In the name of allah

# Linear Control Systems

Hw 02

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Fall 1402

## Theory Assignments

سؤال 1)

$$\dot{x}_1 = x_1 + (-2)x_2 + 2u$$

$$\dot{x}_2 = (-5)x_2 + 4x_1 + u$$

$$y = x_1 + x_2$$

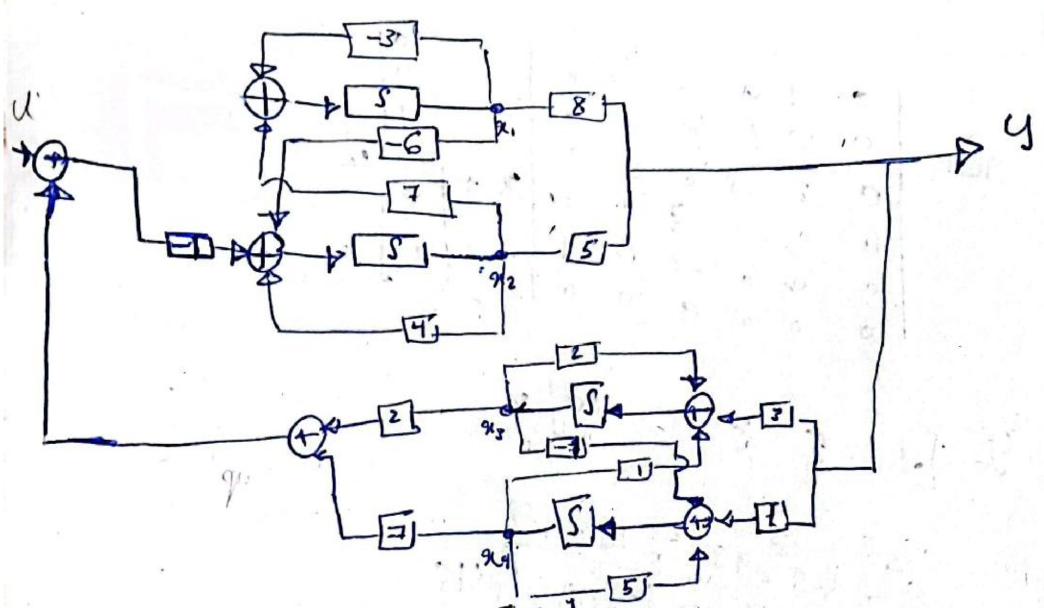
$$A = \begin{bmatrix} 1 & -2 \\ 4 & -5 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}, D = 0$$

① $\to$ $e^{At} = I + At + \frac{A^2t^2}{2} + \_\_\_$

2 $\to$ $e^{At} = \mathcal{L}^{-1}\{(SI - A)^{-1}\} = \mathcal{L}^{-1}\begin{bmatrix} S-1 & 2 \\ -4 & S+5 \end{bmatrix}^{-1} = \frac{1}{(S-1)(S+5)+8}\begin{bmatrix} S+5 & -2 \\ 4 & S-1 \end{bmatrix}$

$$= \mathcal{L}^{-1}\begin{bmatrix} \frac{S+5}{(S+1)(S+3)} & \frac{-2}{(S+1)(S+3)} \\ \frac{4}{(S+1)(S+3)} & \frac{S-1}{(S+1)(S+3)} \end{bmatrix} = \begin{bmatrix} \frac{2}{S+1} + \frac{-1}{S+3} & \frac{1}{S+1} + \frac{-1}{S+3} \\ \frac{2}{S+1} + \frac{-2}{S+3} & \frac{-1}{S+1} + \frac{2}{S+3} \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{-t} - e^{-3t} & e^{-t} - e^{-3t} \\ 2e^{-t} - 2e^{-3t} & -e^{-t} + 2e^{-3t} \end{bmatrix} u(t),$$




$$\dot{x} = \begin{bmatrix} -3 & 7 & 0 & 0 \\ -6 & 4 & -2 & 7 \\ 24 & 15 & 2 & 1 \\ 8 & 5 & -1 & 5 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 8 \\ 5 \\ 0 \\ 0 \end{bmatrix}^T x$$

حالت ج)

$$\Rightarrow \dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -2 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -2 \\ 8 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & -5 & 3 & 1 & 0 & 0 \end{bmatrix} x$$

$$H(s) = \frac{s^4 + 6s^3 + 13s^2 + 12s + 4}{s^5 + 6s^4 + 13s^3 + 12s^2 + 4s}$$

$$\rightarrow \dot{x} = \begin{bmatrix} -6 & -3{,}25 & -1{,}5 & -0{,}5 & 0 \\ 4 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0{,}25 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0{,}5 & 0{,}75 & 0{,}8125 & 0{,}75 & 1 \end{bmatrix} x$$

* یک حالت وابسته داریم ($x_6$) که در ماتریس A اولیه، کاملاً صفر بود
به همین خاطر یک ~~...~~ حالت کاملاً حذف می‌گردد

