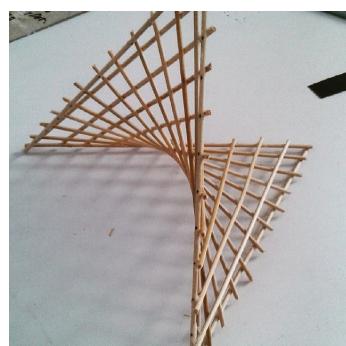
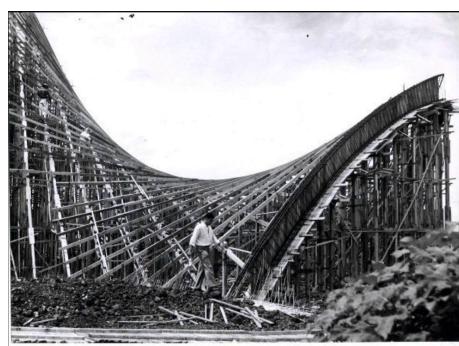


5. Surfaces

Files: The accompanying file `assignment5.html` contains examples showing how to produce parametric surfaces in javascript. The solution to the assignments should be delivered in separate files, named `assignment5_1.html`, `assignment5_2.html`, and `assignment5_3.html`.

Delivery: Upload a zip file with all necessary files to the Racó. All explanations and/or answers to the problems should be included in the HTML file or in a PDF.

Problem 1. Consider the *ruled surface* defined as follows: let π be a plane and let ℓ_1 and ℓ_2 be two lines, none of which lies in π , and such that they do not intersect and they are not parallel (i.e., they are *skew*); the surface is the set of all lines that intersect both ℓ_1 and ℓ_2 and are parallel to π .



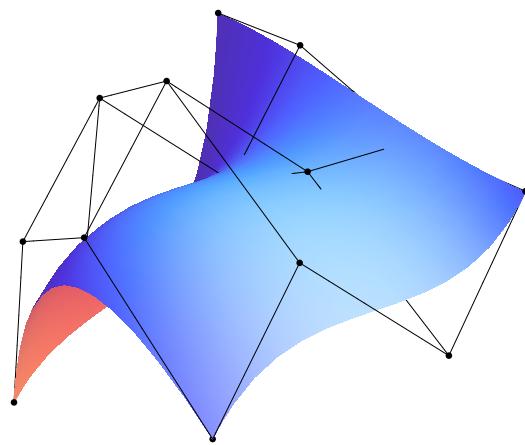
1. Give a parametrization of the surface. **Explain and justify why it represents the desired surface.**
2. Visualize the surface with the help of the code in `assignment5.html`.

5. Surfaces

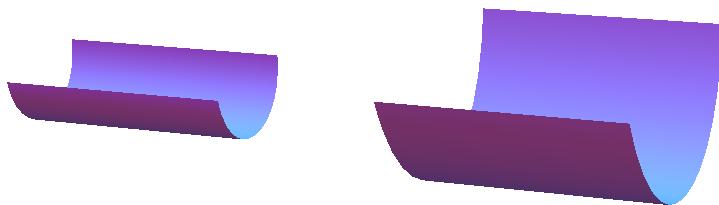
Problem 2. Write a program to draw a Bézier surface of degree 3×3 , and show the result on the following control points (you can translate and scale them, if you wish):

$$\begin{aligned}\{P_{i,0}\} &= \{(0, 0, 0), (3, 0, 3), (6, 0, 3), (9, 0, 0)\} \\ \{P_{i,1}\} &= \{(0, 2, 2), (3, 2, 5), (6, 2, 5), (9, 2, 2)\} \\ \{P_{i,2}\} &= \{(0, 4, 0), (3, 4, 3), (6, 4, 3), (9, 4, 0)\} \\ \{P_{i,3}\} &= \{(0, 6, 6), (3, 6, 5), (6, 6, 3), (9, 6, 2)\}\end{aligned}$$

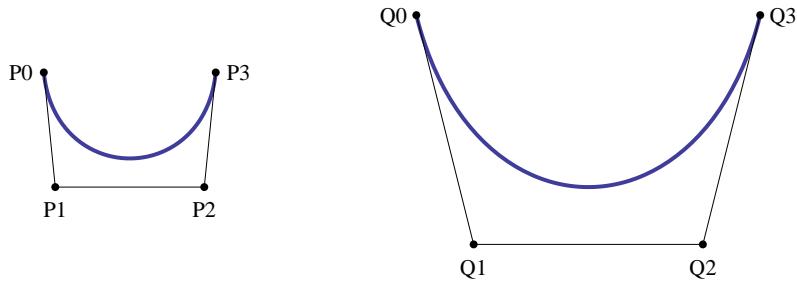
The result should look more or less like this:



Problem 3. We are given two different gutters like the ones in the figure:



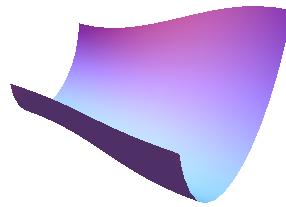
Both of them are (parts of) cylinders. Their profiles, though, are very different, as the following figure shows:



More precisely:

- The thinner gutter has length 6, and its profile is a cubic Bézier curve whose control points are $\{P_i\} = \{(-1.5, 2), (-1.3, 0), (1.3, 0), (1.5, 2)\}$.
- The larger gutter has length 6 too, and its profile is a cubic Bézier curve whose control points are $\{Q_i\} = \{(-3, 3), (-2, -1), (2, -1), (3, 3)\}$.
- Both gutters have the same axis, and the distance between them is equal to 5.

We wish to design a surface to smoothly connect the two gutters, like in the following figure:



Solve the problem using Bézier surfaces of degree 3×3 . Feel free to translate and scale the profile control points, and choose the common axis of the two cylinders at your ease.

These two views show how the final result should more or less look like:

