In the name of allah

Linear Control Systems

Hw 06

ali sadeghian 400101464

Fall 1402

Theory Assignments

(سوال ۱)

انجام شده

علی ... 40001464

a)

$$G_1(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\xrightarrow{abs}|G_1(j\omega)|=\frac{\omega_n^2}{((\omega_n^2-\omega^2)^2+4\xi^2\omega_n^2\omega^2)^{0.5}}$$

$$\Rightarrow \frac{d}{d\omega}|G_1(j\omega)|=\frac{-\omega_n^2\left(\frac{1}{2}(4\omega^3-4\omega_n^2\omega+8\xi^2\omega_n^2\omega)((\omega_n^2-\omega^2)^2+4\xi^2\omega_n^2\omega^2)^{-0.5}\right)}{((\omega_n^2-\omega^2)^2+4\xi^2\omega_n^2\omega^2)^{0.5})^2}=0$$

$$4\omega(\omega^2-\omega_n^2+2\xi^2\omega_n^2)=0\Rightarrow \omega_r=0 \text{ یا } \omega_r=\omega_n\sqrt{1-2\xi^2}$$

$$(\xi>\tfrac{1}{\sqrt2})\qquad(\xi\le\tfrac{1}{\sqrt2})$$

$\rightarrow M_p\ (\xi\le\frac{1}{\sqrt2})$

$$\frac{\omega_n^2}{((\cancel{\omega_n^2}-\cancel{\omega_n^2}(1-2\xi^2))^2+4\xi^2\omega_n^2\omega_n^2(1-2\xi^2))^{1/2}}=\frac{\omega_n^2}{\omega_n^2 2\xi\sqrt{1-\xi^2}}=(2\xi\sqrt{1-\xi^2})^{-1}$$

$4\omega_n^4\xi^4$

$\rightarrow M_p\ (\xi>\frac{1}{\sqrt2})=1$

b)

$$G_2(s)=\frac{\omega_n(s+\omega_n)}{s^2+2\xi\omega_n s+\omega_n^2}\xrightarrow{abs}|G_2(j\omega)|=\frac{\sqrt{\omega_n^2+\omega_n\omega}}{((\omega_n^2-\omega^2)^2+4\xi^2\omega_n^2\omega^2)^{0.5}}$$

$$\frac{d}{d\omega}|G_2(j\omega)|=\frac{\frac{\omega_n}{2\sqrt{\omega_n^2+\omega_n\omega}}\sqrt{(\omega_n^2-\omega^2)^2+4\xi^2\omega_n^2\omega^2}-\sqrt{\omega_n^2+\omega_n\omega}\,\frac{4\omega(\omega^2-\omega_n^2+2\xi^2\omega_n^2)}{2\sqrt{(\omega_n^2-\omega^2)^2+4\xi^2\omega_n^2\omega^2}}}{(\omega_n^2-\omega^2)^2+4\xi^2\omega_n^2\omega^2}=0$$

$$\cancel{\omega_n}((\omega_n^2-\omega^2)^2+4\xi^2\omega_n^2\omega^2)-\cancel{\omega_n}(\omega_n+\omega)4\omega(\omega^2-\omega_n^2-2\xi^2\omega_n^2)=0$$

$$\omega^4-(2\omega_n^2+4\xi^2\omega_n^2)\omega^2+\omega_n^2-4\omega^4+4\omega_n^2\omega^2-8\xi^2\omega_n^2\omega^2-4\omega_n\omega^3+4\omega_n^3\omega-8\xi^2\omega_n^3\omega=0$$

$$-3\omega^4-4\omega_n\omega^3+(2\omega_n^2-12\xi^2\omega_n^2)\omega^2+(4\omega_n^3-8\xi^2\omega_n^3)\omega+\omega_n^2=0$$

$\omega_p=$

$\rightarrow M_p=$

c)

اگر به $\omega_2=\omega_1$ باشد جای $|G_2|$ برابر 0 باشد که منطقی نیست

$$\frac{\max|G_1|}{\max|G_2|}=\frac{1}{\sqrt2}:=\frac{|\omega_n^2|}{|j\omega_n\omega_2+\omega_n^2|}\times\frac{|-\omega_2^2+2j\xi\omega_n\omega_2+\omega_n^2|}{|-\omega_1^2+2j\xi\omega_n\omega_1+\omega_n^2|}$$

$$\overset{\xi\ll 1}{=}\frac{|\omega_n^2|}{|\omega_n^2||1+j\frac{\omega_2}{\omega_n}|}\times\frac{|\omega_n^2-\omega_2^2|}{|\omega_n^2-\omega_1^2|}\overset{\omega_1=\omega_2=\omega_n}{=}\frac{1}{|1+j|}=\frac{1}{\sqrt2}$$

b) $G_2(s) = \dfrac{\omega_n(s+\omega_n)}{s^2+2\xi\omega_n+\omega_n^2} \xrightarrow[\ln]{abs} \ln(\omega_n) + \frac{1}{2}\ln(\omega^2+\omega_n^2) - \frac{1}{2}\ln((\omega_n^2-\omega^2)^2+4\xi^2\omega_n^2\omega^2)$

$$\frac{d}{d\omega}\ln|G(i\omega)| = \frac{1}{\cancel{2}}\frac{\cancel{2}\omega}{\omega^2+\omega_n^2} - \frac{1}{\cancel{2}}\frac{\overset{2}{\cancel{4}}\omega^3+(\overset{4}{\cancel{8}}\xi^2\omega_n^2-\overset{2}{\cancel{4}}\omega_n^2)\omega}{(\omega_n^2-\omega^2)^2+4\xi^2\omega_n^2\omega^2} = 0$$

$\boxed{\omega=0}$

$$\frac{\cancel{\omega}}{\omega^2+\omega_n^2} = \frac{2\cancel{\omega}(\omega^2-\omega_n^2+2\xi^2\omega_n^2)}{(\omega_n^2-\omega^2)+4\xi^2\omega_n^2\omega^2} = \frac{\omega^2-\omega_n^2+2\xi^2\omega_n^2}{\omega^4+(4\omega_n^2\xi^2-4\omega_n^2)\omega^2+\omega_n^2}$$

$$\Rightarrow \cancel{\omega^4}+(4\xi^2\omega_n^2-4\omega_n^2)\omega^2+\omega_n^4 = \cancel{\omega^4}+2\xi^2\omega_n^2\omega^2 \quad -\omega_n^4+2\xi^2\omega_n^4$$

$$(2\xi^2\omega_n^2-4\omega_n^2)\omega^2 = 2\omega_n^4(2\xi^2-1)$$

$$\omega = \sqrt{\frac{2\omega_n^4(2\xi^2-1)}{2\omega_n^2(\xi^2-2)}}$$

$$\rightarrow \boxed{\omega_r = \omega_n\sqrt{\frac{2\xi^2-1}{\xi^2-2}}} \quad \xi < \sqrt{2}^{-1}$$

$$\boxed{\omega_r = 0 \qquad \xi > \sqrt{2}^{-1}}$$

$mp \rightarrow \xi > \sqrt{2}^{-1} \Rightarrow mp = 1$

$\xi < \sqrt{2}^{-1}$:

$$\frac{\cancel{\omega_n^2}\sqrt{1+\frac{2\xi^2-1}{\xi^2-2}}}{\cancel{\omega_n^2}\sqrt{\left(1-\frac{2\xi^2-1}{\xi^2-2}\right)^2+4\xi^2\frac{2\xi^2-1}{\xi^2-2}}}$$

$$\frac{\sqrt{\frac{3\xi^2-3}{\xi^2-2}}}{\sqrt{\frac{(\xi^2-2)^2+(2\xi^2-1)^2-2(\xi^2-2)(2\xi^2-1)+4\xi^2(2\xi^2-1)(\xi^2-2)}{(\xi^2-2)^2}}}$$

$$\boxed{M_p = \sqrt{\frac{8(\xi^2-1)(\xi^2-2)}{(\xi^2-2)^2+(2\xi^2-1)(2\xi^2-1-2\xi^2+4+8\xi^4-4\xi^2)}}}$$

سوال 2)

a) $G_1(S) = \dfrac{k}{\tau S+1} \xrightarrow[dB]{abs} 20\log k - 10\log 1+\tau^2\omega_B^2 = 20\log k - 3dB$

$$\Rightarrow 10\log 1+\tau^2\omega_B^2 = 3$$
$$1+\tau^2\omega_B^2 = 2$$
$$\omega_B = \frac{1}{\tau}$$

b) $G_2(S) = \dfrac{k}{S^2+2\zeta\omega_n S+\omega_n^2} \xrightarrow[dB]{abs} 20\log k - 10\log((\omega_n^2-\omega_B^2)^2+4\zeta^2\omega_n^2\omega_B^2) = 20\log k - 20\log\omega_n^2 - 3dB$

$$\Rightarrow 10\log((\omega_n^2-\omega_B^2)^2+4\zeta^2\omega_n^2\omega_B^2) = 20\log\sqrt{2}\,\omega_n^2$$

$$\omega_B^4 - (2\omega_n^2-4\zeta^2\omega_n^2)\omega_B^2 + \omega_n^4 = 2\omega_n^4$$

$$\left(\frac{\omega_B}{\omega_n}\right)=\omega \longrightarrow \omega^4+(4\zeta^2-2)\omega^2+\overset{-1}{\cancel{1}}=\overset{0}{\cancel{2}}$$

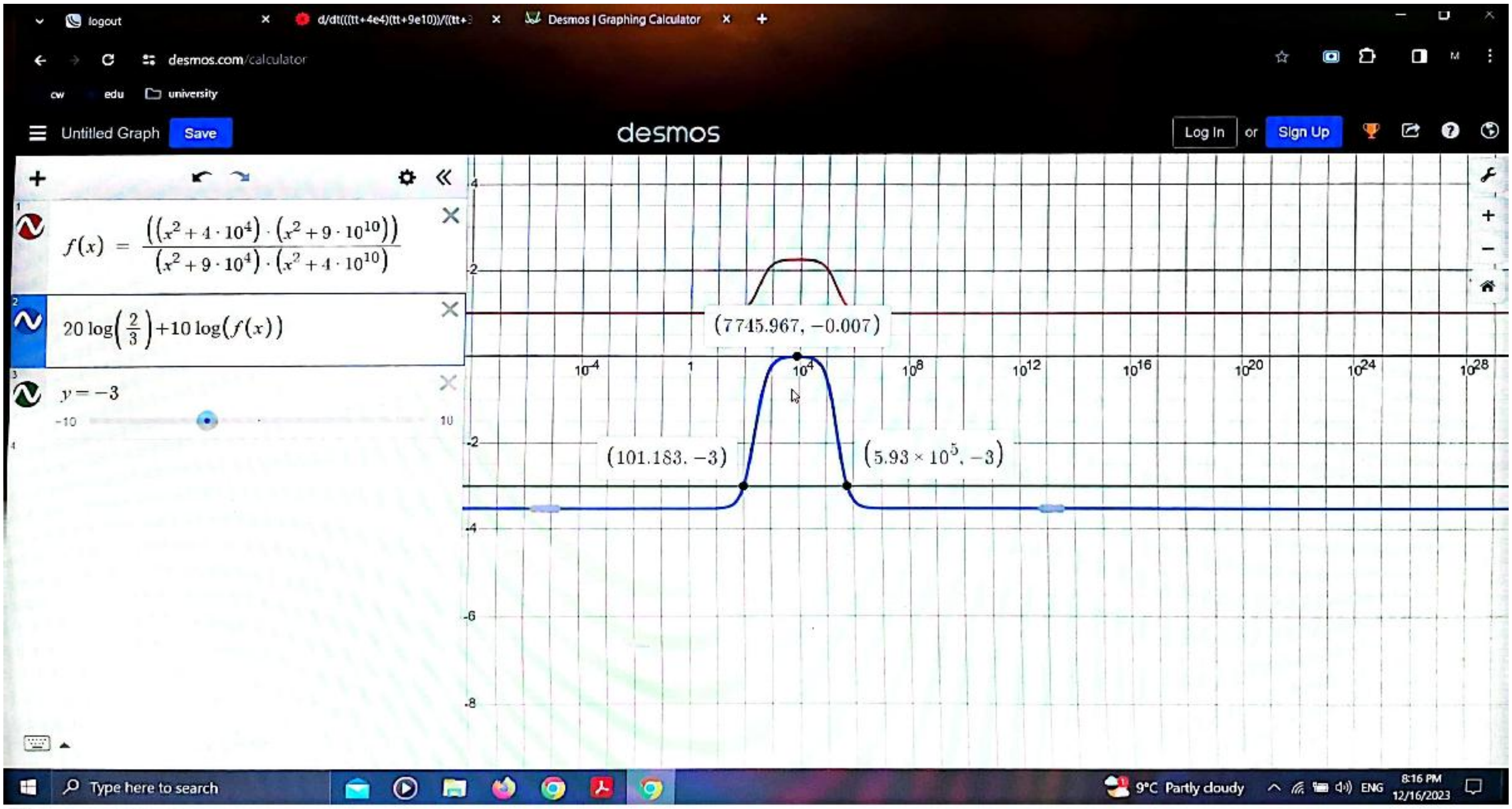
$$\Rightarrow \omega^2 = \frac{2-4\zeta^2 \pm \sqrt{(4\zeta^2-2)^2+4}}{2}$$

$$\rightarrow \omega_B = \omega_n\sqrt{1-2\zeta^2+\sqrt{4\zeta^4-4\zeta^2+2}}$$

c) $G_3(S) = \dfrac{2}{3}\,\dfrac{(S+200)(S+300k)}{(S+300)(S+200k)} \xrightarrow[dB]{abs} 20\log\frac{2}{3} + 10\log\underbrace{\dfrac{(\omega^2+4\,10^4)(\omega^2+9\,10^{10})}{(\omega^2+9\,10^4)(\omega^2+4\,10^{10})}}_{\text{برای Parseval جا بر صفر خواهد بود (در لو صفش)}}$

غلط چون که فرکانس های Hz 100 تا 16 kHz بابت فرکانس تبدیل شده و محدوده شنوایی انسان است

$$\Rightarrow 10\log\frac{(\omega^2+4\,10^4)(\omega^2+9\,10^{10})}{(\omega^2+9\,10^4)(\omega^2+4\,10^5)} = 3dB - 20\log\frac{2}{3} \Rightarrow \omega_B \begin{cases} \rightarrow 100 \\ \rightarrow 600k \end{cases}$$



Scanned with CamScanner

سؤال 3)

a) $\frac{1}{\tau s+1} \xrightarrow[dB]{abs} -10\log \tau^2\omega^2+1 = 0$

$\to \tau^2\omega^2+1 = 1$

$\to \omega_c = 0$

b) $\frac{\omega_n^2}{s(s+2\xi\omega_n)} \xrightarrow[dB]{abs} 20\log\omega_n^2 - 20\log\omega - 10\log\omega^2+4\xi^2\omega_n^2 = 0$

$20\log\omega_n^2 = 10\log\omega^2(\omega^2+4\xi^2\omega_n^2)$

$\omega_n^4 = \omega^4 + 4\omega_n^2\xi^2\omega^2$

$\xrightarrow{\frac{\omega}{\omega_n}=W} W^4 + 4\xi^2W^2 - 1 = 0$

$W^2 = \frac{-4\xi^2 \pm \sqrt{16\xi^2+4}}{2} = \pm\sqrt{2\xi^2+1} - 2\xi^2$

$W_c = \omega_n\sqrt{\sqrt{2\xi^2+1} - 2\xi^2}$

c) $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \xrightarrow[dB]{abs} 20\log\omega_n^2 - 10\log(\omega_n^2-\omega^2)^2+4\xi^2\omega_n^2\omega^2 = 0$

$20\log\omega_n^2 = 10\log(\omega_n^2-\omega^2)^2+4\xi^2\omega_n^2\omega^2$

$\cancel{\omega_n^4} = \omega^4 + (4\xi^2\omega_n^2 - 2\omega_n^2)\omega^2 + \cancel{\omega_n^4}$

$\omega^2(\omega^2 - 2\omega_n + 4\xi^2\omega_n^2) = 0$

$\omega_c = 0 , \omega_c = \omega_n\sqrt{2(1-2\xi^2)}$

4(1b)

a)

$$y(t) = \int_0^t u(\alpha) g(t-\alpha)\, d\alpha = u(t) * g(t)$$

$$\|y\|_2 = \sqrt{\int_0^\infty (u(t) * g(t))^2\, dt}$$

$$= \sqrt{\frac{1}{2\pi}\int^\infty F(j\omega) F^*(j\omega)\, d\omega}$$

$$= \sqrt{\frac{1}{2\pi}\int_{-\infty}^\infty U(j\omega) G(j\omega)\, U(-j\omega)^* G(-j\omega)^*\, d\omega}$$

$$\leq \sqrt{\frac{1}{2\pi} G_{max}^2 \int_{-\infty}^\infty U(j\omega)\, U^*(-j\omega)\, d\omega}$$

$$\leq |G_{max}| \sqrt{\frac{1}{2\pi}\int_{-\infty}^\infty U(j\omega) U^*(j\omega)\, d\omega}$$

$$\|y\|_2 \leq |G_{max}|\, \|u\|_2$$

b)

$$u = e^{j\omega_m} \rightarrow y = G(j\omega_m) e^{j\omega_m}$$

$$U(f) \leftrightarrow \delta(f - f_m)$$

$$\Rightarrow$$

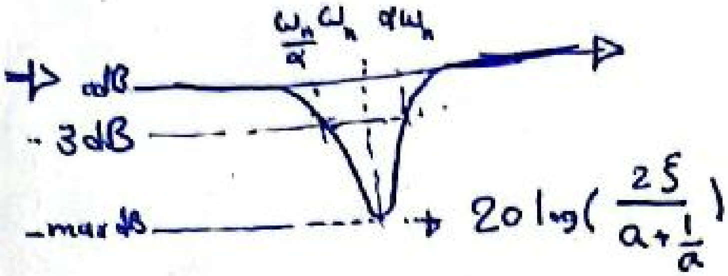
$$\|y\|_2 = \sqrt{\int_{-\infty}^\infty \delta(f - f_m) G(f) \cdot \delta(f - f_m) \overline{G(f)}\, df}$$

$$= \sqrt{G_m^2 \int_{-\infty}^\infty (\delta(f - f_m))^2\, df}$$

$$\rightarrow \|y\|_2 = |G_m|\, \|u\|_2 \Rightarrow \|S\| = \max_\omega |G(j\omega)|$$

سوال 5)

$$G_n(s) = \frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{\omega_n^2} \times \frac{a\omega_n}{s + a\omega_n} \times \frac{\frac{\omega_n}{a}}{s + \frac{\omega_n}{a}} = \frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{s^2 + s\omega_n(a + \frac{1}{a}) + \omega_n^2}$$



$$G_n(\beta\omega_n) = G_n(\frac{\omega_n}{\beta})$$

$$\alpha = \frac{50}{47}, \quad \omega_n = 2\pi\, 50, \quad \zeta = 0{,}5 \Rightarrow -max = -6dB$$

برای نصف شدن توان با دامنه 3dB کاهش یابد.

