may naturally be incorporated into Kerr-Schild formalism as a master equation controlling the Kerr-Newman geometry. As a result, the Dirac electron acquires an extended space-time structure of the Kerr-Newman geometry - singular ring of the Compton size and twistorial polarization of the gravitational and electromagnetic fields.

Behavior of this Dirac – Kerr-Newman system in the weak and slowly changed electromagnetic fields is determined by the wave function of the Dirac equation, and is indistinguishable from the behavior of the Dirac electron. The wave function of the Dirac equation plays in this model the role of an “order parameter” which controls dynamics, spin-polarization and twistorial structure of space-time.

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I. INTRODUCTION

The discovery of the Kerr and Kerr-Newman solutions [1, 2] had fundamental importance for all the areas of theoretical physics from cosmology and astrophysics to superstring theory. Interest to these solutions in the physics of elementary particles has been raised recently by the conjecture that black holes may be created in the laboratory conditions.

On the other hand, the treatment of the Kerr-Newman solution as a model of electron have been considered many times [3–16] after the well known Carter remark [3] that the Kerr-Newman solution has gyromagnetic ratio $g = 2$ as that of the Dirac electron. If this coincidence is not occasional, there appears a fundamental question - what is the relation of the Dirac equation to the structure of Kerr-Newman solution?

The Dirac equation is also the discovery of fundamental importance. Practically, most of the theoretical and experimental results in the particle physics are based on the Dirac equation and its consequences. Nevertheless, the Dirac description of electron can not be considered as complete for two reasons:

- it does not take into account gravitational field of electron,

- the Dirac wave function carries a very obscure information on the space-time position and, especially, on the structure of electron.

Forming the wave packets one can only show that electron cannot be localized inside the Compton region. Meanwhile, the multi-particle QED description showed that the “naked” electron is point-like and the electron “dressed” by virtual photons is smeared over the Compton region. The point-like structure of electron is also confirmed by the deep inelastic scattering. There were a lot of attempts to describe the extended structure of spinning particle on the base of field models, and some of them were related to gravity.

The Kerr solution gives us a natural description of spinning particle with gravity, and moreover, it hints us about the relation to electron by the double gyromag-

netic ratio [3, 5], by the extension on the Compton region [4, 6, 10], by the wave properties [6], and by the reach spinor and twistorial structures [7, 8, 12–15].

The aim of this paper is to consider a model which sets a relation between spinor solutions of the Dirac equation and the spinor (twistorial) structure of the Kerr-Newman solution. In this paper we suggest a new approach which is based on the assumption that *the Dirac equation is complimentary to the Kerr-Newman spinning particle and plays the role of a master equation controlling the motion and orientation of the Kerr-Newman twistorial space-time structure*. In this model the Dirac spinors are matched to the spinor (twistorial) structure of the Kerr-Newman spinning particle, and the Dirac wave function plays the role of an order parameter controlling this system.

On the other hand, since *the behavior of this combined Dirac – Kerr-Newman system turns out to be determined by the Dirac wave function, it is undistinguishable from the behavior of the Dirac electron*, which allows one to interpret this model in the frame of one-particle quantum theory.

Plane of the work is following. To realize our aim we have to find a bridge between the Dirac theory and the Kerr-Newman structure, i.e. to obtain some objects which are common for both these structures.

In sec.II we consider the real and complex structures of the Kerr-Newman geometry and obtain that the motion and polarization of the Kerr-Newman spinning particle are controlled by two null vectors k_L and k_R which are related to a “point-like” complex representation of the Kerr-Newman geometry.

In sec.III, analyzing the structure of Dirac equation in the Weyl basis, we obtain two similar null vectors k_L and k_R which are determined by the Dirac wave function Ψ and control the dynamics and polarization of the Dirac electron.

It allows as to set a link between the Dirac and Kerr-Newman spinning particle, which is performed in sec.IV. As a result, the the Dirac – Kerr-Newman electron acquires a definite extended space-time structure of the

Kerr-Newman geometry, and the Dirac wave function takes the role of an “order parameter” which controls polarization and dynamics of this structure. In particular, the Kerr-Newman twistorial structure acquires a dependence on the vector potential of the external electromagnetic field via the Dirac equation.

In the following sections we discuss the properties and consequences of the Dirac – Kerr-Newman model in comparison with the properties of electron in Dirac theory and QED.

In particular, in sec.V we show that renormalization of the self-energy of the Dirac – Kerr-Newman electron may be performed by gravity for different distributions of the mechanical mass and charge, and we describe the approach which allows one to get the regular “dressed” solutions for different values of charge, mass and spin.

In sec.VI, following to the known approach to quantum fields in curved spaces [17, 18], we consider regularization of the stress-energy tensor, which has a specific realization in the Kerr-Newman geometry.

In sec.VII, treating the obtained recently multi-particle Kerr-Schild solutions [19], we show that the Dirac – Kerr-Newman electron model takes an intermediate position between the one-particle Dirac electron and the multi-particle structure of electron “dressed” by virtual photons in QED.

Sec.VIII contains a discussion on the space-time structure of electron and the role of wave function in the Dirac theory, QED and in the Dirac – Kerr-Newman model.

In Conclusion we return to the origin of mass term in the Dirac equation and discuss some possible ways for modification of this model.

We use the spinor notations of the book [20] (see Appendix A). However our spinors are commuting, which changes some relations, and we give them in the Appendix B. Matching notations to the paper [5], we use signature $(-+++)$, Cartesian coordinates $x^0 = t$, $x^\mu = (t, x, y, z)$ and $E = T_0^0 = p_0 = -p^0 > 0$.

II. REAL AND COMPLEX STRUCTURES OF THE KERR GEOMETRY

The real structure of the Kerr-Newman geometry, Kerr congruence and the Kerr theorem were discussed many times, and we refer readers to our previous papers, for example [13–15, 19, 21].

Recall, that angular momentum $J = \hbar/2$ for parameters of electron is so high that the black hole horizons disappear and the source of the Kerr-Newman spinning particle represents a naked singular ring.

It was suggested in [6, 7] that the Kerr-Newman singular ring represents a string which may have some excitations generating the spin and mass of the extended particle-like object - “microgeon”. The Kerr-Newman singular ring is a focal line of the principal null congruence which is a bundle of the lightlike rays – twistors. The null vector field $k^\mu(x)$, which is tangent to these

rays, determines the form of metric

$$g^{\mu\nu} = \eta^{\mu\nu} + 2Hk^\mu k^\nu \quad (1)$$

(where $\eta^{\mu\nu}$ is the auxiliary Minkowski metric) and the form of vector potential

$$A_\mu = \mathcal{A}(x)k_\mu \quad (2)$$

for the charged Kerr-Newman solution.

The Kerr congruence is twisting (see figures in our previous works [13–15, 21]). It is determined by the Kerr theorem [21–23] which constructs the Kerr-Schild ansatz (1), skeleton of the Kerr-Newman geometry, as a bundle of twistors [14, 15, 21].

Point-like complex representation of the Kerr-Newman geometry.

Complex representation of the Kerr-Newman geometry is important for our treatment here since the complex Kerr-Newman source is “point-like”, which corresponds to a “naked” electron in our treatment.