Scanned with CamScanner

```
A1 = [0 1 0 ;0 0 1;0 -2 -3];
A2 = [-3 1 0;-2 0 1;0 0 0];
A = [A1 zeros([3 3]);zeros([3 3]) A2];

B = [0 0 1 -2 8 1]';
C = [1 -5 3 1 0 0];
D = [0];
sys = ss(A,B,C,D)
```

```
sys =
```

```
  A =
        x1  x2  x3  x4  x5  x6
   x1    0   1   0   0   0   0
   x2    0   0   1   0   0   0
   x3    0  -2  -3   0   0   0
   x4    0   0   0  -3   1   0
   x5    0   0   0  -2   0   1
   x6    0   0   0   0   0   0

  B =
        u1
   x1    0
   x2    0
   x3    1
   x4   -2
   x5    8
   x6    1

  C =
        x1  x2  x3  x4  x5  x6
   y1    1  -5   3   1   0   0

  D =
        u1
   y1    0

Continuous-time state-space model.
```

```
sys_tf=tf(sys);
```

```
sys_tfs = tf([1 6 13 12 4],[1 6 13 12 4 0])
```

```
sys_tfs =

    s^4 + 6 s^3 + 13 s^2 + 12 s + 4
  -----------------------------------
  s^5 + 6 s^4 + 13 s^3 + 12 s^2 + 4 s

Continuous-time transfer function.
```

```
sys_s = ss(sys_tfs)
```

```
sys_s =

  A =
            x1      x2      x3      x4      x5
   x1       -6   -3.25    -1.5    -0.5       0
   x2        4       0       0       0       0
   x3        0       2       0       0       0
   x4        0       0       1       0       0
   x5        0       0       0    0.25       0

  B =
        u1
   x1    2
   x2    0
   x3    0
   x4    0
   x5    0
```

```
C =
         x1       x2       x3       x4       x5
 y1     0.5     0.75   0.8125     0.75        1

D =
      u1
 y1    0

Continuous-time state-space model.
```

$$y(t) = C e^{At} x(0) + \int_0^t C e^{A\tau} B u(t-\tau)\, d\tau \qquad (4\ \text{[illegible]}$$

$$e^{At} = \mathcal{L}^{-1}\left[(SI - A)^{-1}\right] = \begin{bmatrix} S+1 & +2 \\ -1 & S+4 \end{bmatrix}^{-1} = \begin{bmatrix} S+4 & -2 \\ 1 & S+1 \end{bmatrix} \frac{1}{\cancel{(S+1)(S+4)+2}}$$

$$(S+2)(S+3)$$

$$\therefore = \mathcal{L}^{-1}\begin{bmatrix} \dfrac{S+4}{(S+2)(S+3)} & \dfrac{2}{(S+2)(S+3)} \\ \dfrac{1}{(S+2)(S+3)} & \dfrac{S+1}{(S+2)(S+3)} \end{bmatrix} = \begin{bmatrix} \text{—} & \text{—} \\ e^{-2t} - e^{-3t} & -e^{-2t} + 2e^{-3t} \end{bmatrix}$$

$$\frac{1}{S+2} - \frac{1}{S+3} \qquad \frac{-1}{S+2} + \frac{2}{S+3}$$

$$\cancel{u(t)}\, u(t-\tau)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \text{—} & \text{—} \\ e^{-2t} - e^{-3t} & \text{—} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \int_0^t \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \text{—} & \text{—} \\ \text{—} & -e^{-2t} + 2e^{-3t} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} d\tau$$

$$(e^{-2t} - e^{-3t})u(t) + u(t) \int_0^t -e^{-2\tau} + 2e^{-3\tau}\, d\tau$$

$$\frac{e^{-2t}}{2} - \frac{2}{3} e^{-3t}$$

$$= \left(e^{-2t} - e^{-3t} + \frac{e^{-2t}}{2} - \frac{2}{3} e^{-3t} + \frac{1}{6}\right) u(t)$$

$$= \left(\frac{3}{2} e^{-2t} - \frac{5}{3} e^{-3t} + \frac{1}{6}\right) u(t)$$

(5

$$s_1'' + 3s_2' + 2s_1 = 3u' - u \qquad s^2 S_1 + 3s S_2 + 2S_1 = 3sU - U$$

$$s_2' - 2s_1' + s_2 = 0 \quad \longrightarrow \quad sS_2 - 2sS_1 + S_2 = 0$$

$$y = s_1' + s_2' \qquad Y = sS_1 + sS_2$$

$$S_2 = \frac{-2sS_1}{s+1}$$

$$\Rightarrow U = \frac{1}{3s-1}\left(s^2 + \frac{-6s^2}{s+1} + 2\right) S_1$$

$$Y = \left(s - \frac{2s^2}{s+1}\right) S_1$$

$$\Rightarrow \frac{Y}{U} = \frac{\frac{-s^2+s}{s+1}(3s-1)}{\frac{s^3-5s^2+2s+2}{s+1}} = \frac{-3s^3+4s^2-s}{s^3-5s^2+2s+2}$$

$$= \boxed{-3} + \frac{-11s^2+6s+6}{s^3-5s^2+2s+2}$$

①

$$z''' - 5z'' + 2z' + 2z = u \qquad z = x_1$$

$$z'' - z' + 6z = Y \qquad z' = x_2$$

$$z'' = x_3$$

$$\rightarrow x' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -2 & 5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -11 & -6 & 6 \end{bmatrix} x + \begin{bmatrix} -3 \end{bmatrix} u$$