فیلتر بالا در اصلاح استاد آمده است (α و ω_n و ζ پارامترهای طراحی هستند)

چون فرکانس مرکزی $50_{Hz} \Rightarrow \omega_n = 100\pi$

چون می‌خواهیم فرکانس 51 _ 49 حذف شود $\Rightarrow 47 \to 51{,}02 \Leftarrow \alpha = \frac{50}{47}$ با دامنه بیش از 3dB تضعیف

با ζ عمق ناچ تعین می‌شود هرچه کمتر باشد تاثیر بر فرکانسهای دیگر کمتر خواهد بود ولی باید مقدار معقول
خواهند شد

انتخاب شود $\Rightarrow \zeta = 0{,}5$

$$G(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)} \Rightarrow T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

سوال 6)

a) phase margine $\omega_\phi = 9\,rad \Rightarrow \phi_m = 42{,}5$
gain margine $\omega_g = \infty \Rightarrow g_m = \infty$
corresponding crossover $f. = 9\,rad$

b)

به ازای تمام kهای جنگتر از 1، g_m برابر ∞ خواهد بود، همچنین ϕ_m نیز همواره
بزرگتر از 0 خواهد بود و صرفاً با افزایش k، ϕ_m کاهش می‌یابد ولی تغییر علامت نمی‌دهد.

d) $K_p = \lim_{s \to 0} G(s) = \frac{\omega_n^2}{0} = \infty \Rightarrow e = \frac{1}{1+K_p} = 0$

e)

$$\text{Phase} = -135 \Rightarrow \omega = 2\xi\omega_n\left(-(\angle \overset{90}{\cancel{j\omega}} + \angle \overset{-45}{\cancel{j\omega + 2\xi\omega_n}}) = -135\right)$$

$$\Rightarrow \omega = 8$$

$$\omega = 10^0 \Rightarrow 22{,}5 = 20 \log \omega_n^2 - \overset{0}{\cancel{20 \log \omega}} - 10 \log \omega^2 + 64$$

$$20 \log \omega_n^2 = 22{,}5 - 10 \log 65$$

$$\log \omega_n = \frac{22{,}5 - 10 \log 65}{40}$$

$$\omega_n = 10{,}368 \Rightarrow \xi = \frac{0{,}772}{2} = 0{,}386$$

$$P.O. = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = 26{,}9\,\%$$

$$T_r = \frac{2{,}16 * \xi + 0{,}6}{\omega_n} = 0{,}138$$

$$T_s = \frac{4}{\xi\omega_n} = 0{,}5$$

f)

$$\omega_B = \omega_n \sqrt{1 - 2\xi^2 + \sqrt{4\xi^4 - 4\xi^2 + 2}} = 14{,}540$$

$$M_p = \left(2\xi\sqrt{1-\xi^2}\right)^{-1} = 1{,}1591$$

$$\omega_r = \omega_n \sqrt{1 - 2\xi^2} = 8{,}853$$

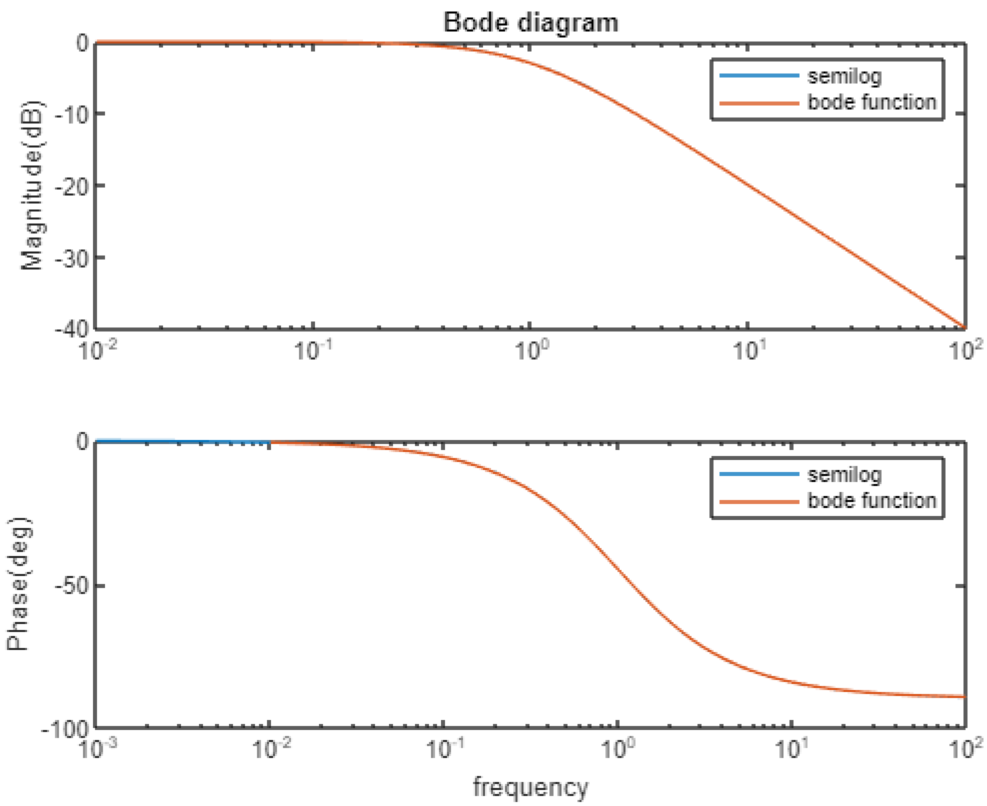
```
clc
clear all
close all
```

MATLAB Assignment

7 Bode Diagram Plot

a)

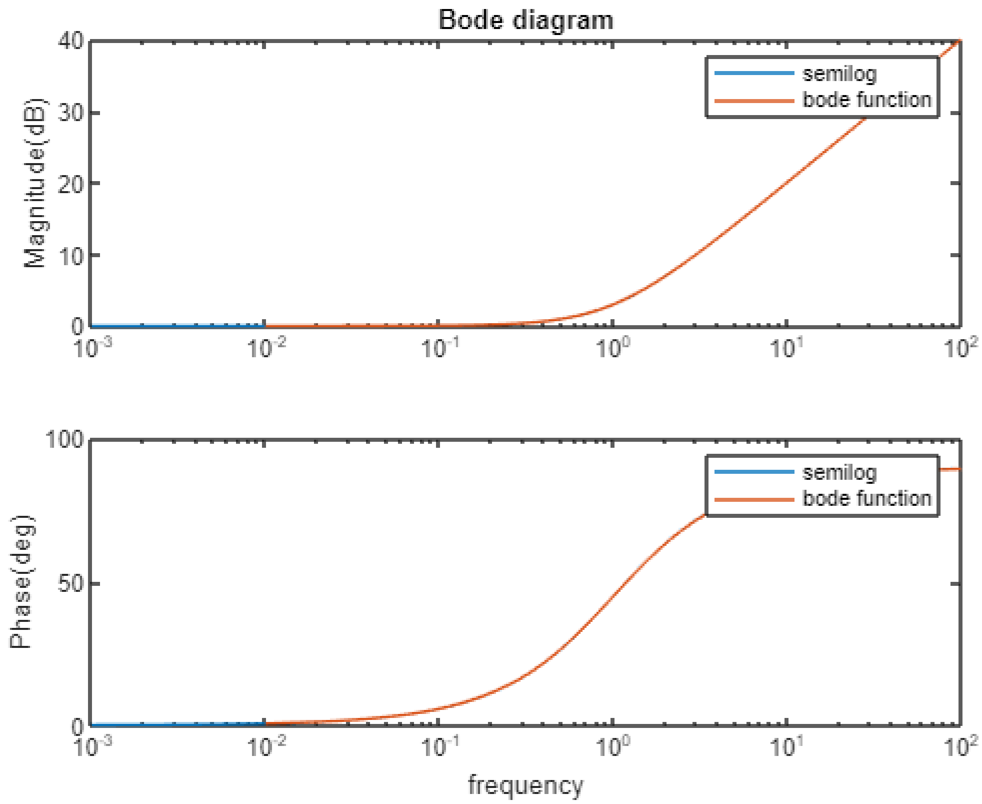
```
w = logspace(-3,2,200);

s = tf("s");
Gb = 1/(s+1);
syms s
Gl = @(s) 1./(s+1);
Gl = Gl(1i*w);


[mag,pha,wout]=bode(Gb);

Glm = 20*log10(abs(Gl));
Glp = atan2d(imag(Gl),real(Gl));

magi = 20*log10(mag(1,1,1));
for i=2:size(mag,3)
    magi = [magi,20*log10(mag(1,1,i))];
end
phai = pha(1,1,1);
for i=2:size(pha,3)
    phai = [phai,pha(1,1,i)];
end

subplot(2,1,1)
semilogx(w,Glm,wout,magi)
xlim([0.01 100])

ylabel("Magnitude(dB)")
title("Bode diagram")
legend("semilog","bode function")

subplot(2,1,2)
semilogx(w,Glp,wout,phai)
xlabel("frequency")
ylabel("Phase(deg)")
legend("semilog","bode function")
```



b)

```
s = tf("s");
Gb = (s+1);
syms s
Gl = @(s) (s+1);
Gl = Gl(1i*w);

[mag,pha,wout]=bode(Gb);

Glm = 20*log10(abs(Gl));
Glp = atan2d(imag(Gl),real(Gl));

magi = 20*log10(mag(1,1,1));
for i=2:size(mag,3)
    magi = [magi,20*log10(mag(1,1,i))];
end
phai = pha(1,1,1);
for i=2:size(pha,3)
    phai = [phai,pha(1,1,i)];
end

subplot(2,1,1)
semilogx(w,Glm,wout,magi)
ylabel("Magnitude(dB)")
title("Bode diagram")
legend("semilog","bode function")
```