Applying the complex shift $(x, y, z) \rightarrow (x, y, z + ia)$ to the source $(x_0, y_0, z_0) = (0, 0, 0)$ of the Coulomb solution q/r , Appel in 1887(!) considered the resulting solution

$$\phi(x, y, z) = \Re q/\tilde{r}, \quad (3)$$

where $\tilde{r} = \sqrt{x^2 + y^2 + (z - ia)^2}$ turns out to be complex. On the real slice (x, y, z) , this solution acquires a singular ring corresponding to $\tilde{r} = 0$. It has radius a and lies in the plane $z = 0$. The solution is conveniently described in the oblate spheroidal coordinate system r, θ , where $\tilde{r} = r + ia \cos \theta$, and one can see that the space is twofolded having the ring-like singularity as the branch line. Therefore, for each real point $(t, x, y, z) \in \mathbf{M}^4$ we have two points, one of them is lying on the positive sheet, corresponding to $r > 0$, and another one lies on the negative sheet, where $r < 0$.

It was obtained that Appel potential corresponds exactly to electromagnetic field of the Kerr-Newman solution written in the Kerr-Schild form [6]. The vector of complex shift $\vec{a} = (a_x, a_y, a_z)$ corresponds to angular momentum of the Kerr-Newman solution.

Newman and Lind [24] suggested a description of the Kerr-Newman geometry in the form of a retarded-time construction, where it is generated by a complex source, propagating along a *complex world line* $\overset{\circ}{X}(\tau)$ in a complexified Minkowski space-time \mathbf{CM}^4 . The rigorous substantiation of this representation can be achieved in the Kerr-Schild approach [5] which is based on the Kerr theorem and the Kerr-Schild form of metric (1) which are related to the auxiliary \mathbf{CM}^4 [21, 25, 26]. In the rest frame of the considered Kerr-Newman particle, one can form two null 4-vectors $k_L = (1, 0, 0, 1)$ and $k_R = (1, 0, 0, -1)$, and represent the 3-vector of complex shift $i\vec{a} = i\Im m \overset{\circ}{X}(\tau)$ as the difference $i\vec{a} = \frac{ia}{2}\{k_L - k_R\}$. The straight complex world line corresponding to a free particle may be decomposed into the form

$$\overset{\circ}{X}{}^\mu(\tau) = \overset{\circ}{X}{}^\mu(0) + \tau u^\mu + \frac{ia}{2}\{k_L^\mu - k_R^\mu\}, \quad (4)$$

where the time-like 4-vector of velocity $u^\mu = (1, 0, 0, 0)$ can also be represented via vectors k_L and k_R

$$u^\mu = \partial_t \Re e \overset{\circ}{X}{}^\mu(\tau) = \frac{1}{2}\{k_L^\mu + k_R^\mu\}. \quad (5)$$

It allows one to describe the complex shift in the Lorentz covariant form.

One can form two complex world lines related to the complex Kerr-Newman source, $\overset{\circ}{X}_+(t+ia) = \Re e \overset{\circ}{X}{}^\mu(\tau) + iak_L^\mu$ and $\overset{\circ}{X}_-(t-ia) = \Re e \overset{\circ}{X}{}^\mu(\tau) - iak_R^\mu$. This representation will allow us to match the Kerr-Newman geometry to the solutions of the Dirac equation.

Indeed, since the complex world line $\overset{\circ}{X}{}^\mu(\tau)$ is parametrized by the complex time parameter $\tau = t + i\sigma$, it represents a stringy world sheet. However, this string is very specific, it is extended along the imaginary time parameter σ . The world lines $\overset{\circ}{X}_+ = \overset{\circ}{X}{}^\mu(t+ia)$ and $\overset{\circ}{X}_- = \overset{\circ}{X}{}^\mu(t-ia)$ are the end points of this open complex twistor-string [12–15], see fig.1. By analogue with the real strings where the end points are attached to quarks, one can conventionally call these complex point-like sources as ‘quarks’.

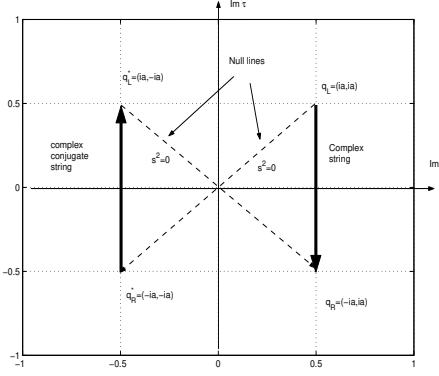


FIG. 1: Positions of the complex point-like sources (conventionally “quarks”) at the ends of the open complex Kerr-Newman string. The “quarks” form two pairs which are related by the light-like intervals $s^2 = 0$. Two complex conjugated strings are to be joined forming orientifold, i.e. a folded closed string.

III. DIRAC EQUATION IN THE WEYL BASIS.

In the Weyl basis the Dirac equation

$$(\gamma^\mu \hat{\Pi}_\mu + m)\Psi = 0, \quad (6)$$

where $\Psi = \begin{pmatrix} \phi_\alpha \\ \chi_{\dot{\alpha}} \end{pmatrix}$, and $\hat{\Pi}_\mu = -i\partial_\mu - eA_\mu$, splits into the system [27, 28]

$$\sigma_{\alpha\dot{\alpha}}^\mu(i\partial_\mu + eA_\mu)\chi^{\dot{\alpha}} = m\phi_\alpha, \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha}(i\partial_\mu + eA_\mu)\phi_\alpha = m\chi^{\dot{\alpha}}, \quad (7)$$

The Dirac current

$$J_\mu = e(\bar{\Psi}\gamma_\mu\Psi) = e(\bar{\chi}\sigma_\mu\chi + \bar{\phi}\bar{\sigma}_\mu\phi), \quad (8)$$

where $\bar{\Psi} = (\chi^+, \phi^+)$, can be represented as a sum of two lightlike components of opposite chirality

$$J_L^\mu = e\bar{\chi}\sigma^\mu\chi, \quad J_R^\mu = e\bar{\phi}\bar{\sigma}^\mu\phi. \quad (9)$$

Forming the null vectors $k_L^\mu = \bar{\chi}\sigma^\mu\chi$, and $k_R^\mu = \bar{\phi}\bar{\sigma}^\mu\phi$, one can obtain for their product[44]

$$k_L^\mu k_{R\mu} = (\bar{\chi}\sigma^\mu\chi)(\bar{\phi}\bar{\sigma}^\mu\phi) = -2(\bar{\phi}\bar{\chi})(\chi\bar{\phi}) = 2(\bar{\chi}\phi)(\bar{\chi}\phi)^+. \quad (10)$$

One can also form two more vector combinations from the Dirac spinors, $m^\mu = \phi\sigma^\mu\chi$, and $\bar{m}^\mu = (\phi\sigma^\mu\chi)^+ = (\bar{\chi}\sigma^\mu\bar{\phi})$, which are complex conjugated and have also the scalar product

$$m^\mu \bar{m}_\mu = 2(\bar{\chi}\phi)(\bar{\chi}\phi)^+. \quad (11)$$

All the other products of the vectors k_L , k_R , m , \bar{m} are null.

The normalized four null vectors

$$n^a = \frac{1}{\sqrt{2(\bar{\chi}\phi)(\bar{\chi}\phi)^+}}(m, \bar{m}, k_L, k_R), \quad a = 1, 2, 3, 4, \quad (12)$$

form a field of the natural quasi orthogonal null tetrad which is determined by the given solution $\Psi(x)$ of the Dirac equation.

