

In the name of allah

Linear Control Systems



Hw 04

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Fall 1402

Theory Assignments

دول 1

1. $G(s) = \frac{5s+5}{s^4 + 3s^3 + 3s^2 + 3s+2}$ \rightarrow ریشه مد خروج
حالت میانه
وقتی تغییر است.

s^4	1	3	2	
s^3	3	3		
s^2	2	2		$2s^2+2 \rightarrow s= \pm j$
s^1	4	0		\rightarrow حالت 2
s^0	2			

marginally stable \rightarrow ریشه خنثی (تغییر است) \rightarrow سیستم

2. $\frac{8s+1}{s^4 + 3s^3 + 3s^2 + 3s+1}$

s^4	1	3	1	
s^3	3	3		
s^2	8	0		\rightarrow حالت خاص 1
s^1	$\frac{24-s}{\epsilon}$	0		
s^0	1			$\epsilon \rightarrow 0^+$

unstable \rightarrow تغییر است \rightarrow ریشه خنثی یا باردار است \rightarrow سیستم

3. $\frac{4}{s^4 + 4s^3 + 3s^2 - 4s - 4}$

s^4	1	3	-4	
s^3	4	-4		
s^2	4	-4		$4s^2-4 \rightarrow s= \pm 1$
s^1	8	0		\rightarrow حالت خاص 2
s^0	-4			

unstable \rightarrow تغییر است \rightarrow ریشه خنثی یا باردار است \rightarrow سیستم

4. $\frac{2s-1}{s^5 + 2s^4 + 2s^3 + 4s^2 + 11s+10}$ \rightarrow ریشه خنثی یا باردار است
حالت میانه
وقتی تغییر است.

s^5	1	2	11	
s^4	2	4	10	
s^3	4	0		\rightarrow حالت خاص 1
s^2	$\frac{12-4\epsilon}{\epsilon}$	10		
s^1	$\frac{2}{\epsilon}$			
s^0	10			

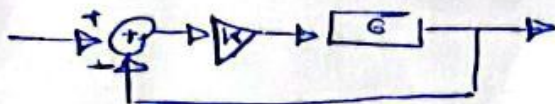
Stable \rightarrow تغییر است \rightarrow ریشه خنثی یا باردار است \rightarrow سیستم

5. $\frac{2}{s^6 + 7s^5 + 20s^4 + 30s^3 + 25s^2 + 11s+2}$

s^6	1	20	25	2
s^5	7	30	11	
s^4	110	184	12	$\times 7$
s^3	2152	1112		
s^2	230608	14		$\times 110$
s^1	2452	14		$\times 2152$
s^0	230608	14		

Stable \rightarrow تغییر است \rightarrow ریشه خنثی یا باردار است \rightarrow سیستم

2. حل



$$G(s) = \frac{2s^3 + s^2 + s - 1}{s^3 - 5s^2 + 2s + 8}$$

$$H(s) = \frac{KG}{1+KG} \Rightarrow G = \frac{Z}{P} \Rightarrow \frac{KZ}{P+KZ} = H$$

$$\Rightarrow s^3 - 5s^2 + 2s + 8 + 2Ks^3 + Ks^2 + Ks - K$$

$$= (1+2K)s^3 + (K-5)s^2 + (2+K)s + (8-K)$$

$$s^3 \quad 1+2K \quad 2+K \quad 1$$

$$s^2 \quad K-5 \quad 8-K$$

$$s^1 \quad \frac{-(1+2K)(8-K) + (2+K)(K-5)}{K-5} \Rightarrow K^2 - 9K - 10 - (-2K^2 + 15K + 8)$$

$$s^0 \quad 8-K \quad 3K^2 - 18K - 18$$

$$\Rightarrow \frac{1+2K}{K-5} \quad \frac{60}{K-5} \quad \frac{K \pm \sqrt{36+24}}{2} \quad s \pm \sqrt{5} \quad 6,87$$

$$3K^2 - 18K - 18 = K^2 - 6K - 6 = \frac{(K-3-\sqrt{5})(K-3+\sqrt{5})}{K-5}$$

8K

↑

نقطة الاستقرار

$$\ominus \quad K < -\frac{1}{2}$$

$$K < 5$$

$$K < 6,87$$

$$K < 8$$

$$\oplus \quad K > -\frac{1}{2}$$

$$K > 5$$

$$K > 6,87 \quad -9,87 < K < 5$$

$$K < 8$$

نقطة الاستقرار

$$8 > K > 6,87$$

```

t = 0:0.01:10000;
u = Step01(t);

K = 7;
num = [2*K K K -K];
den = [ 2*K+1 K-5 K+2 8-K];
sys = tf(num, den);
y1 = lsim(sys, u, t);

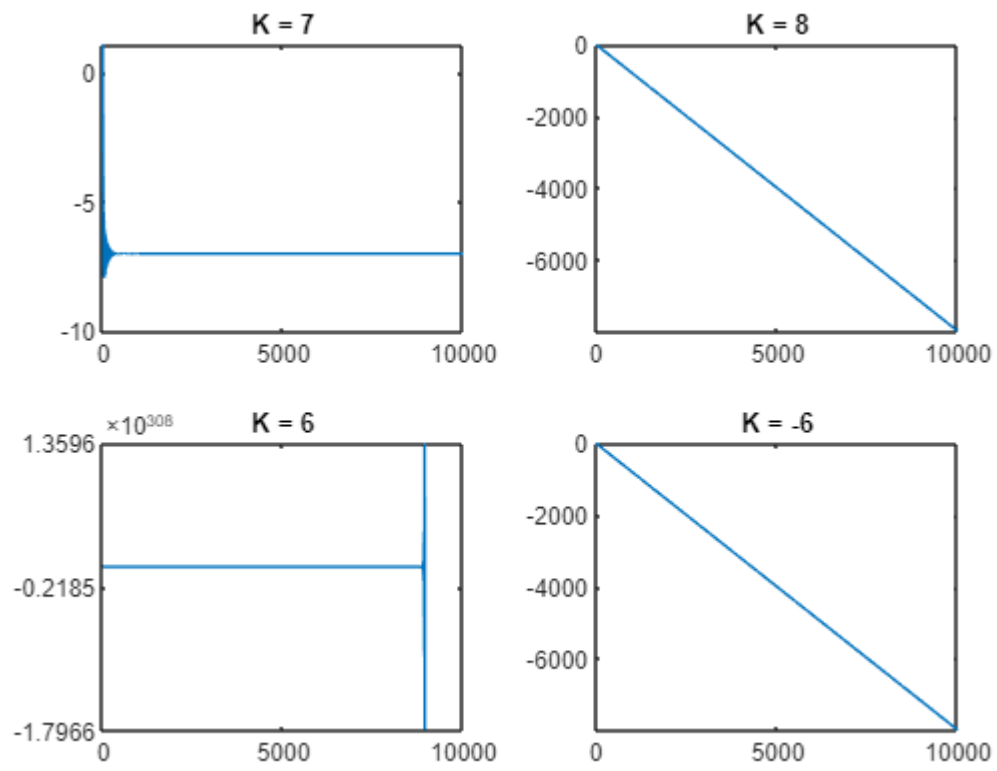
K = 8;
num = [2*K K K -K];
den = [ 2*K+1 K-5 K+2 8-K];
sys = tf(num, den);
y2 = lsim(sys, u, t);

K = 6;
num = [2*K K K -K];
den = [ 2*K+1 K-5 K+2 8-K];
sys = tf(num, den);
y3 = lsim(sys, u, t);

K = -6;
num = [2*K K K -K];
den = [ 2*K+1 K-5 K+2 8-K];
sys = tf(num, den);
y4 = lsim(sys, u, t);

figure
subplot(2,2,1)
plot(t,y1)
title("K = 7")
xlim([0 10000])
subplot(2,2,2)
plot(t,y2)
title("K = 8")
xlim([0 10000])
subplot(2,2,3)
plot(t,y3)
title("K = 6")
xlim([0 10000])
subplot(2,2,4)
plot(t,y4)
title("K = -6")
xlim([0 10000])

```



As we see, for $6.87 < K < 8$ system is stable


```

t = 0:0.01:100;
u = Step01(t);

num = [1] ;
den = [1 1];
sys = tf(num, den);
y1 = lsim(sys, u, t);

num = [1] ;
den = [1 -1];
sys = tf(num, den);
y2 = lsim(sys, u, t);

num = [1] ;
den = [1 0 1];
sys = tf(num, den);
y3 = lsim(sys, u, t);

num = [1] ;
den = [1 2 1];
sys = tf(num, den);
y4 = lsim(sys, u, t);

syms e
y5 = (-1/(e+1)^2); % s(t) = d/dt h(t)

```

$$y5 = -\frac{1}{(e+1)^2}$$

```

y6 = (-2*e/(e^2+1)^2); % s(t) = d/dt h(t)

```

$$y6 = -\frac{2e}{(e^2+1)^2}$$

```

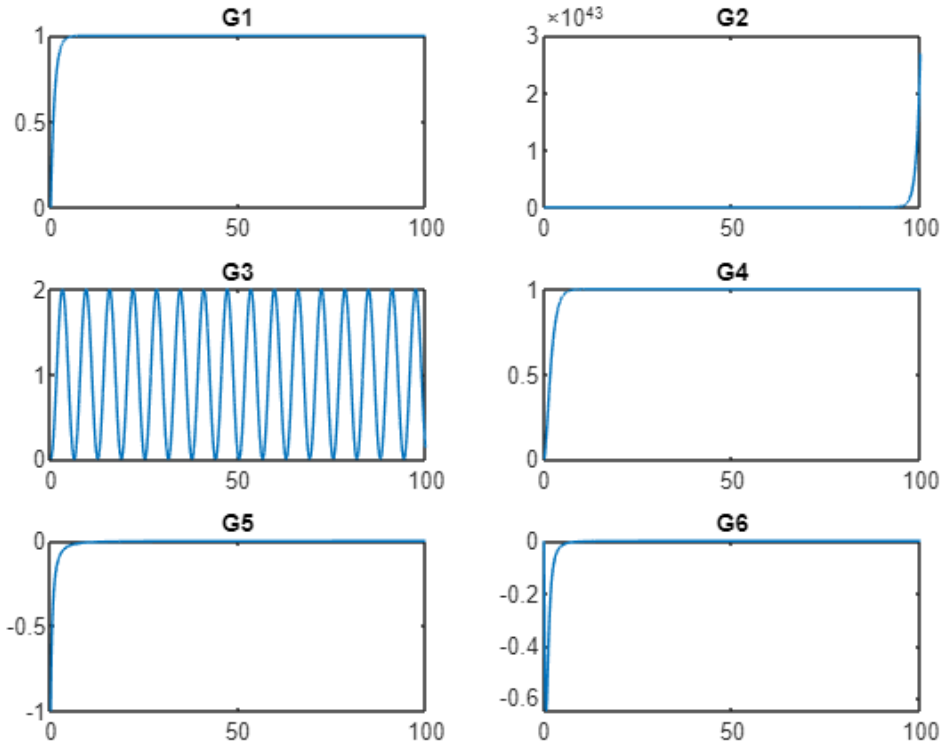
figure
subplot(3,2,1)
plot(t,y1)
title("G1")
xlim([0 100])
subplot(3,2,2)
plot(t,y2)
title("G2")
xlim([0 100])
subplot(3,2,3)
plot(t,y3)
title("G3")
xlim([0 100])
subplot(3,2,4)

```

```

plot(t,y4)
title("G4")
xlim([0 100])
subplot(3,2,5)
fplot(y5)
title("G5")
xlim([0 100])
subplot(3,2,6)
fplot(y6)
title("G6")
xlim([0 100])

```



9065)

$$1) \quad -0,02 < \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \overset{\text{max}}{\sin \omega_d t + \phi} < 0,02$$

$$-0,02 < \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} < 0,02$$

$$t = \frac{\eta}{\xi \omega_n} : \quad -0,02 < \frac{e^{-\eta}}{\sqrt{1-\xi^2}} < 0,02$$

$$e^{-\eta} < 0,02 \sqrt{1-\xi^2}$$

$$\frac{e^{-\eta}}{0,02} < \sqrt{1-\xi^2}$$

$$\frac{e^{-2\eta}}{4 \cdot 10^{-4}} < 1 - \xi^2$$

$$\xi^2 < 1 - \frac{e^{-2\eta}}{4 \cdot 10^{-4}}$$

$$\xi < \sqrt{1 - \frac{e^{-2\eta}}{4 \cdot 10^{-4}}}$$

$$\eta = 4 \rightarrow \xi < 0,401$$

$$\eta = 4,2 \rightarrow \xi < 0,441$$

$$\eta = 4,5 \rightarrow \xi < 0,481$$

$$\eta = 4,75 \rightarrow \xi < 0,501$$

$$\eta = 5 \rightarrow \xi < 0,541$$

$$\eta = 6 \rightarrow \xi < 0,992290$$

$$\xi < 0,9$$

در گاهی کوچک،

در حفظ کم است و

مقدار زمان نشست بزرگ است

نه گاهی بزرگ در حفظ زیاد

و اختلاف زمان نشست کم است.

$$T_d = \frac{6}{\omega_n} \leftarrow \xi = 1$$

2)

2)

$$\frac{d}{dt} \left(1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \sin(\omega_n \sqrt{1-\xi^2} t + \cos^{-1} \xi) \right)$$

$$= \frac{\xi \omega_n}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \sin \omega_d t + \phi - \frac{\omega_d}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \cos \omega_d t + \phi$$

$$= \frac{\omega_n e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \left(\xi \sin \omega_d t + \phi - \sqrt{1-\xi^2} \cos \omega_d t + \phi \right)$$

$$= \frac{\omega_n e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin \omega_d t + \phi \rightarrow \cos^{-1}(\xi)$$

$$\Rightarrow t = \frac{k\pi}{\omega_n \sqrt{1-\xi^2}} \xrightarrow[\text{Short}]{\text{more over } k=1} t = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$$

$$\Rightarrow 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi \omega_n \frac{\pi}{\omega_n \sqrt{1-\xi^2}}} \sin \left(\omega_d \frac{\pi}{\omega_d} + \cos^{-1}(\xi) \right)$$

$$1 + \frac{e^{-\frac{\xi \pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \underbrace{\sin(\cos^{-1}(\xi))}_{\sqrt{1-\xi^2}} = 1 + \frac{e^{-\frac{\xi \pi}{\sqrt{1-\xi^2}}}}{mp}$$

→ دایره بین ۵ جا باشد تا اینکه پله به صورت جا باشد، هیچ شرط اضافی روی ξ لازم نیست.