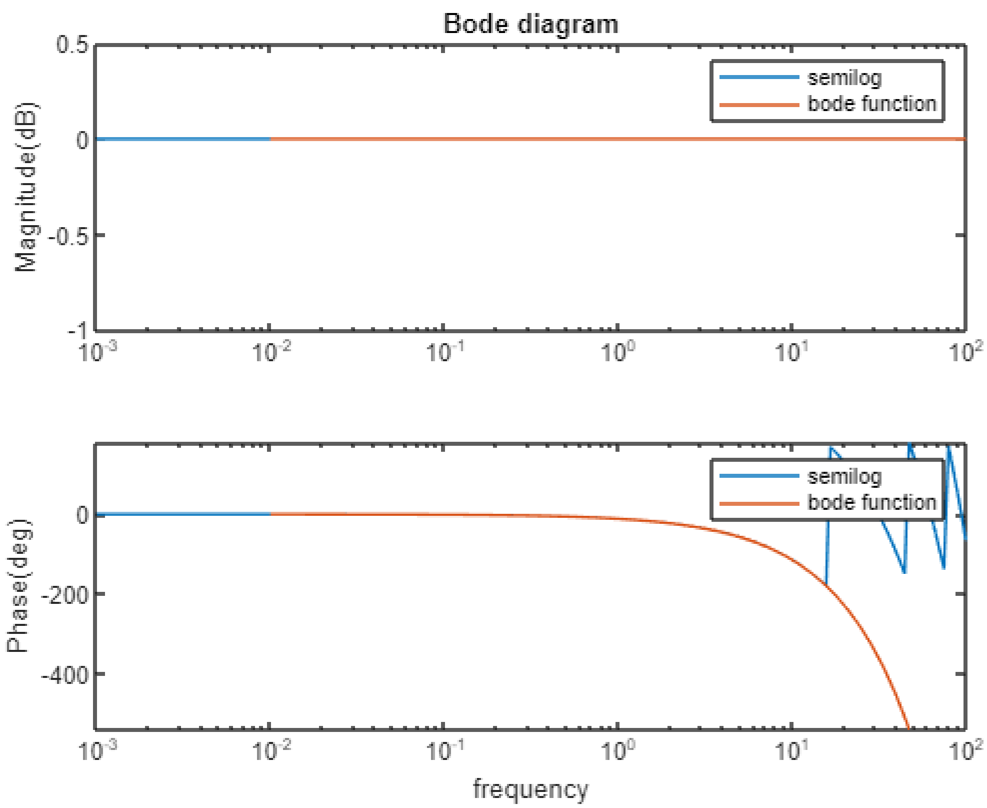
```
subplot(2,1,2)
semilogx(w,Glp,wout,phai)
xlabel("frequency")
ylabel("Phase(deg)")
legend("semilog","bode function")
```



c)

```
s = tf("s");
Gb = exp(-0.2*s);
syms s
Gl = @(s) exp(-0.2*s);
Gl = Gl(1i*w);

[mag,pha,wout]=bode(Gb,{0.01 100});

Glm = 20*log10(abs(Gl));
Glp = atan2d(imag(Gl),real(Gl));

magi = 20*log10(mag(1,1,1));
for i=2:size(mag,3)
    magi = [magi,20*log10(mag(1,1,i))];
end
phai = pha(1,1,1);
for i=2:size(pha,3)
    phai = [phai,pha(1,1,i)];
end
```

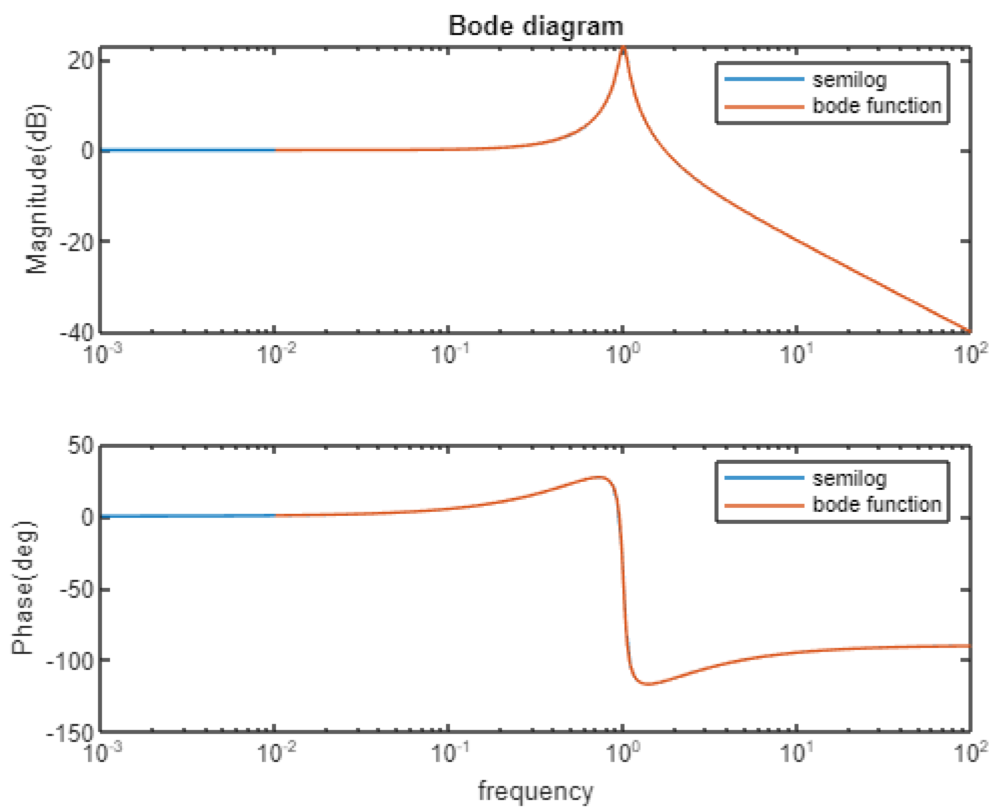
```
subplot(2,1,1)
semilogx(w,Glm,wout,magi)
ylim([-1 0.5])
ylabel("Magnitude(dB)")
title("Bode diagram")
legend("semilog","bode function")

subplot(2,1,2)
semilogx(w,Glp,wout,phai)
xlabel("frequency")
ylabel("Phase(deg)")
legend("semilog","bode function")
ylim([-540 180])
```



d)

```
s = tf("s");
Gb = (s+1)/(s^2+0.1*s+1);
syms s
Gl = @(s) (s+1)./(s.^2+0.1*s+1);
Gl = Gl(1i*w);

[mag,pha,wout]=bode(Gb);

Glm = 20*log10(abs(Gl));
Glp = atan2d(imag(Gl),real(Gl));
```

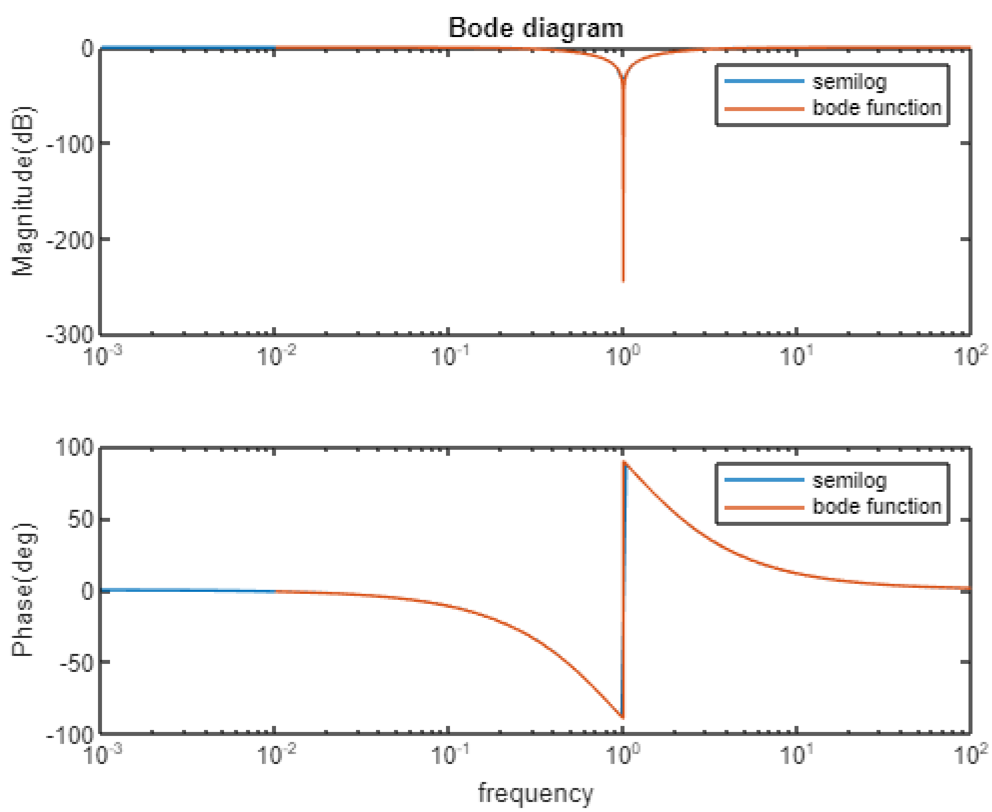
```
magi = 20*log10(mag(1,1,1));
for i=2:size(mag,3)
    magi = [magi,20*log10(mag(1,1,i))];
end
phai = pha(1,1,1);
for i=2:size(pha,3)
    phai = [phai,pha(1,1,i)];
end

subplot(2,1,1)
semilogx(w,Glm,wout,magi)
ylabel("Magnitude(dB)")
title("Bode diagram")
legend("semilog","bode function")

subplot(2,1,2)
semilogx(w,Glp,wout,phai)
xlabel("frequency")
ylabel("Phase(deg)")
legend("semilog","bode function")
```



e)

```
s = tf("s");
Gb = (s^2+1)/(s+1)^2;
syms s
Gl = @(s) (s.^2+1)./(s+1).^2;
Gl = Gl(1i*w);
```

```
[mag,pha,wout]=bode(Gb);

Glm = 20*log10(abs(Gl));
Glp = atan2d(imag(Gl),real(Gl));

magi = 20*log10(mag(1,1,1));
for i=2:size(mag,3)
    magi = [magi,20*log10(mag(1,1,i))];
end
phai = pha(1,1,1);
for i=2:size(pha,3)
    phai = [phai,pha(1,1,i)];
end

subplot(2,1,1)
semilogx(w,Glm,wout,magi)
ylabel("Magnitude(dB)")
title("Bode diagram")
legend("semilog","bode function")

subplot(2,1,2)
semilogx(w,Glp,wout,phai)
xlabel("frequency")
ylabel("Phase(deg)")
legend("semilog","bode function")
```



```
clc
clear all
close all
```