Notice, that the complex vectors m and \bar{m} turn out to be modulated by the phase factor $\exp\{2ip_\mu x^\mu\}$ coming from the Dirac spinors and carry oscillations and also de Broglie periodicity for the moving particle. Meanwhile, this phase factor cancels for the real null vectors k_L and k_R .

If Ψ is a plane wave

$$\Psi = \begin{pmatrix} \phi_\alpha \\ \chi_{\dot{\alpha}} \end{pmatrix} = \begin{pmatrix} \overset{\circ}{\phi}_\alpha \\ \overset{\circ}{\chi}_{\dot{\alpha}} \end{pmatrix} \exp\{ip_\mu x^\mu\}, \quad (13)$$

the Dirac equations take the form

$$m\phi_\alpha = -\Pi_\mu \sigma_{\alpha\dot{\alpha}}^\mu \chi^{\dot{\alpha}}, \quad m\chi^{\dot{\alpha}} = -\Pi_\mu \bar{\sigma}^{\mu\dot{\alpha}\alpha} \phi_\alpha, \quad (14)$$

where

$$\Pi_\mu = p_\mu - eA_\mu. \quad (15)$$

Multiplying these equation by the spinors $\phi, \chi, \bar{\phi}, \bar{\chi}$ to the left, one obtains the null tetrad components of Π_μ

$$\Pi_\mu m^\mu = -m\phi\phi = 0, \quad \Pi_\mu \bar{m}^\mu = -m\chi\chi = 0. \quad (16)$$

$$\Pi_\mu k_L^\mu = -m\bar{\chi}\phi, \quad \Pi_\mu k_R^\mu = -m\bar{\phi}\chi. \quad (17)$$

It shows that the vector Π_μ is spanned by the real vectors k_L and k_R , i.e. $\Pi^\mu = ak_L^\mu + bk_R^\mu$, where $a = b = -\frac{m}{2(\phi\bar{\chi})}$. Therefore,

$$\Pi^\mu = -\frac{m}{2\bar{\phi}\chi}(k_L^m + k_R^m). \quad (18)$$

In the rest frame

$$k_L^0 = k_R^0, \quad k_L^i = -k_R^i, \quad (19)$$

and the space components of Π , ($i = 1, 2, 3$) are cancelled.

The spin of electron is determined by the polarization vector which has the form [28]

$$S^\mu = i\bar{\Psi}\gamma^\mu\gamma^5\Psi = k_L^\mu - k_R^\mu, \quad (20)$$

and therefore, in the rest frame $S^0 = 0$ and $S = (0, S^i)$.

For normalized spinors

$$|\bar{\phi}\chi| = |\phi_1|^2 + |\phi_2|^2 = 1. \quad (21)$$

For $E > 0$

$$\bar{\phi}\chi = -1, \quad (22)$$

which yields $\Pi^0 > 0$. For $E < 0$

$$\bar{\phi}\chi = 1, \quad (23)$$

and $\Pi^0 < 0$.

IV. DIRAC EQUATION AS A MASTER EQUATION FOR THE KERR-NEWMAN TWISTORIAL STRUCTURE

In previous treatment we have seen that the complex Kerr-Newman geometry is related to two null vectors k_L and k_R which determine the momentum and angular momentum of Kerr-Newman particle. We have also seen that the momentum and spin of the Dirac electron in the Weyl basis are also expressed via two null vectors k_L and k_R . It allows us to connect the solutions of the Dirac equation to twistorial structure of the Kerr-Newman spinning particle by setting an equivalence for these null vectors.

The Kerr-Schild ansatz for metric(1) is fixed by the null vector field $k_\mu(x)$, which is tangent to the Kerr principal null congruence (PNC) and is determined by a complex function $Y(x)$

$$k_\mu dx^\mu = P^{-1}(du + \bar{Y}d\zeta + Yd\bar{\zeta} - Y\bar{Y}dv), \quad (24)$$

where $P(Y)$ is a normalizing factor and $\{u, v, \zeta, \bar{\zeta}\}$ are the null Cartesian coordinates

$$\begin{aligned} 2^{\frac{1}{2}}\zeta &= x + iy, & 2^{\frac{1}{2}}\bar{\zeta} &= x - iy, \\ 2^{\frac{1}{2}}u &= z + t, & 2^{\frac{1}{2}}v &= z - t. \end{aligned} \quad (25)$$

The Kerr-Schild formalism is based on the null congruences which are geodesic and shear-free.

The Kerr Theorem [22, 23] claims that all the geodesic and shear-free congruences are determined by the function $Y(x)$ which is a solution of the algebraic equation

$$F = 0, \quad (26)$$

where the generating function F is arbitrary holomorphic function of the projective twistor variables

$$Y, \quad \lambda_1 = \zeta - Yv, \quad \lambda_2 = u + Y\bar{\zeta}. \quad (27)$$

Recall, that twistor is the pair $Z^a = \{\psi_\alpha, \mu^{\dot{\alpha}}\}$, where $\mu^{\dot{\alpha}} = x^\mu \bar{\sigma}_\mu \psi_\alpha$, and projective twistor is

$$Z^a/\psi_1 = \{1, Y, \lambda_1, \lambda_2\}. \quad (28)$$

Therefore, the target function $Y(x)$ is a projective spinor coordinate $Y = \psi_2/\psi_1$, and function F may be chosen as homogenous function of Z^a .

The complex world line $\overset{\circ}{X}_+$ can be used as a complex Kerr-Newman source for generating function F of the Kerr theorem.

$$\overset{\circ}{X}_+^\mu = \Re e \overset{\circ}{X}_+^\mu + i a k_L^\mu = \Re e \overset{\circ}{X}_+^\mu + i a \bar{\chi} \sigma^\mu \chi. \quad (29)$$

Indeed, as it was shown in [7, 8, 21, 25, 26], the complex time parameter τ is cancelled in the Kerr generating function F . Because of that, the world line $\overset{\circ}{X}_-$, having the same complex shift in the space-like direction and the same 4-velocity, may also be used on the equal reason and yields the same result.

The Kerr generating function F may be represented in the form [7, 8, 21, 25, 26]

$$F = (\lambda_{\dot{\alpha}} - \overset{\circ}{\lambda}_{\dot{\alpha}}) \check{K} \lambda^{\dot{\alpha}}, \quad (30)$$

where

$$\overset{\circ}{\lambda}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}} \overset{\circ}{X}_+^{\dot{\beta}\mu} \bar{\sigma}_\mu^{\dot{\beta}\alpha} \psi_\alpha. \quad (31)$$

are the values of twistor parameters at the complex world line of the Kerr-Newman source $\overset{\circ}{X}_+$. Operator $\check{K} =$

$u^\mu \partial_\mu$ is related to momentum of the complex particle and expressed, in accordance with (5), via vectors k_L and k_R . Setting the equivalence of these vectors for the Kerr-Newman and Dirac particles, one matches their spin and momentum.