زمان پیک برابر $\frac{\pi}{\omega_n \sqrt{1-\xi^2}}$ است، جری کاهش زمان پیک (منتیم افزایش سرعت پیک) در ξ ثابت

جای $\sqrt{1-\xi^2}$ همیشه شود و ξ جا کاهش ξ مقدار overshoot افزایش می‌یابد

هرچه ξ کمتر overshoot بیشتر باشد جای ξ باشد هرچه ξ کمتر سرعت پیک بیشتر

در ξ کمتر هرچه ξ کمتر باشد هرچه ξ کمتر سرعت پیک کمتر خواهد بود.

MATLAB Assignments

```
clc  
close all  
clear all
```

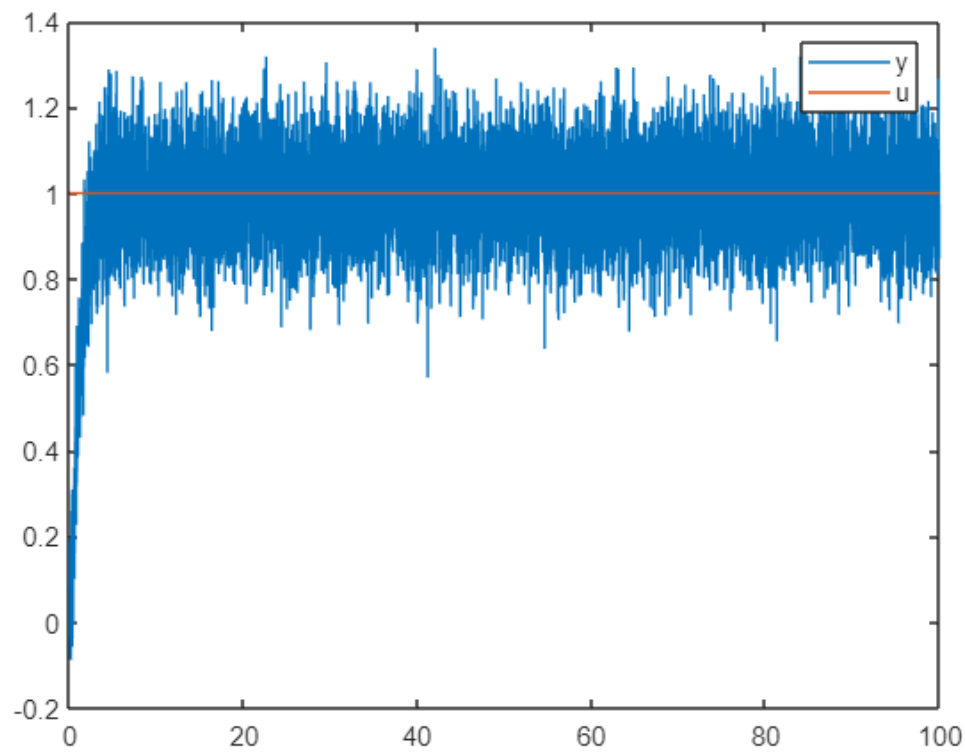
5 Curve Fitting Toolbox

a)

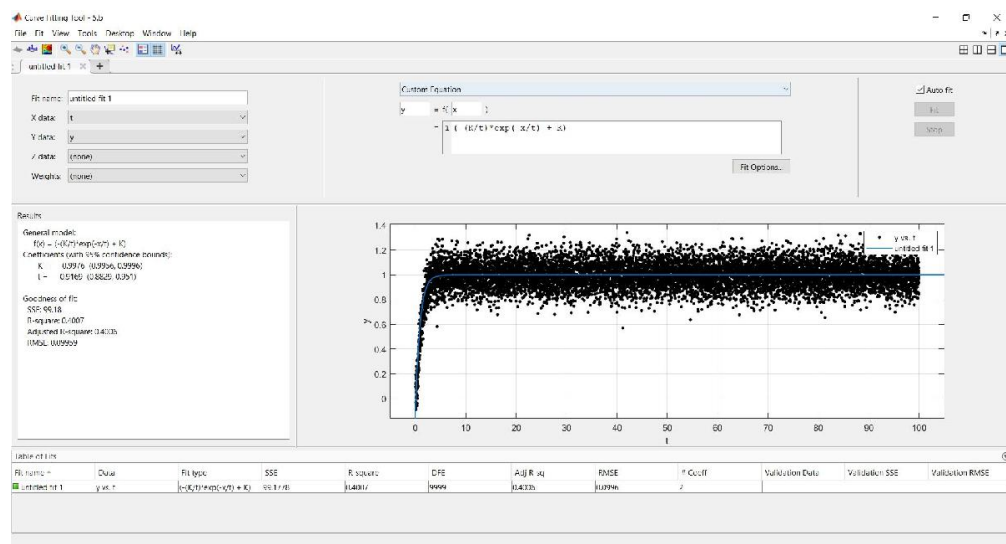
$$G(s) = \frac{K}{\tau s + 1} \Rightarrow \text{step response : } \frac{K}{\tau s + 1} * \frac{1}{s} = \frac{-\frac{K}{\tau}}{\tau s + 1} + \frac{K}{s}$$
$$\Rightarrow = Ku(t) - \frac{K}{\tau} e^{-\frac{t}{\tau}}$$

b)

```
t = out.tout;  
x = out.x;  
y = out.y;  
  
figure  
plot(t,y,t,x)  
legend("y","u")
```



c)



$K = 0.9976$, $\tau = 0.9169$

d)

$K = 0.9976;$
 $T = 0.9169;$
 $u = \text{Step01}(t);$

```

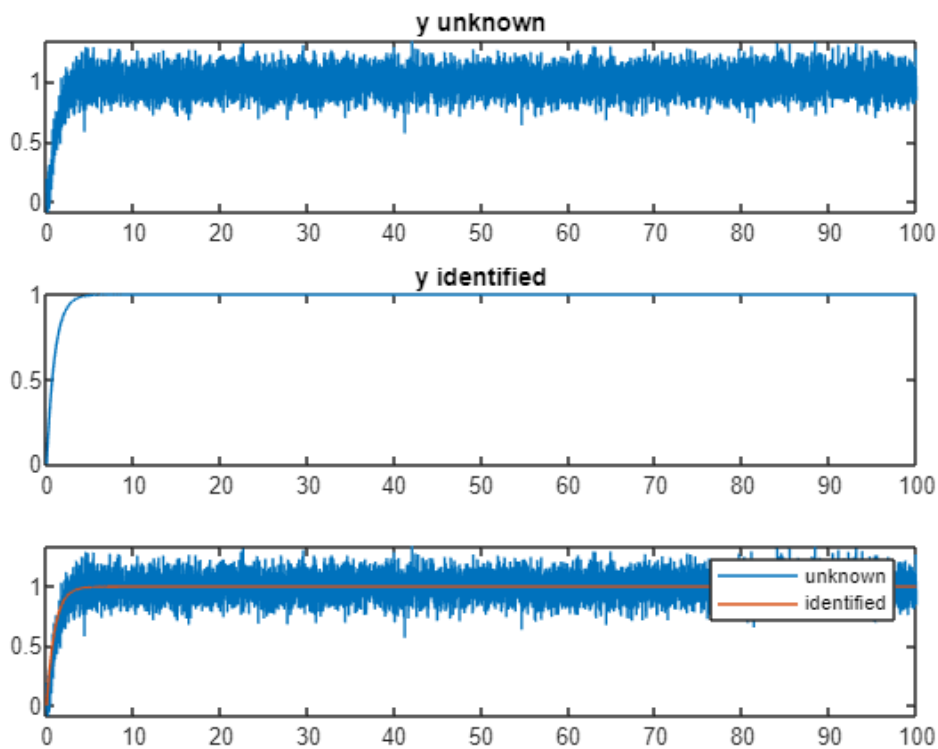
num = [K] ;
den = [T 1];
sys = tf(num, den);
y1 = lsim(sys, u, t);

```

```

figure
subplot(3,1,1)
plot(t,y)
title("y unknown")
subplot(3,1,2)
plot(t,y1)
title("y identified")
subplot(3,1,3)
plot(t,y,t,y1)
legend("unknown","identified")

```



e)

$$y(t) = Ku(t) - \frac{K}{\tau} e^{-\frac{t}{\tau}}$$

as we see, K is almost equal to 1.

Also, in $A = B$, the output reaches its final value of 0.95.