8 Convolution

a)

```
Tmax = 15;
Samples = 1000;
T  = linspace(-1,Tmax,Samples);
TY = linspace(-1,2*Tmax,2*Samples-1);
syms t
h1 = piecewise( t<2,0 , 2<=t<3,1 , ...
     3<=t<4,-2 ,4<=t<5,1 , t>=5,0 )
```

h1 =

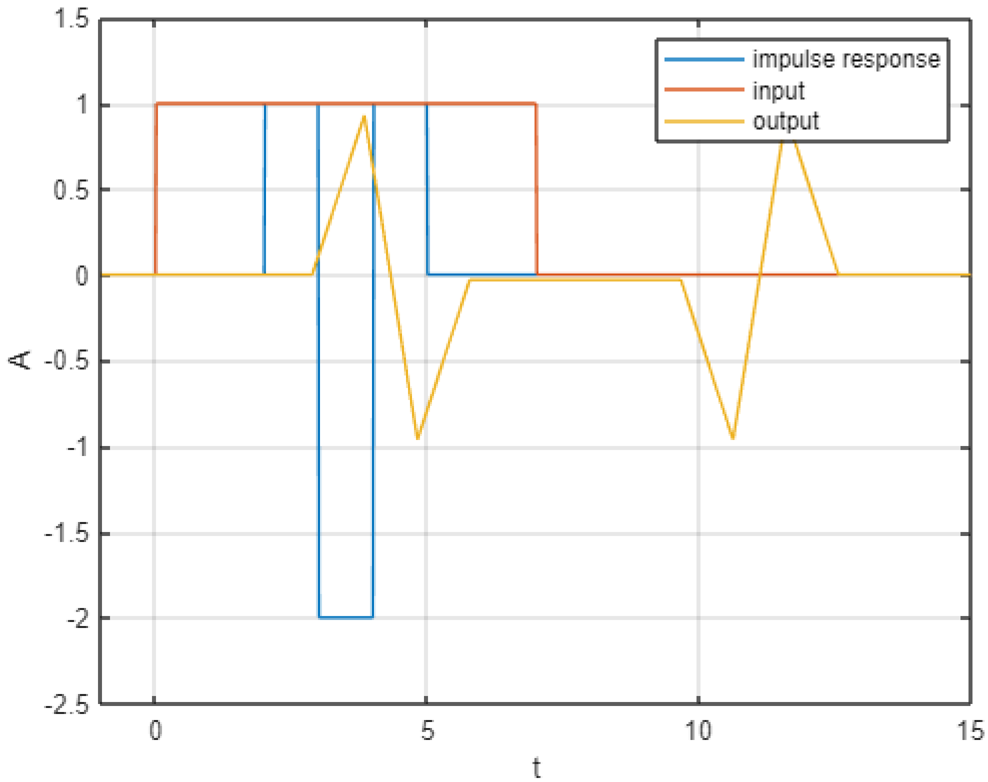
$$\begin{cases} 0 & \text{if } t < 2 \\ 1 & \text{if } t \in [2,3) \\ -2 & \text{if } t \in [3,4) \\ 1 & \text{if } t \in [4,5) \\ 0 & \text{if } 5 \leq t \end{cases}$$

```
u1 =  piecewise(t<0,0 , 0<=t<7,1 , t>=7,0)
```

u1 =

$$\begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \in [0,7) \\ 0 & \text{if } 7 \leq t \end{cases}$$

```
ht = eval(subs(h1,t,T));

ut = eval(subs(u1,t,T));
yt = Tmax/Samples*conv(ht,ut);

plot(T,ht,T,ut,TY,yt)
ylim([-2.5 1.5])
xlim([-1 15])
grid on
legend("impulse response","input","output")
xlabel("t")
ylabel("A")
```



b)

```
Tmax = 6;
Samples = 1000;
T  = linspace(-1,Tmax,Samples);
TY = linspace(-1,2*Tmax,2*Samples-1);
syms t
h2 = piecewise( t<0,0 , 0<=t<1,t , ...
     1<=t<2,1 ,2<=t<3,3-t , t>=3,0 )
```

h2 =

$$\begin{cases} 0 & \text{if } t < 0 \\ t & \text{if } t \in [0,1) \\ 1 & \text{if } t \in [1,2) \\ 3-t & \text{if } t \in [2,3) \\ 0 & \text{if } 3 \le t \end{cases}$$

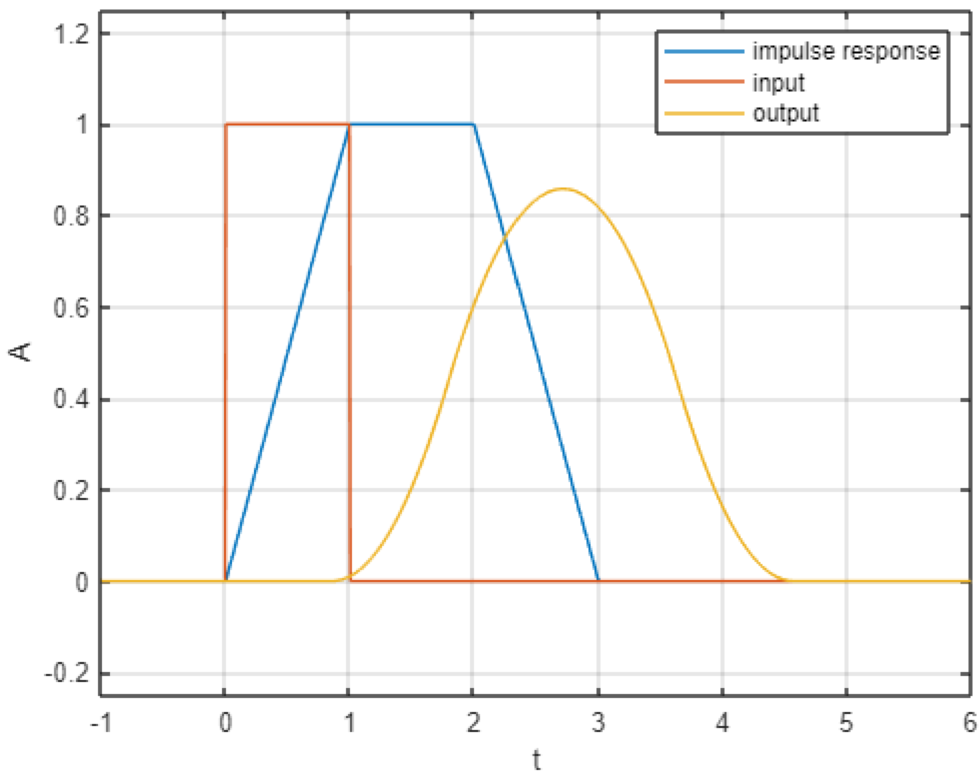
```
u2 =  piecewise(t<0,0 , 0<=t<1,1 , t>=1,0)
```

u2 =

$$\begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \in [0,1) \\ 0 & \text{if } 1 \le t \end{cases}$$

```
ht = eval(subs(h2,t,T));
```

```
ut = eval(subs(u2,t,T));
yt = Tmax/Samples*conv(ht,ut);

plot(T,ht,T,ut,TY,yt)
ylim([-0.25 1.25])
xlim([-1 6])
grid on
legend("impulse response","input","output")
xlabel("t")
ylabel("A")
```



c)

```
Tmax = 5;
Samples = 1000;
T  = linspace(-1,Tmax,Samples);
TY = linspace(-1,2*Tmax,2*Samples-1);
syms t
h3 = piecewise( t<0,0 , 0<=t<1,t , ...
    1<=t<2,1-t , t>=2,0 )
```

h3 =

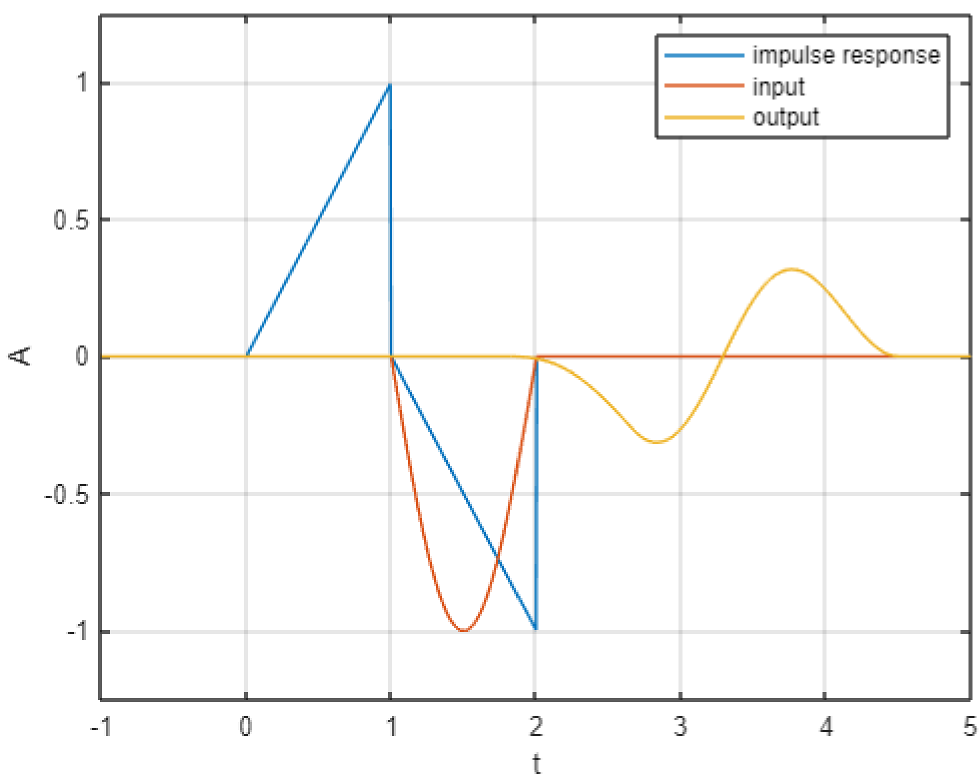
$$\begin{cases} 0 & \text{if } t < 0 \\ t & \text{if } t \in [0, 1) \\ 1 - t & \text{if } t \in [1, 2) \\ 0 & \text{if } 2 \le t \end{cases}$$

```
u3 =  piecewise(t<1,0 , 1<=t<2,sin(pi*t) , t>=2,0)
```

u3 =

$$\begin{cases} 0 & \text{if } t < 1 \\ \sin(\pi\, t) & \text{if } t \in [1, 2) \\ 0 & \text{if } 2 \le t \end{cases}$$

```
ht = eval(subs(h3,t,T));

ut = eval(subs(u3,t,T));
yt = Tmax/Samples*conv(ht,ut);

plot(T,ht,T,ut,TY,yt)
ylim([-1.25 1.25])
xlim([-1 5])
grid on
legend("impulse response","input","output")
xlabel("t")
ylabel("A")
```