# MATLAB Assignments

```
clc
close all
clear all
```

# 6 Canonical Form Representation of Transfer Functions

```
% Define a transfer function
num = [11 12 13];
den = [1 2 3 4 5];
sys = tf(num, den)
```

```
sys =

        11 s^2 + 12 s + 13
  -----------------------------
  s^4 + 2 s^3 + 3 s^2 + 4 s + 5

Continuous-time transfer function.
```

```
[A, B, C] = canonicalForms(sys, "C")
```

```
A = 4×4
     0     1     0     0
     0     0     1     0
     0     0     0     1
    -5    -4    -3    -2
B = 4×1
     0
     0
     0
     1
C = 1×4
    13    12    11     0
```

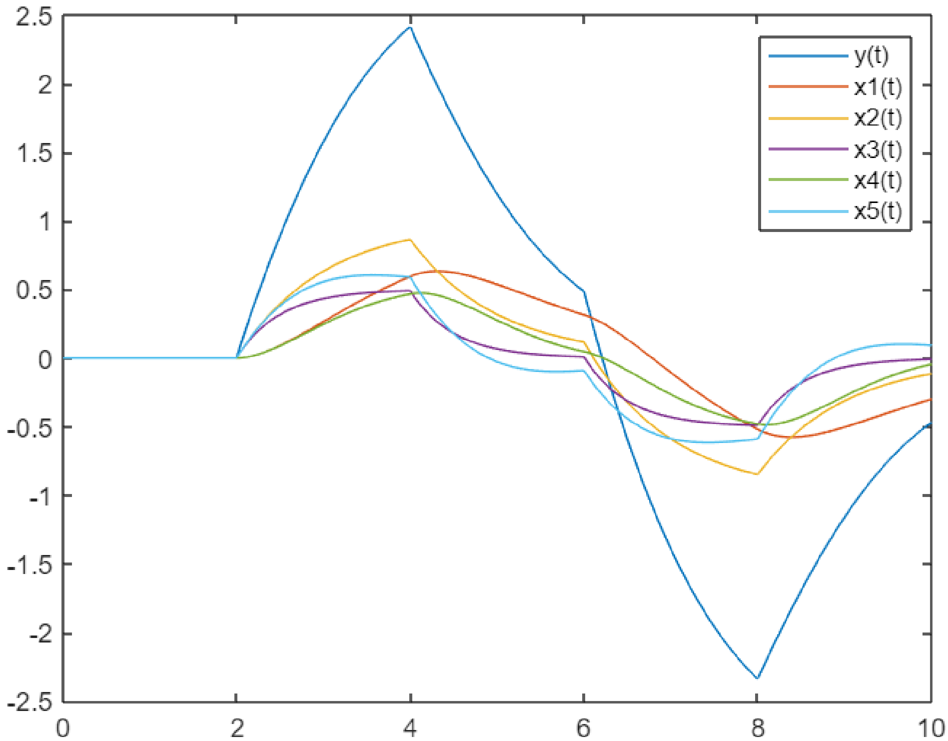
```
[A, B, C] = canonicalForms(sys, "O")
```

```
A = 4×4
    -2     1     0     0
    -3     0     1     0
    -4     0     0     1
    -5     0     0     0
B = 4×1
     0
    11
    12
    13
C = 1×4
     1     0     0     0
```

# 7 Response of LTI State-Space Models using lsim

```
A = [-1 1 0 0 0; 0 -1 0 0 0; 0 0 -2 0 0; 0 0 0 -1 1; 0 0 0 -1 -1];
B = [0 1 1 0 1]';
C = [1 1 1 1 0];
D = [0];
```

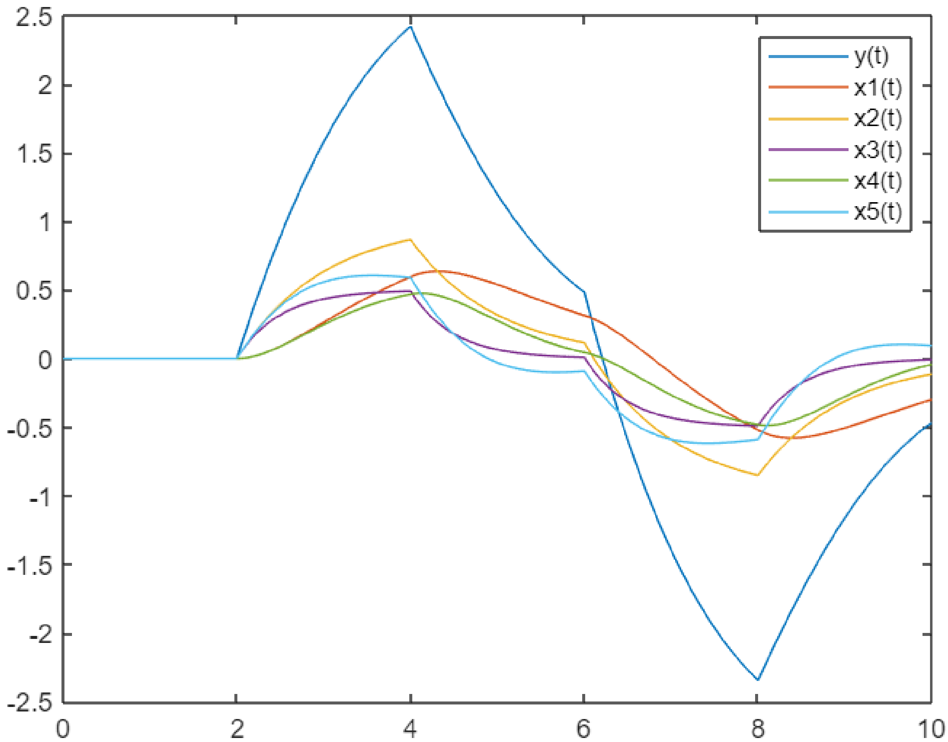
```
sys = ss(A, B, C, D);
t = 0:0.01:10;
u =  Step01(t-2)-Step01(t-4)-Step01(t-6)+Step01(t-8);
x0 = [0, 0, 0, 0, 0];
[y, tOut, x] = lsim(sys, u, t, x0);

figure
plot(t,y,t,x)
legend("y(t)","x1(t)","x2(t)","x3(t)","x4(t)","x5(t)")
```




