ECE250: Signals and Systems
Assignment 4

Issued on: December 1, 2022

Max-Marks : 80

Due by: December 6, 2022 (5:30 pm)

Guidelines for submission

- Submit a hard copy of your solutions in the wooden box kept on the 3rd Floor of Old Academic Block (right side of the lift).
- Write your Name, Roll No. and Group No. (as assigned for your tutorials) on the hard copy of your solutions.
- Do all questions in sequence.
- Use A4 sheets (Plain).
- Staple your sheets properly.
- Please Provide proper mathematical justifications with your answers. No marks will be awarded without a valid justification.
- Institute's plagiarism policy will be strictly followed.

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1) (CO5) (12 points) A signal $x(t) = 4e^{3t}u(t) + 5e^{4t}u(t)$ is given. Using the concepts of Fourier and Laplace transform answer the following questions:

   (a) (2 pts) Will the Fourier transform of the given signal converge? Justify.

   (b) (6 pts) Will the Fourier transform of $x(t)e^{-(2\alpha-\frac{7}{2})t}$ converge for the following values of α? Justify.

      i. $\alpha = 2.5$

      ii. $\alpha = 3.5$

      iii. $\alpha = 4.5$

   (c) (4 pts) Compute the Laplace transform $X(s)$ of $x(t)$ and determine the ROC. Sketch the obtained ROC depicting poles and zeros of $X(s)$. Using ROC, comment on the convergence results of Fourier transform in parts-(a) & (b).

2) (CO5) (12 points) Compute the Laplace transform, if it exists, for each the following functions. Clarify your reasoning on why the Laplace transform should exist or not in each case:

   (a) (4 pts) $f_1(t) = t^2 sin(2t)$

   (b) (4 pts) $f_2(t) = e^{t^2}$

   (c) (4 pts) $f_3(t) = |2 - t|[u(t-1) - u(t-3)]$, where $u(t)$ denotes the unit step function.

3) (CO5) (15 points) Find the inverse Laplace transform of the following signals:

   (a) (5 pts) $F_1(s) = \frac{2+2se^{-2s}+4e^{-4s}}{s^2+4s+3}$, $Re(s) > -1$.

   (b) (5 pts) $F_2(s) = \frac{5s+13}{s(s^2+4s+13)}$, $Re(s) > 0$.

   (c) (5 pts) $F_3(s) = log\left(\frac{s+4}{s+5}\right)$

4) (CO5) (9 points) Using initial value theorem and final value theorem, find $x(0^+)$ and $x(\infty)$ of the following Laplace transforms:

(a) (3 pts) $\frac{1}{s^2(s+1)}$

(b) (3 pts) $\frac{(s+1)^2-1}{[(s+1)^2+1]^2}$

(c) (3 pts) $\frac{1-e^{-sT}}{s}$

5) (CO5) (12 points) Consider four pole-zero configurations as shown in Fig. 1. What should be the possible ROC for each of these configurations if the related time domain function $x(t)$ follows the given criteria?

(a) (6 pts) The Fourier transform of $x(t)e^{-t}$ converges.

(b) (6 pts) $x(t) = 0$ for $t > 5$.

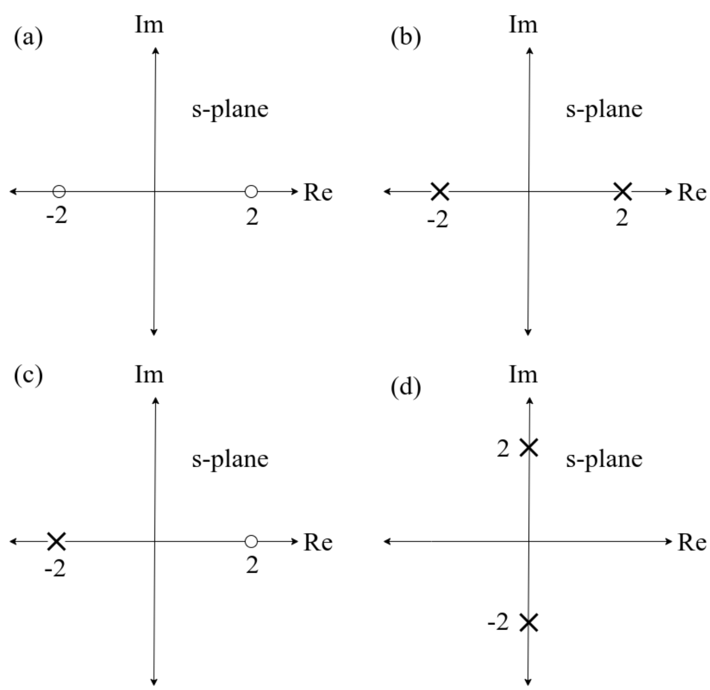

Figure 1: Pole-zero configurations (problem-5).

6) (CO5) (8 points) Consider an LTI system for which the input $x(t)$ and output $y(t)$ satisfy the linear differential equation as defined below:

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t) \tag{1}$$

(a) (4 pts) Compute the system function $H(s)$, where $H(s)$ denotes the Laplace transform of system's impulse response. Assume all initial conditions to be zero.

(b) (4 pts) Evaluate $y(t)$ if the input $x(t) = e^{-t}$.

7) (CO5) (8 points) Evaluate $y(t)$ by solving the following differential equation using the Laplace transform:

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} = cos(t-3) + 4t \tag{2}$$

Assume $y(3) = 0$ and $\frac{dy(3)}{dt} = 7$.

8) (CO5) (4 points) For a system S, as depicted in Fig. 2, input signal $x(t)$ is of the form $x(t) = cos(2\pi f_0 t + \theta)$. Signal $x(t)$ is sampled as $x_p(t)$, using a periodic impulse train $p(t)$. Compute $X_p(j\omega)$, where $X_p(j\omega)$ denotes the Fourier spectrum of sampled signal $x_p(t)$.

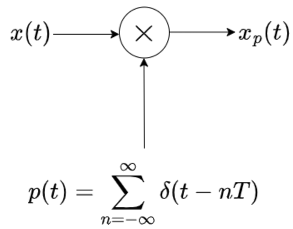

Figure 2: System S (problem-8).