Without loss of generality one can set $\Re e X_+^\mu = 0$. Taking into account the relations (64) and (66) one obtains for the commuting spinors

$$\overset{\circ}{\lambda}{}^\alpha = -2ia(\bar{\chi}\psi)\chi^\alpha, \quad (32)$$

and

$$K\lambda^\alpha = \Pi^\mu \bar{\sigma}_\mu \psi = -\frac{m}{(\bar{\phi}\chi)}[(\phi\psi)\bar{\phi}^\alpha + (\bar{\chi}\psi)\chi^\alpha]. \quad (33)$$

As a result, function F acquires the form (up to the nonessential factor m)

$$F(\psi, x^\mu) = x^\mu[(\phi\psi)(\bar{\phi}\bar{\sigma}_\mu\psi) + (\bar{\chi}\psi)(\chi\bar{\sigma}_\mu\psi)]/(\bar{\phi}\chi) - 2ia(\phi\psi)(\bar{\chi}\psi). \quad (34)$$

One sees that it is the function which possesses all the properties which are necessary for the Kerr generating function. It is holomorphic in ψ in accord with the conditions of the Kerr theorem, and it is homogenous in ψ , which allows one to transform it to the standard Kerr-Schild form by the replacement $Y = \psi_2/\psi_1$. Finally, it is quadratic in ψ , which corresponds to the studied before case yielding the Kerr congruence up to the shifts and Lorentz transformations [7, 21, 26, 29].

The Dirac spinor solutions ϕ, χ depend via (15) from the vector potential A_μ which is an external electromagnetic field. Therefore, the Dirac wave function $\Psi = (\phi, \chi)$ plays in this model the role of an order parameter which controls dynamic of the Dirac – Kerr-Newman particle, spin-polarization, momentum and deformation of the Kerr congruence caused by external electromagnetic field.

On the other hand, *the behavior of the Dirac – Kerr-Newman spinning particle turns out to be fully determined by the Dirac wave function and will be indistinguishable from the behavior of Dirac electron*, at least in the weak and slowly changed on the Compton lengths electromagnetic field. It allows also to use the standard stochastic interpretation of the wave function $\Psi(x)$ corresponding to one-particle quantum theory.

V. RENORMALIZATION BY GRAVITY AND REGULARIZATION OF SELF-ENERGY

The mass renormalization is the most complicate and the most vulnerable procedure in QED. Gravitational field is ignored in QED, relying on the argument that its local action is negligible. However, gravity has a strong

non-local action which automatically provides the self-energy renormalization for island sources.

Indeed, mass of an isolated source is determined by only asymptotic gravitational field, and therefore, it depends only on the mass parameter m which survives in the asymptotic expansion for the metric. On the other hand, the total mass can be calculated as a volume integral which takes into account densities of the electromagnetic energy ρ_{em} , material (mechanical mass) sources ρ_m and energy of gravitational field ρ_g . The last term is not taken into account in QED, but namely this term provides perfect renormalization. For a spherically symmetric system, the expression may be reduced to an integral over radial distance r [45]

$$m = 4\pi \int_0^\infty \rho_{em} dr + 4\pi \int_0^\infty \rho_m dr + 4\pi \int_0^\infty \rho_g dr. \quad (35)$$

Some of these terms may be divergent, but the total result will not be changed, since divergences always will be compensated by contribution from gravitational term. Therefore, in the island systems *gravity performs perfectly the mass-energy renormalization for arbitrary distributions of the charges and matter*.

It shows that, due to the strong non-local action, gravity turns out to be essential for elementary particles, on the distances which are very far from the Planck scale.

The Kerr-Schild form of metric allows one to consider a broad class of regularized solutions which remove the Kerr singular ring, covering it by a matter source. There is a long-term story of the attempts to find some interior regular solution for the Kerr or Kerr-Newman solutions [4, 10, 11, 30, 32]. [46] Usually, the regularized solutions have to retain the Kerr-Schild form of metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_n \quad (36)$$

and the form of Kerr principal null congruence $k_\mu(x)$, as well as its property to be geodesic and shear-free. The space part \vec{n} of the Kerr congruence $k_\mu = (1, \vec{n})$ has the form of a spinning hedgehog. In the case $a = 0$, it takes the form of the spherically symmetric hedgehog which is usually considered as an ansatz for the solitonic models of elementary particles and quarks. It suggests that this model may also have relation to the other elementary particles. Indeed, the Kerr-Schild class of metric has a remarkable property, allowing us to consider a broad class of the charged and uncharged, the spinning and spinless solutions from an unified point of view.

Our treatment will be based on the approach given in [30, 32], where the *smooth* regularized sources were obtained for the rotating and non-rotating solutions of the Kerr-Schild class.[47] These smooth and regular solutions are based on the Gürsey and Gürses ansatz [34], in which the Kerr-Schild scalar function H of the general form

$$H = f(r)/(r^2 + a^2 \cos^2 \theta). \quad (37)$$

For the Kerr-Newman solution function $f(r)$ has the form

$$f(r) \equiv f_{KN} = mr - e^2/2. \quad (38)$$

Regularized solutions have tree regions:

i) the Kerr-Newman exterior, $r > r_0$, where $f(r) = f_{KN}$,

ii) interior $r < r_0 - \delta$, where $f(r) = f_{int}$ and function $f_{int} = \alpha r^n$, and $n \geq 4$ to suppress the singularity at $r = 0$, and provide the smoothness of the metric up to the second derivatives.

iii) a narrow intermediate region $r \in [r_0 - \delta, r_0]$ which allows one to get a smooth solution interpolating between regions i) and ii).

It is advisable to consider first the non-rotating cases, since the rotation can later be taken into account by an easy trick. In this case, taking $n = 4$ and the parameter $\alpha = 8\pi\Lambda/6$, one obtains for the source (interior) a space-time of constant curvature $R = -24\alpha$ which is generated by a source with energy density

$$\rho = \frac{1}{4\pi}(f'r - f)/\Sigma^2, \quad (39)$$

and tangential and radial pressures

$$p_{rad} = -\rho, \quad p_{tan} = \rho - \frac{1}{8\pi}f''/\Sigma, \quad (40)$$

where $\Sigma = r^2$. It yields for the interior the stress-energy tensor $T_{\mu\nu} = \frac{3\alpha}{4\pi}diag(1, -1, -1, -1)$, or

$$\rho = -p_{rad} = -p_{tan} = \frac{3\alpha}{4\pi}, \quad (41)$$

which generates a de Sitter interior for $\alpha > 0$, anti de Sitter interior for $\alpha < 0$. If $\alpha = 0$, we have a flat interior which corresponds to some previous classical models of electron, in particular, to the Dirac model of a charged sphere and to the López model in the form of a rotating elliptic shell [10].

The resulting sources may be considered as the bags filled by a special matter with positive ($\alpha > 0$) or negative ($\alpha < 0$) energy density.[48]

The transfer from the external electro-vacuum solution to the internal region (source) may be considered as a phase transition from ‘true’ to ‘false’ vacuum in a supersymmetric $U(1) \times \tilde{U}(1)$ Higgs model [30, 31].

Assuming that transition region iii) is very thin, one can consider the following graphical representation which turns out to be very useful.