# 8 Response of LTI State-Space Models using Euler Method

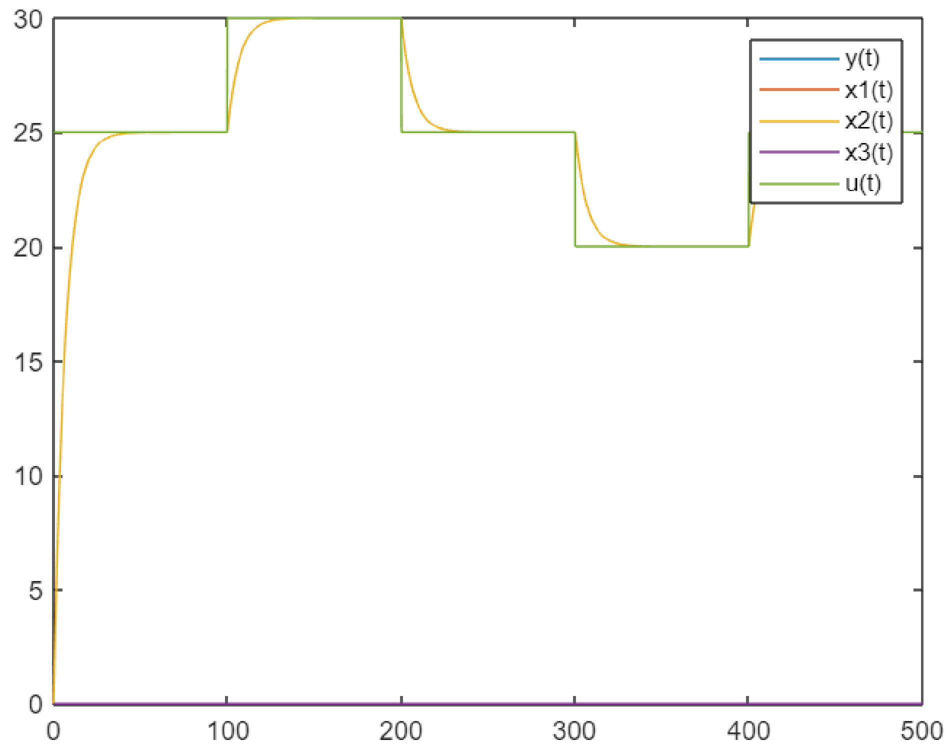
```
A = [-1 1 0 0 0; 0 -1 0 0 0; 0 0 -2 0 0; 0 0 0 -1 1; 0 0 0 -1 -1];
B = [0 1 1 0 1]';
C = [1 1 1 1 0];
D = [0];
Ts = 0.01;
t = 0:0.01:10;
u =  Step01(t-2)-Step01(t-4)-Step01(t-6)+Step01(t-8);
X = zeros([length(A), length(t)]);
x0 = [0;0;0;0;0];
X(:, 1) = x0;
Y = zeros(size(t));
Y(1) = C * x0;
for i=2:length(t)
X(:, i) = X(:, i-1) + Ts * (A * X(:, i-1) + B * u(i-1));
```

```
Y(i) = C * X(:, i) + D * u(i);
end

figure
plot(t,Y,t,X)
legend("y(t)","x1(t)","x2(t)","x3(t)","x4(t)","x5(t)")
```

