Beta Distribution

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1 PDF

$$f_X(x \mid \alpha, \beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathcal{Z}},\ x \in [0,1],\ \alpha, \beta > 0 \tag{1}$$

The normalization constant $\mathcal{Z}$ will be,

$$\mathcal{Z} = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} = B(\alpha, \beta) \tag{2}$$

2 CDF

$$\begin{aligned} F_X(x \mid \alpha, \beta) &= \int_0^x \frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha,\beta)}dt && (3) \\ &= \frac{1}{B(\alpha,\beta)}\int_0^x t^{\alpha-1}(1-t)^{\beta-1}dt && (4) \\ &= \frac{B(x;\alpha,\beta)}{B(\alpha,\beta)} && (5) \\ &= I_x(\alpha,\beta) && (6) \end{aligned}$$

Where, $B(x;\alpha,\beta)$ is the incomplete beta function and $I_x(\alpha,\beta)$ is the regularized incomplete beta function.

3 Inverse CDF

$$F_X^{-1}(x \mid \alpha, \beta) = I_x^{-1}(\alpha, \beta) \tag{7}$$

$f_X(x)$	$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathcal{Z}}$	$\log(f_X(x))$	$(\alpha-1)\log(x) + (\beta-1)\log(1-x) - \log(\mathcal{Z})$
$\mathcal{Z}(\beta)$	$B(\alpha,\beta)$	$\log(\mathcal{Z}(\beta))$	$\log(B(\alpha,\beta))$
$F_X(x)$	$I_x(\alpha,\beta)$	$\log(F_X(x))$	$\log(I_x(\alpha,\beta))$
$F_X^{-1}(x)$	$I_x^{-1}(\alpha,\beta)$	$\log\left(F_X^{-1}(x)\right)$	$I_x^{-1}(\alpha,\beta)$