1 Key ideas about problem 1 of Lab 5

- Take plane π as the plane $z = 0$ (e.g., the xy-plane).
- ℓ_1 is parametrized by $\ell_1(\lambda) = \lambda \cdot v + p_1$, for (fixed) vector $v = (v_x, v_y, v_z)$ and point $p_1 = (x_1, y_1, z_1)$, and where $\lambda \in \mathbb{R}$ is the parameter.
- ℓ_2 is parametrized by $\ell_2(t) = t \cdot w + p_2$, for (fixed) vector $w = (w_x, w_y, w_z)$, and where $t \in \mathbb{R}$ is the parameter.
- Now we want to connect ℓ_1 and ℓ_2 with lines that are **parallel to** π (i.e., they are horizontal lines).
- We want lines defined by one point on ℓ_1 and one point on ℓ_2. To be parallel to π, those two points need to have the same z-coordinate. That is, for each point on ℓ_1, say, $\ell_1(\lambda)$, we want the point on ℓ_2 that has the same z-coordinate as $\ell_1(\lambda)$. Note that, in general, $\lambda \neq t$, that is, it is not true that the z-coordinate of $\ell_1(\lambda)$ is the same as the z-coordinate of $\ell_2(\lambda)$.
- The z-coordinate of the point $\ell_1(\lambda)$ is $\lambda \cdot v_z + z_1$.
- Similarly, the z-coordinate of a generic point on ℓ_2, $\ell_2(t)$, is $t \cdot w_z + z_2$.
- To force both points to have the same z-coordinate, we find the value of t that guarantees that $\lambda \cdot v_z + z_1 = t \cdot w_z + z_2$. That gives us $t = \frac{\lambda \cdot v_z + z_1 - z_2}{w_z}$.
- So for each point $\ell_1(\lambda)$, the corresponding point on ℓ_2 that has the same z-coordinate is $\ell_2(\frac{\lambda \cdot v_z + z_1 - z_2}{w_z})$.
- This leads to the following parametrization of the surface:

$$S(\lambda, s) = (1 - s) \cdot \ell_1(\lambda) + s \cdot \ell_2(\frac{\lambda \cdot v_z + z_1 - z_2}{w_z})$$

where λ and s are real numbers.