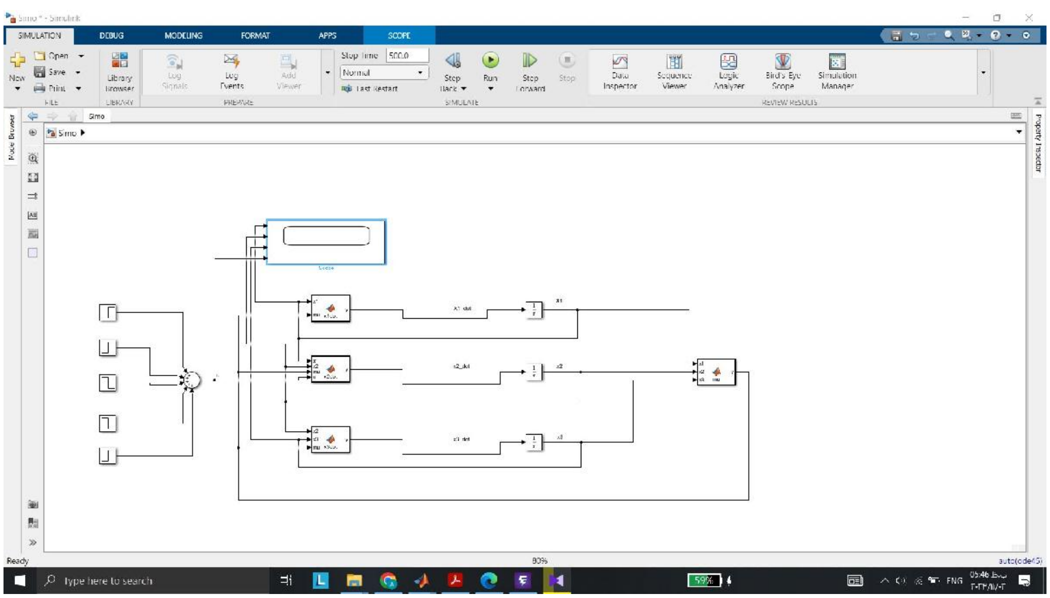



# 9 Response of Nonlinear State-Space Models

```
Ts = 0.01;
t = 0:0.01:500;
x0 = [0; 0; 0];
X = zeros([length(x0), length(t)]);
X(:, 1) = x0;
u = 25+5*(Step01(t-100)-Step01(t-200))-5*(Step01(t-300)-Step01(t-400));

for i=2:length(t)
X(:, i) = X(:, i-1) + Ts * f(X(:, i-1), u(i-1));
Y(i) = g(X(:, i));
end
figure
plot(t,Y,t,X,t,u)
legend("y(t)","x1(t)","x2(t)","x3(t)","u(t)")
```