$$y(4) = 1 - \frac{1}{\tau} e^{-\frac{4}{\tau}} = 0.95$$

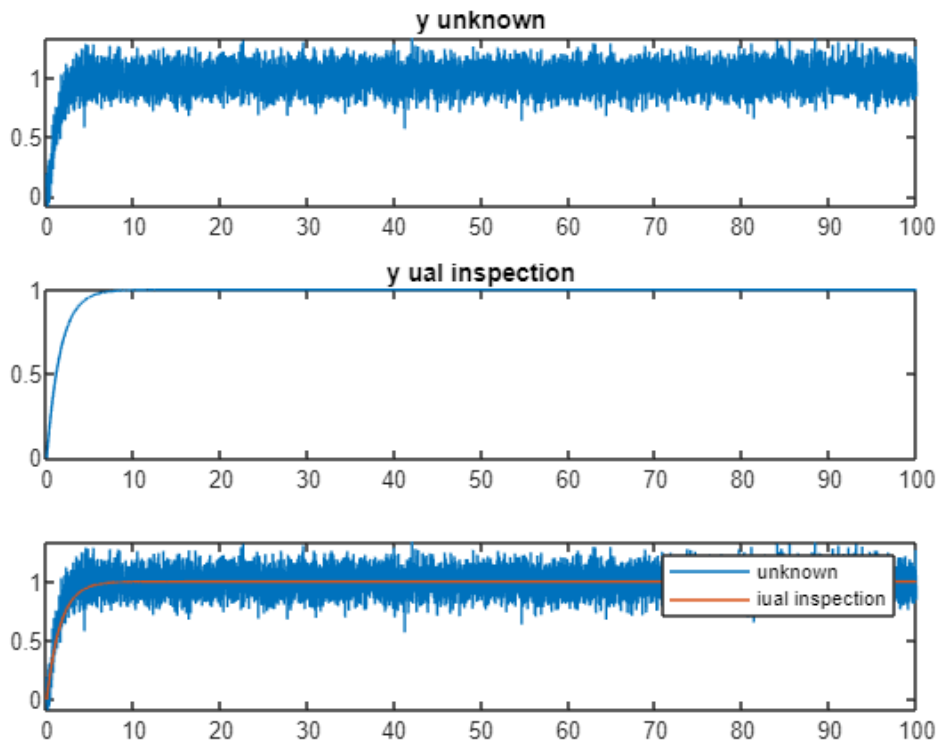
$K = 1$

$\tau = 1.573$

```
K = 1;
T = 1.573;
u = Step01(t);

num = [K] ;
den = [T 1];
sys = tf(num, den);
y2 = lsim(sys, u, t);

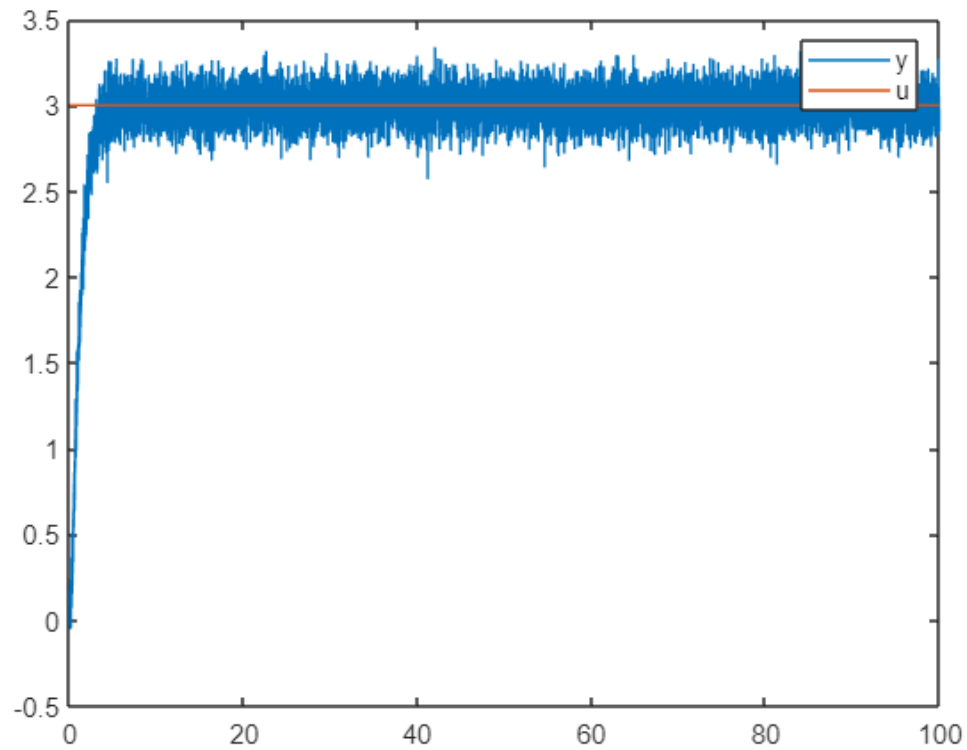
figure
subplot(3,1,1)
plot(t,y)
title("y unknown")
subplot(3,1,2)
plot(t,y2)
title("y ual inspection")
subplot(3,1,3)
plot(t,y,t,y2)
legend("unknown","iual inspection")
```

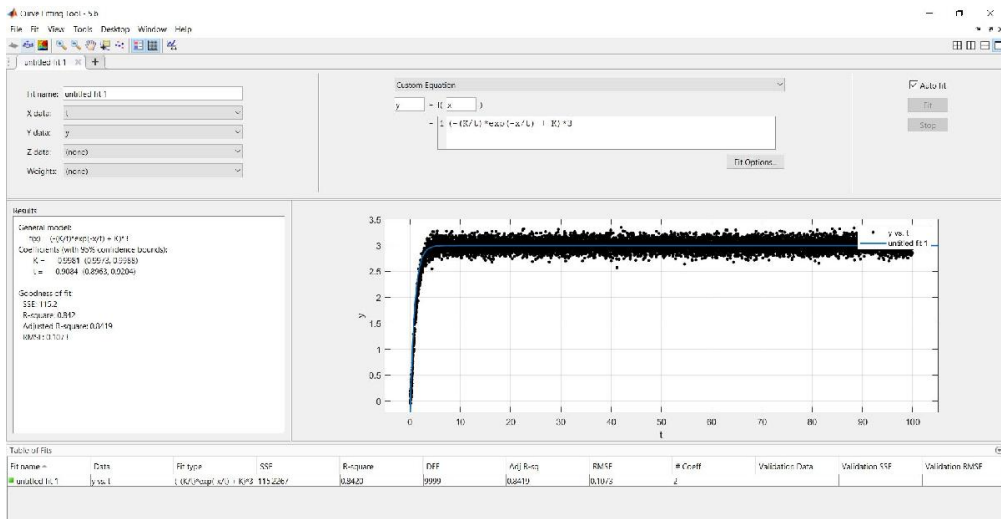


f)

$$y(t) = AKu(t) - \frac{AK}{\tau} e^{-\frac{t}{\tau}}, u(t) = Au(t), A = 3$$

```
t = out.tout;  
x = out.x;  
y = out.y;  
  
figure  
plot(t,y,t,x)  
legend("y","u")
```



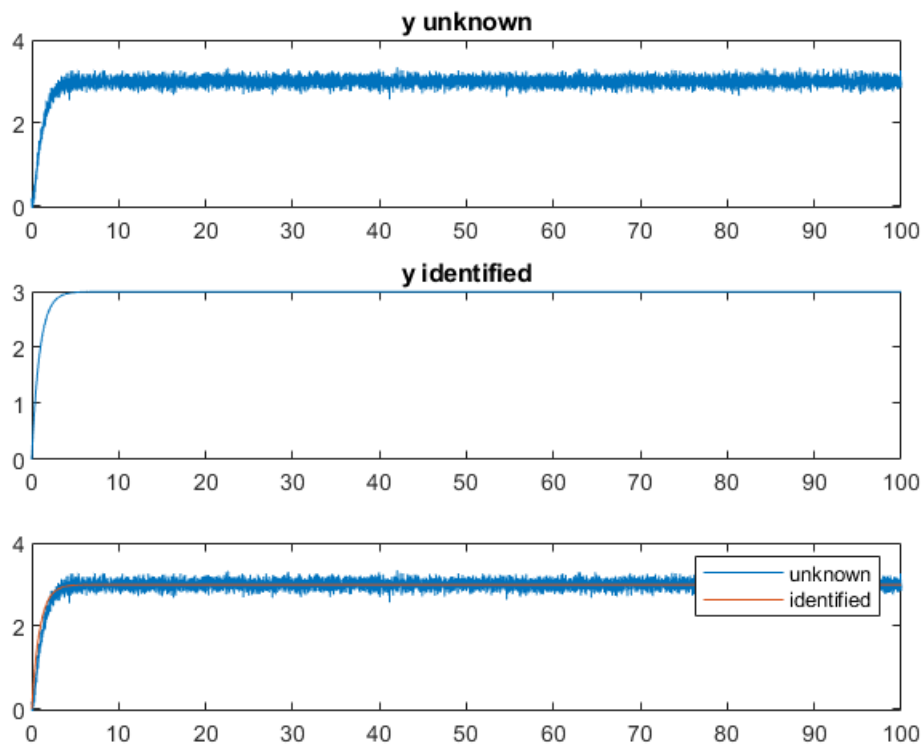


$K = 0.9981, \tau = 0.9084$

```
K = 0.9981;
T = 0.9084;
u = Step01(t);

num = [K] ;
den = [T 1];
sys = tf(num, den);
y1 = lsim(sys, 3*u, t);
```

```
figure
subplot(3,1,1)
plot(t,y)
title("unknown")
subplot(3,1,2)
plot(t,y1)
title("y identified")
subplot(3,1,3)
plot(t,y,t,y1)
legend("unknown","identified")
```



g)

sinusoidal respons

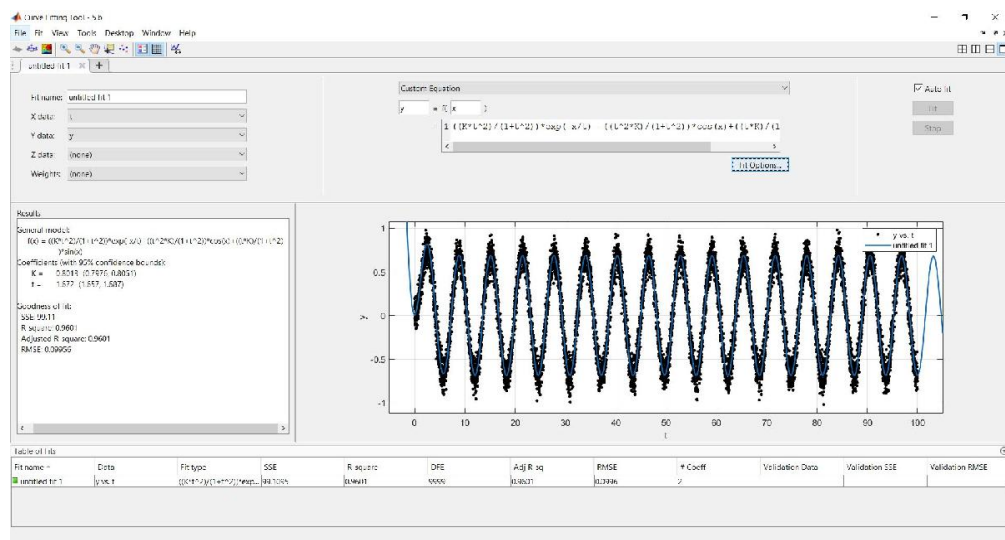
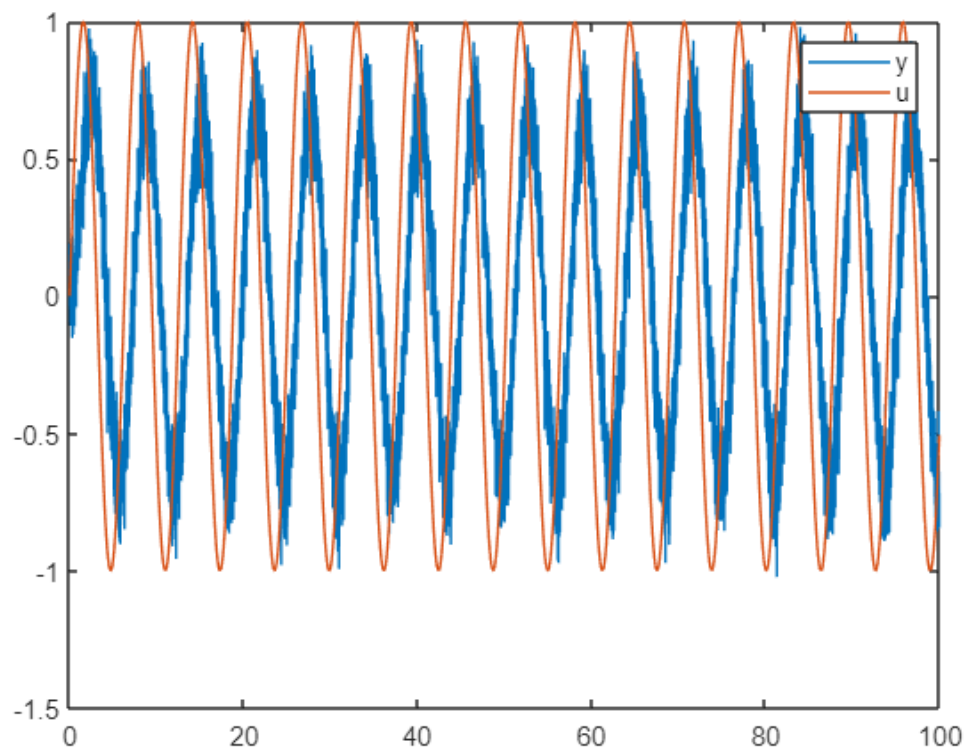
$$\frac{K}{\tau s + 1} \cdot \frac{a}{s^2 + a^2} = \frac{\frac{K\tau^2 a}{1 + \tau^2 a}}{\tau s + 1} + \frac{-\frac{a\tau^2 K}{1 + \tau^2 a}s + \frac{a\tau K}{1 + \tau^2 a}}{s^2 + a^2}$$

$$\Rightarrow \frac{K\tau^2 a}{1 + \tau^2 a^2} e^{-\frac{t}{\tau}} - \frac{a\tau^2 K}{1 + \tau^2 a^2} \cos(at) + \frac{\tau K}{1 + \tau^2 a^2} \sin(at)$$

$$a = 1 \Rightarrow \frac{K\tau^2}{1 + \tau^2} e^{-\frac{t}{\tau}} - \frac{\tau^2 K}{1 + \tau^2} \cos(t) + \frac{\tau K}{1 + \tau^2} \sin(t)$$

```
t = out.tout;
x = out.x;
y = out.y;

figure
plot(t,y,t,x)
legend("y","u")
```



$K = 0.8013$, $\tau = 1.672$

step respons

```
t = out.tout;
x = out.x;
y = out.y;
```

```
K = 0.8013;
T = 1.672;
```



