Course: Discrete Mathematics

Faculty: Dr. Venkatakrishnan Ramaswamy
Module 1: Proof Methods
Lesson 1: Basics of Logic

Topic: Logical Equivalences

Reading Objective

This reading supplements the concepts you have learned in the video. In this reading, you will learn about definitions of tautologies and contradictions. Following this, you will learn how to define a logical equivalence with the help of several examples.

1 Tautology and Contradiction

Tautology: A compound proposition that is true for all combinations of truth values of its proposition variables is called a tautology. All rows of the last column of the truth table will be 'T' for a tautology.
Example :

$p \to p \lor q$:

p	**q**	**p ∨ q**	**p → p ∨ q**
F	F	F	T
F	T	T	T
T	F	T	T
T	T	T	T

Contradiction: A compound proposition that is false for all combinations of truth values of its proposition variables is called a contradiction. All rows of the last column of the truth table will be 'F' for a contradiction.
Example:

$p \land \neg p$:

p	**¬p**	**p ∧ ¬p**
F	T	F
T	F	F

2 Logical equivalence

The compound propositions p and q are logically equivalent if their truth tables are the same. It is denoted by $p \equiv q$. **Some equivalences:** (*Verify the given equivalences by constructing their truth tables!*)

- *Involving Conditionals:*
  - $p \to q \equiv \neg q \to \neg p$ (Conditional statement is logically equivalent to its contrapositive)
  - $(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$
  - $(p \to r) \lor (q \to r) \equiv (p \land q) \to r$
- *DeMorgan's Laws:*
  - $\neg(p \land q) \equiv \neg p \lor \neg q$
  - $\neg(p \lor q) \equiv \neg p \land \neg q$
  - $\neg p \lor q \equiv p \to q$

- *Distributivity:*
  - $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
  - $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
- *Associativity:*
  - $p \lor (q \lor r) \equiv (p \lor q) \lor r$
  - $p \land (q \land r) \equiv (p \land q) \land r$

Nonequivalence: Two compound propositional statements are said to be nonequivalent if they differ in truth values for at least one combination of truth values of the propositional variables, that is, at least one row of the last columns do not match in the truth table.
To prove nonequivalence, show assignment to the propositional variables such that one of the statements evaluates to T and the other to F. This is called a counterexample.
Example: $p \to q$ is not equivalent to $p \land q$. This is because there exists an assignment p: F and q: T for which the first statement evaluates to T, while the second statement evaluates to F. Thus, we can say that such an assignment is the counterexample.

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3 Reading Summary

In this reading, you have learned the following:

- Definition of tautology and contradiction
- Logical equivalence and verifying it with truth tables
- Nonequivalent propositional statements