Variant 10.

$x \in (x_0, X)$; $y(x_0) = y_0$;

		y_0	x_0	X
10	$-y^2/3 - 2/(3x^2)$	2	1	5

This is a $\boxed{\text{Ricatti eq.}}$ Consider: $y_1 = 1/x$

Substitution: $y = y_1 + z(x) = 1/x + z$; So, $y' = -1/x^2 + z'$

$z = y - \frac{1}{x}$

$$\frac{-1/x^2 - 2z/x - z^2}{3} - \frac{2}{3x^2} = -\frac{1}{x^2} + z';$$

$$\frac{-1 - 2zx - x^2z^2 - 2 + 3}{3x^2} = z';$$

$$-2zx - x^2z^2 = 3z'x^2;$$

$$z^2 + 2z/x + 3z' = 0; \quad //: z^2$$

Now, this is a $\boxed{\text{Bernoulli equation}}$ [n=2]

$$1 + \frac{2}{zx} + \frac{3z'}{z^2} = 0; \Rightarrow \frac{1}{3} + \frac{2}{3x}\cdot\frac{1}{z} = -\frac{z'}{z^2};$$

Substitution: $t = 1/z$; $t' = -z'/z^2$

$$1/3 + 2t/3x = t'; \Rightarrow t' - 2t/3x = 1/3;$$

Solve it as a $\boxed{\text{Linear FO DE}}$:

1) $t' - 2t/3x = 0$; $\Rightarrow 3x\,dt = 2t\,dx$;

$$\frac{dx}{3x} = 2\frac{dt}{t}; \Rightarrow \frac{1}{3}\int\frac{dx}{x} = 2\int\frac{dt}{t} \Rightarrow t = \sqrt[3]{x^2}\cdot C;$$

2) $C \to b(x)$; $\Rightarrow t = \sqrt[3]{x^2}\cdot b$; $t' = \frac{2b}{3\sqrt[3]{x}} + b'\cdot\sqrt[3]{x^2}$

$$\frac{2b}{3\sqrt[3]{x^2}} + b'\cdot\sqrt[3]{x^2} - \frac{2\cdot\sqrt[3]{x^2}\,b}{3x} = \frac{1}{3};$$

$$\cancel{\frac{b}{x^{2/3}}} + 3b'\cdot\sqrt[3]{x^2} - \cancel{\frac{b}{x^{2/3}}} = 1;$$

$$3b'\cdot\sqrt[3]{x^2} = 1; \Rightarrow b' = \frac{1}{3\sqrt[3]{x^2}};$$

$$b = \frac{1}{3}\int x^{-2/3} \Rightarrow b = x^{1/3} + C$$

Thus, $t = \sqrt[3]{x^2}\cdot b = x + C\cdot\sqrt[3]{x^2}$;

Substitute back t:

$$\frac{1}{z} = x + C\cdot\sqrt[3]{x^2}; \Rightarrow z = \frac{1}{(x + C_1\cdot\sqrt[3]{x^2})}$$

Substitute back z:

$$y - \frac{1}{x} = \frac{1}{(x + C_1\cdot\sqrt[3]{x^2})} \Rightarrow \boxed{y = \frac{1}{x} + \frac{1}{(x + C_1\cdot\sqrt[3]{x^2})}}$$

Discontinuity, if $x = 0$ or $x = -C_1^3$

Solution of the IVP:

$$y = \frac{1}{x} + \frac{1}{(x + C_1\cdot\sqrt[3]{x^2})}$$

$$2 = 1 + \frac{1}{1 + C_1};$$

$$1 = \frac{1}{1 + C_1};$$

$$1 + C_1 = 1;$$

$$\boxed{C_1 = 0}$$

General solution:

Suppose $y(a) = b$

$$b = \frac{1}{a} + \frac{1}{a + C_1\cdot a^{2/3}},$$

$$b(a + C_1\cdot a^{2/3}) = 1 + \frac{a + C_1\cdot a^{2/3}}{a},$$

$$b(a + C_1\cdot a^{2/3}) = 1 + 1 + \frac{C_1}{\sqrt[3]{a}},$$

$$C_1 = b(a^{4/3} + C_1 a) - 2a^{1/3}$$

$$C_1 = b a^{4/3} + C_1 a b - 2a^{1/3}$$

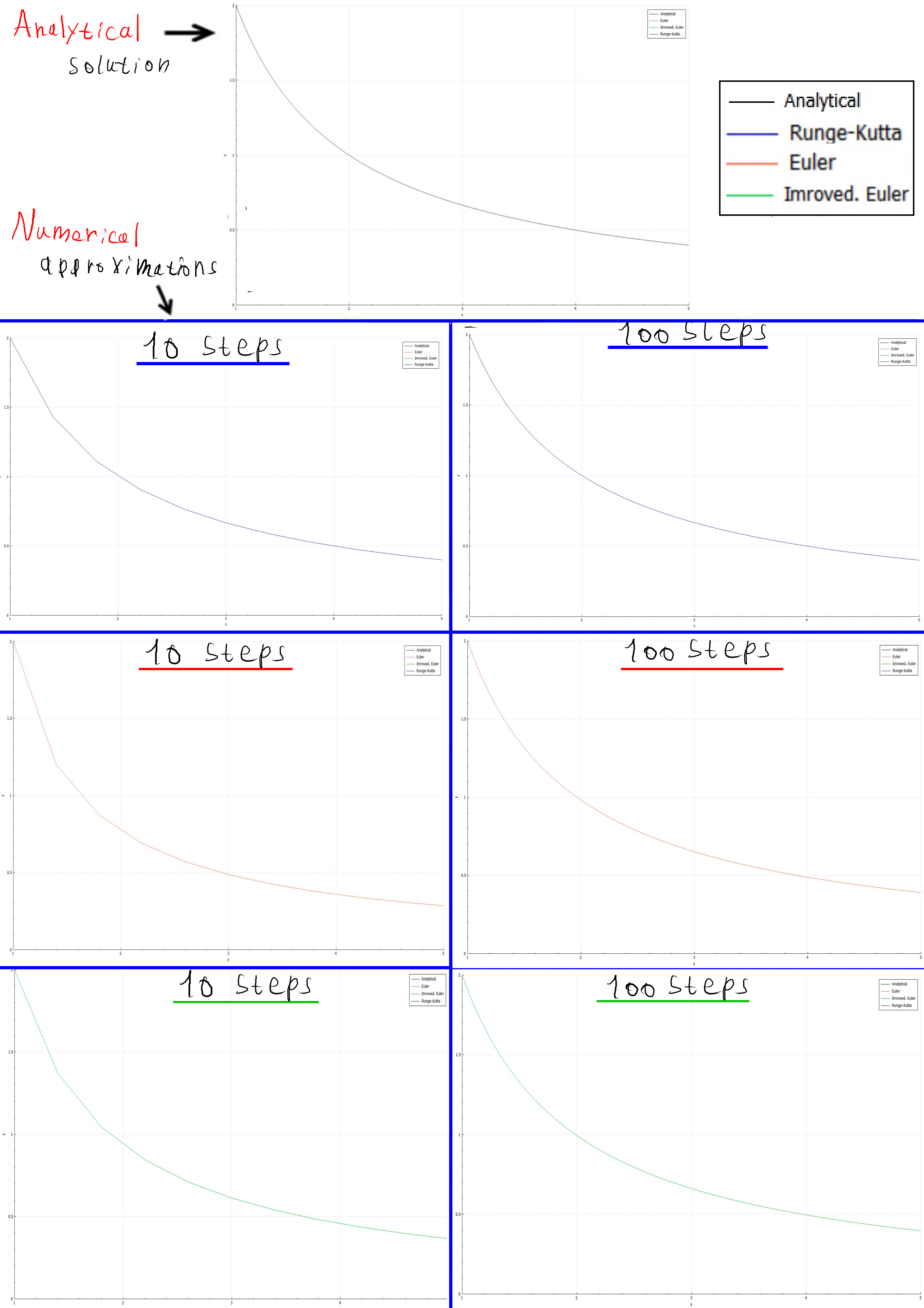
$$C_1(1 - ab) = b a^{4/3} - 2a^{1/3}$$

$$C_1 = \frac{b\cdot a^{4/3} - 2a^{1/3}}{1 - a\cdot b},$$

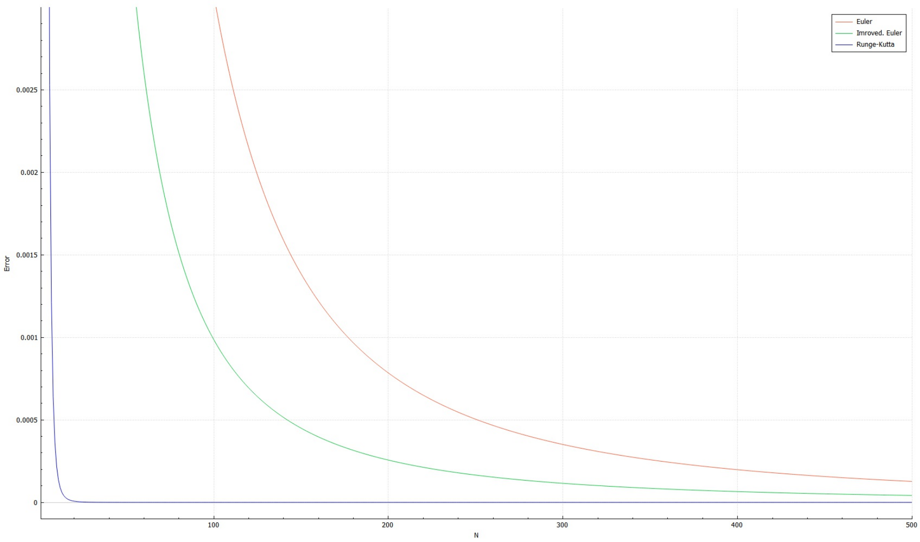
Answer:

$$y = \frac{2}{x}, \quad y(5) = 0{,}4$$

$$\boxed{y = \frac{1}{x} + \frac{1}{(x + C_1\cdot\sqrt[3]{x^2})}}$$



Analytical solution
Numerical approximations
Analytical
Runge-Kutta
Euler
Imroved. Euler
10 steps
100 steps
10 steps
100 steps
10 steps
100 steps