The point of phase transition r_0 is determined by the equation $f_{int}(r_0) = f_{KN}(r_0)$ which yields $\alpha r_0^4 = mr_0 - e^2/2$. From (41), we have $\rho = \frac{3\alpha}{4\pi}$ and obtain the equation

$$m = \frac{e^2}{2r_0} + \frac{4}{3}\pi r_0^3 \rho. \quad (42)$$

In the first term on the right side, one can easily recognize the electromagnetic mass of a charged sphere with radius r_0 , $M_{em}(r_0) = \frac{e^2}{2r_0}$, while the second term is the

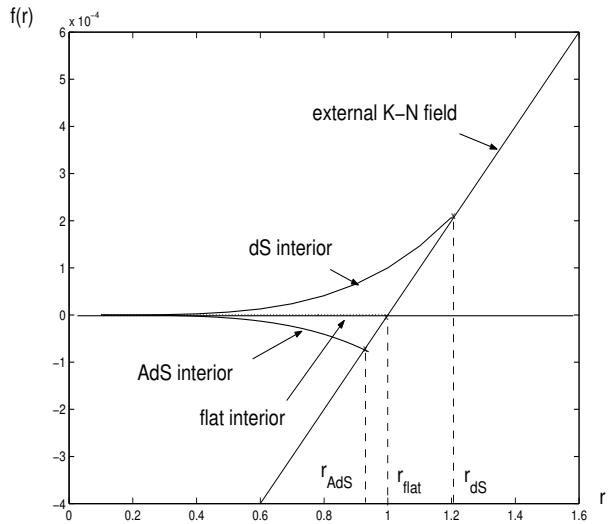


FIG. 2: Regularization of the Kerr-Newman spinning particle by matching the external field with dS, flat or AdS interior.

mass of this sphere filled by a material with a homogeneous density ρ , $M_m = \frac{4}{3}\pi r_0^3 \rho$. Thus, the point of intersection r_0 acquires a deep physical sense, providing an energy balance by the mass formation. In particular, for the classical Dirac model of a charged sphere with radius $r_0 = r_e = \frac{e^2}{2m}$, the balance equation yields the flat internal space with $\rho = 0$. If $r_0 > r_e$, a material mass of positive energy $M_m > 0$ gives a contribution to total mass m . If $r_0 < r_e$, this contribution has to be negative $M_m < 0$, which is accompanied by the formation of an AdS interior.

All the above treatment retains valid for the rotating cases, and for the passage to a rotating case, one has to set

$$\Sigma = r^2 + a^2 \cos^2 \theta, \quad (43)$$

and consider r and θ as the oblate spheroidal coordinates.

The Kerr-Newman spinning particle with a spin $J = \frac{n}{2}\hbar$, acquires the form of a relativistically rotating disk. The corresponding stress-energy tensor (41) describes in this case the matter of source in a co-rotating with this disk coordinate system. Disk has the form of a highly oblate ellipsoid with thickness r_0 and radius $a = \frac{n}{2}\hbar/m$ which is of order the Compton length. Interior of the disk represents a ‘false’ vacuum having superconducting properties, so the charges are concentrated on the surface of this disk, at $r = r_0$. Inside the disk the local gravitational field is negligible.

VI. NON-STATIONARITY AND REGULARIZATION OF THE ZERO-POINT FIELD.

Classical models of spinning particle encounter an unavoidable contradiction with quantum theory.

Metric of the Kerr-Newman solution is only a stationary approximation for the metric which may be related to spinning particle. In the old Kerr's microgeon model [6], the Kerr singular ring acquires electromagnetic wave excitations which were interpreted as excitations of a circular string [7, 8, 13]. Such excitations break axial symmetry of the Kerr-Newman solution and stationarity. As a result, only an average metric takes the Kerr-Newman form. Integration of the Kerr-Schild equations in the general non-stationary case [5] leads to the appearance of an extra electromagnetic field depending on some function $\gamma(x)$ which describes electromagnetic radiation along the Kerr congruence k_μ [39]. In [5], authors set the restriction $\gamma = 0$ to obtain the final, stationary form of the Kerr-Newman solution. The exact Einstein-Maxwell solutions with $\gamma \neq 0$ have not been obtained so far and represent the old and very hard problem.[49] There are known the Vaidya "shining star" solutions [21, 23], in which $\gamma \neq 0$, but the Maxwell equation have been switched out there, since the non-coherent radiation is only considered. These solutions show that radiation leads also to the non-stationarity via the loss of mass.

Note, that in the one-particle quantum theory oscillations are stationary and absence of radiation caused by oscillations is postulated. Contrary to quantum physics, in classical theory non-stationarity entails radiation, and here lies a rather sharp boundary between the classical and quantum theories.

It should be mentioned that radiation is present in QED too, since the structure of free particles in QED is related to radiative corrections caused by vacuum field of virtual photons: zero point field and vacuum polarization.

By application of the quantum field theory in the curved spaces [17, 18], quantum effects are concentrated in the stress-energy tensor which is divergent due to the vacuum zero point fields. By a transfer to the classical Einstein-Maxwell theory, the quantum vacuum fields have to be subtracted, which means that the classical stress-energy tensor has to be regularized with respect to the vacuum fluctuations [18]

$$T_{\mu\nu}^{(reg)} = T_{\mu\nu} - <0|T_{\mu\nu}|0>, \quad (44)$$

under the condition

$$T^{(reg)\, \mu\nu}_{\,\,\, ,\mu} = 0. \quad (45)$$

Since the non-stationarity of the Kerr-Schild solutions is related to the field γ , it was conjectured [7, 8, 14, 21] that this field has to be subtracted by some regularization. Indeed, the field γ is an electromagnetic radiation

propagating along the Kerr congruence k_μ , and it has to involve a loss of mass. However, *twofoldedness* of the Kerr geometry leads to a very specific effect: *the outgoing radiation on the "positive" out-sheet of the metric is compensated by an ingoing radiation on the "negative" in-sheet*, and therefore, the solution has to be stationary, as if the loss of mass is absent. It shows, that the field γ has to be identified with the vacuum zero-point field and it has to be subtracted from the stress-energy tensor by means of a procedure of regularization which has to satisfy the condition (45). Such regularization may be performed, [8, 13], and leads to some modified Kerr-Schild equations (see Appendix C). Although the equations are essentially simplified, there is still a remnant of the field γ in the Maxwell equations, which reflects a relation between the real electromagnetic excitations and vacuum fields. By such a regularization, electromagnetic excitations may be interpreted as a resonance of the zero-point fluctuations on the (superconducting) source of the Kerr spinning particle [8, 13, 15].

Although, the exact nontrivial solutions of the regularized system were not obtained so far, there were obtained corresponding exact solutions of the Maxwell equations which showed that any excitation of the Kerr geometry leads to the appearance of some extra "axial" singular line (string) which is semi-infinite and modulated by de Broglie periodicity [13–15, 39]. [50] There are exact Kerr-Schild solutions of this type containing the ring-like singular string and semi-infinite "axial" string. They corresponds to the end points of the complex Kerr-Newman string and appear as the real images of the complex point-like "quarks" which were mentioned in the end of section II. The obtained recently multiparticle Kerr-Schild solutions [19] supported this point of view, showing that interaction occurs via pp-strings of this type, which we shall discuss in the next section.

VII. DIRAC-KERR ELECTRON AND MULTIPARTICLE KERR-SCHILD SOLUTIONS.

Some evidences that the Dirac – Kerr-Newman model is related to a multi-particle representation may be extracted from the obtained recently multiparticle Kerr-Schild solutions. It was shown in [19] that taking the generating function of the Kerr theorem F in the form of the product of partial functions for i-th particle

$$F = \prod_i F_i(Y|q_i), \quad (46)$$

where q_i is the set of parameters of motion and orientation of particle i, one can obtain the multi-particle Kerr-Schild solutions of the Einstein-Maxwell system in the assumption that particles are stationarily moving along some different trajectories.

The main equation of the Kerr theorem for twistorial structure (26) is satisfied by any partial solution $F_i(Y) =$

0. It means that the twistorial multi-particle space-time splits on the sheets corresponding to different roots of the equation $F(Y) = 0$, similar to the sheets of a Riemann surface.

