Linear Control Systems
25411

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Fall 1402 | **Due Date: 1402/10/1**

Note 1: ***purple*** *problems are bonus ones.*

1 Maximum Gain

Determine the maximum gain M_p and the frequency ω_r at which the maximum gain occurs for each given transfer function. It is required to solve the problem parametrically providing the details of derivation.

a) $G_1(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

b) $G_2(s) = \dfrac{\omega_n(s + \omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

c) Show the correctness of the subsequent relation for small values of ζ. An intuitive explanation is sufficient. Hint: there is no necessity to solve part b.

$$\max |G_1(j\omega)| \approx \frac{\max |G_2(j\omega)|}{\sqrt{2}}$$

2 Cutoff Frequency and Bandwidth

Obtain the bandwidth of the given systems.
Hint: In the context of low-pass systems, the cutoff frequency refers to the frequency at which the system's gain equals $\frac{1}{\sqrt{2}}$ times its DC gain (or almost the magnitude is attenuated by 3 dB respecting its value at low frequencies). However, in pass-band filters the cutoff frequency refers to the frequency at which the system response is attenuated by 3 dB relative to its value at passband frequencies.

a) $G_1(s) = \dfrac{k}{\tau s + 1}$

b) $G_2(s) = \dfrac{k}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

c) $G_3(s) = \dfrac{2}{3} \times \dfrac{(s + 200)(s + 300000)}{(s + 300)(s + 200000)}$

Is the auditory experience of songs processed through the filter in part c enjoyable?

3 Gain Crossover Frequency

The gain crossover frequency ω_c is the frequency at which the gain of the system equals 1. In other words, in the Bode diagram, it is the frequency at which the gain becomes equal to 0 dB.

$$|G(j\omega_c)| = 1$$

Determine ω_c of the following systems:

a) $G_1(s) = \dfrac{1}{\tau s + 1}$

b) $G_2(s) = \dfrac{\omega_n^2}{s(s + 2\zeta\omega_n)}$

c) $G_3(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

4 Stability and l^2 Norm

Consider the set of bounded continuous-time signals. The l^2 norm for this set is defined as follows:

$$||x||_2 = \sqrt{\int_0^\infty x^2(t)dt}$$

Using the Parseval's theorem we have:

$$||x||_2 = \sqrt{\int_0^\infty x^2(t)dt} = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)X(-j\omega)d\omega}$$

System S is an arbitrary LTI system with transfer function $G(s)$ and $G(s)$ has no poles in the closed right half-plane. $||S||$ is defined as follows:

$$||S|| = \sup_{||u||_2 \neq 0} \frac{||y||_2}{||u||_2}$$

a) Prove the following equation:

$$||y||_2 \leq ||u||_2 \max_{\omega} |G(j\omega)|$$

b) Prove the following equation:

$$||S|| = \max_{\omega} |G(j\omega)|$$

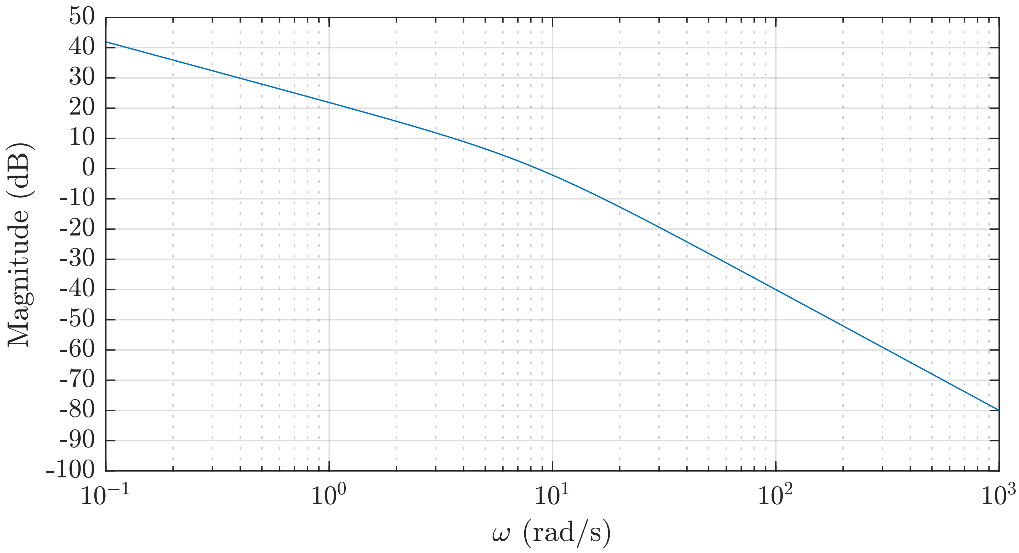
Hint: find a signal that satisfies the equality (=) in part a.