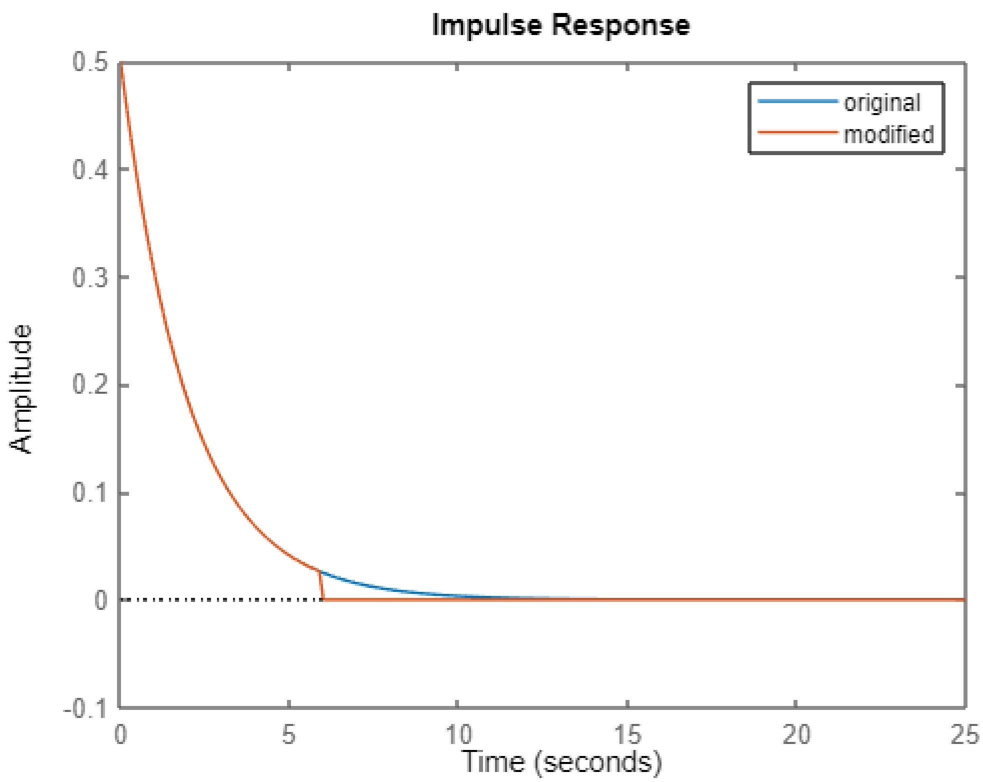

```
clc
clear all
close all
```

9 Impulse Response Truncation

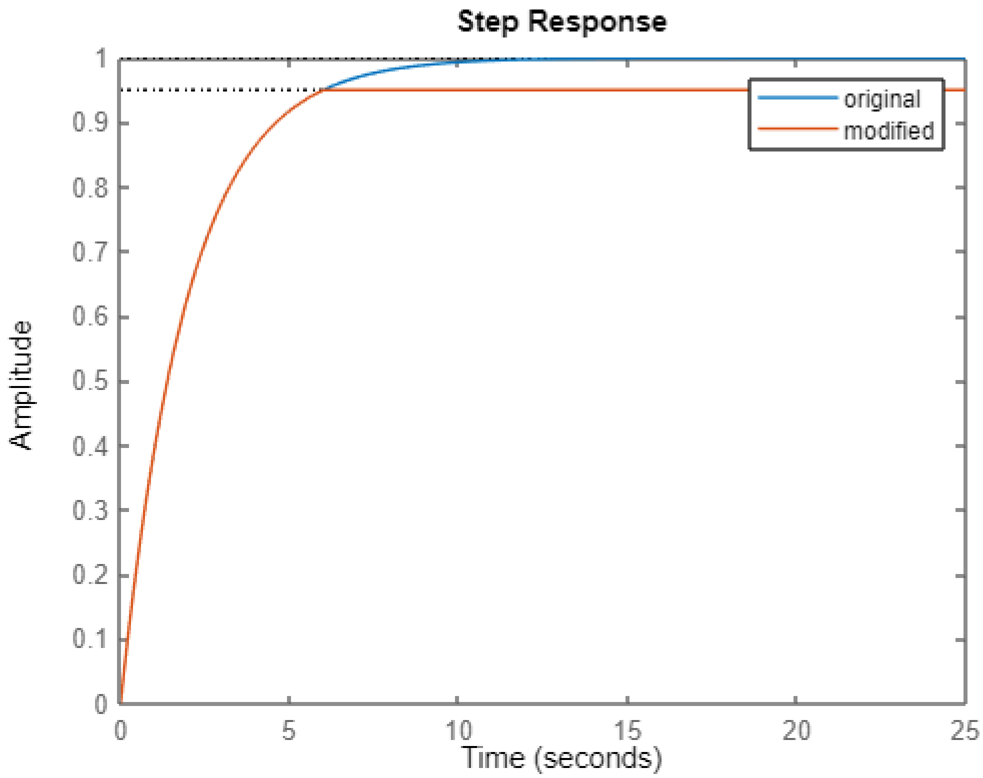
a)

$$\frac{1-e^{-\frac{T_s}{2}}e^{-T_s s}}{2s+1} = \frac{1}{2s+1} - \frac{e^{-\frac{T_s}{2}}e^{-T_s s}}{2s+1} \Rightarrow 0.5e^{-0.5t}u(t) - 0.5e^{-\frac{T_S}{2}}e^{-0.5(t-T_S)}u(t-T_s) = 0.5e^{-0.5t}u(t) - 0.5e^{-0.5(t)}u(t-T_s) = \begin{cases} t < T_s \\ t \geq T_s \end{cases}$$

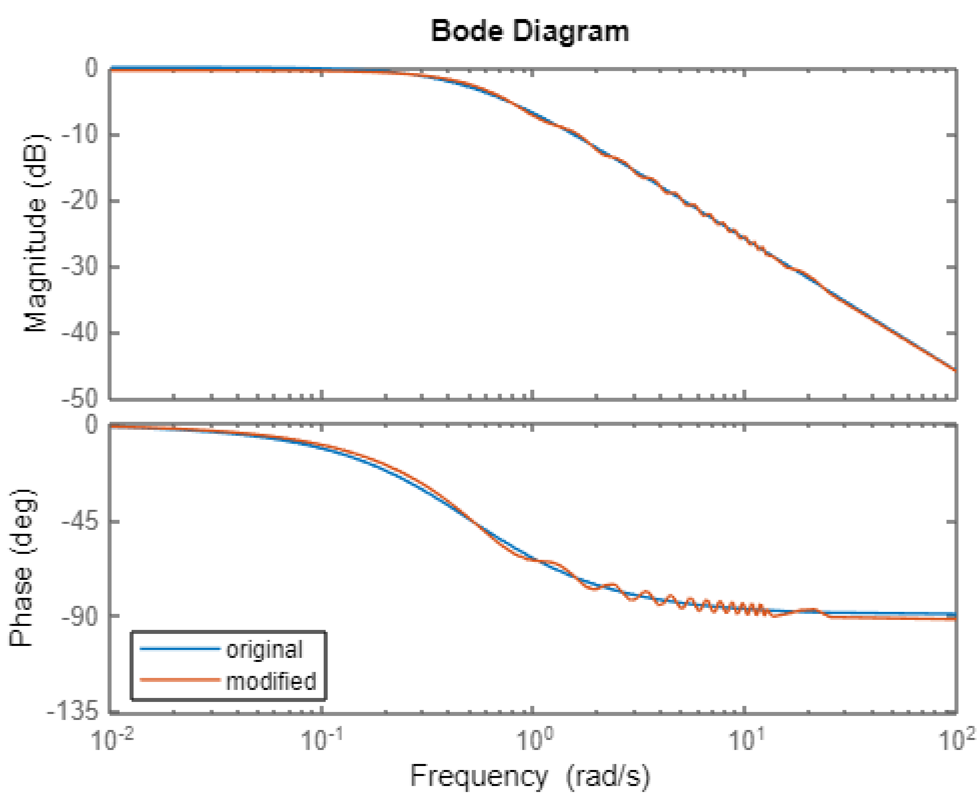
```
s = tf("s");
Ts = 6;
T1 = 1/(2*s+1);
T2 = (1-exp(-Ts/2)*exp(-Ts*s))/(2*s+1);
impulse(T1,T2)
legend("original","modified")
```




```
step(T1,T2)
legend("original","modified")
```




```
bode(T1,T2)
legend("original","modified",'Location','southwest')
```



b)