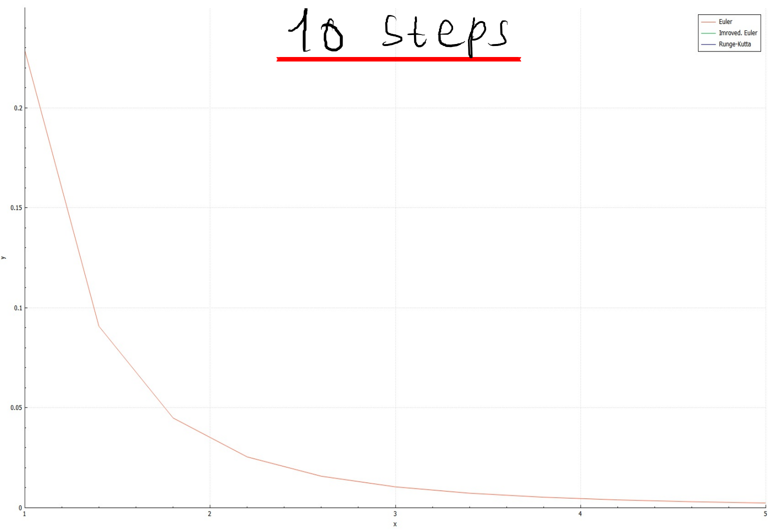
Total errors for $N \in [1, 500]$

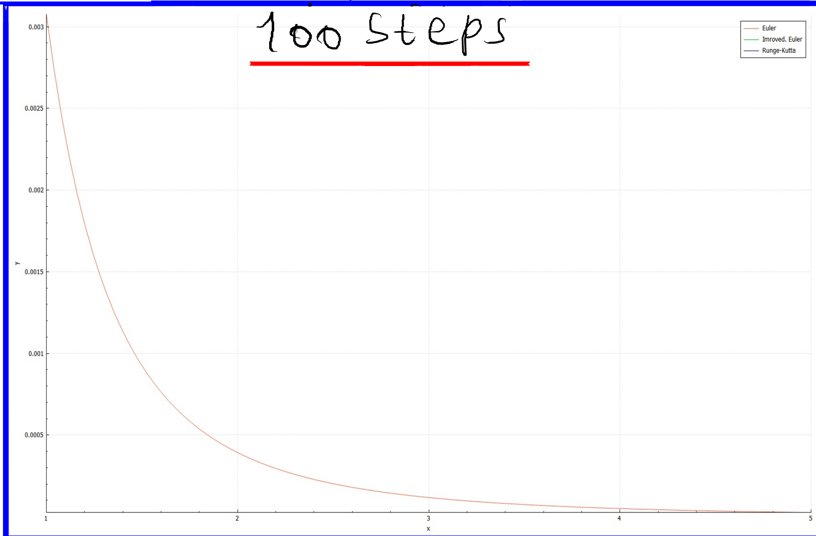