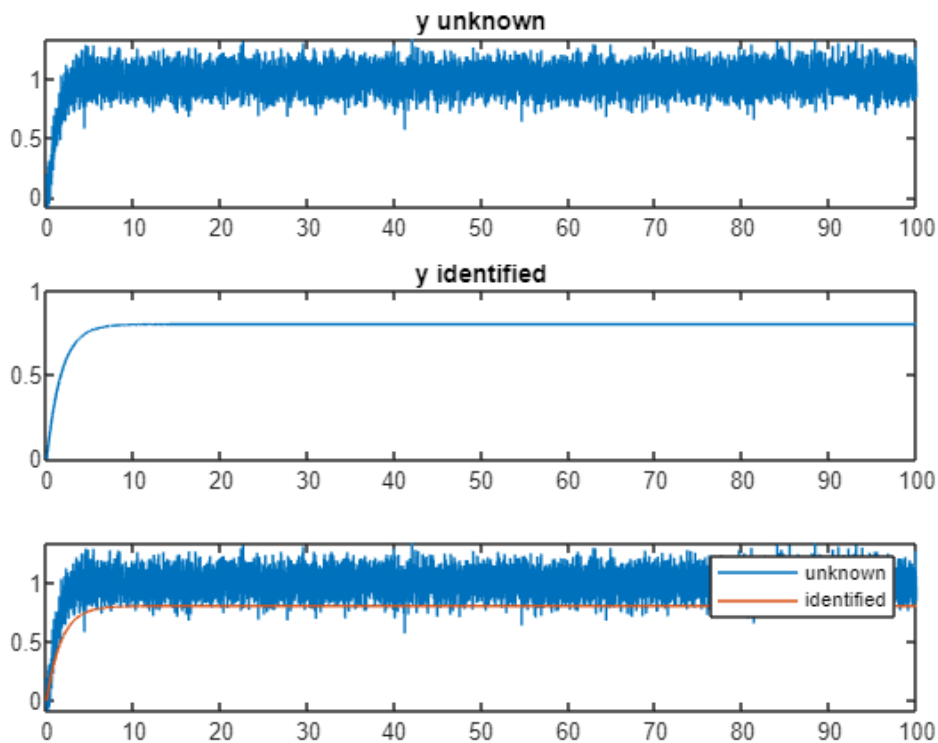
```

u = Step01(t);

num = [K] ;
den = [T 1];
sys = tf(num, den);
y1 = lsim(sys, u, t);

figure
subplot(3,1,1)
plot(t,y)
title("y unknown")
subplot(3,1,2)
plot(t,y1)
title("y identified")
subplot(3,1,3)
plot(t,y,t,y1)
legend("unknown","identified")

```



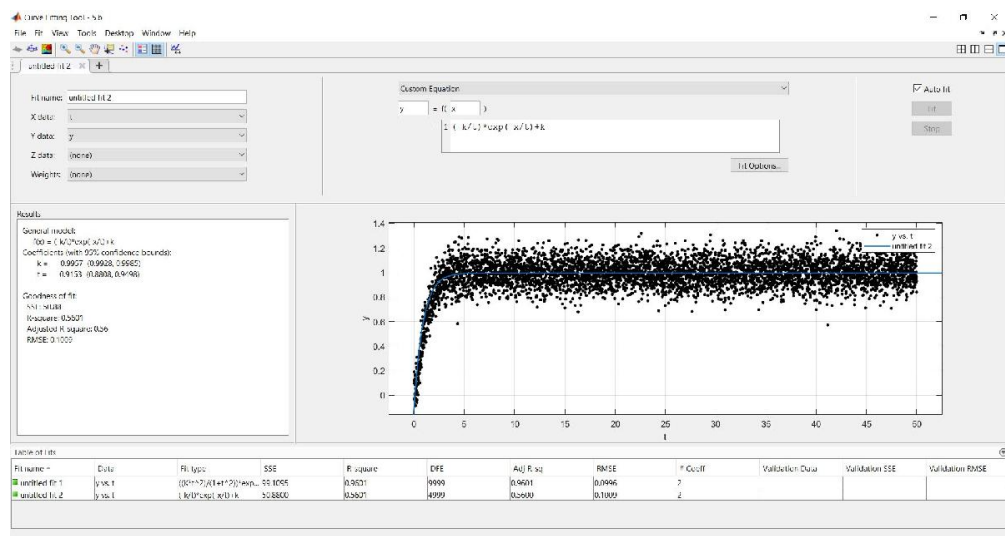
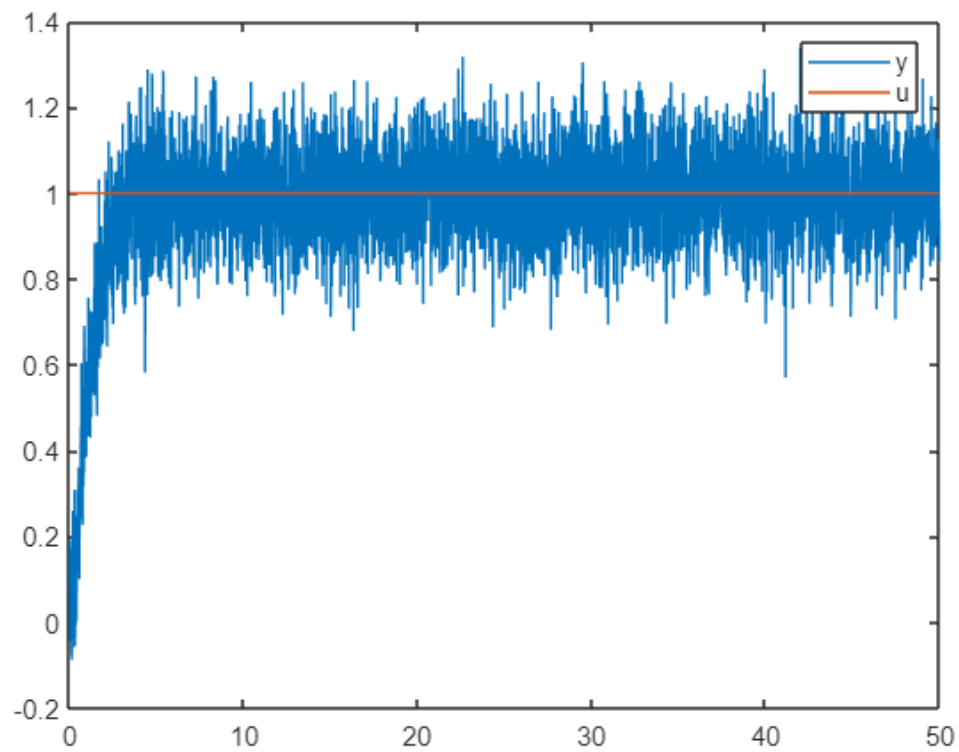
h)

```

t = out.tout;
x = out.x;
y = out.y;

figure
plot(t,y,t,x)
legend("y","u")

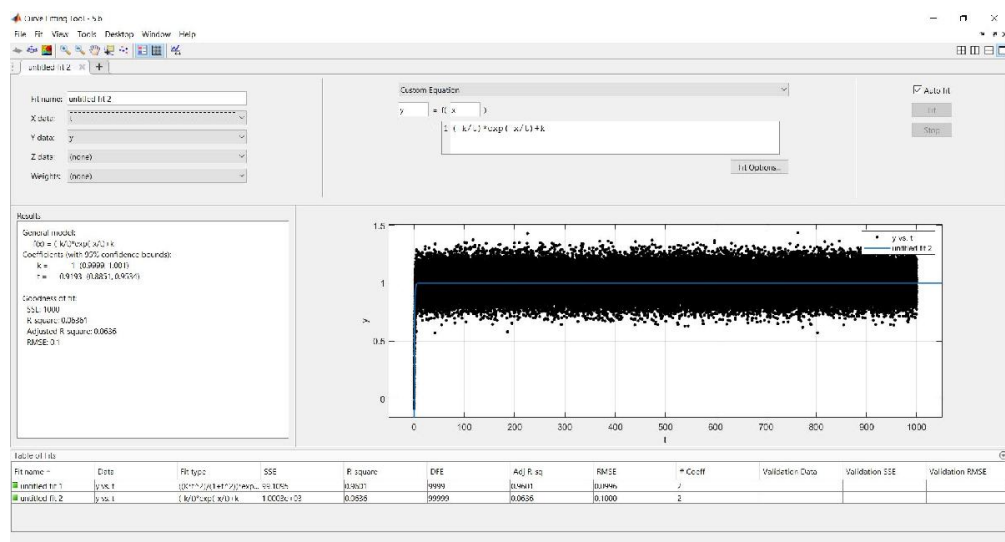
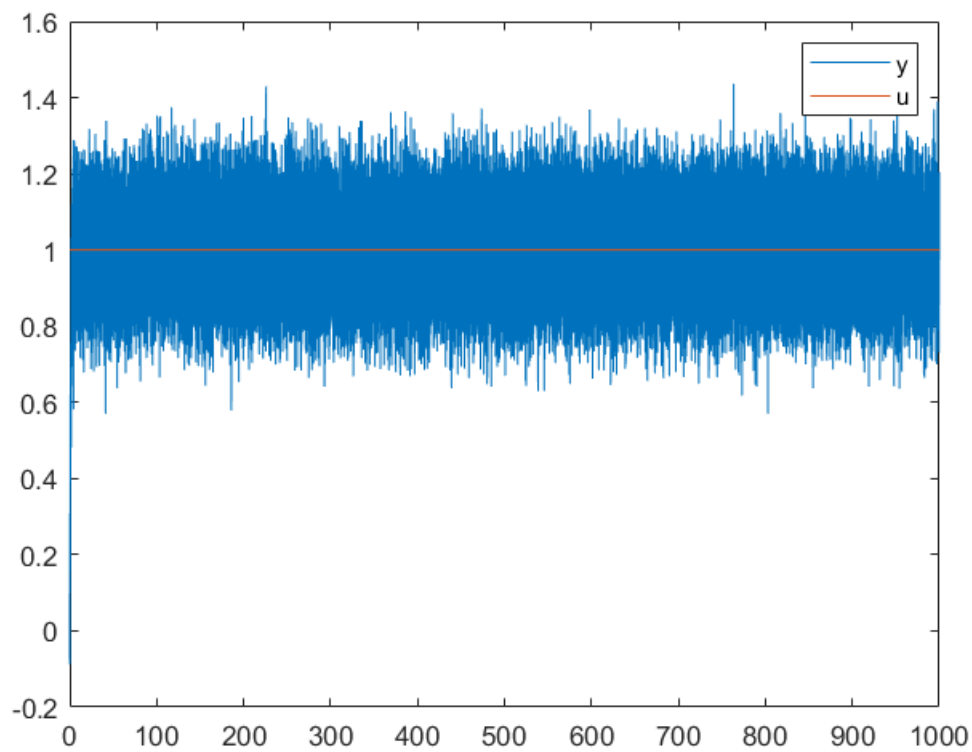
```



$K = 0.9957, \tau = 0.9153$

```
t = out.tout;
x = out.x;
y = out.y;
```

```
figure
plot(t,y,t,x)
legend("y","u")
```



$K = 1, \tau = 0.9193$

j)

$u = \text{Step01}(t);$

$K = 0.9976;$

$T = 0.9169;$

$\text{num} = [K];$

```

den = [T 1];
sys = tf(num, den);
y1 = lsim(sys, u, t);

K = 1;
T = 1.573;
num = [K] ;
den = [T 1];
sys = tf(num, den);
y2 = lsim(sys, u, t);

K = 0.9981;
T = 0.9084;
num = [K] ;
den = [T 1];
sys = tf(num, den);
y3 = lsim(sys, u, t);

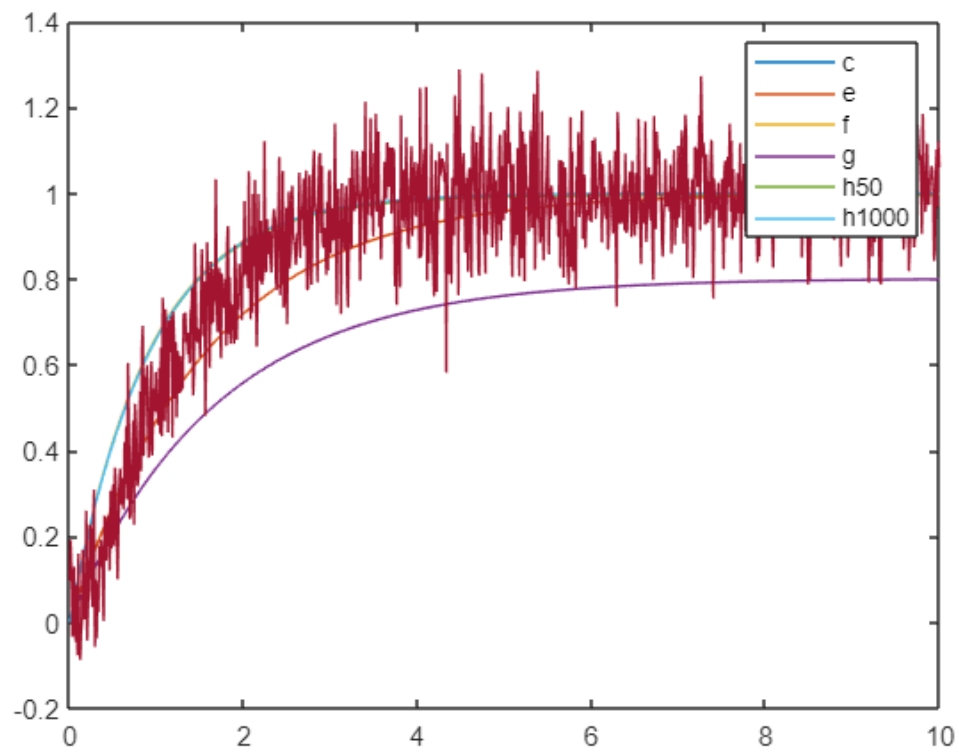
K = 0.8013;
T = 1.672;
num = [K] ;
den = [T 1];
sys = tf(num, den);
y4 = lsim(sys, u, t);

K = 0.9957;
T = 0.9153;
num = [K] ;
den = [T 1];
sys = tf(num, den);
y5 = lsim(sys, u, t);

K = 1;
T = 0.9193;
num = [K] ;
den = [T 1];
sys = tf(num, den);
y6 = lsim(sys, u, t);

figure
plot(t,y1,t,y2,t,y3,t,y4,t,y5,t,y6,t,y)
legend("c","e","f","g","h50","h1000")
xlim([0 10])