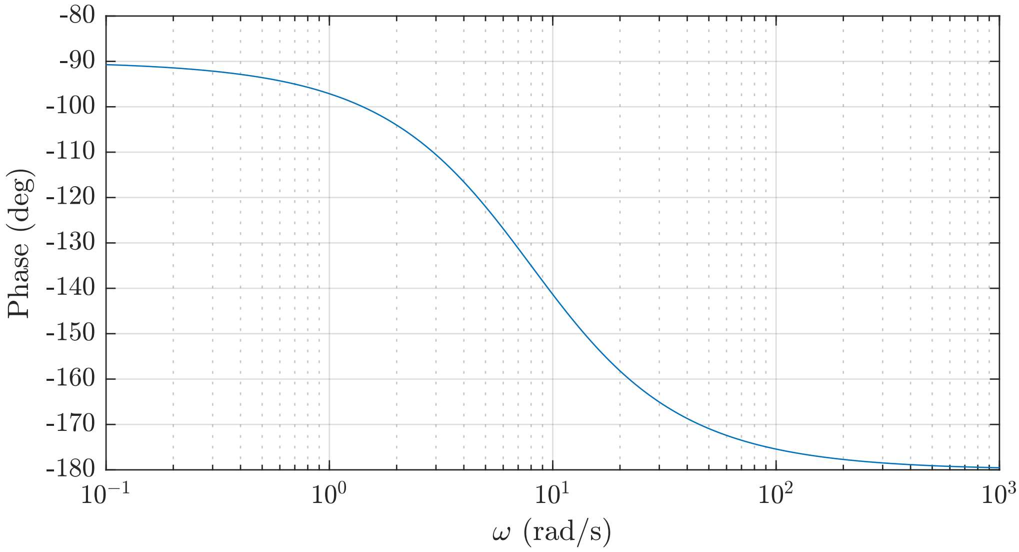
5 Notch Filter

In a closed-loop control system, a 50 Hz city electricity frequency has caused a problem in the control process. The task is to design a notch filter that eliminates the effect of the 50 Hz frequency, reduces the signal power of frequencies between 49-51 Hz by half, and maximizes the transmission of other frequencies. Describe the design process step by step, including the parameters and characteristics of the notch filter and plot the Bode diagram of the designed filter.

6 Frequency-Domain and Time-Domain Responses

The Bode plots of $G(s) = \dfrac{\omega_n^2}{s(s+2\zeta\omega_n)}$ in a unity feedback control system $T(s) = \dfrac{G(s)}{1+G(s)}$ are shown below:





a) Find the gain and phase margins and the corresponding crossover frequencies.

b) Replace $G(s)$ by $kG(s)$ with $k \geq 1$. Find the range of k for closed-loop stability.

c) Replace $G(s)$ by $e^{-\tau s}G(s)$ with $\tau \geq 0$. Find the range of τ for closed-loop stability.

d) Find the steady-state error to a unit step input.

e) Estimate the percent overshoot, rise time, and settling time of the closed-loop step response.

f) Estimate the bandwidth, peak amplitude and peak frequency of the closed-loop frequency response.