Euler

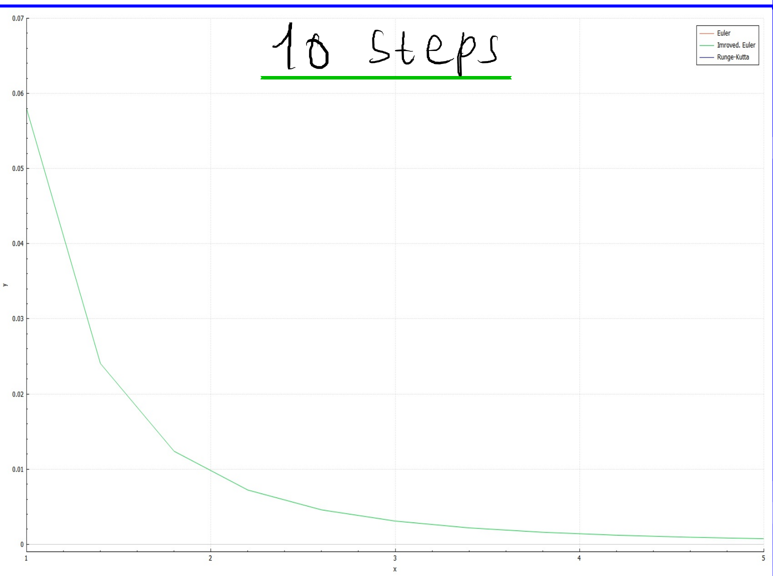
Imroved. Euler

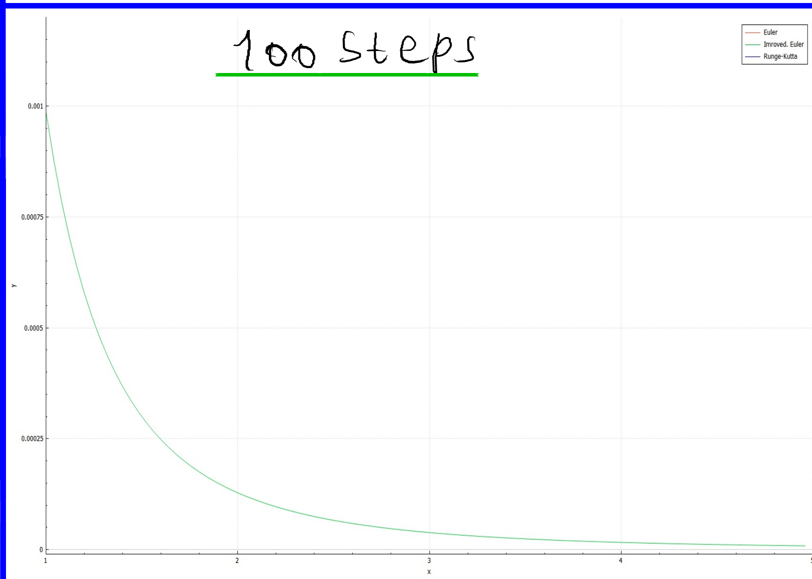
Runge-Kutta