```



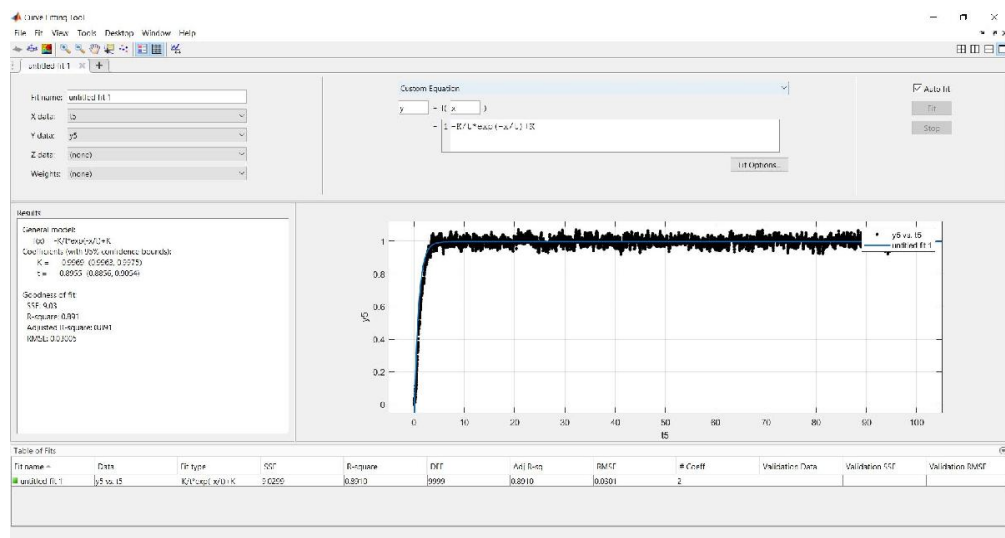
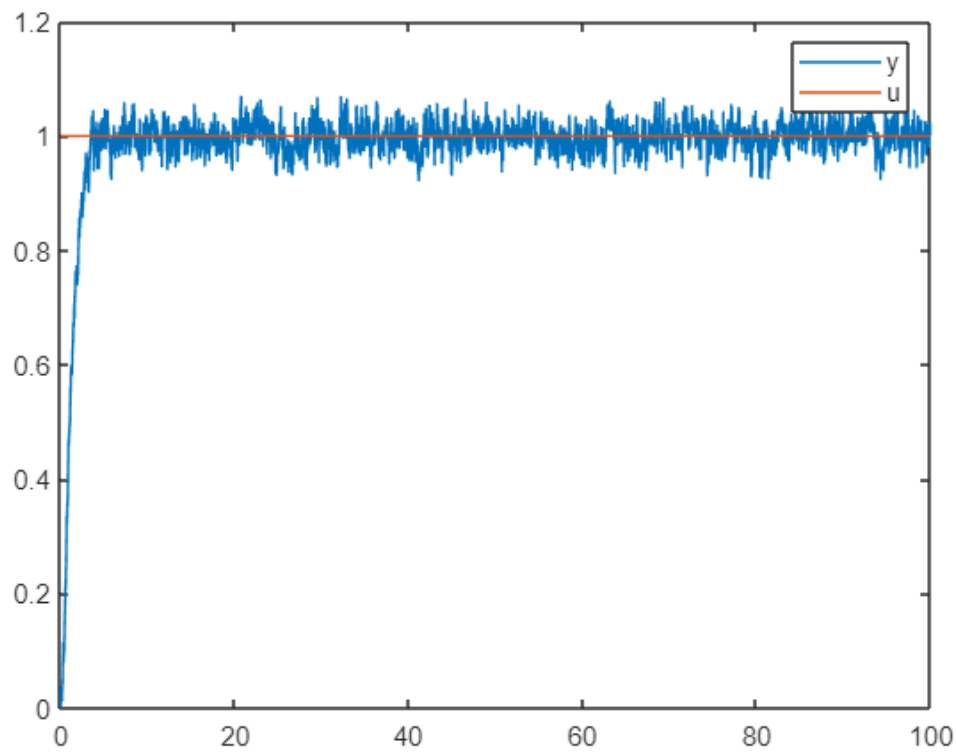
as we see, c, f, h50 & h1000 are good approximation.

Bonus

i)

```
% t5 = out.tout;
% x5 = out.x;
% y5 = out.y;
```

```
figure
plot(t5, y5, t5, x5)
legend("y", "u")
```

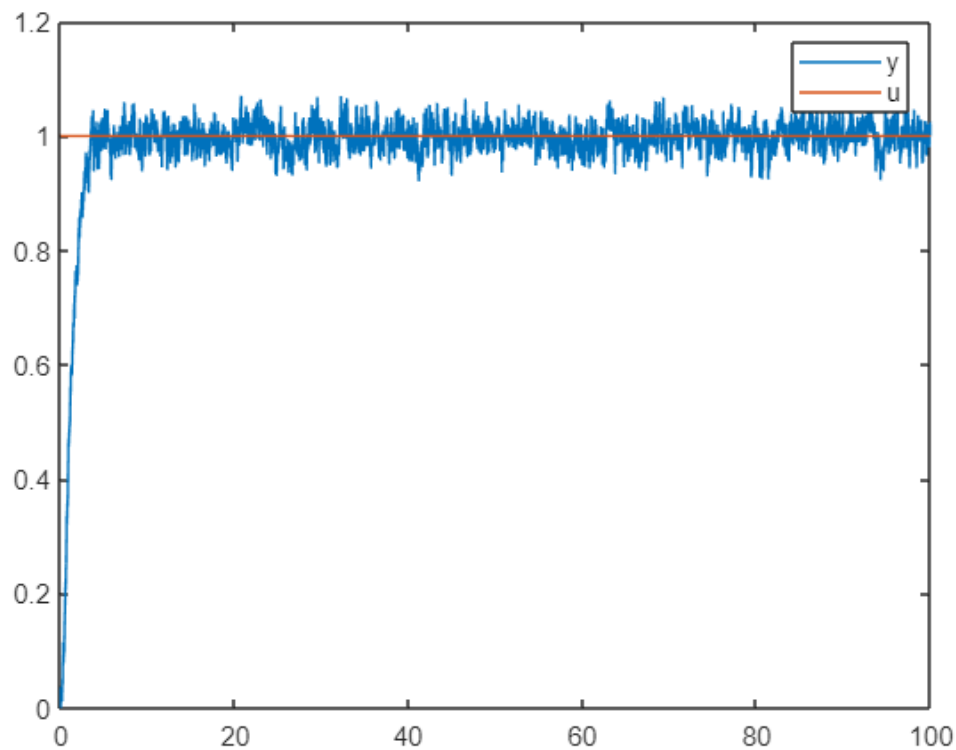
$K = 0.9969, \tau = 0.8955$

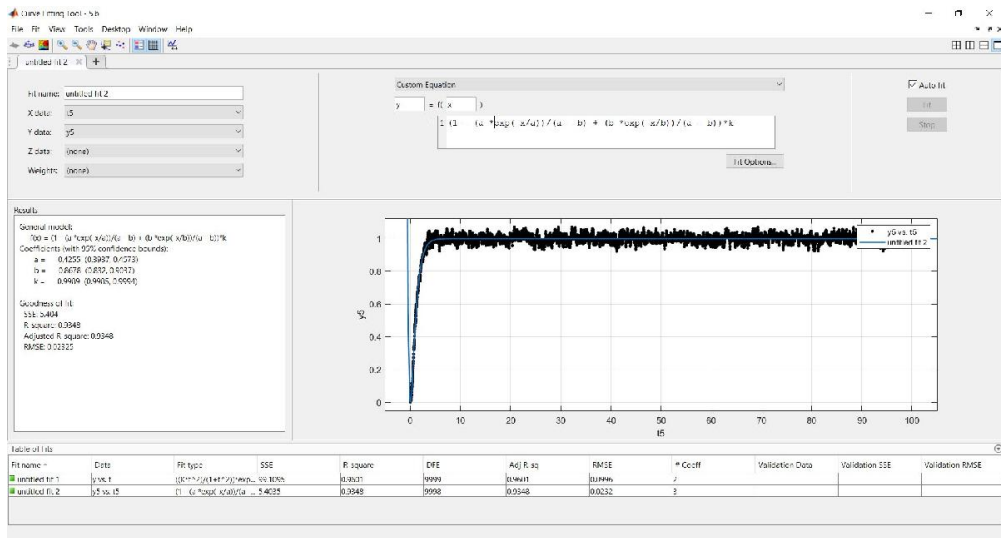
k)

$$G(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} \Rightarrow \text{step response : } \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} * \frac{1}{s} = -\frac{\frac{K\tau_1}{1 - \frac{\tau_2}{\tau_1}}}{\tau_1 s + 1} - \frac{\frac{K\tau_2}{1 - \frac{\tau_1}{\tau_2}}}{\tau_2 s + 1} + \frac{K}{s}$$

$$\Rightarrow = Ku(t) - \frac{K\tau_1}{\tau_1 - \tau_2} e^{-\frac{t}{\tau_1}} - \frac{K\tau_2}{\tau_2 - \tau_1} e^{-\frac{t}{\tau_2}}$$

```
figure
plot(t5,y5,t5,x5)
legend("y","u")
```





$K = 0.9989, \tau_1 = 0.8678, \tau_2 = 0.4255$

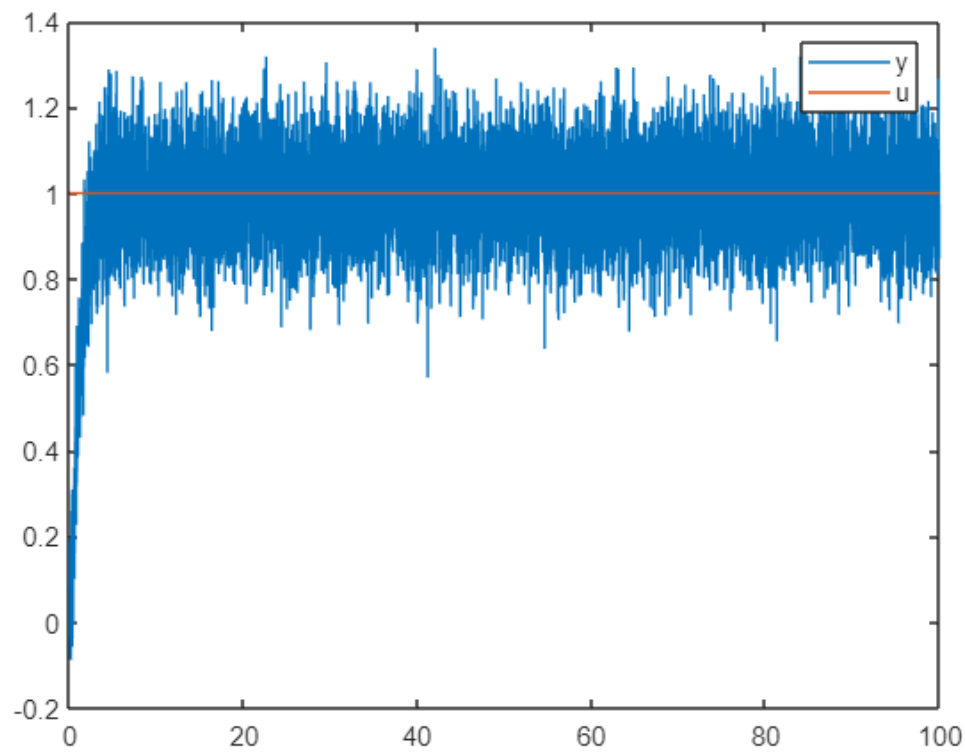
6 System Identification Toolbox

```
clc
close all
clear all
```

a)

```
t = out.tout;
x = out.x;
y = out.y;

figure
plot(t,y,t,x)
legend("y","u")
```



b)

