Linear Control Systems
25411

Instructor. Behzad Ahi — **Assignment 5**

Fall 1402 — **Due Date: 1402/9/12**

Note 1: ***purple*** *problems are bonus ones.*

1 Root Locus

The transfer functions of two open-loop control systems are given below:

$$G_1(s) = \frac{s^2 - 2s + 4}{(s+1)(s+10)(s+30)}$$
$$G_2(s) = \frac{s + 4}{s(s+6)(s+8)(s^2 + 3s + 4)}$$

For each system, sketch the root locus plot as proportional gain K is varied from 0 to ∞. Specifically determine:

a) Angle of asymptotes and the centroid point

b) The breaking-away points (if any)

c) The range of K such the closed-loop system becomes unstable

d) Angle of departure (from complex poles) and angle of arrival (to complex zeros) (if any)

e) The value of K such that the closed-loop system is critically damped (if any)

2 Centroid Point Formula

Prove the introduced formula for calculation of centroid point.

3 Root Locus Shape

Show that for $0 < K < 4$, the root locus of the given open-loop transfer function, lies on a circle.

$$G(s) = \frac{s + 2}{(s+1)^2}$$

4 Full Range Root Locus

The transfer function of a open-loop control system is given below:

$$G(s) = \frac{s + 0.1}{s(s - 0.2)(s^2 + s + 0.6)}$$

Sketch the root locus plot as K is varied from $-\infty$ to $+\infty$ and determine:

a) Angle of asymptotes and the centroid point

b) The breaking-away points (if any)

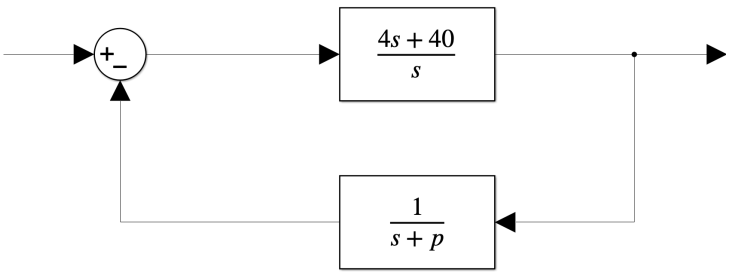
c) The range of K such the closed-loop system becomes unstable

d) Angle of departure (from complex poles) and angle of arrival (to complex zeros) (if any)

e) The value of K such that the closed-loop system is critically damped (if any)

5 Root Contours

A closed-loop control system is shown below, wherein the feedback path has a pole located at $s = -p$. Sketch the root contour plot when the parameter p is varied from 0 to ∞ and examine the closed-loop stability.



6 Effect of Noise and Uncertainty

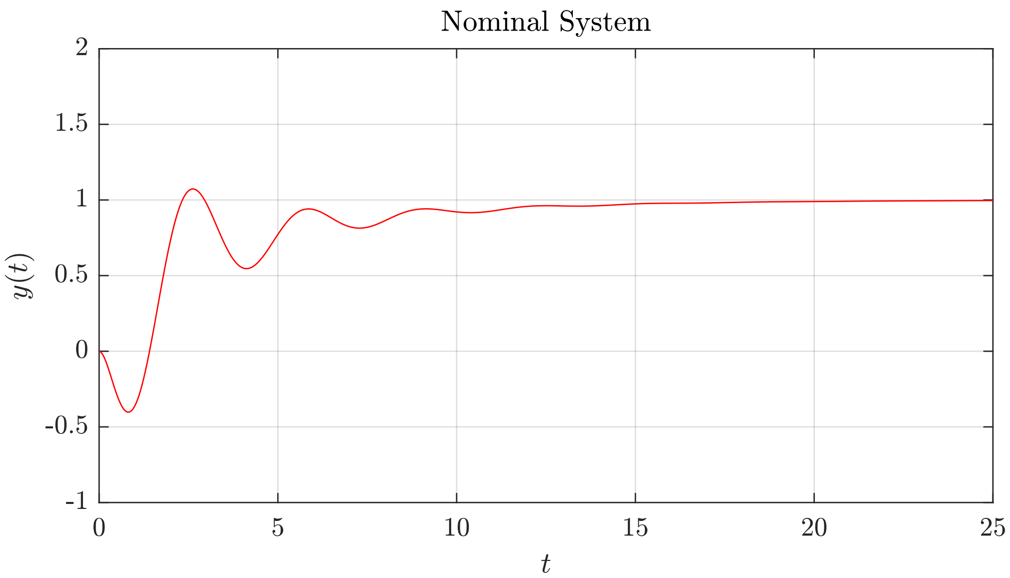
Consider the transfer function below:

$$G(s) = \frac{k(b_1 s + 1)(1 - b_2 s)\omega_n^2}{(\tau_1 s + 1)(\tau_2 s + 1)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

where the nominal values of the system parameters are given as follows:

$\hat{k}$	1
$\hat{\tau}_1$	0.5
$\hat{\tau}_2$	5
$\hat{\zeta}$	0.2
$\hat{\omega}_n$	2
$\hat{b}_1$	3
$\hat{b}_2$	1

The nominal step response of system is as follows:



Determine the corresponding plot for each one of the following items. Provide explanations in each case.

1) Input noise

2) Output noise

3) $k = \hat{k} \pm 50\%$

4) $\tau_1 = \hat{\tau}_1 \pm 50\%$

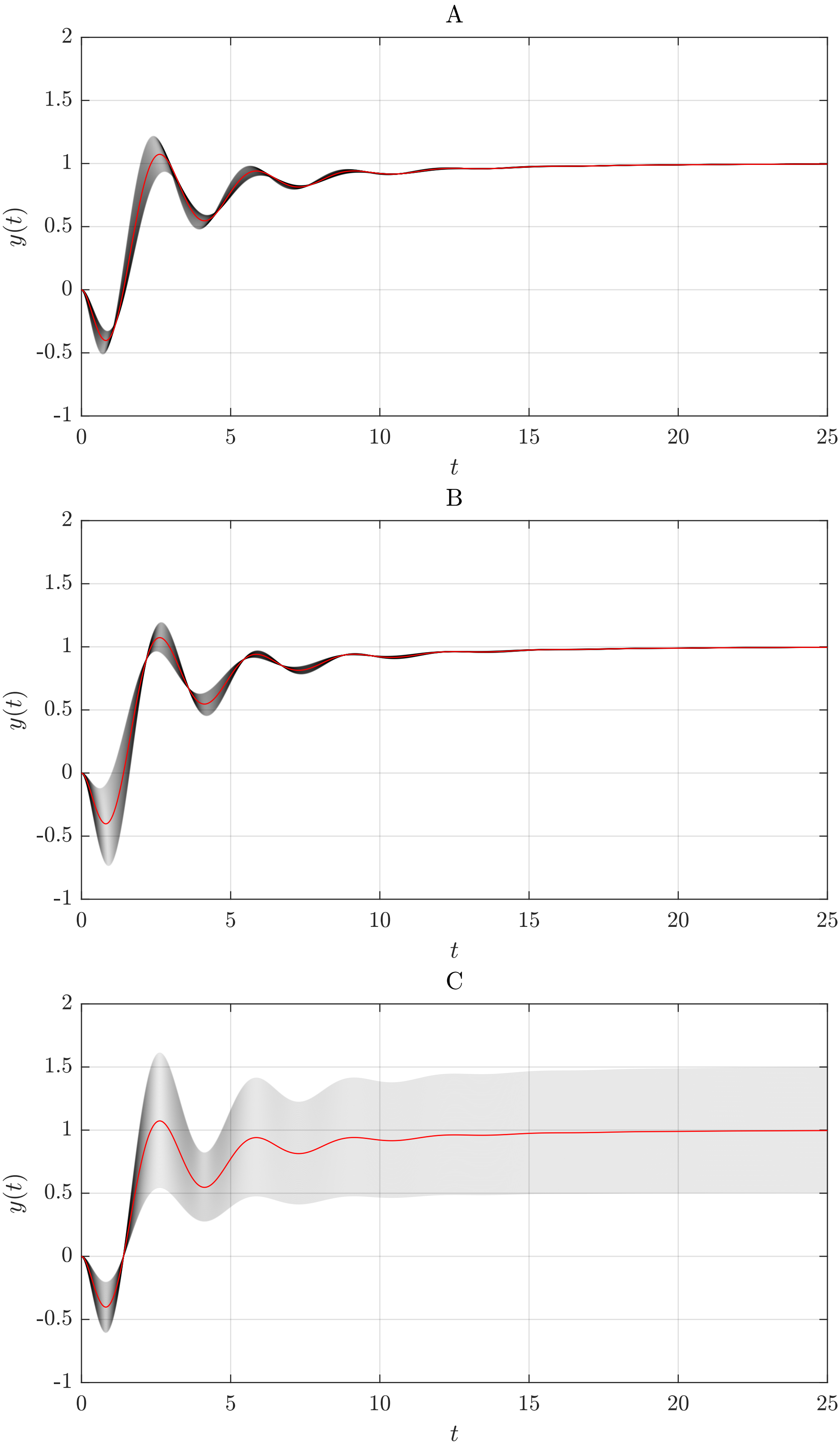
5) $\tau_1 = \hat{\tau}_1 \pm 50\%$

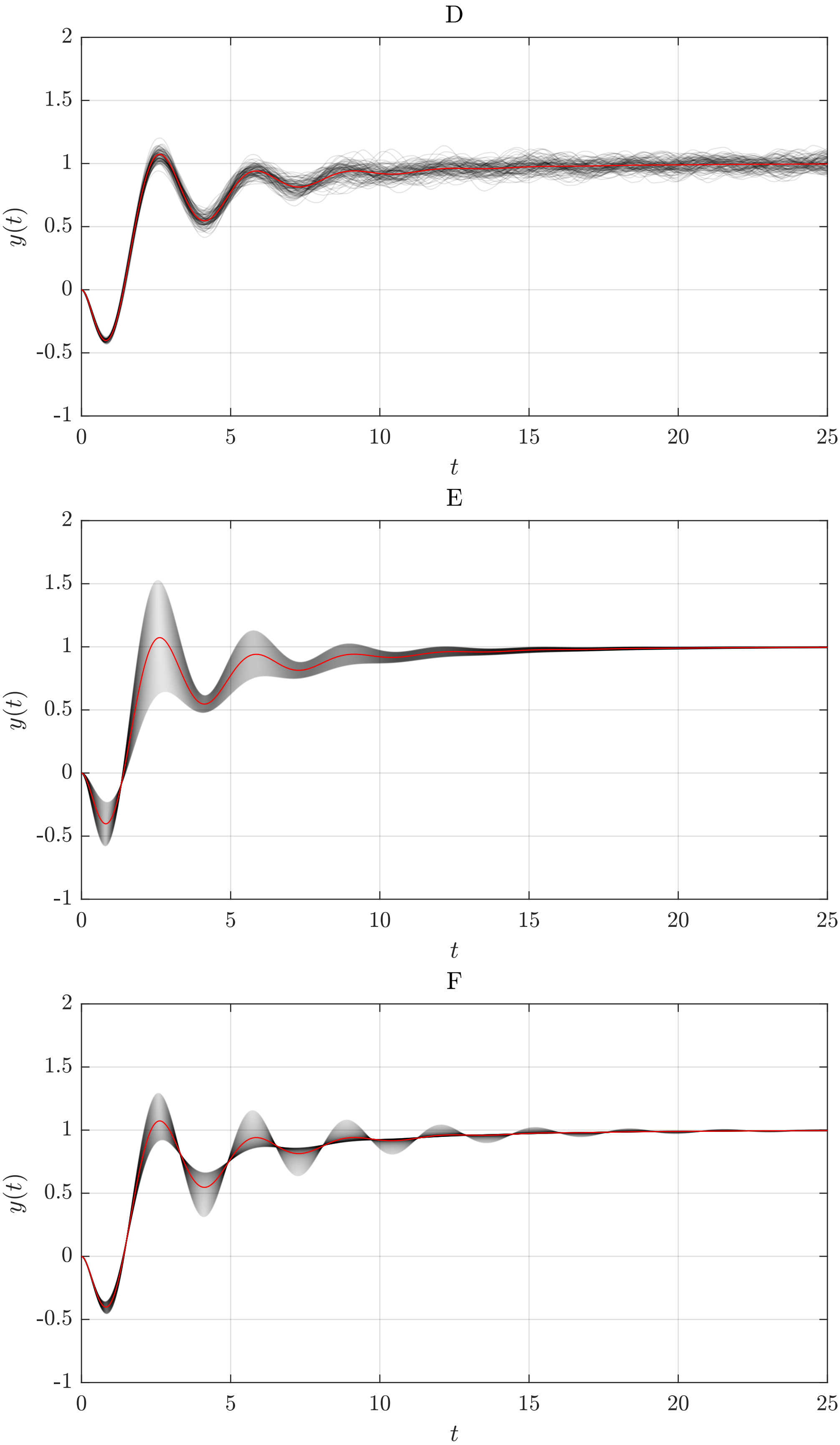
6) $\zeta = \hat{\zeta} \pm 50\%$

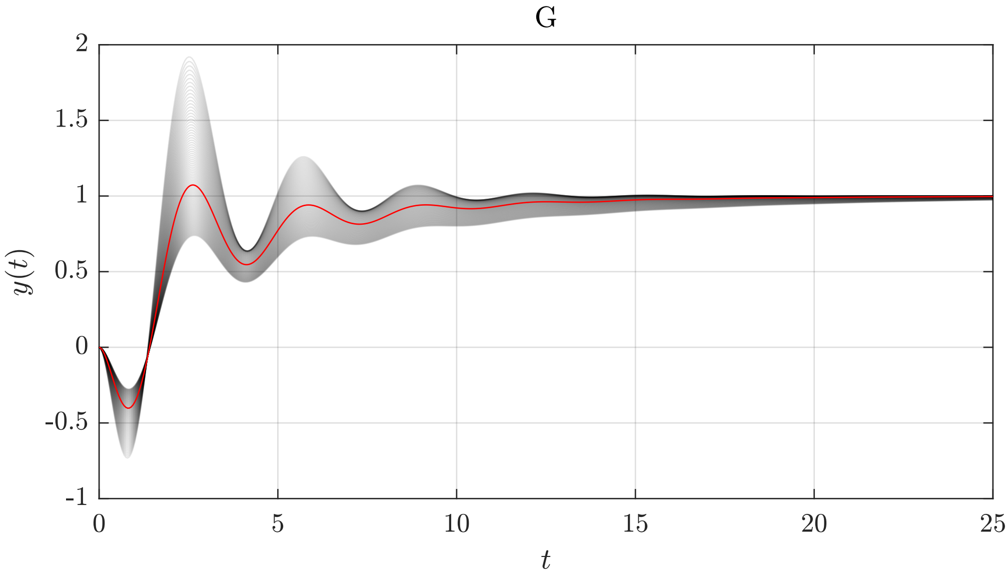
7) $\omega_n = \hat{\omega}_n \pm 50\%$

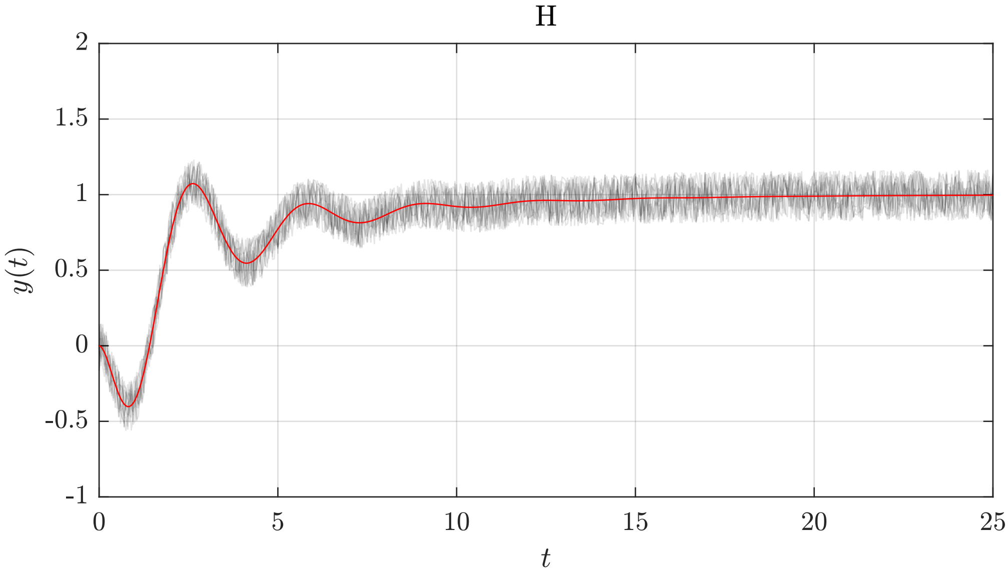
8) $b_1 = \hat{b}_1 \pm 50\%$

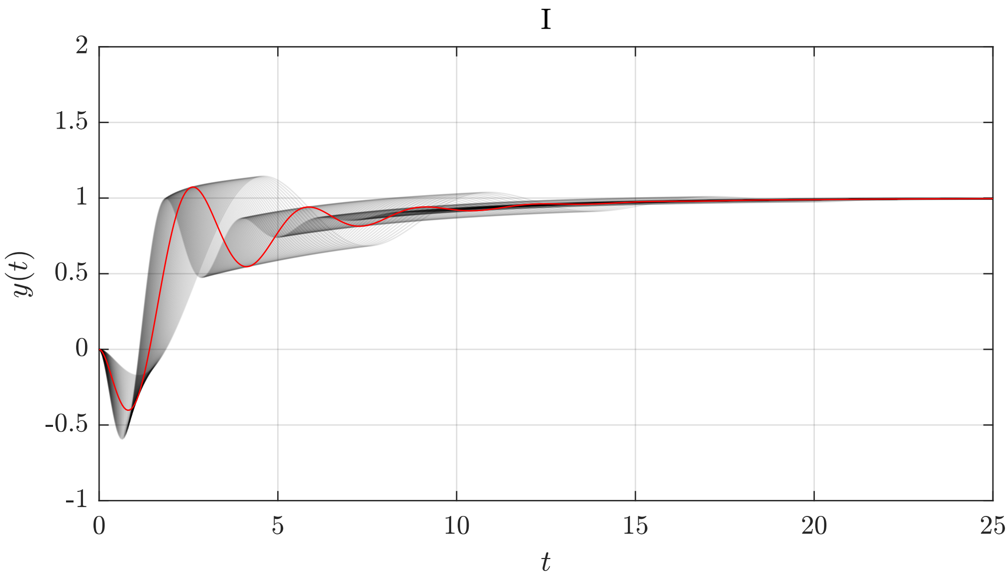
9) $b_2 = \hat{b}_2 \pm 50\%$


A
B
C
y(t)
t
2
1.5
1
0.5
0
-0.5
-1
0
5
10
15
20
25


D
y(t)
t
E
y(t)
t
F
y(t)
t


G
2
1.5
1
0.5
0
-0.5
-1
$y(t)$
0
5
10
15
20
25
t


H
2
1.5
1
0.5
0
-0.5
-1
$y(t)$
0
5
10
15
20
25
t


I
2
1.5
1
0.5
0
-0.5
-1
$y(t)$
0
5
10
15
20
25
t

MATLAB Assignments

7 Breakaway Points

Write a MATLAB function that takes a transfer function as input and,

a) Returns the candidate breakaway points. Ckeck your function by sketching root-locus plot of following transfer functions using *rlocus* command.

$$G_1(s) = \frac{s^2 + 2s + 2}{s(s^2 + 0.25)} \ , \quad G_2(s) = \frac{s^2}{(s^2 - 1)^2}$$

Hint: you might want to use *syms*, *diff*, *vpasolve* commands.

b) How can we plot complementary root-locus using matlab functions?

c) Manipulate your function to return the following two outputs

1) The points which belong to root-locus
2) The points which belong to complementary root-locus
3) The candidate points which do not belong to root-locus or its complementary plot