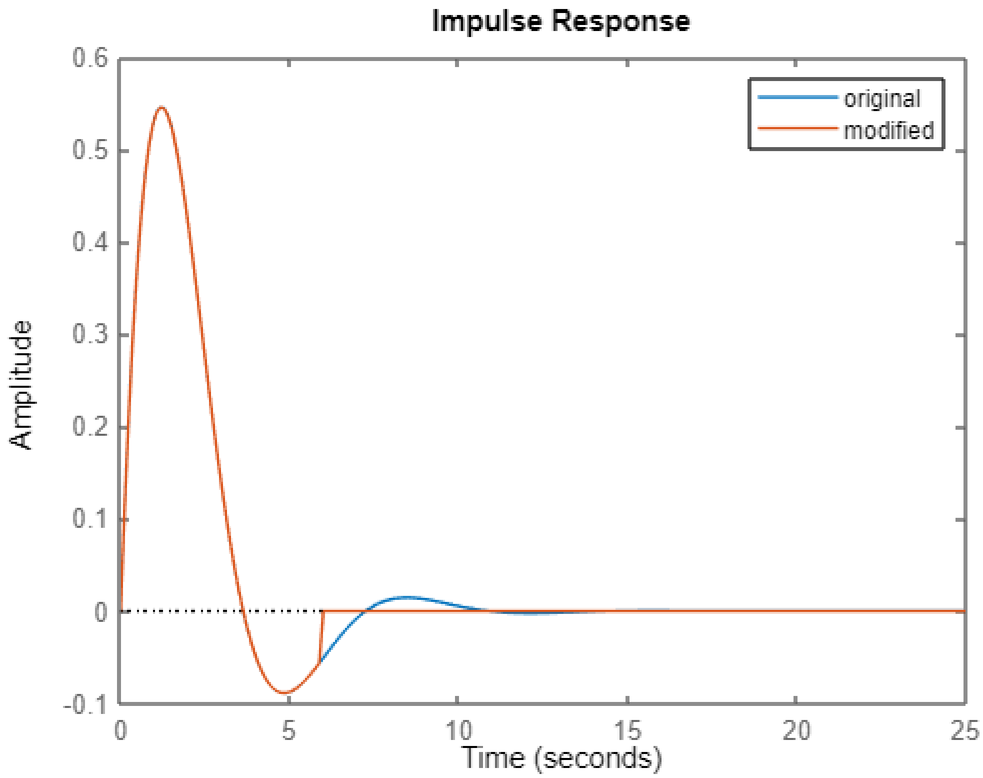
$$\frac{1}{s^2+s+1} \longrightarrow \frac{2}{\sqrt{3}} e^{-0.5t} \sin\left(\frac{\sqrt{3}}{2} t\right) u(t)$$

modified sestem:

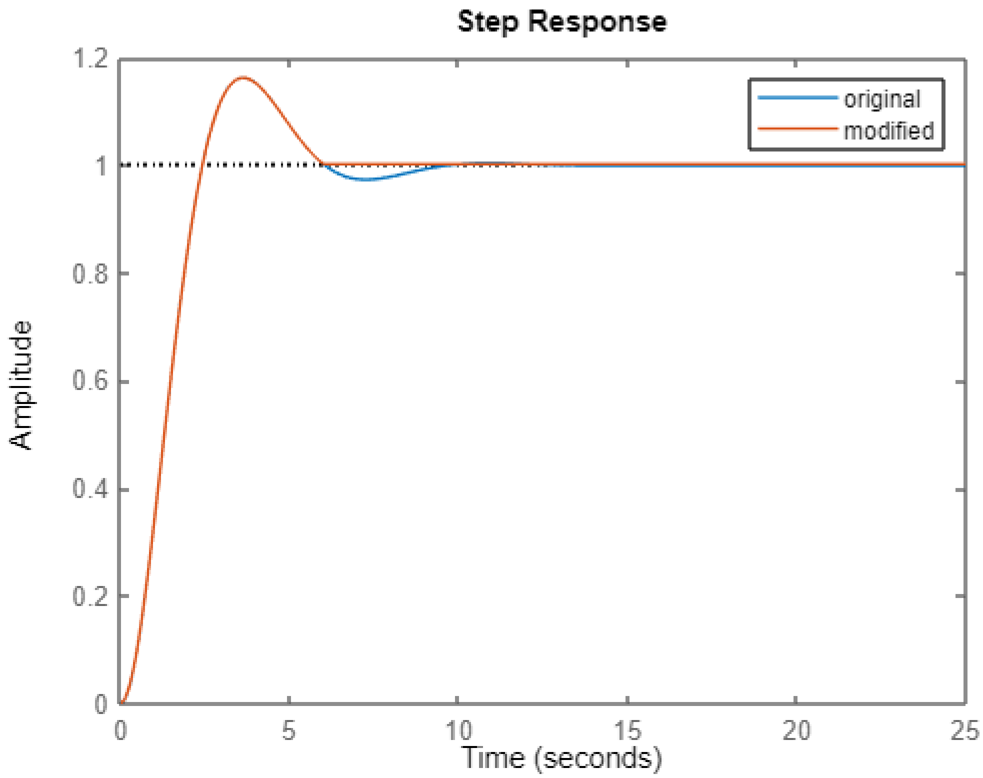
$$\frac{2}{\sqrt{3}} e^{-0.5t} \sin\left(\frac{\sqrt{3}}{2} t\right) (u(t) - u(t - T_s))$$

$$\longrightarrow \frac{1}{s^2+s+1} - \frac{e^{-T_s(s+0.5)}\left((1.1547s + 0.57735)\sin\left(\frac{\sqrt{3}}{2} T_S\right) + \cos\left(\frac{\sqrt{3}}{2} T_s\right)\right)}{s^2+s+1}$$

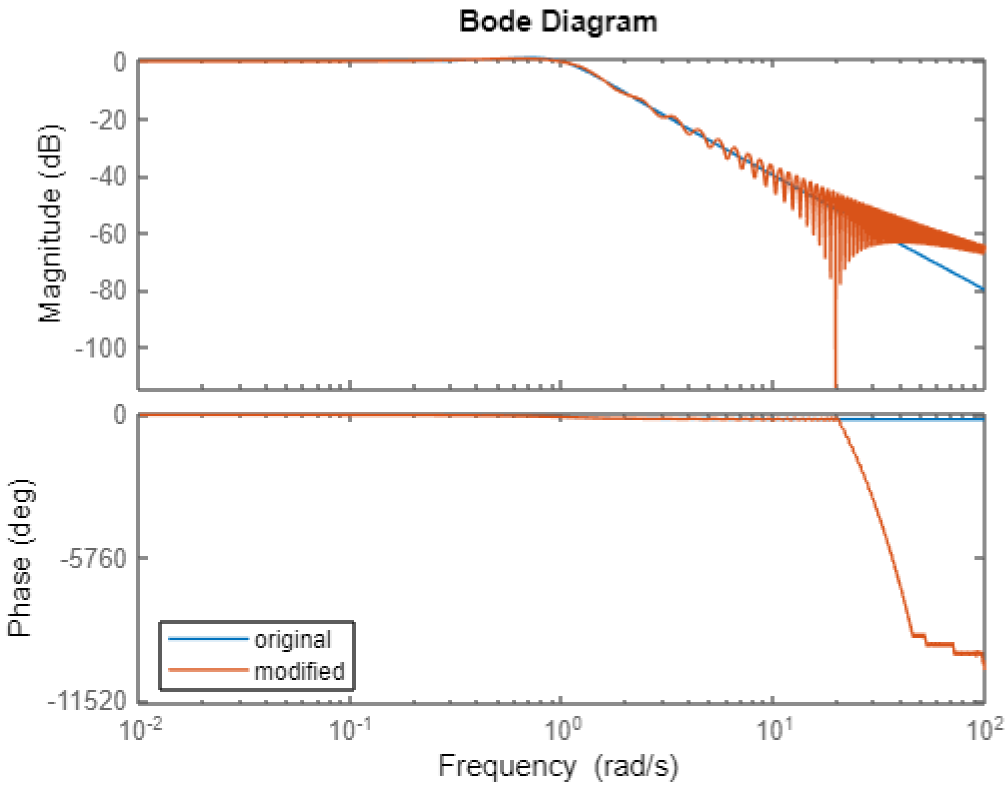
```
s = tf("s");
Ts = 6;
T1 = 1/(s^2+s+1);
T2 = (1 - ( exp(-Ts*s)*exp(-Ts/2) *(((1.1547*s+0.57735)*sin(0.5*sqrt(3)*Ts))+cos(0.5*sqrt(3)*Ts
impulse(T1,T2)
legend("original","modified")
```



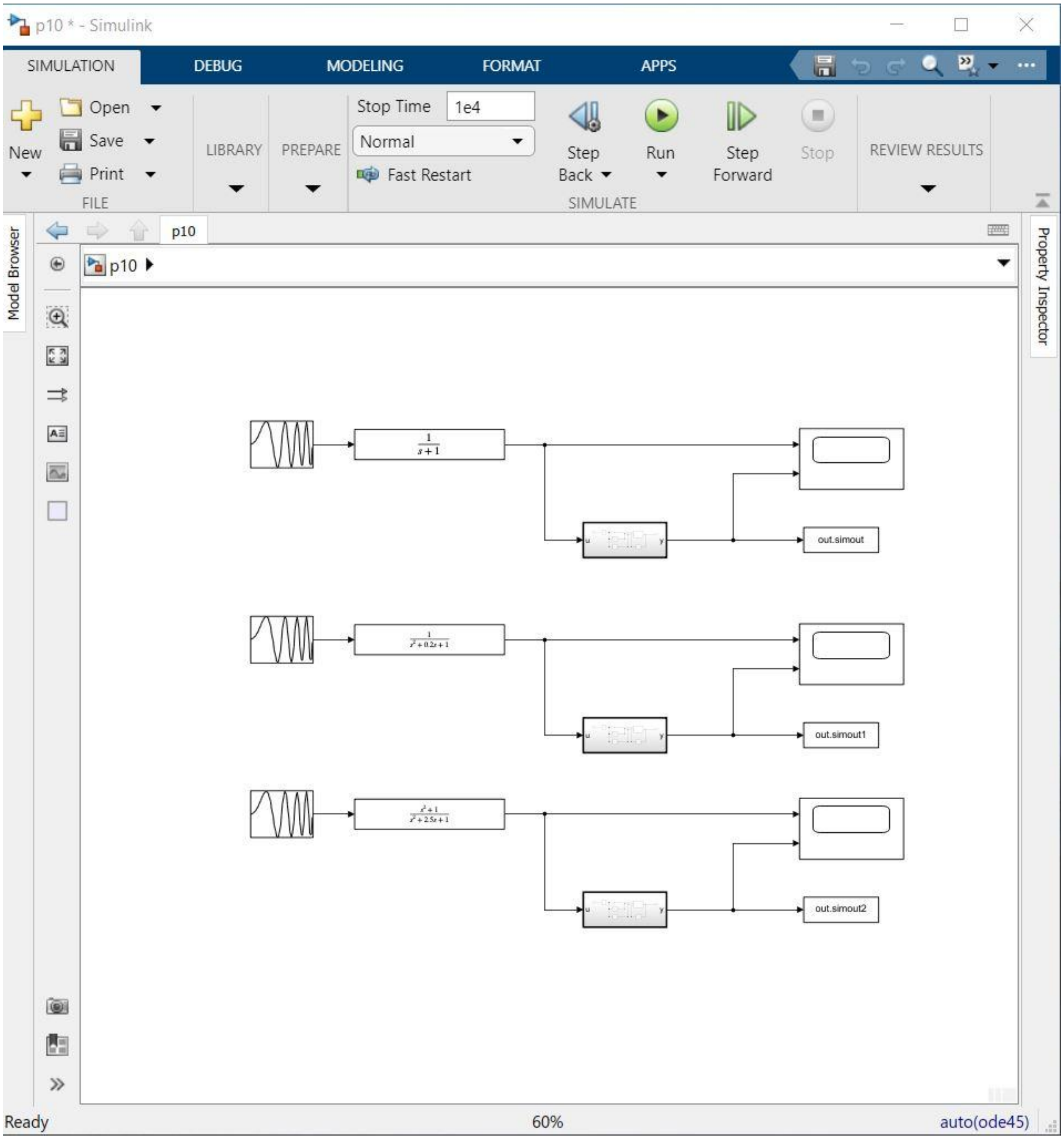

```
step(T1,T2)
legend("original","modified")
```




```
bode(T1,T2)
legend("original","modified",'Location','southwest')
xlim([0.01 100])
```