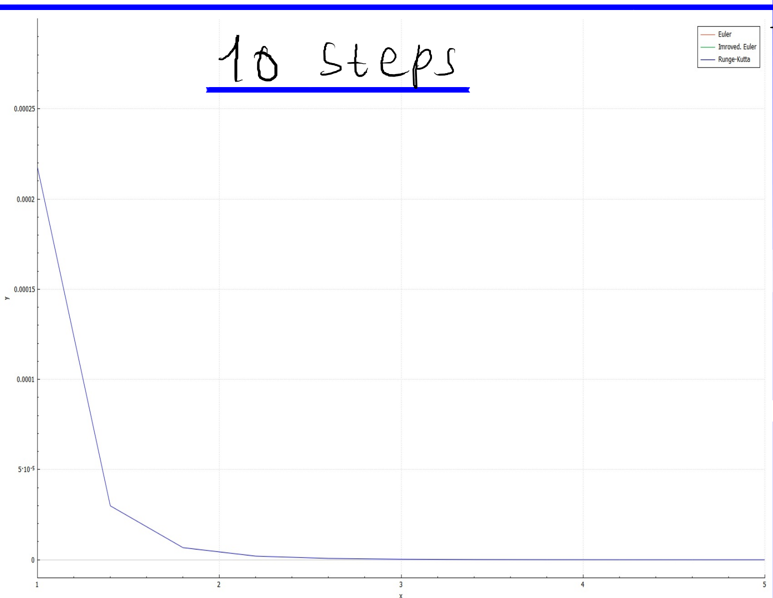
Below are Local errors

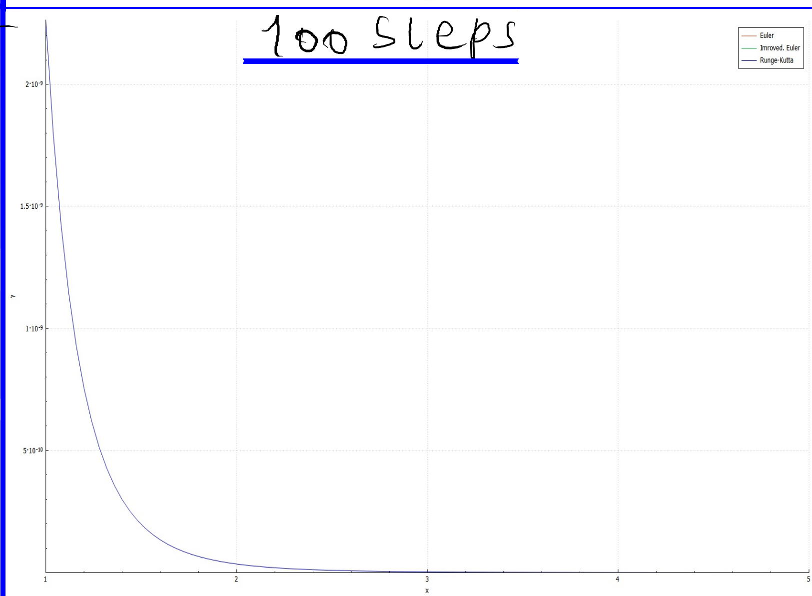