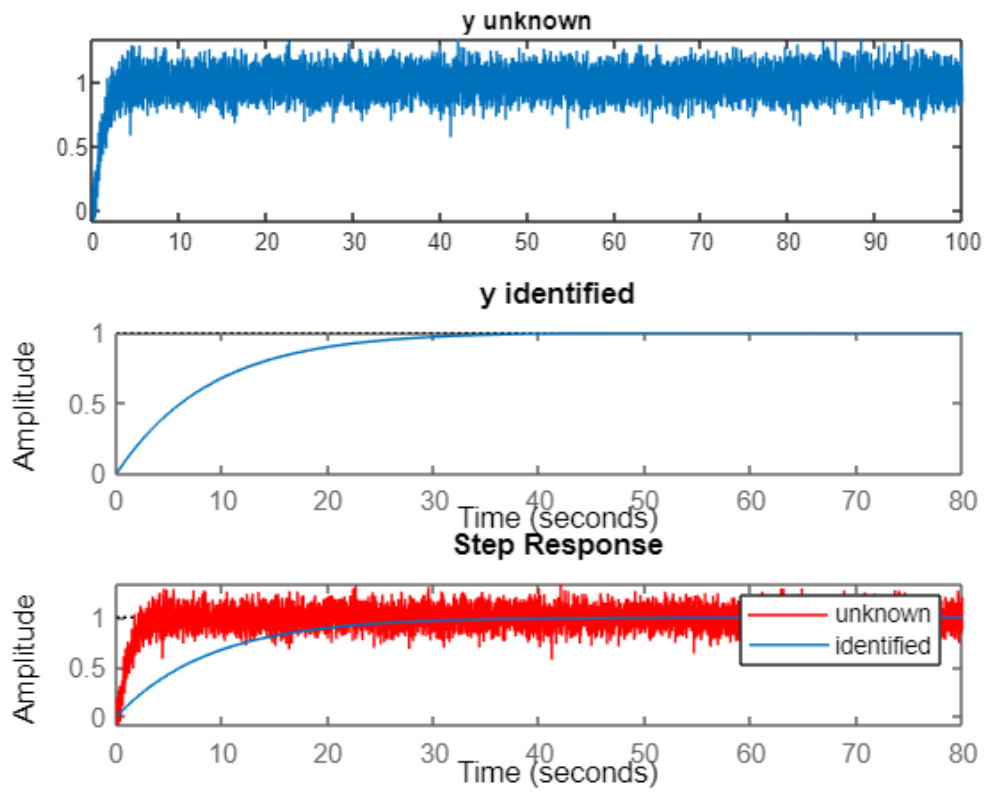
```
Gb = tf(tf1.Numerator , tf1.Denominator)
```

Gb =

```
0.1127
-----
s + 0.1128
```

Continuous-time transfer function.

```
figure
subplot(3,1,1)
plot(t,y)
title("y unknown")
subplot(3,1,2)
step(Gb)
title("y identified")
subplot(3,1,3)
hold on
plot(t,y,"Color","r")
step(Gb)
legend("unknown","identified")
```

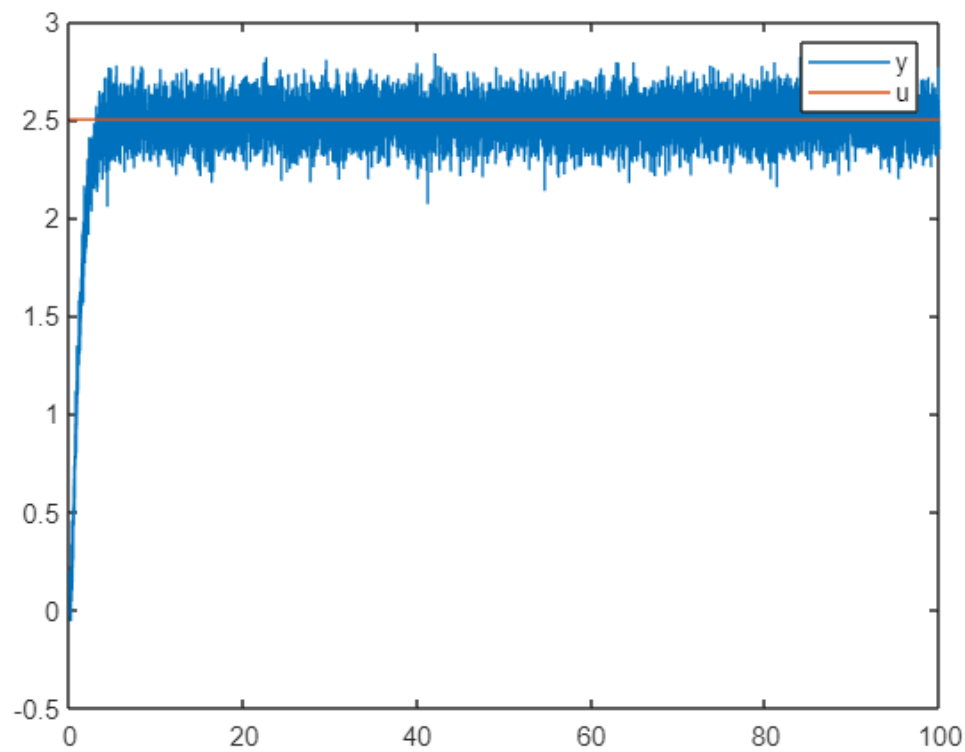


c)

$A = 2.5$

```
% t1 = out.tout;
% x1 = out.x;
% y1 = out.y;

figure
plot(t1,y1,t1,x1)
legend("y","u")
```

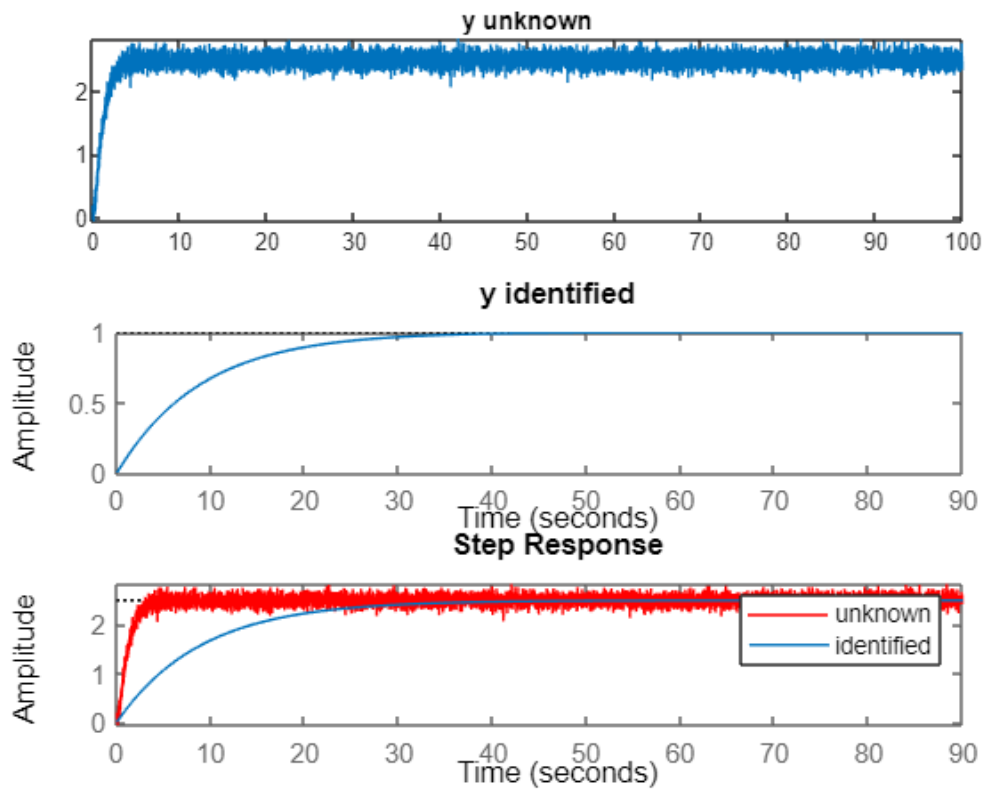
```
Gc1 = tf(tf2.Numerator , tf2.Denominator)
```

Gc1 =

$$\frac{0.1116}{s + 0.1116}$$

Continuous-time transfer function.

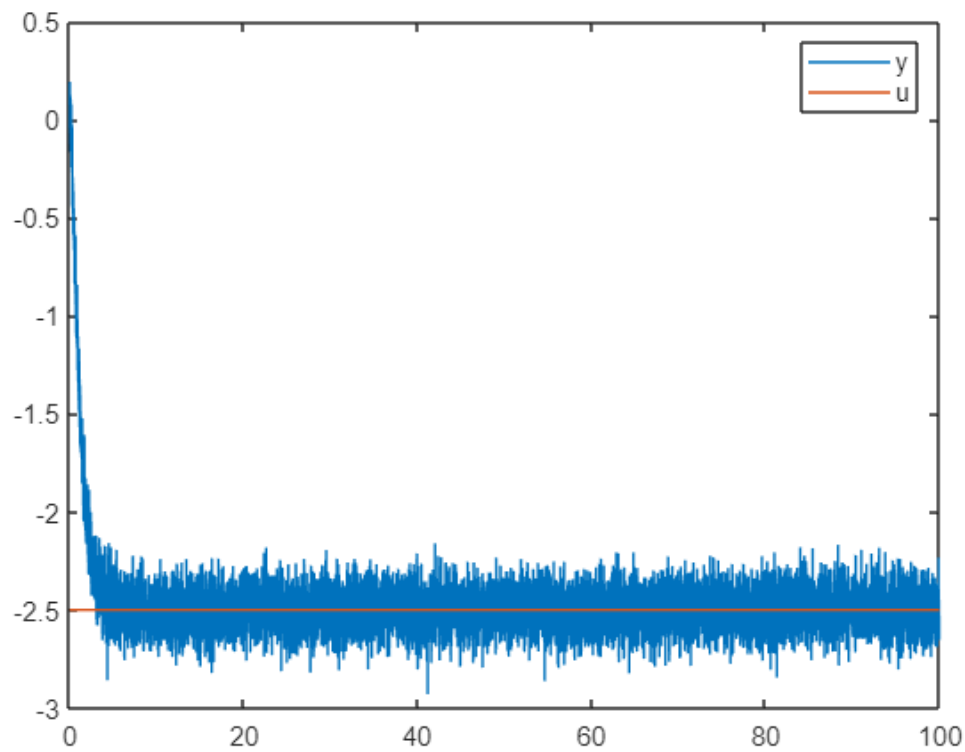
```
figure
subplot(3,1,1)
plot(t1,y1)
title("y unknown")
subplot(3,1,2)
step(Gc1)
title("y identified")
subplot(3,1,3)
hold on
plot(t1,y1,"Color","r")
step(2.5*Gc1)
legend("unknown","identified")
```



$A = -2.5$

```
% t2 = out.tout;
% x2 = out.x;
% y2 = out.y;

figure
plot(t2,y2,t2,x2)
legend("y","u")
```



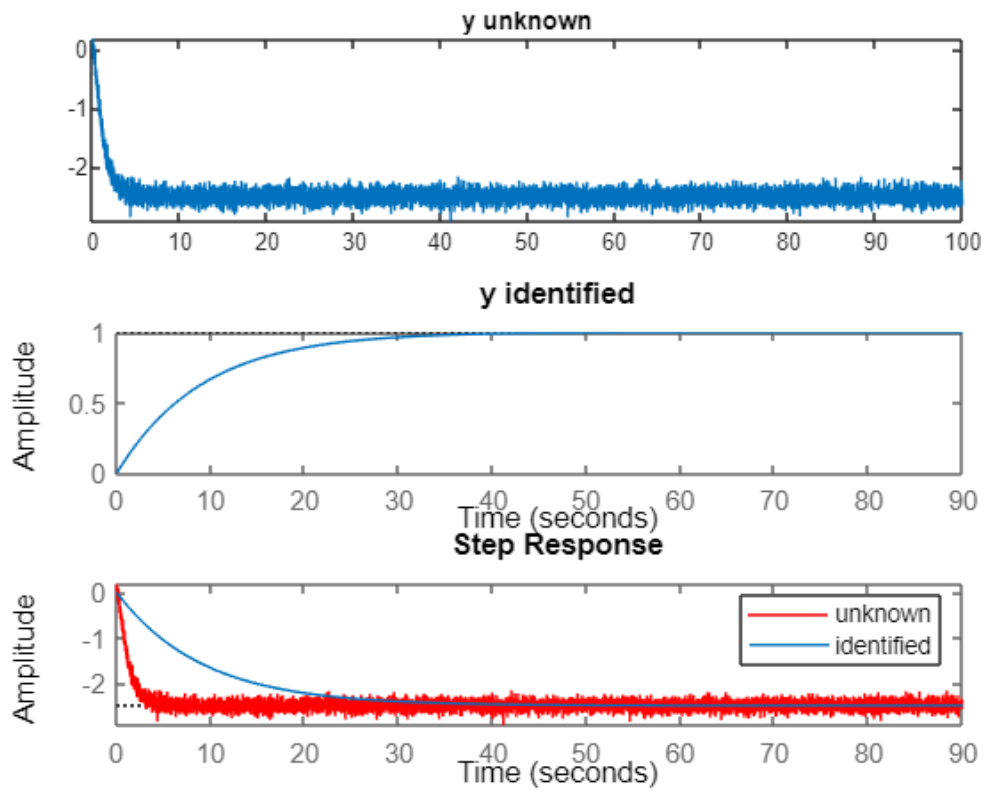
```
Gc2 = tf(tf3.Numerator , tf3.Denominator)
```

Gc2 =

$$\frac{0.1101}{s + 0.11}$$

Continuous-time transfer function.

