Linear Control Systems
25411

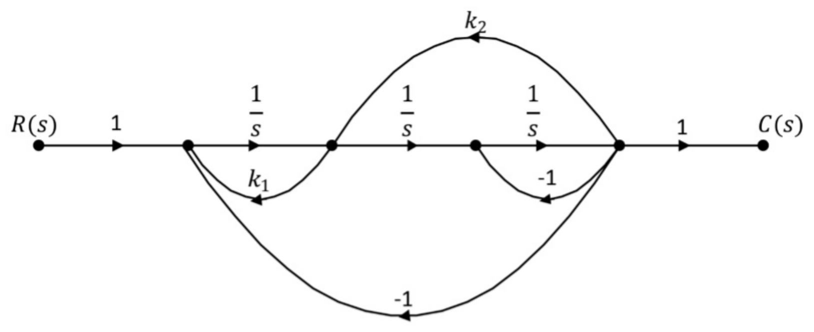
Instructor. Behzad Ahi — **Assignment 3**

Fall 1402 — **Due Date: 1402/8/22**

Note 1: ***purple*** *problems are bonus ones.*

1 Oscillation

Given the system diagram below, identify the values of K_1 and K_2 that cause the system to exhibit oscillatory behavior. Also, calculate the frequency of the oscillations in this case.



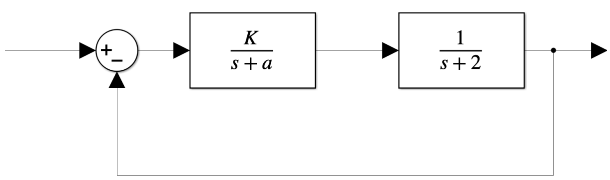
2 Model Order Reduction

Reduce the order of following transfer functions and approximate them using given structures.

1. $\dfrac{20}{(s+3)(s+30)} \rightarrow \dfrac{k}{s+p_1}$
2. $\dfrac{-0.2s+1}{(0.5s+1)(0.25s+1)(0.2s+1)} \rightarrow \dfrac{k}{(s+p_1)(s+p_2)}$

3 Step-Response Characteristics

Given the control system diagram below, determine the values of a and k that satisfy the performance specifications of 2% settling time less than 2 seconds and overshoot less than 5%.



4 Type of the Feedback System and the Steady-State Error

Consider a unity feedback control system with loop gain of $L(s)$, i.e. the closed-loop transfer function is $T(s) = L(s)/(1 + L(s))$. For each of the following transfer functions, determine the type of the feedback system and calculate the steady-state error to the step input $r_1(t) = 1$, the ramp input $r_2(t) = 10t$, and the parabola input $r_3(t) = t^2$.

1. $L_1(s) = \dfrac{20}{s(s+1)(s+3)}$

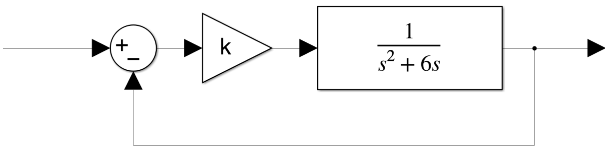
2. $L_2(s) = \dfrac{100}{s(s+2)(s+4)(s+6)}$

3. $L_3(s) = \dfrac{10(s+7)}{(s+1)(s+2)}$

4. $L_4(s) = \dfrac{5(s+1)}{s^2(s+3)}$

5 Performance Optimization

Consider the closed-loop control system with unity feedback shown below. For each of the following optimization problems, determine the optimal value of the proportional controller gain k. The reference signal is the unit step, i.e. $r(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$.



1. minimize $\quad J(k) = \text{settling time}$

2. minimize $\quad J(k) = \text{settling time}$
   subject to $\quad \forall t: \quad y(t) \leq r(t)$

3. minimize $\quad J(k) = \displaystyle\int_0^\infty e^2(t)dt$

4. minimize $\quad J(k) = \displaystyle\int_0^\infty \left(e^2(t) + u^2(t)\right) dt$
   where $u(t)$ denotes the system input.