Twistorial structures on the different sheets turn out to be independent and twistorial structure of i -th particle “does not feel” the structure of particle j , forming a sort of its internal space. This property is direct generalization of the corresponding property of the Kerr-Newman geometry - the twosheeted structure of space-time, which splits up the ‘in’ and ‘out’ fields.

Since function $F(Y)$ for one Kerr-Newman particle is quadratic in Y [9, 21, 25, 29], the equation $F_i(Y|q_i) = 0$ has two roots Y_i^+ and Y_i^- corresponding to the positive (‘out’) and negative (‘in’) sheets. In terms of these roots one can express F_i in the form [19]

$$F_i(Y) = A_i(x)(Y - Y_i^+)(Y - Y_i^-). \quad (47)$$

One sees that metric of a multi-particle solution will depend on the solution $Y_i(x)$ on the considered sheet of i -th particle. Indeed, substituting the (+) or (-) roots $Y_i^\pm(x)$ in the relation (24), one determines the Kerr congruence $k_\mu^{(i)}(x)$ and corresponding function h_i of the Kerr-Schild ansatz (1) on the i -th sheet

$$H_i = \frac{m}{2} \left(\frac{1}{\mu_i \tilde{r}_i} + \frac{1}{\mu_i^* \tilde{r}_i^*} \right) + \frac{(e/\mu_i)^2}{2|\tilde{r}_i|^2}. \quad (48)$$

Electromagnetic field generated by electric charge is given by the vector potential

$$A_\mu^{(i)} = \Re e \left(\frac{e}{\mu_i \tilde{r}_i} \right) k_\mu^{(i)}. \quad (49)$$

The complex radial distance \tilde{r}_i and function $\mu_i(Y)$ are also determined from the extended version of the Kerr theorem [19],

$$\tilde{r}_i = -d_Y F_i, \quad (50)$$

$$\mu_i(Y_i) = \prod_{j \neq i} A_j(x)(Y_i - Y_j^+)(Y_i - Y_j^-). \quad (51)$$

Contrary to independence of twistorial structures for different particles, there is an interaction between them, since the function $\mu_i(Y)$ acquires a pole $\mu_i \sim A(x)(Y_i^+ - Y_j^-)$ on the twistor line which is common for the particles i and j . The metric and electromagnetic field will be singular along the common twistor lines. For example, a light-like interaction occurs along the line which connects the *out* - sheet of particle i to the *in* - sheet of particle j , see fig.3. In the Witten twistor-string model [37, 38] such a pole is considered as propagator for a free fermion.

This singular line is extended to infinity, since it will also be common for the *out* - sheets of the both particles. The field structure of this line is similar to singular

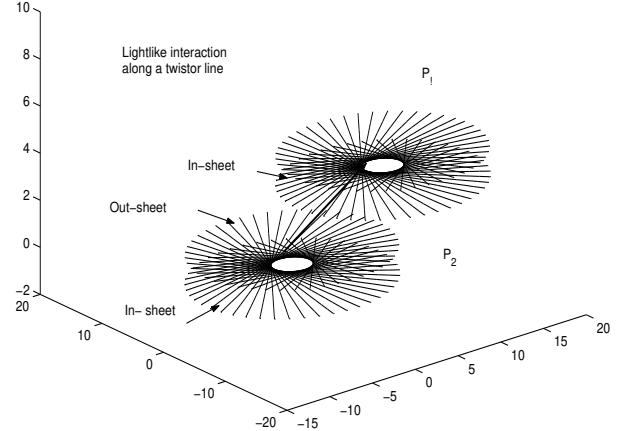


FIG. 3: The light-like interaction via a common twistor line connecting out-sheet of one particle to in-sheet of another.

pp-wave solutions [23, 39]. Analysis of some simple cases shows that each particle has a pair of semi-infinite singular lines which is caused by interaction with some external particle. [51] As it was discussed in [14, 15, 39] such a pair of strings turns out to be a carrier of de Broglie periodicity.

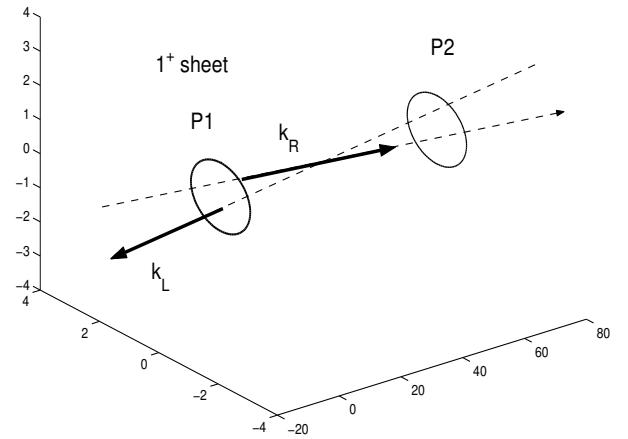


FIG. 4: Two outgoing semi-infinite singular lines of the particle P1, caused by its interaction with external particle P2.

In the limit of infinitely many external particles, the selected Kerr’s particle will be connected by singular twistor lines with many other external particle, and the singular twistor lines will have even dense distribution among the twistor lines, covering the principal null congruence of the selected particle.

The multiparticle Kerr-Schild solutions show us that the usual Kerr-Newman solution is the solution for an

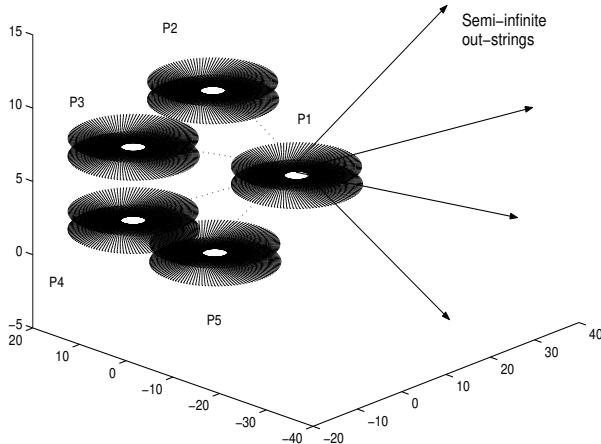


FIG. 5: Formation of the outgoing semi-infinite singular lines by interaction via the common ‘out-sheet’-‘out-sheet’ twistor lines.

isolated particle, while there is a series of the exact corresponding solutions, in which the selected Kerr particle is surrounded by other particles, perhaps on the very far distances. This new type of the Kerr-Newman solution differs from the initiate one in one respect only - some of the twistor lines of the Kerr principal null congruence turn out to be *singular*. These singular lines are the light-like Schild strings which are described by singular pp-waves [14, 39]. It shows, that the Dirac-Kerr model is multiparticle indeed, because of the stringy inter-particle interactions, and also that the singular twistor lines are analogs of the virtual photons which provide vacuum polarization and radiative corrections by the mass and charge renormalization in QED [27]. Therefore, we arrive at the conclusion that the Dirac – Kerr-Newman electron takes an intermediate position between the one-particle Dirac model and the multiparticle QED.

Returning now to the discussed in sec.VII regularization of the stress-energy tensor for the wave solutions, one can see that the multiparticle Kerr-Schild solutions confirm the above conjecture that the wave field γ propagating along the Kerr congruence have to be identified with the field of virtual photons and have to be removed by regularization. On the other hand, it allows one to suppose that *the singular pp-strings of the multiparticle Kerr-Schild solutions represent elements of the vacuum structure, suggesting that vacuum has apparently a twistor-string texture*.