MATLAB Assignments

7 Bode Diagram Plot

Plot the Bode diagram of following transfer function using `semilogx` command and compare your results with the built-in `bode` MATLAB function.

a) $G_1(s) = \dfrac{1}{s+1}$

b) $G_2(s) = s+1$

c) $G_3(s) = e^{-0.2s}$

d) $G_4(s) = \dfrac{s+1}{s^2+0.1s+1}$

e) $G_5(s) = \dfrac{s^2+1}{(s+1)^2}$

8 Convolution

Determine the output of the following systems in response to the specified inputs using convolution. Plot the input signal, impulse response and the output signal.

$$y(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau = \int_{-\infty}^{\infty} u(\tau)h(t-\tau)d\tau$$

a)

$$h_1(t) = \begin{cases} 1 & 2 \le t < 3 \\ -2 & 3 \le t < 4 \\ 1 & 4 \le t < 5 \\ 0 & \text{O.W.} \end{cases} \qquad u_1(t) = \begin{cases} 1 & 0 \le t < 7 \\ 0 & \text{O.W.} \end{cases}$$

b)

$$h_2(t) = \begin{cases} t & 0 \le t < 1 \\ 1 & 1 \le t < 2 \\ 3-t & 2 \le t < 3 \\ 0 & \text{O.W.} \end{cases} \qquad u_2(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & \text{O.W.} \end{cases}$$

c)

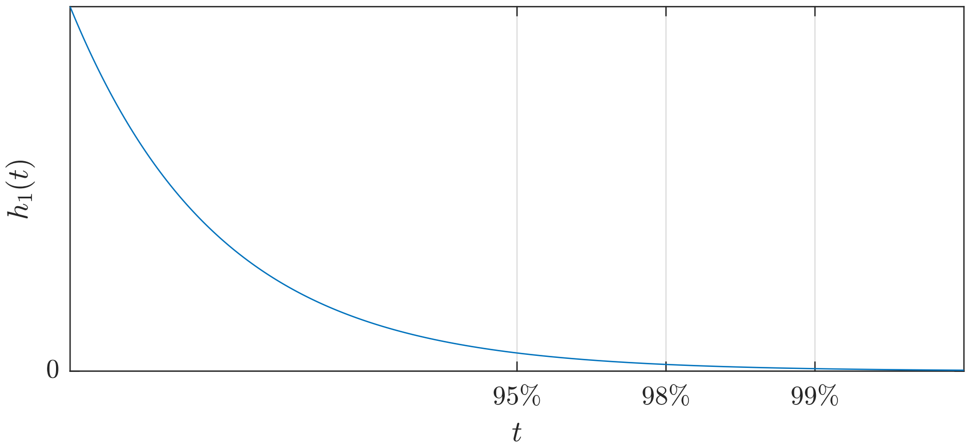
$$h_3(t) = \begin{cases} t & 0 \le t < 1 \\ 1-t & 1 \le t < 2 \\ 0 & \text{O.W.} \end{cases} \qquad u_3(t) = \begin{cases} \sin(\pi t) & 1 \le t < 2 \\ 0 & \text{O.W.} \end{cases}$$

Hint: You might use `conv` command by choosing an appropriate sampling-time T_s. Furthermore, be careful upon the start time of output signal.