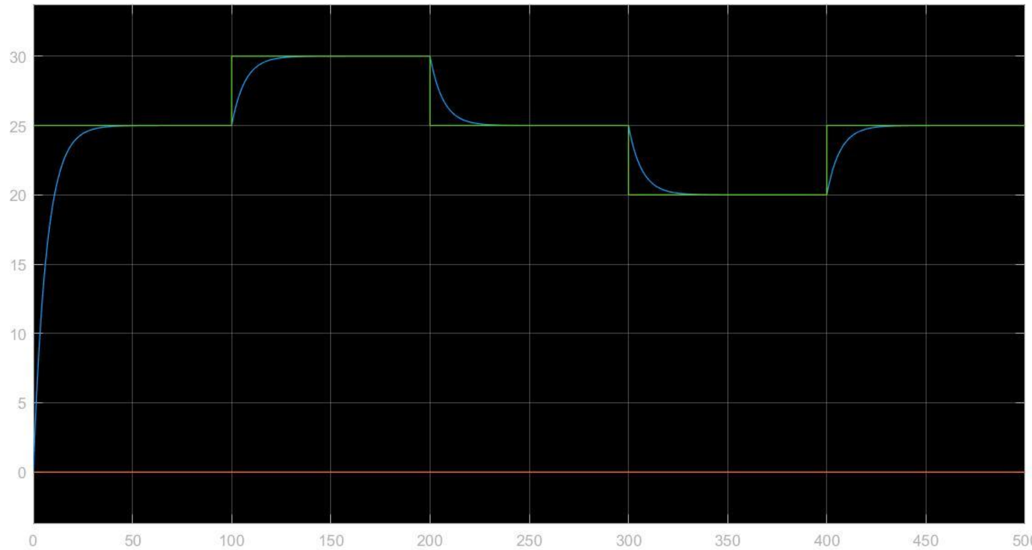


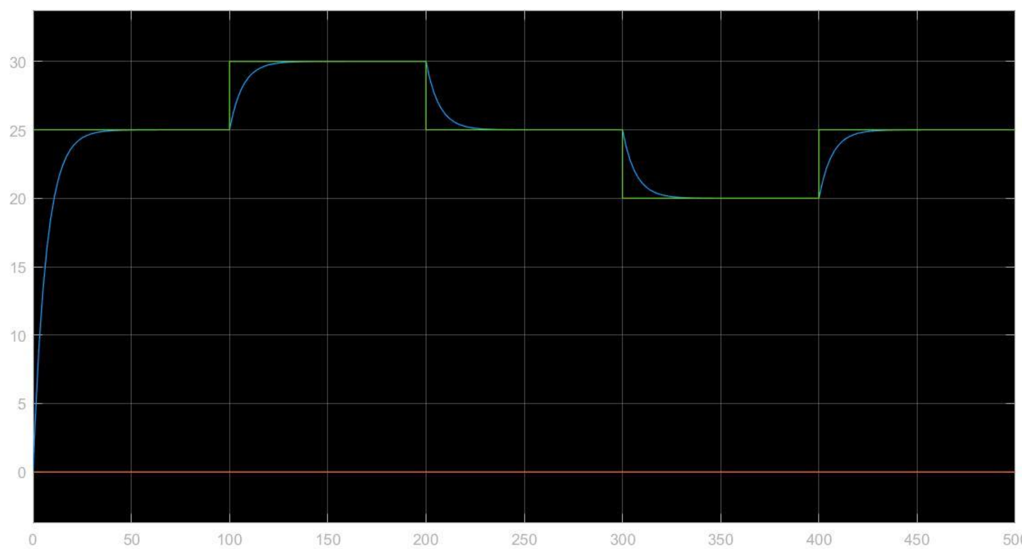

## 10 State-Space Implementation using Simulink

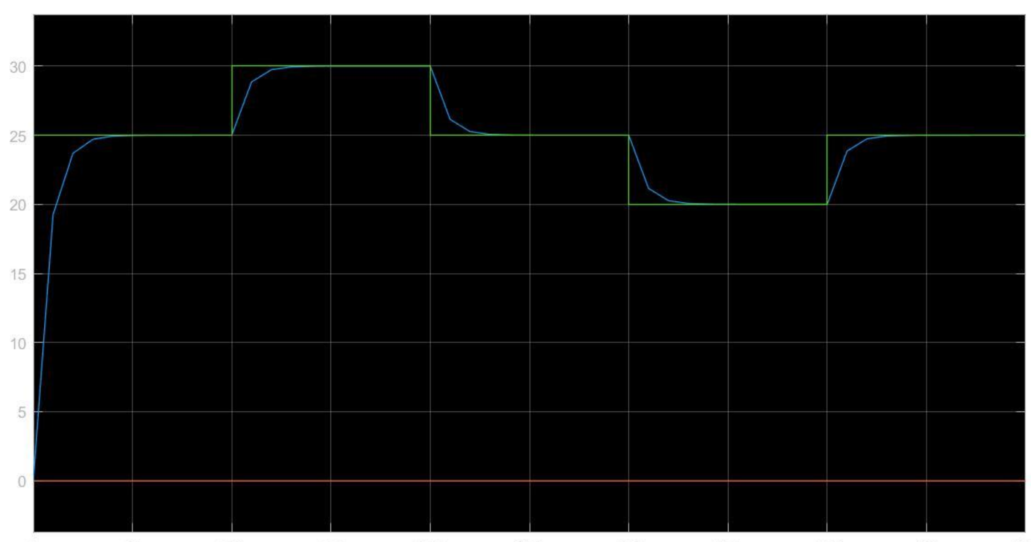


min step size(slow=0.1 suitable=1 fast=10)

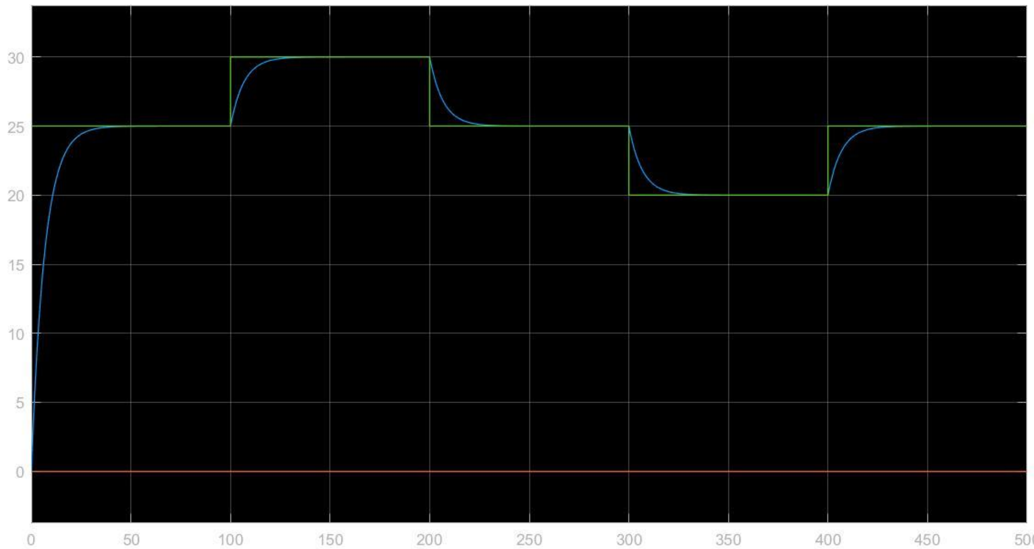
ode 45 (slow suitable fast)