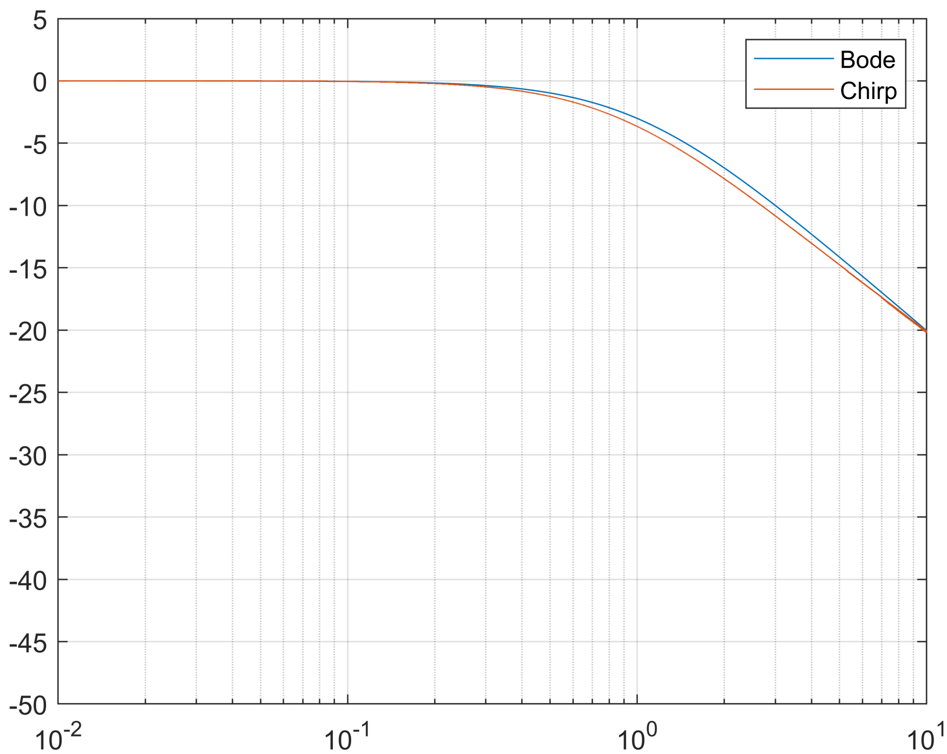


10 Chrip Signal and Frequency Response



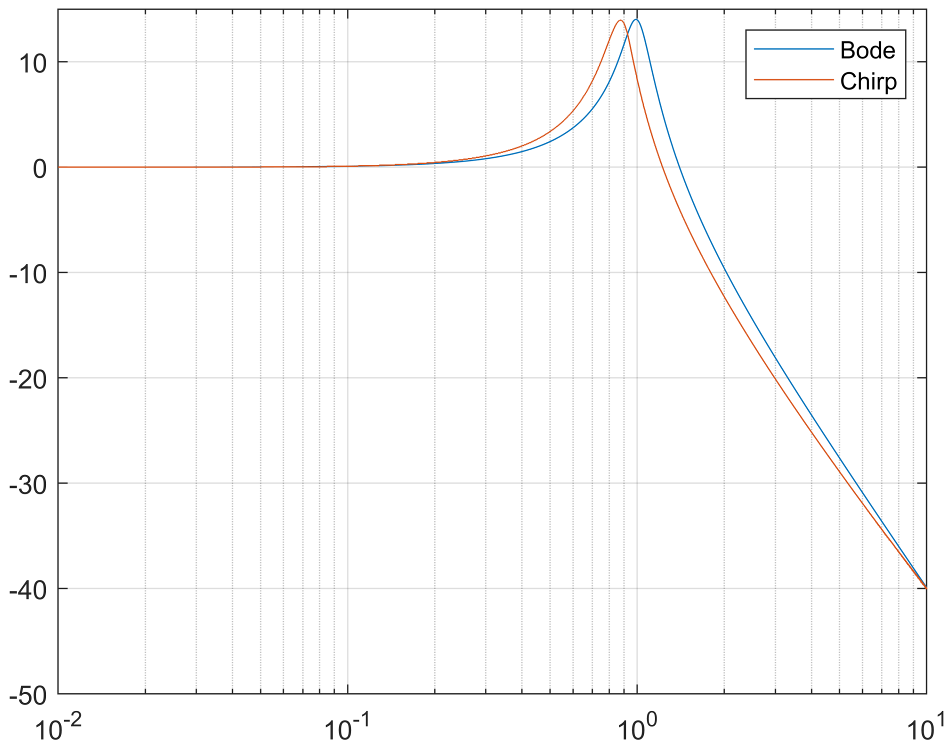
a)

```
s = tf("s");
G1 = 1/(s+1);

envelope = out.simout.Data;
w = linspace(0.01,10,length(envelope));
h = reshape(bode(G1,w),size(w));
figure();
semilogx(w,20*log10(h))
hold on
semilogx(w,20*log10(envelope));
grid on
ylim([-50 5]);
legend({'Bode',"Chirp"});
```



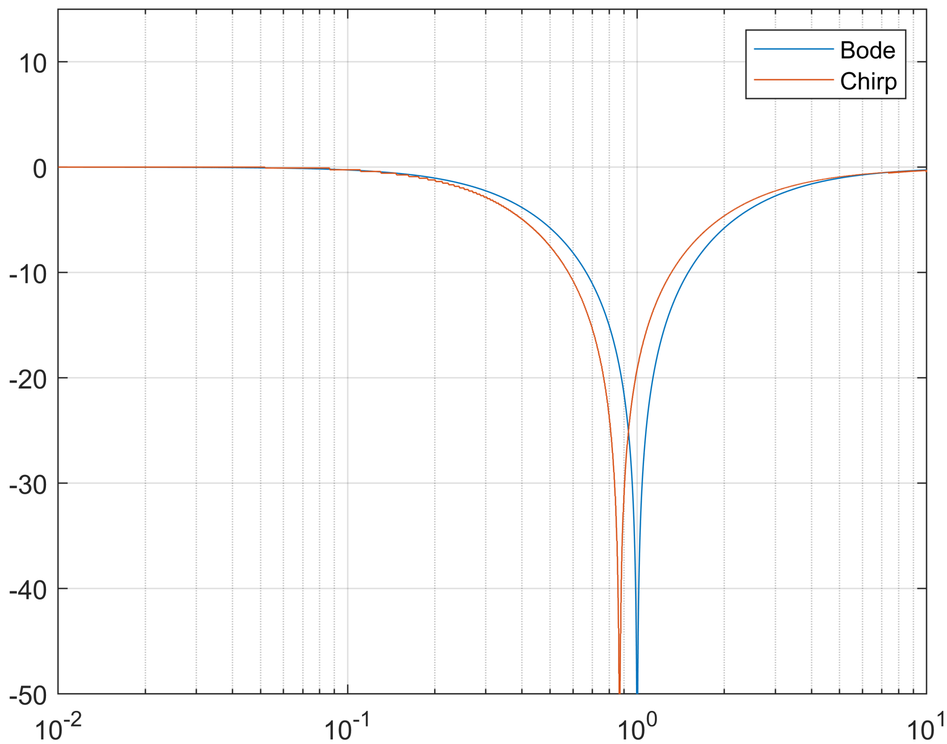
b)

```
s = tf("s");
G1 = 1/(s^2+0.2*s+1);

envelope = out.simout1.Data;
w = linspace(0.01,10,length(envelope));
h = reshape(bode(G1,w),size(w));
figure();
semilogx(w,20*log10(h))
hold on
semilogx(w,20*log10(envelope));
grid on
ylim([-50 15]);
legend({'Bode',"Chirp"});
```



c)

```
s = tf("s");
G1 = (s^2+1)/(s^2+2.5*s+1);

envelope = out.simout2.Data;
w = linspace(0.01,10,length(envelope));
h = reshape(bode(G1,w),size(w));
figure();
semilogx(w,20*log10(h))
hold on
semilogx(w,20*log10(envelope));
grid on
ylim([-50 15]);
legend({'Bode',"Chirp"});
```



11 Simulink System Implementation using Impulse Response