MATLAB Assignments

6 Symbolic e^{At}

Given the system transfer function $G(s)$ below

$$G(s) = \frac{1-s}{(s+1)(2s+1)}$$

1. Generate the impulse signal, then use the `lsim` function to find the impulse response of the system. (hint: you shouldn't use the `dirac` function)
2. Use `ilaplace` function to find the impulse response of the system directly from the given transfer function.
3. Find the state-space representation of $G(s)$ using `ss` function, then use the `expm` function to find the e^{At} matrix, then use B, C and e^{At} matrices to find the impulse response of the system from state-space representation of the system.
4. Use the `impulse` function to find the impulse response of the system.
5. Plot and compare all of the impulse responses.
6. Use the `ilaplace` function and the $e^{At} = \mathcal{L}^{-1}\{(sI - A)^{-1}\}$ formula to find the e^{At}, then compare the result to the e^{At} matrix from problem 3.

Simulation Assignments

Note 2: *Required files to run simulation of system 1 and system 2 are provided in separate folders.*
Note 3: ***Blue*** *problems require screenshots or graphs.*

7 System 1

State-space representation of the open-loop system is

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \dfrac{u(t) - kx_1(t) - x_2(t)}{m} \\ y(t) = x_1(t) \end{cases}$$

where $u(t)$ is the applied force to moving box and $x_1(t)$ is the position, $x_2(t)$ is the velocity and m is the mass of the moving box and k is the spring constant.

Assignment:

1. Is the described system LTI?
2. Write the state-space representation of the open-loop system using the standard matrix form.
3. Find the open-loop transfer function $G(s) = \dfrac{Y(s)}{U(s)}$.
4. Determine $u(t)$ in the closed-loop system regarding proportional controller.
5. Write the state-space representation of the closed-loop system regarding proportional controller.
6. Find the closed-loop transfer function $T(s) = \dfrac{Y(s)}{R(s)}$ regarding proportional controller.
7. Given $m = 0.25$ and $k = 0.25$, determine the range of values for K_p that ensure closed-loop stability.
8. Using simulation, set the values of $m = 0.25$, $k = 0.25$ and $Kp = 200$ and observe the behavior of the closed-loop system. Explain whether the system is stable or not and provide a reason for your answer.
9. Implement the system in Simulink, and assign the values of $m = 0.25$, $k = 0.25$ and $Kp = 200$. Select the fixed-step size solver Euler (ode1) and set $T_s = 0.01$. Analyze the stability of the closed-loop system and justify your conclusion.
10. Modify the solver and the sampling-time T_s and perform the same simulation in Simulink. Is it possible to stabilize the closed-loop system by changing these parameters?
11. Determine the steady-state error of the system, then simulate different situations (change m, k and K_p) and compare the results to the first answer.
12. Given $m = 0.25$ and $k = 0.25$, calculate the value of K_p that results in a critically damped closed-loop system.
13. Assuming $m = 0.25$ and $k = 0.25$, determine the optimal value of K_p that minimizes the settling time of the system.
14. Find the condition on the parameters m and k that guarantees the existence of oscillations in the closed-loop system.

8 System 2

State-space representation of the open-loop system is

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \dfrac{2k(x_3(t) - x_1(t)) - x_2(t)}{m} \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = \dfrac{u(t) - 2k(x_3(t) - x_1(t)) - x_4(t)}{M} \\ \\ y_1(t) = x_1(t) \\ y_2(t) = x_2(t) \end{cases}$$

where $u(t)$ is the applied force to the moving frame and $x_1(t)$ is the position, $x_2(t)$ is the velocity and m is the mass of the moving box and $x_3(t)$ is the position, $x_4(t)$ is the velocity and M is the mass of the moving frame and k is the spring constant.

Assignment:

1. Write the state-space representation of the open-loop system using the standard matrix form.
2. Find the open-loop transfer functions $G_1(s) = \dfrac{Y_1(s)}{U(s)}$ and $G_2(s) = \dfrac{Y_2(s)}{U(s)}$.
3. Find the closed-loop transfer function $T_1(s) = \dfrac{Y_1(s)}{R(s)}$ while $u(t) = K_p(r(t) - y_1(t))$.
4. Find the closed-loop transfer function $T_2(s) = \dfrac{Y_2(s)}{R(s)}$ while $u(t) = K_p(r(t) - y_2(t))$.
5. Determine the steady-state error of the system, then simulate different situations (change m, k and K_p) and compare the results to the first answer.