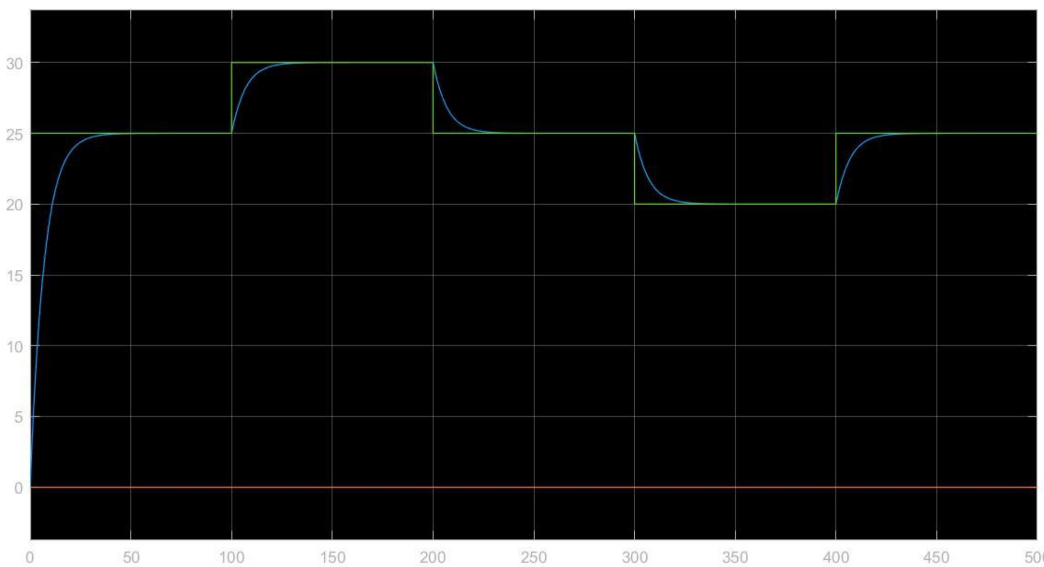

30
25
20
15
10
5
0
0
50
100
150
200
250
300
350
400
450
500

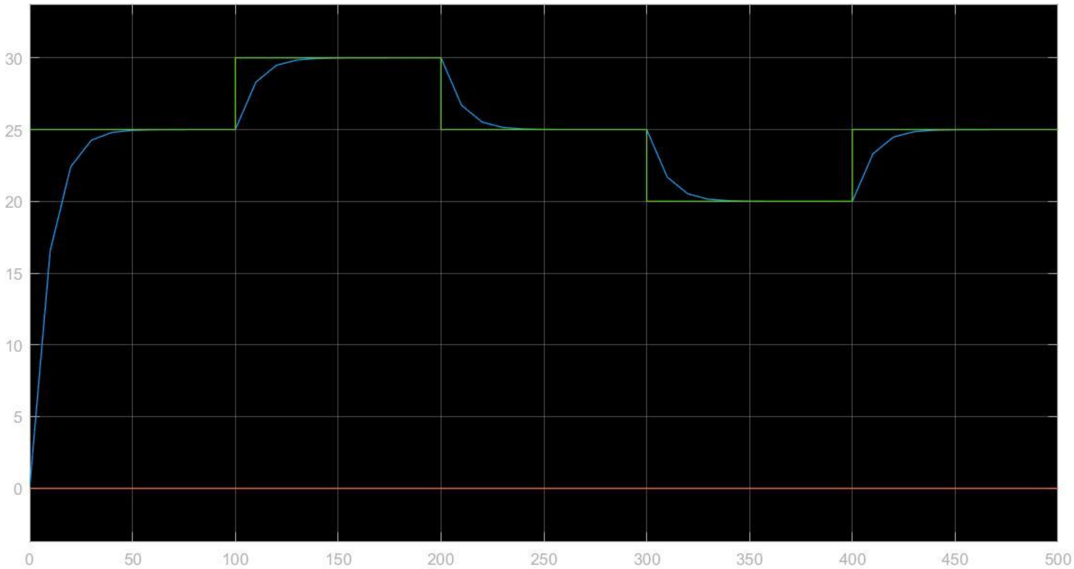

30
25
20
15
10
5
0
0
50
100
150
200
250
300
350
400
450
500


30
25
20
15
10
5
0
0
50
100
150
200
250
300
350
400
450
500

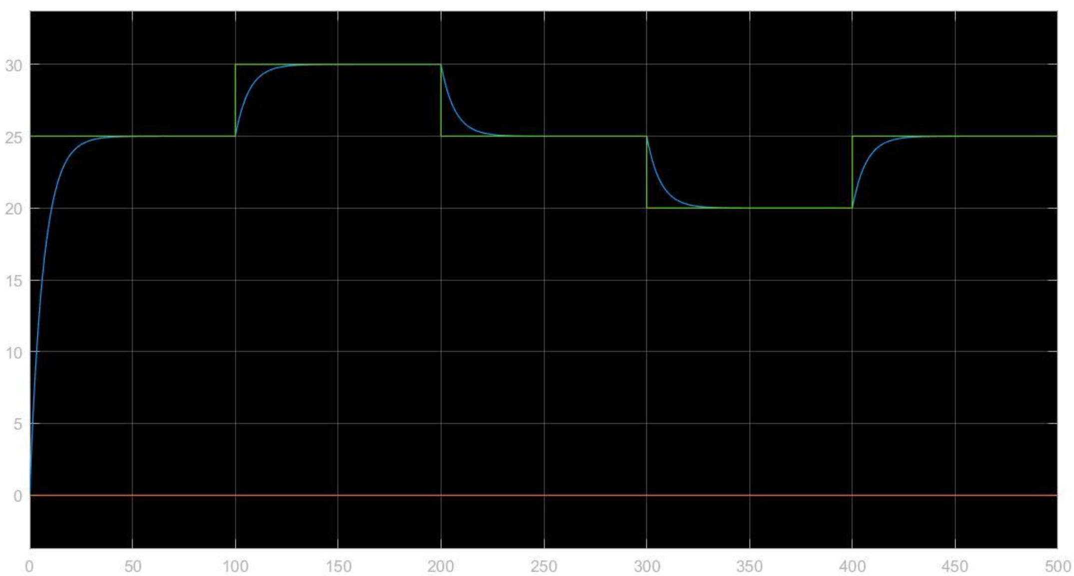
ode 15 (slow suitable fast)