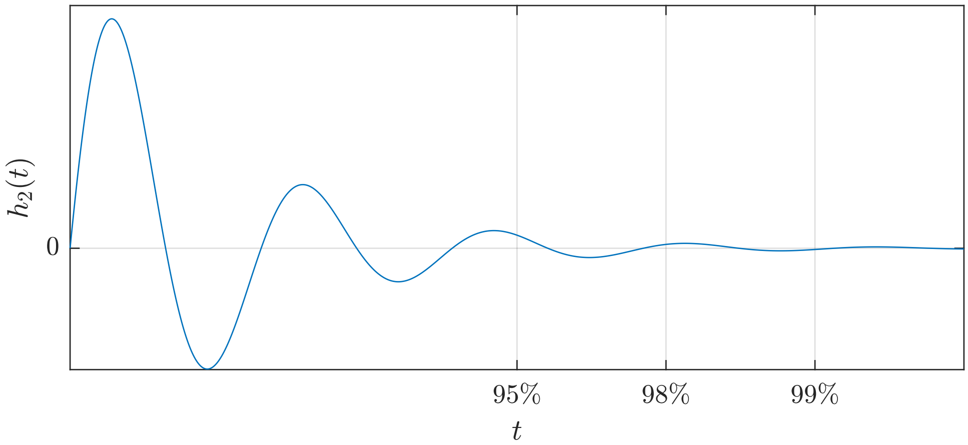
9 Impulse Response Truncation

Given a dynamic system and its step response reaching a steady state after the settling time, we are interested in investigating the effects of truncating the impulse response after the settling time. This will be tested using different settling time percentages (95%, 98%, and 99%) with a comparison of the results.

$$G_1(s) = \frac{1}{\tau s + 1} \quad \rightarrow \quad \begin{cases} 95\% & \rightarrow \quad T_s = 3\tau \\ 98\% & \rightarrow \quad T_s = 4\tau \\ 99\% & \rightarrow \quad T_s = 5\tau \end{cases}$$



$$G_2(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \rightarrow \quad \begin{cases} 95\% & \rightarrow \quad T_s \approx \dfrac{3}{\zeta\omega_n} \\ 98\% & \rightarrow \quad T_s \approx \dfrac{4}{\zeta\omega_n} \\ 99\% & \rightarrow \quad T_s \approx \dfrac{5}{\zeta\omega_n} \end{cases}$$



Compare the original system's impulse response, step response, and Bode diagram with the modified ones obtained through the truncation of the impulse response.

a) $T_1(s) = \dfrac{1}{2s+1}$ (Hint: First prove the transfer function of the modified system is $M_1(s) = \dfrac{1 - e^{-T_s/2}\, e^{-T_s s}}{2s+1}$)

b) $T_2(s) = \dfrac{1}{s^2 + s + 1}$

10 Chirp Signal and Frequency Response

Using Simulink, implement the given systems individually. Next, employ a suitable chirp signal and an appropriate envelope detector to obtain the magnitude of frequency response of each system in a single simulation. Subsequently, plot the obtained result and compare it with the magnitude plot obtained from the Bode diagram.

a) $G_1(s) = \dfrac{1}{s+1}$

b) $G_2(s) = \dfrac{1}{s^2 + 0.2s + 1}$

c) $G_3(s) = \dfrac{s^2+1}{s^2 + 2.5s + 1}$

11 Simulink System Implementation using Impulse Response

You are given systems defined by their impulse responses. Implement these systems in Simulink and apply the specified input $u(t)$ to obtain the system response $y(t)$ for the time interval $0 \leq t \leq 10$. Then obtain the frequency response using a desired method such as using a chirp signal or testing different frequencies separately.

$$u(t) = \begin{cases} 1 & 2 \leq t \leq 4 \\ -1 & 6 \leq t \leq 8 \\ 0 & O.W. \end{cases}$$

a) $h_1(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{O.W.} \end{cases}$

b) $h_2(t) = \begin{cases} 1-t & 0 \leq t \leq 1 \\ 0 & \text{O.W.} \end{cases}$

c) $h_3(t) = \begin{cases} 1-t^2 & 0 \leq t \leq 1 \\ 0 & \text{O.W.} \end{cases}$

d) $h_4(t) = \begin{cases} 1-\sqrt{t} & 0 \leq t \leq 1 \\ 0 & \text{O.W.} \end{cases}$

e) $h_5(t) = \begin{cases} \dfrac{2t-2}{t-2} & 0 \leq t \leq 1 \\ 0 & \text{O.W.} \end{cases}$

f) $h_6(t) = \begin{cases} 2-2^t & 0 \leq t \leq 1 \\ 0 & \text{O.W.} \end{cases}$

Hint: use a known and fixed sampling time T_s for the implementations and simulations.