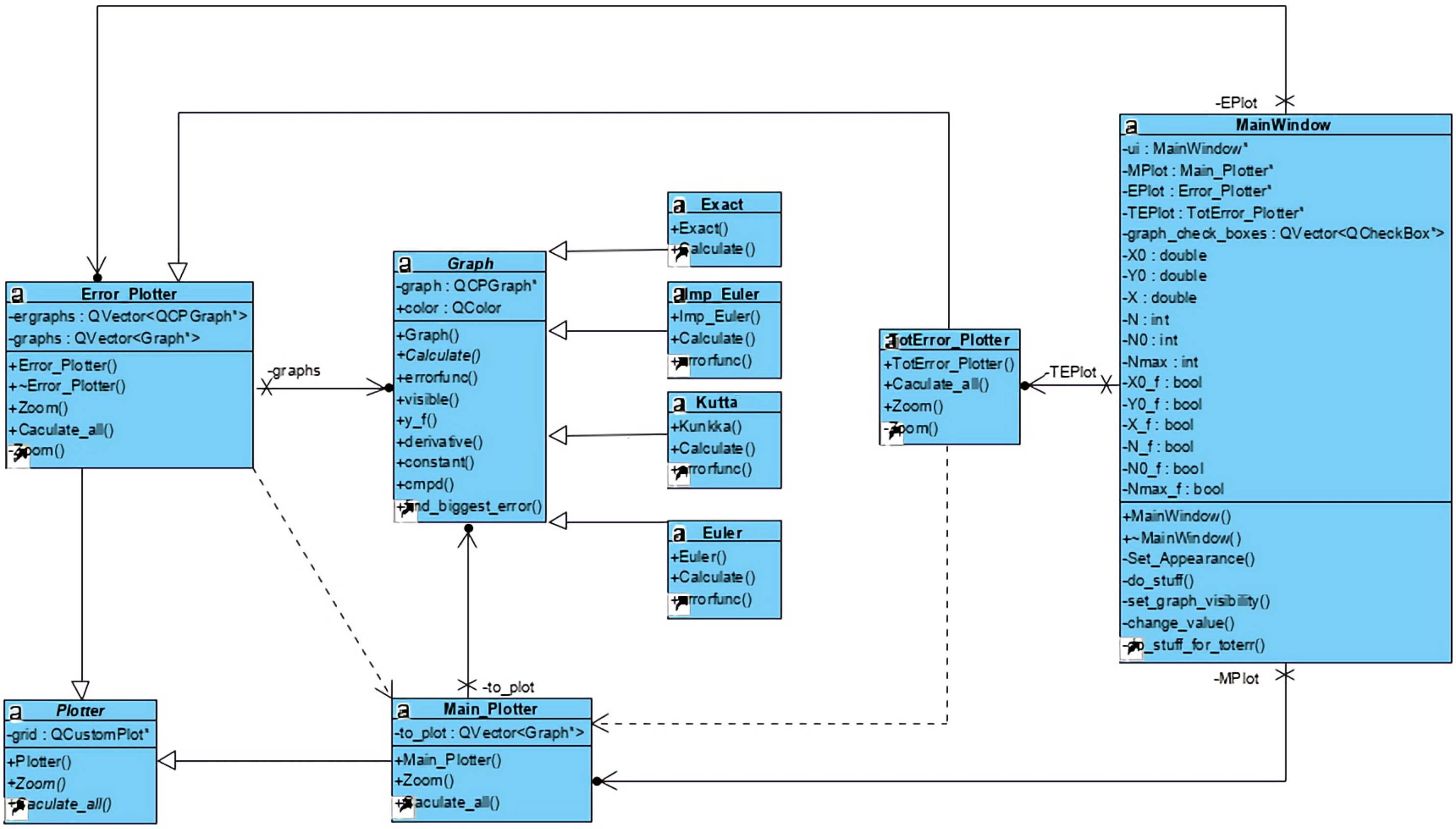






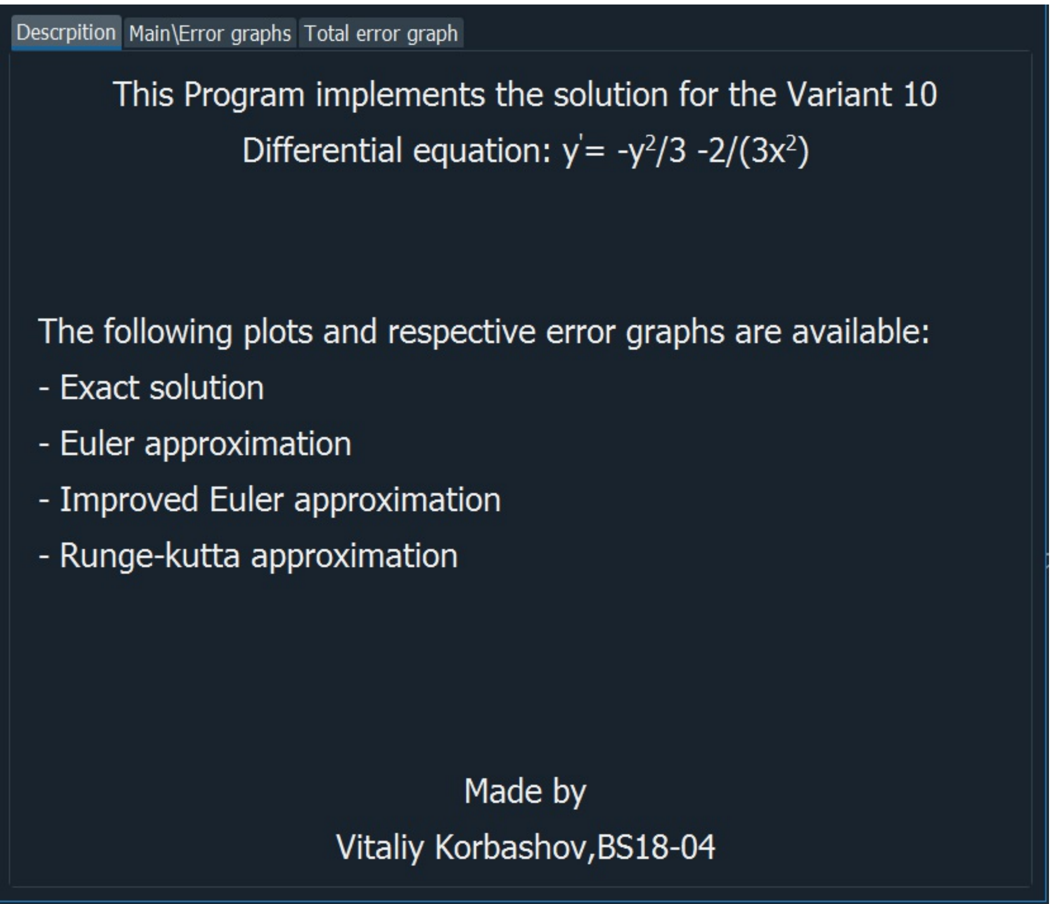


UML DIAGRAM



GUI Description:

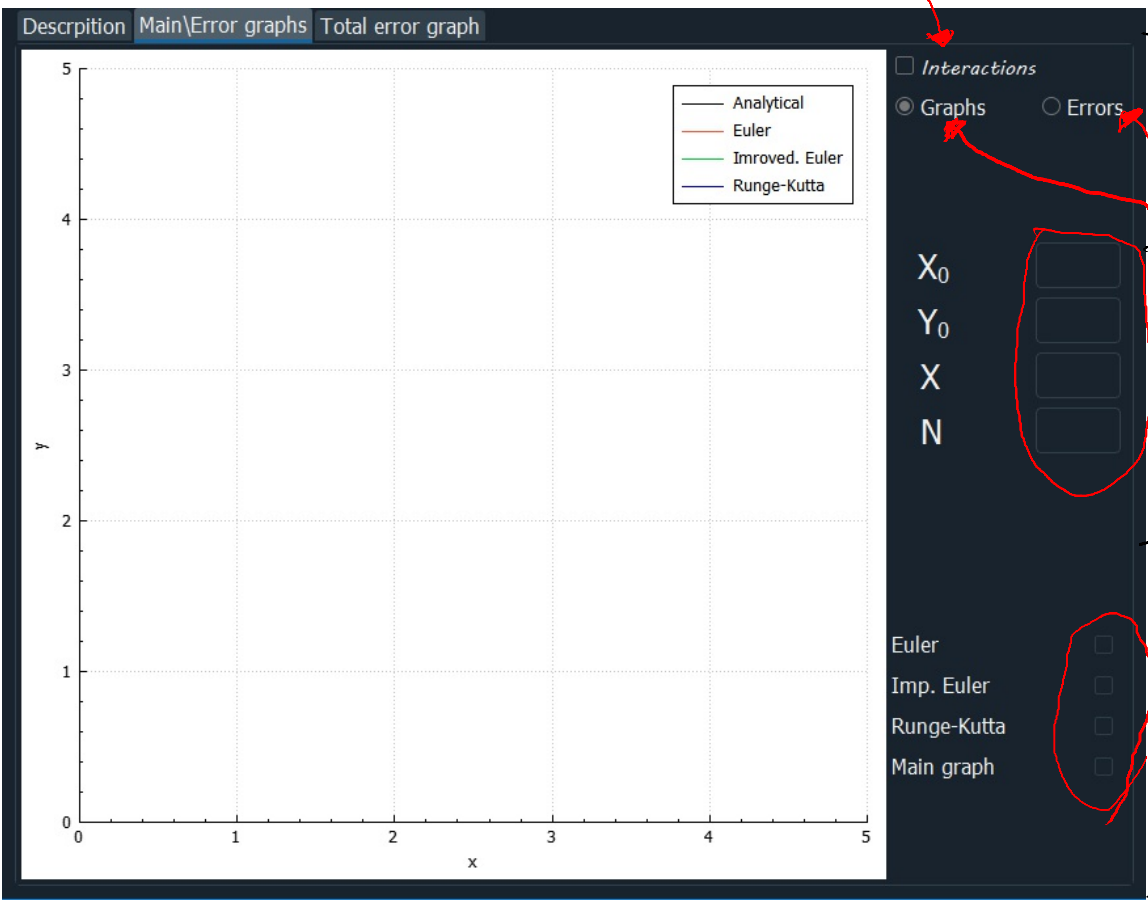
Tab I



In the first tab you
Can see the program
Description and the
Differential equation
It is supposed to
Solve and approximate

Tab II

Zoom/drag features via the mouse scroll/drag can be enabled if desired



One can switch betwen 2 types of graphs.

Graphs get automatically built upon input, not the "Enter" press

Toggle the graphs you want to see

If the user enters Invalid data, some of the field labels light up red to indicate the mistake; Thus, only the valid input is passed down to the functions.