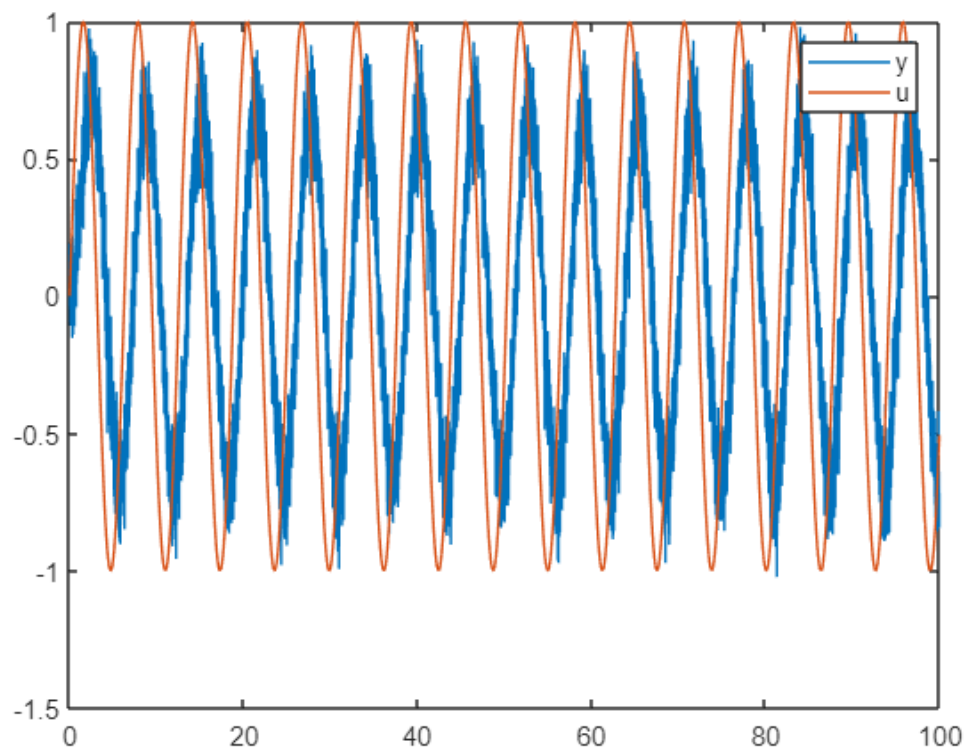
```
figure
subplot(3,1,1)
plot(t2,y2)
title("y unknown")
subplot(3,1,2)
step(Gc2)
title("y identified")
subplot(3,1,3)
hold on
plot(t2,y2,"Color","r")
step(-2.5*Gc2)
legend("unknown","identified")
```



d)

```
% t3 = out.tout;
% x3 = out.x;
% y3 = out.y;

figure
plot(t3,y3,t3,x3)
legend("y","u")
```



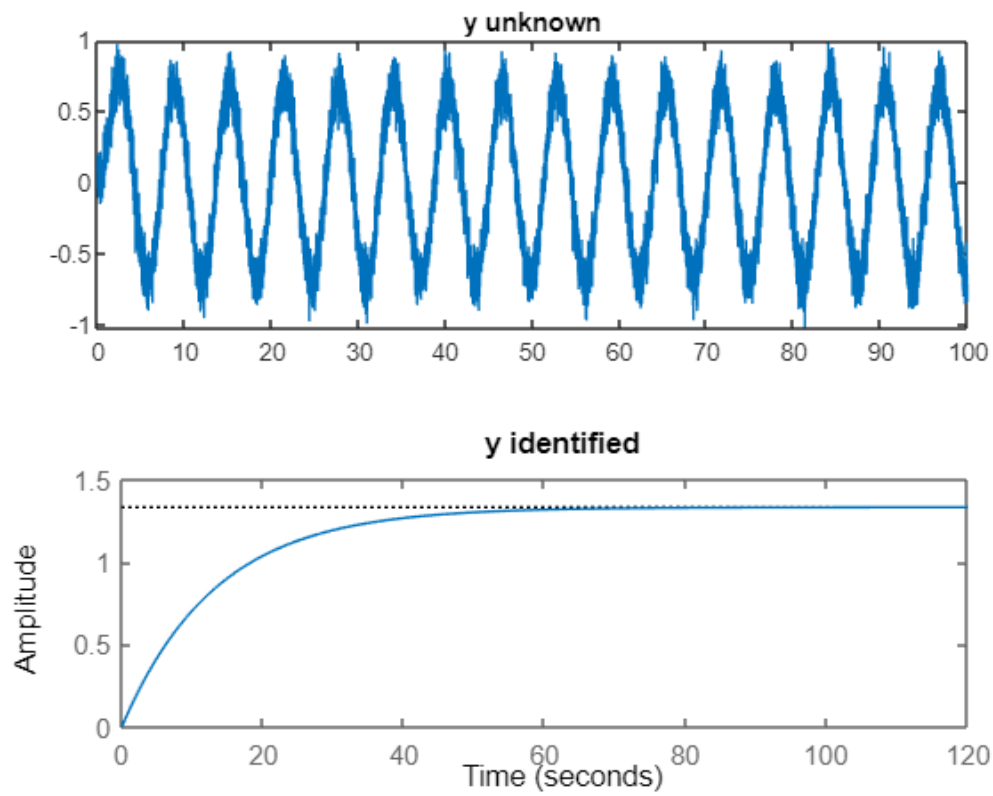
```
Gd = tf(tf4.Numerator , tf4.Denominator)
```

Gd =

$$\frac{0.1006}{s + 0.07537}$$

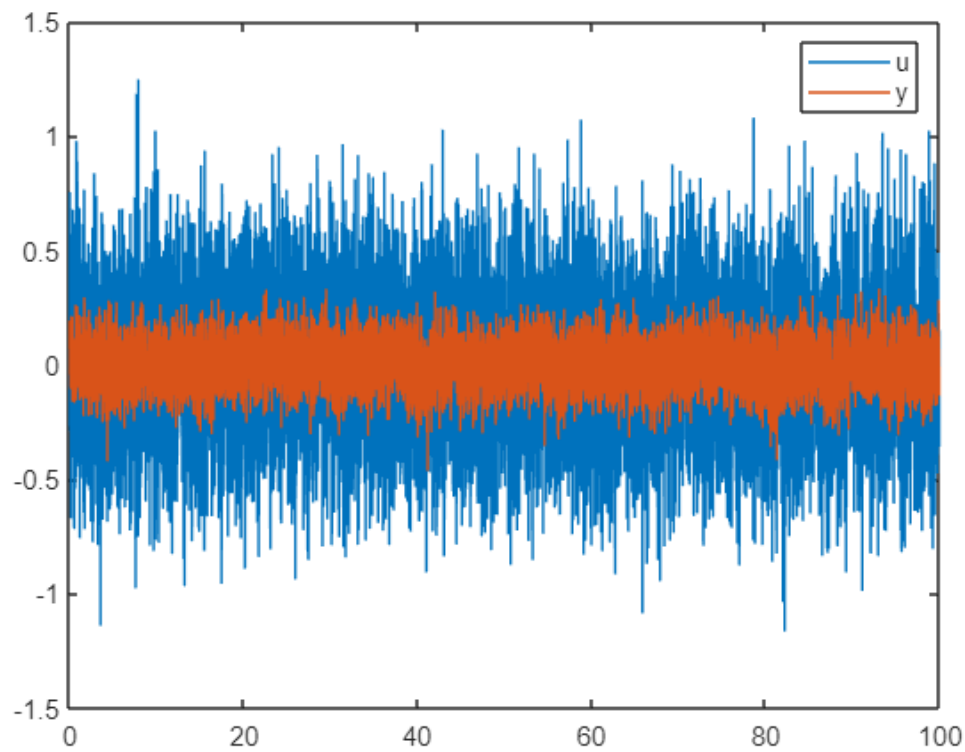
Continuous-time transfer function.

```
figure
subplot(2,1,1)
plot(t3,y3)
title("y unknown")
subplot(2,1,2)
step(Gd)
title("y identified")
```



e)

```
% t4 = out.tout;  
% x4 = out.x;  
% y4 = out.y;  
  
figure  
plot(t4,x4,t4,y4)  
legend("u","y")
```



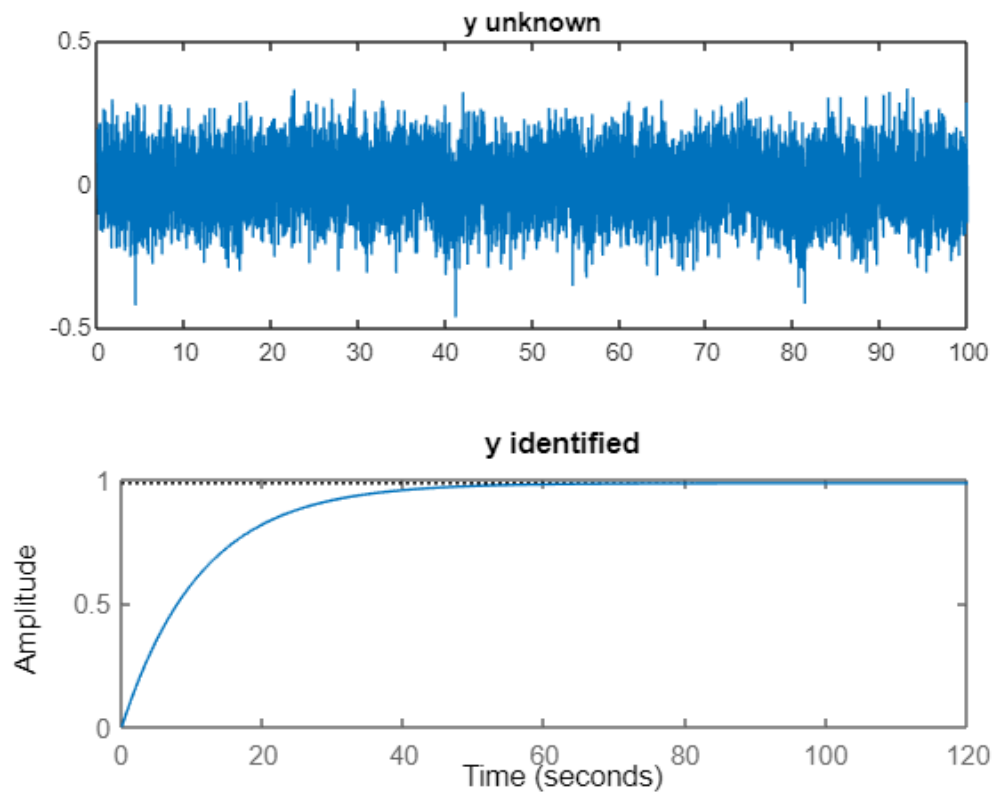
```
Ge = tf(tf5.Numerator , tf5.Denominator)
```

Ge =

$$\frac{0.08783}{s + 0.08898}$$

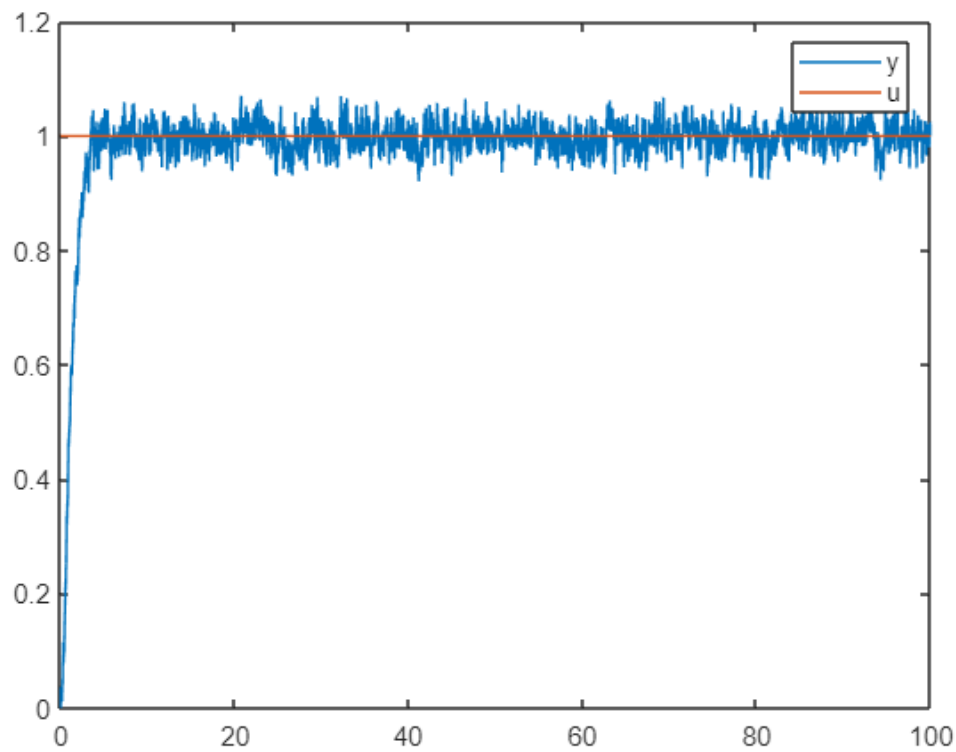
Continuous-time transfer function.

```
figure
subplot(2,1,1)
plot(t4,y4)
title("y unknown")
subplot(2,1,2)
step(Ge)
title("y identified")
```



f)

```
% t5 = out.tout;  
% x5 = out.x;  
% y5 = out.y;  
  
figure  
plot(t5,y5,t5,x5)  
legend("y","u")
```