VIII. DISCUSSION: THE WAVE FUNCTION AND SPACE-TIME STRUCTURE OF ELECTRON

The ‘point-like’ and ‘extended’ electron.

One of the main consequences of the considered here

Dirac – Kerr-Newman model is that electron acquires a definite space-time structure of the Kerr-Newman geometry. In particular, it acquires a twistorial structure having a caustic on the Kerr-Newman ring of the Compton size, and the wave excitations of this ring-like string are related with the wave properties of electron [14, 15].

On the other hand, the Dirac equation controls the motion of the Kerr spinning particle, and consequently, the behavior of this model does not differ from the corresponding behavior of the Dirac electron, in particular, in the external electromagnetic field. It is known that the external field in the one-particle Dirac model has to be restricted by the conditions to be small enough and weakly changing on the Compton distances [27, 40, 41]. These are exactly the conditions, which allow us to consider the Dirac – Kerr-Newman spinning particle as a united system. Note that similar restrictions exist also for the multi-particle treatments in QED, as the conditions for decomposition on the creation and annihilation operators. Therefore, the model may be interpreted in terms of the one-particle Dirac theory which contains, however, a hidden space-time structure, and in this respect it is also close to the local QED.

In accordance with QED, the naked electron is point-like, while the dressed by virtual photons electron is deformed and turns out to be smeared over the Compton region [27, 43]. Therefore, calculation of this effect in QED is based on the Coulomb solution which is used as a first approximation in coordinate representation [27]. It is clear that these calculations do not take into account the spin of electron which may be described in the first approximation by the Kerr-Newman field. Although the Kerr-Newman gravitational field is very weak and may apparently be neglected, the corresponding electromagnetic field cannot be considered as small. The extremely high spin of electron polarizes the space-time, and the aligned to the Kerr congruence electromagnetic field turns out to be polarized along the Kerr-Newman twistorial structure, acquiring singularity at the ring of Compton radius. This effect of polarization has also to be acting for the field of virtual photons. Therefore, the use of the Kerr-Newman electromagnetic field instead of the Coulomb field would be more correct in this case. It is clear that the resulting field of virtual photons shall again be concentrated near the Kerr-Newman singular ring of the Compton size forming its stringy excitation.

In is known that the relativistic quantum theory, in spite of the great success in the experimental confirmations, has brought some problems of theoretical consistency [27, 40, 41]. In particular, the Dirac wave solutions cannot be considered consequently as the wave functions, since the relativistic quantum theory cannot be considered consistently as one-particle theory [27, 40, 41]. There appears the problem of localization of electron and stochastic interpretation of the Dirac wave function. The problems exhibited manifestly in coordinate representation, leading experts to the conclusion (see [27]) that “... the wave functions $\psi(q)$ as the carriers of unobservable

information cannot be used in the consequent relativistic theory ...". There appears even the extreme points of view that the modern relativistic quantum theory has to refuse to deal with the wave function at all and also with the treatment of processes in space-time, and that the Feinman graphs and S-matrix may represent itself the final form of the quantum theory [27, 40].

The considered here combined Dirac – Kerr-Newman model of electron realizes point of view that the spinning particle has some distinct extended space-time structure which is represented in a Lorentz-covariant form and does not contradict to the main conclusions of the one-particle Dirac theory and to the structure of QED. Besides, the combined Dirac – Kerr-Newman model turns out to be naturally related with gravitational field.

Wave function as an order parameter

The presented here combined Dirac - Kerr-Newman model may be considered in the frame of the usual one-particle quantum theory with the interpretation of the Dirac wave function as a density of probability for the position of electron. There appears however an extra local information on the space-time polarization for each point $x \in M^4$, which is determined by the Dirac wave function $\Psi(x)$. This information may also be interpreted in the spirit of the local quantum field theory, in which the role of wave function may be more essential.

In sec. III we obtained that there is a natural null tetrad n^a related to the Dirac wave function. This tetrad is a direct generalization of the null tetrad $\{d\zeta, d\bar{\zeta}, du, dv\}$ which is determined by the Cartesian null coordinates and plays important role in the Kerr-Schild formalism and twistor theory. The spin-direction of the Kerr's particle in a standard representation is determined by the real direction $du + dv$ which has for the Dirac electron the analog $(k_{L\mu} - k_{R\mu})dx^\mu$. Similar, the complex forms $d\zeta, d\bar{\zeta}$ are analogs of the Dirac null vectors $m_\mu dx^\mu$ and $\bar{m}_\mu dx^\mu$. These complex vectors of the Dirac tetrad have a very important peculiarity. They are oscillating with the double Compton frequency and are carrying de Broglie periodicity for the moving electron.

There is especial interest to use corresponding generalization of the Kerr-Schild formalism to the case of oscillating tetrad. Formally, the choice of the tetrad is only a gauge transformation which will not result to any change in the nature of the corresponding solutions of the Einstein-Maxwell system of equation. However, two factors are important:

i - it may simplify the equation, opening a way for obtaining some new solutions (for example oscillating ones) which were not obtained so far in the Kerr-Schild formalism, and

ii - there may be a topologically non-trivial situation, in which the new tetrad will not be equivalent to initiate one. An important example of this situation is the twist of spin-structure[52] which mixes the internal and orbital degrees of freedom and has found important applications in the solitonic models, topological sigma-models and twistor-string theory [37, 38].

In this case, the Dirac wave function acquires again the role of an order parameter (Goldstone fermion) controlling the broken symmetry which is related to fixation of the tetrad. Therefore, in the Dirac – Kerr-Newman model, the Dirac wave function may acquire the role of an ‘order parameter’ which controls the twistorial polarization of space-time, fixing the broken symmetry of the surrounding vacuum.

It should also be noted, that the twistorial structure of the Dirac – Kerr-Newman model allows one to transform the Dirac wave function from coordinate representation to the projective twistor space. The twistor-space representation turns out to be more natural for the Kerr-Newman geometry than the both coordinate and momentum representations. Taking into account that S-matrix and scattering amplitude can naturally be represented in twistor space too, as it has been shown for the more simple case of gauge scattering amplitudes in twistor-string model [37], there appears apparently some new approach to QED which could resolve the existing problems with the wave function in coordinate representation.

IX. CONCLUSION

The considered Dirac – Kerr-Newman model of electron has a few attractive properties:

- electron acquires an extended space-time structure which does-not contradict to conclusions of the Dirac model and QED,

- the Kerr-Newman twistorial structure is controlled by the Dirac equation, so all the consequences of this model have to match to the reach experimental data obtained from the Dirac equation, and since the Feinman rules are also based on the Dirac equation, one can expect that this model will not conflict with the results of QED too,

- the model shows that gravity performs renormalization and regularization of the particle-like models in a very elegant manner,

- the model has a non-trivial geometrical structure which displays a relationship to quantum theory. It opens a way to a geometrical view on the formal quantum prescriptions.

A peculiarity of this model is that the Dirac equation takes an especial role with respect to the other fields - it is considered on the auxiliary Minkowski space-time, while the other fields are considered on the Kerr-Schild background. One could argue that this difference is not essential, since the local gravitational field of electron is extremely small and will not affect on the solutions of the Dirac equation. However, it is not correct, indeed. The Kerr solution has the nontrivial twofold topology which is retained even in the limit of the infinitely small mass. Because of that the aligned to the Kerr congruence electromagnetic and spinor solutions, which could lead to the self-consistent solutions, turn out to be essentially different from the plane wave solutions on Minkowski space-time, which is illustrated by the appearance of the extra

axial singular filament. So, the plane wave solutions are deformed with concentration of the fields near singular lines.