Valid ranges:

X = { -999.00 , 999.00 }

Y_0= { -999.00 , 999.00 }

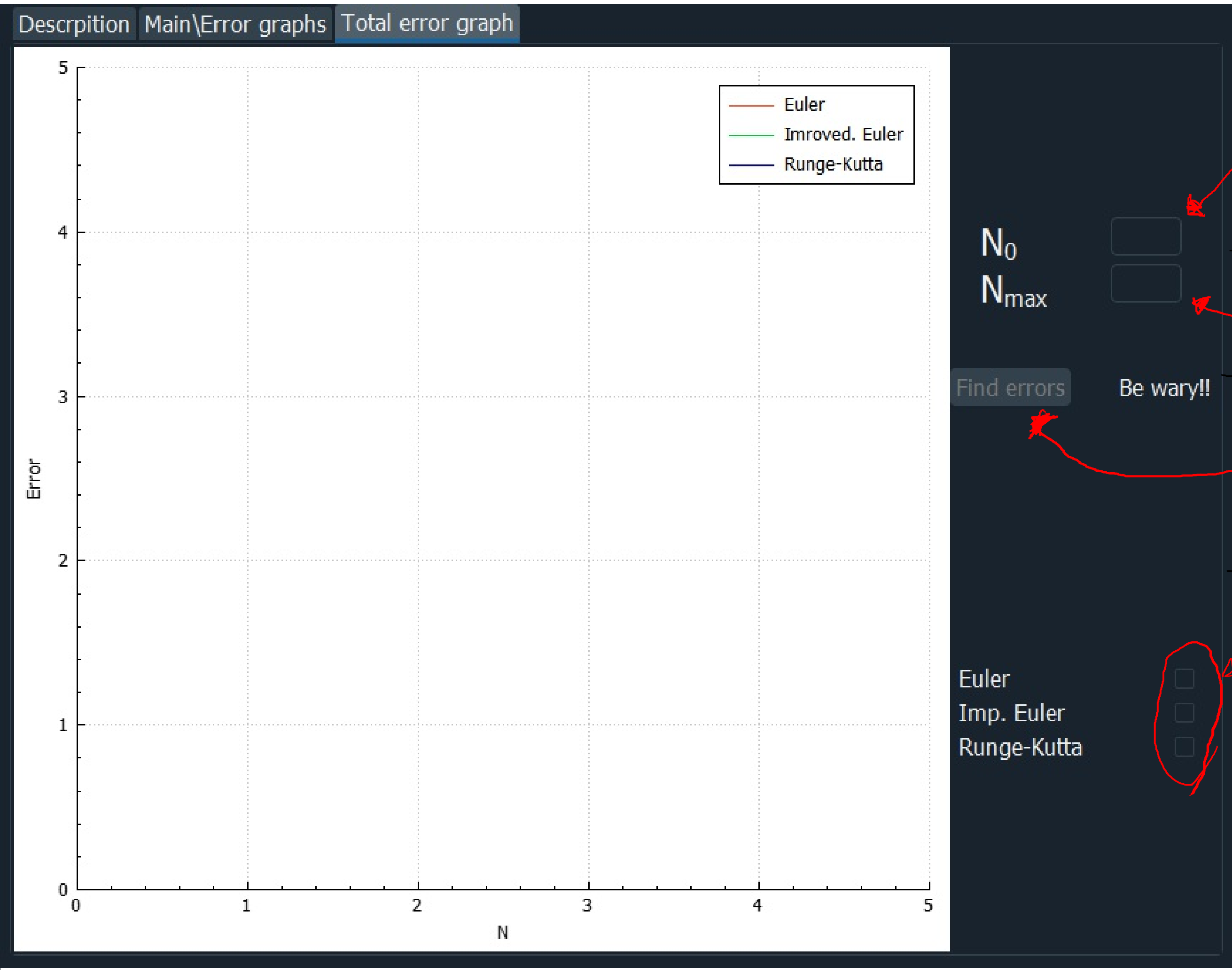
X_0= { -999.00 , 999.00 }

N = { 1, 99999}

$X > X_0$

The exact, "Main graph", is built on the bigger range than its approximation to allow for the ability to better investigate it; (All asymptotes are handled correctly)
Also, some constraints were put on the possible zoom ranges to avoid graph being too small;

Tab III



Start value

Final value

Press to plot

Toggle the graphs you want to see

Here, one can view how the error of each approximation graph changes with the number of steps;
The “Find” button was added to remove lag, caused by auto calculations on input.
The field labels light up if the following rules are violated:

$$N = \{1, 9999\}$$

$$N_0 = \{1, 9999\}$$

$$N > N_0$$

To view all the code on githue repository click the text

Code

github.com/BlackNuziGuy/Differ_eq