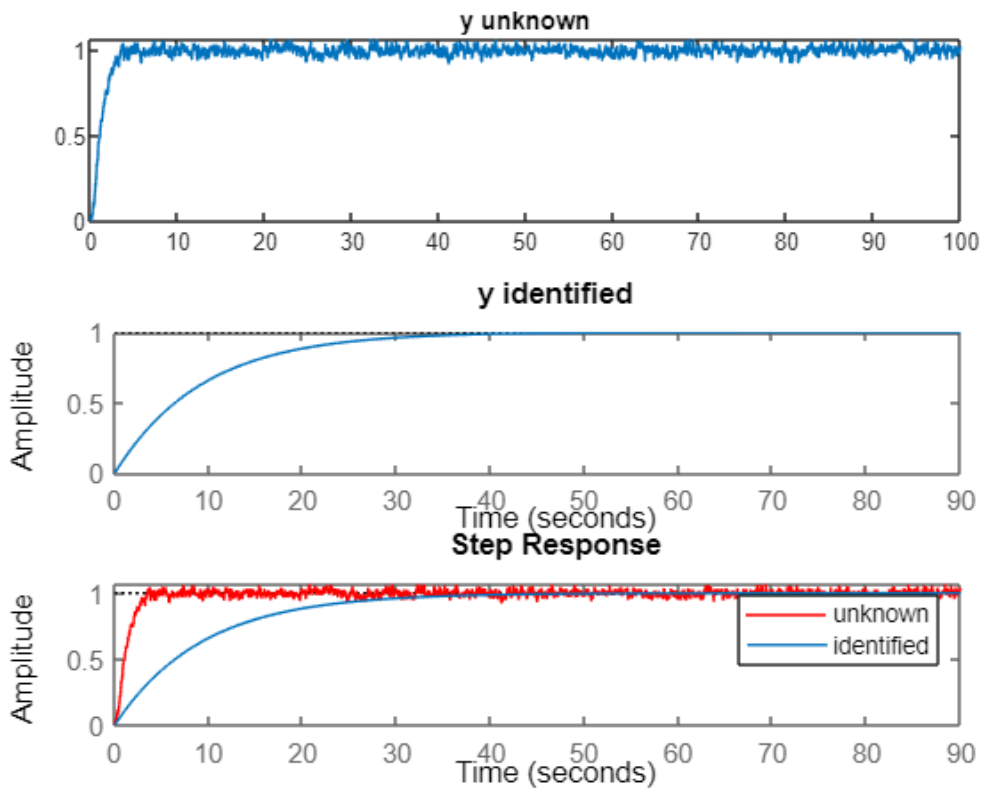
```
Gf = tf(tf6.Numerator , tf6.Denominator)
```

Gf =

$$\frac{0.1076}{s + 0.1077}$$

Continuous-time transfer function.

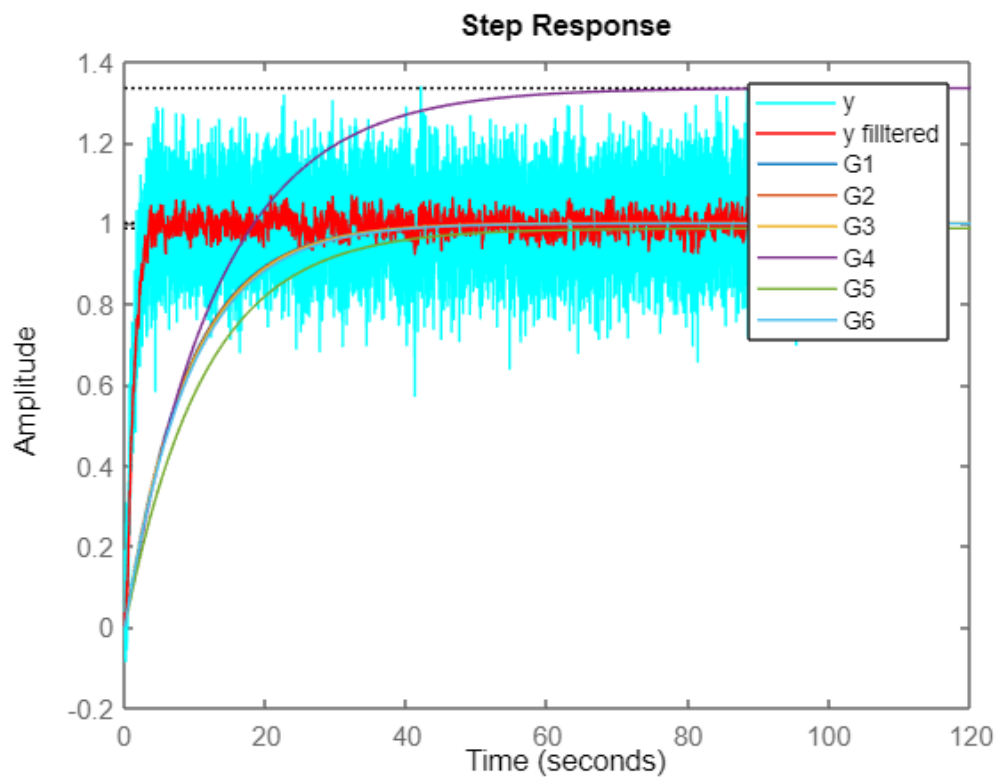
```
figure
subplot(3,1,1)
plot(t5,y5)
title("y unknown")
subplot(3,1,2)
step(Gf)
title("y identified")
subplot(3,1,3)
hold on
plot(t5,y5,"Color","r")
step(Gf)
legend("unknown","identified")
```



g)

```
G1 = tf(tf1.Numerator , tf1.Denominator);
G2 = tf(tf2.Numerator , tf2.Denominator);
G3 = tf(tf3.Numerator , tf3.Denominator);
G4 = tf(tf4.Numerator , tf4.Denominator);
G5 = tf(tf5.Numerator , tf5.Denominator);
G6 = tf(tf6.Numerator , tf6.Denominator);

close all
hold on
plot(t,y,'c')
plot(t5,y5,'r')
step(G1)
step(G2)
step(G3)
step(G4)
step(G5)
step(G6)
legend("y","y filltered","G1","G2","G3","G4","G5","G6")
```

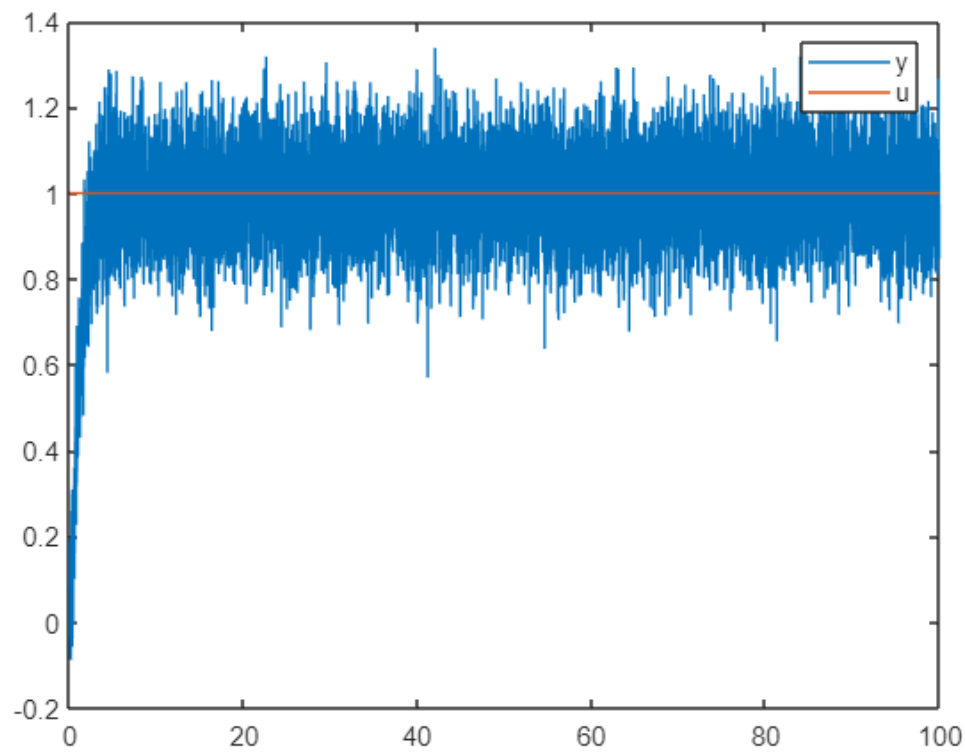


Except for the two approximations G5 and G4, the other approximations have the same and better performance.

h)

the best method is G1(part b):

```
figure
plot(t,y,t,x)
legend("y","u")
```



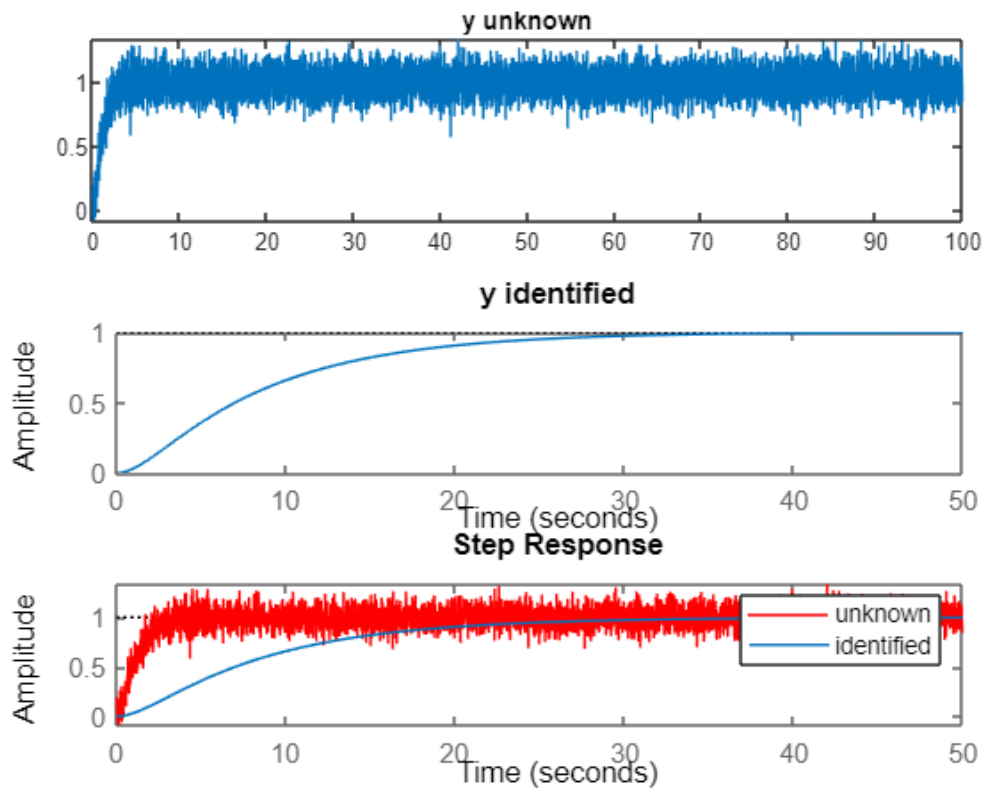
```
Gh = tf(tf7.Numerator , tf7.Denominator)
```

Gh =

$$\frac{0.08766}{s^2 + 0.8126 s + 0.08775}$$

Continuous-time transfer function.

```
figure
subplot(3,1,1)
plot(t,y)
title("y unknown")
subplot(3,1,2)
step(Gh)
title("y identified")
subplot(3,1,3)
hold on
plot(t,y,"Color","r")
step(Gh)
legend("unknown","identified")
```



```
function [u] = Step01(C)
u = C;
s = length(C);
for i=1:s
    if C(i)>=0
        u(i)=1;
    else
        u(i)=0;
    end
end
end
function [u] = Step00(C)
u = C;
s = length(C);
for i=1:s
    if C(i)>0
        u(i)=1;
    else
        u(i)=0;
    end
end
end
end
```