ECE250: Signals and Systems
Assignment 3

Issued on: October 15, 2022

Max-Marks : 55

Due by: October 22, 2022 (5:30 pm)

Guidelines for submission

Theory Problems:

- Submit a hard copy of your solutions in the wooden box kept on the 3rd Floor of Old Academic Block (right side of the lift).
- Write your Name, Roll No. and Group No. (as assigned for your tutorials) on the hard copy of your solutions.
- Do all questions in sequence.
- Use A4 sheets (Plain)
- Staple your sheets properly

Programming Problems:

- Use Matlab or python to solve the programming problems.
- For your solutions, you need to submit a zipped file on Google classroom with the following:
  - program files (.m) or (.ipynb) with all dependencies.
  - a report (.pdf) with your coding outputs and generated plots. The report should be self-complete with all your assumptions and inferences clearly specified.
- Before submission, please name your zipped file as: "A3_GroupNo_RollNo_Name.zip".
- Codes/reports submitted without a zipped file or without following the naming convention will NOT be checked.

Theory Problems:

1) (8 points) Consider a periodic signal $s(t)$ with fundamental time-period T. A portion of that signal, i.e., for $0 < t < T/4$ is known and depicted in Fig. 1.
Determine the complete signal $s(t)$, i.e., for the interval $0 < t < T$, if the signal follows the following conditions:
   (a) (4 pts) $s(t)$ is an even function, and its Fourier series comprises of only odd-harmonics.
   (b) (4 pts) $s(t)$ is an odd function, and its Fourier series comprises of only odd-harmonics.

2) (14 points) A rectangular pulse $p(t)$ of total width T_1 and height unity is given. It is also known that the pulse is symmetric about the origin.
   (a) (1 pt) Using the given information, plot the signal $p(t)$.
   (b) (1 pt) A new signal $q(t)$ is defined as a periodic repetition of $p(t)$ with time-period $T_0 = 3T_1/2$. Plot the signal $q(t)$.
   (c) (4 pts) Compute $P(j\omega)$, i.e. the Fourier transform of $p(t)$ and sketch the magnitude $|P(j\omega)|$ for $|\omega| \leq 6\pi/T_1$.
   (d) (4 pts) Compute the Fourier series coefficients a_k of the periodic signal $q(t)$ and sketch the coefficients a_k for $k = 0,\ \pm 1,\ \pm 2,\ \pm 3$.

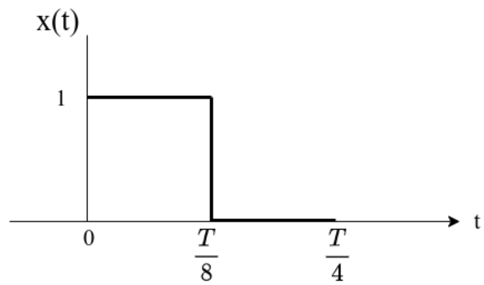

Figure 1: Signal $s(t)$ (problem-1).

(e) (4 pts) Using the obtained relations, specify the relationship between $X(\omega)$ and a_k. Justify how the Fourier series for signal $q(t)$ can be determined considering that the Fourier transform of $p(t)$ is known. Note that $p(t)$ is one time-period of the periodic signal $q(t)$.

3) (6 points) A system S generates the output signal $y(t)$ related to Fourier transform of the provided input $x(t)$ as depicted in the Fig. 2(a). If the same input is passed through the system configuration as shown in Fig. 2(b), compute the output $v(t)$.

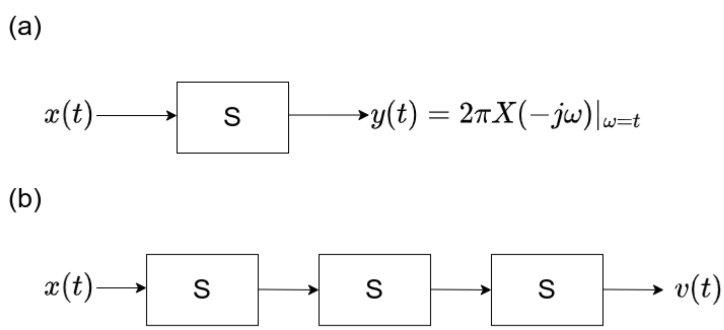

Figure 2: Block diagram for system S (problem-3).

4) (8 points) Consider a continuous-time LTI system described by:

$$\frac{dy(t)}{dt} + 2y(t) = x(t) \tag{1}$$

Using the Fourier transform, find the output $y(t)$ to each of the following input signals;

(a) (4 pts) $x(t) = e^{-t}u(t)$

(b) (4 pts) $x(t) = u(t)$

5) (12 points) Compute the Fourier transform of the following signals:

(a) (4 pts) $[e^{-\alpha t}cos(\omega_0 t)]u(t)$, $\alpha > 0$

(b) (4 pts) $e^{-3|t|}sin(2t)$

(c) (4 pts) $\left(\frac{sin\pi t}{\pi t}\right)\left(\frac{sin2\pi t}{\pi t}\right)$

Programming Problem:

1. (7 points) For the discrete time signals defined below, compute the Discrete Time Fourier Transform (DTFT) and plot the following:

   - the signal $x[n]$
   - real part of the complex DTFT signal
   - imaginary part of the complex DTFT signal
   - magnitude spectrum of the DTFT signal

   (a) (3pts) Signal-1: Unit impulse signal or $x_1[n] = 1$ for the value of $n = [0]$ and zero otherwise.

   (b) (4pts) Signal-2: $x_2[n] = 1$ for the values of $n = [-4, -3, -2, -1, 0, 1, 2, 3, 4]$ and zero otherwise.

   For plots use $n \in [-1000, 1000]$ and $\omega \in [-2\pi, 2\pi]$. Add all relevant plots in your report and comment about the periodicity of the obtained DTFT signals.

   Hint: To simulate continuous signals use appropriate discretization (wherever required).