In the considered Dirac – Kerr-Newman model the Dirac wave function plays the especial role of a ‘master equation’ which controls the polarization and set a synchronization of tetrad. The perspective models of another sort, in which the Dirac equation ‘feels’ the Kerr geometry, i.e. has a back reaction from the Kerr geometry, may be based on the stringy structures of the Kerr geometry. It has to use some initially massless Dirac solutions on the Kerr space-time, forming a foliation over the complex Kerr string. The non-zero mass term of the Dirac equation appears in this model by averaging over the string length, similar to the appearance of the mass in the massless dual string models. We expect to consider such a model elsewhere.

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Appendix A: Spinor notations

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad (52)$$

where

$$\bar{\sigma}^{\mu\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\alpha\beta}\sigma_{\beta\dot{\beta}}^\mu, \quad (53)$$

and

$$\sigma_0 = \bar{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = -\bar{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$\sigma_2 = -\bar{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = -\bar{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$\epsilon^{12} = 1, \quad \epsilon^{21} = \epsilon_{12} = -1.$$

Also,

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad (54)$$

$$\gamma_5 = \gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}. \quad (55)$$

Appendix B: Some relations for commuting spinors

$$\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \delta_\alpha^\gamma, \quad (56)$$

$$\epsilon^{12} = \epsilon_{21} = 1, \quad \epsilon^{21} = \epsilon_{12} = -1. \quad (57)$$

$$\psi_\alpha\chi^\alpha = \epsilon_{\alpha\beta}\psi^\beta\epsilon^{\alpha\gamma}\chi_\gamma = -\psi^\beta\epsilon_{\beta\alpha}\epsilon^{\alpha\gamma}\chi_\gamma = -\psi^\beta\chi_\beta = -\psi\chi. \quad (58)$$

It yields

$$\psi\chi = -\psi_\alpha\chi^\alpha = -\chi^\alpha\psi_\alpha = -\chi\psi \Rightarrow \psi\psi = 0, \quad (59)$$

Complex conjugation changes the order of spinors without change of sign, which yields

$$(\chi\psi)^+ = (\chi^\alpha\psi_\alpha)^{*T} = (\bar{\chi}^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}})^T = (\bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}), \quad (60)$$

and due to (59)

$$(\chi\psi)^+ = (\chi^\alpha\psi_\alpha)^+ = \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = \bar{\psi}\bar{\chi} = -\bar{\chi}\bar{\psi}. \quad (61)$$

Next, using the relation

$$\sigma_{\alpha\dot{\alpha}} = \epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\sigma}^{\dot{\beta}\beta} \quad (62)$$

and taking into account that $\epsilon\psi = -\psi\epsilon$, one obtains

$$\chi^\alpha\sigma_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} = \chi_\beta\bar{\psi}_{\dot{\beta}}\bar{\sigma}^{\dot{\beta}\beta}, \quad (63)$$

which yields (assuming that $\chi^\alpha \rightarrow \chi_\alpha$ and $\bar{\psi}^{\dot{\alpha}} \rightarrow \bar{\psi}_{\dot{\alpha}}$)

$$\chi\sigma\bar{\psi} = \bar{\psi}\bar{\sigma}\chi. \quad (64)$$

Basing on the relation

$$\sigma_{\alpha\dot{\alpha}}^{\mu}\bar{\sigma}_{\mu}^{\dot{\beta}\beta} = -2\delta_\alpha^\beta\delta_{\dot{\alpha}}^{\dot{\beta}}, \quad (65)$$

and taking into account the order of the co- and contravariant spinors, one obtains

$$(\bar{\chi}\sigma^\mu\phi)(\bar{\sigma}_\mu\psi)^{\dot{\alpha}} = -2(\bar{\chi}\psi)\phi^{\dot{\alpha}}, \quad (66)$$

and

$$(\psi\phi)\bar{\chi}_{\dot{\beta}} = -\frac{1}{2}(\phi\sigma^\mu\bar{\chi})(\psi\sigma_\mu)_{\dot{\beta}}. \quad (67)$$

We have also

$$(\chi\sigma\bar{\psi})^+ = \psi\sigma\bar{\chi}, \quad (68)$$

and

$$(\chi\sigma^\mu\bar{\sigma}^\nu\psi)^+ = \bar{\psi}\bar{\sigma}^\nu\sigma^\mu\bar{\chi}. \quad (69)$$

Appendix C: Regularized Einstein-Maxwell equations

The field equations for the Einstein-Maxwell system after preliminary integration in [5] are the following. Electromagnetic field is given by tetrad components of the self-dual tensor \mathcal{F}_{ab} ,

$$\mathcal{F}_{12} = AZ^2, \quad (70)$$

$$\mathcal{F}_{31} = \gamma Z - (AZ)_{,1}, \quad (71)$$

and the equations relating the functions $A(x)$ and $\gamma(x)$ are

$$A_{,2} - 2Z^{-1}\bar{Z}Y_{,3}A = 0, \quad (72)$$

$$\mathcal{D}A + \bar{Z}^{-1}\gamma_{,2} - Z^{-1}Y_{,3}\gamma = 0. \quad (73)$$

Here $Z = P/\tilde{r}$ and $Y_{,3} = -ZP_Y/P$ (notations of [5]).

Gravitational equations take the form

$$M_{,2} - 3Z^{-1}\bar{Z}Y_{,3}M = A\bar{\gamma}\bar{Z}, \quad (74)$$

$$\mathcal{D}M = \frac{1}{2}\gamma\bar{\gamma}, \quad (75)$$

where

$$\mathcal{D} = \partial_3 - Z^{-1}Y_{,3}\partial_1 - \bar{Z}^{-1}\bar{Y}_{,3}\partial_2. \quad (76)$$

Solutions of this system were given in [5] only for the case for $\gamma = 0$. Regularization of the stress-energy tensor

$$T_{reg}^{\mu\nu} = :T^{\mu\nu}: \equiv T^{\mu\nu} - <0|T^{\mu\nu}|0> \quad (77)$$

under the condition $\nabla_\mu T_{reg}^{\mu\nu} = 0$ retains the term γ in the Maxwell equations (70), (71), (72), (73) and in the gravitational equation (74), while the equation (75) takes the simple form

$$\mathcal{D}M = 0, \quad (78)$$

which provides stationarity.

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- [44] We use the relations (64),(66) and (67) for commuting spinors.
- [45] It looks like the expressions in a flat space-time. However, in the Kerr-Schild background, the exact Tolman relations [33] taking into account gravitational field and rotation give just the same result [32].
- [46] Extra references may be found in [11, 30, 32].
- [47] It was also published in the proceedings of conferences, in particular [31].
- [48] It resembles the discussed at present structure of dark energy and dark matter in Universe. The case $\alpha > 0$ is reminiscent of the old Markov suggestions to consider particle as a semi-closed Universe [35].
- [49] The wave electromagnetic solutions on the Kerr-Newman background which are asymptotically exact in the low-frequency limit where obtained recently in [36].
- [50] It was argued in [14, 15], that the “axial” strings may acquire the quark indices and are responsible for the scattering at high energies.
- [51] The light-like strings contain only the one-way light-like modes. On the other hand, the modes of opposite directions link the positive sheet of the second particle to negative sheet of the first one, which is a stringy analog of the photon exchanges.
- [52] Do not mix with twist of the Kerr congruence.