3. Interpolation

Files: The accompanying files for this assignment are `assignment3_1.html`, `assignment3_2.html`, and `assignment3_3.html`.
Delivery: upload the modified HTML files and any other necessary files to the Racó. All explanations and/or answers to the problems should be included in the HTML files.

Problem 1. Write a program to draw a **cubic Lagrange polynomial** that interpolates *any* four points, using:

1. Uniform parameter values.
2. Non-uniform parameter values, using the distance between consecutive interpolated points.

You can draw both curves at the same time, using different colors.

Problem 2. In this problem we want to explore Runge's phenomenon. You will write a program to draw a **Lagrange polynomial** to interpolate $n+1$ points, and use it to interpolate samples from the function $f(x) = \frac{1}{1+25x^2}$, for different values of n. The points to be interpolated will be samples from the graph of the function $f(x) = \frac{1}{1+25x^2}$ taken uniformly from the interval $[-1, 1]$, that is: $x_i = 2i/n - 1$.

1. Write a program to draw a Lagrange polynomial to interpolate $n+1$ points.
2. Use the Lagrange polynomial to interpolate the points for different values of n (make n a user-defined parameter that can be changed interactively).
3. Draw the graph of $f(x)$ in the background, in order to compare better the different interpolating curves.

Note: you should translate and rescale your canvas or curve appropriately to be able to see it in your program.

Problem 3. Write a program to draw a **cubic Hermite polynomial** that interpolates two points P_0, P_1, allowing the user to see and control the two points and the two tangent vectors at P_0 and P_1.

Problem 4. Using the program from Problem 3, consider the particular case $P_0 = (200, 200)$, $P_1 = (400, 300)$, and tangent vectors $\vec{v_0} = (100, 100)$ and $\vec{v_1} = (100, 0)$.

1. Compute the position of the curve at $t = 1/2$.
2. How should the tangent vectors be modified in order to: keep the same directions at P_0 and P_1, and at the same time go through $(300, 300)$ at $t = 1/2$?

Solve the problem first, and then illustrate your result with your program. Write down your answers in `assignment3_3.html`.