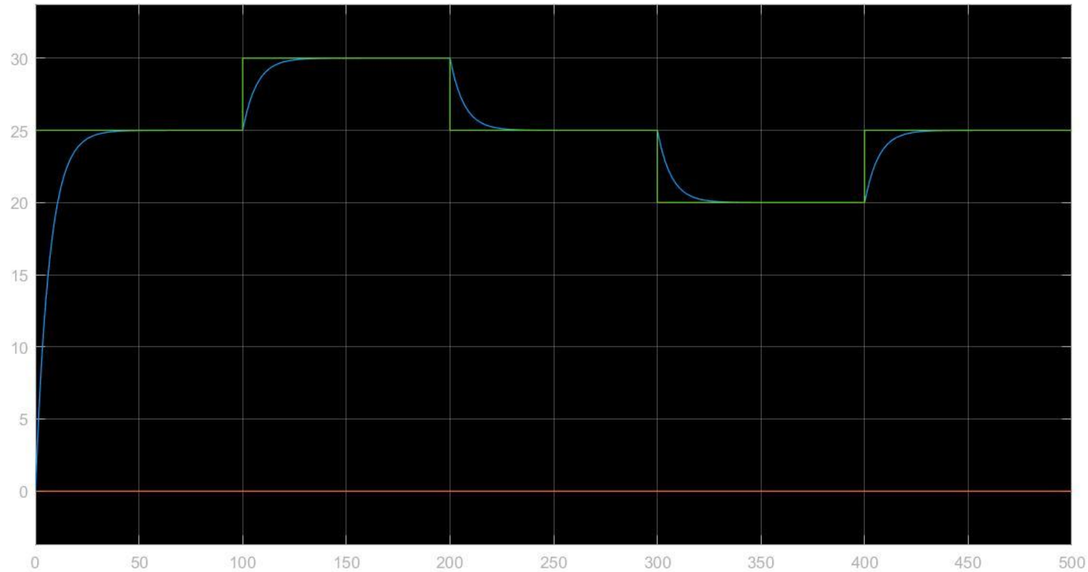


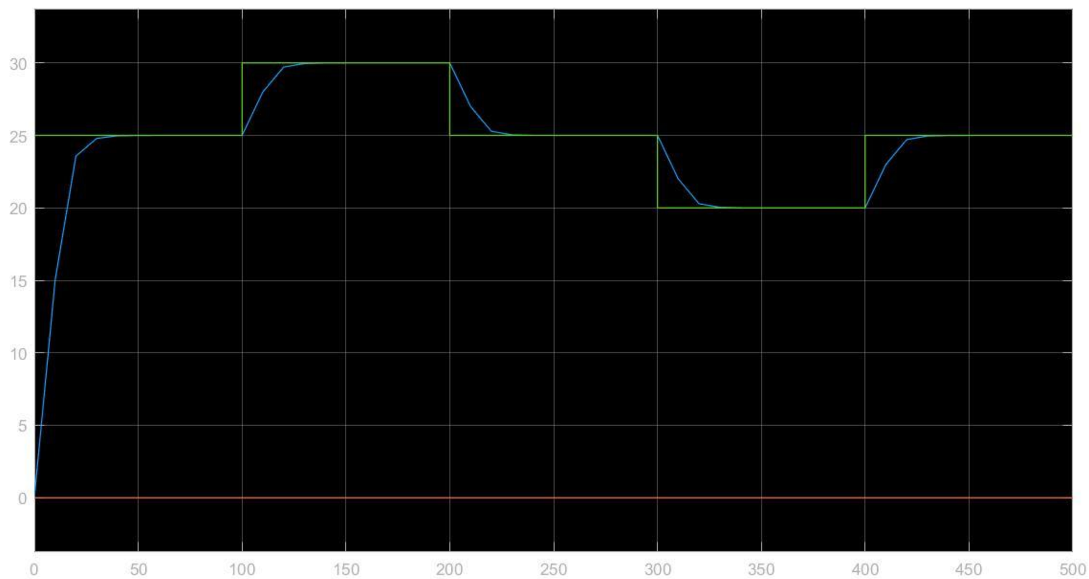






ode 23s (slow suitable fast)






```
function [A, B, C] = canonicalForms(sys, form)

if ~isproper(sys)
    error("The input transfer function must be strictly proper.")
end

switch upper(form)
    case "C"
        [A, B, C] = controlabe(sys);
    case "O"
        [A, B, C] = observable(sys);
```

```
    otherwise
        error("Invalid canonical form. Please enter 'C' for controllable" + ...
            " canonical form or 'O' for observable canonical form.")
end
end
function [A,B,C] = controlabe(sys)
a = sys.Denominator{1,1};
n = size(a,2);
a = a(2:end);
b = sys.Numerator{1,1};
b = b(2:end);

n = n-1 ;
a = flip(a);
b = flip(b);

A = eye(n);
A = circshift(A,1,2);
A(n,1) = 0;
A(n,:) = -a;

B = zeros([n 1]);
B(n) = 1;

C = b;

end
function [A,B,C] = observable(sys)
a = sys.Denominator{1,1};
n = size(a,2);
a = a(2:end);
b = sys.Numerator{1,1};
b = b(2:end);

n = n-1;
A = eye(n);
A = circshift(A,1,2);
A(n,1) = 0;
A(:,1) = -1 * a';

B = zeros([n 1]);
B = b';

C = zeros([1 n]);
C(1) = 1;

end


function y = g(x, u)
y =  x(3);
end
function dxdt = f(x, u)
```

```
mu = ( 0.48*(1-0.02*x(3))*x(2) )/( 1.2+x(2)+( (x(2)^2)/22 ) );

dxdt = [-0.15*x(1)+mu*x(1);
        0.15*(u - x(2))-2.5*mu*x(1);
        -0.15*x(3)+(2.2*mu+0.2)*x(1)];
end


function [u] = Step01(C)
u = C;
s = length(C);
for i=1:s
    if C(i)>=0
        u(i)=1;
    else
        u(i)=0;
    end
end
end
function [u] = Step00(C)
u = C;
s = length(C);
for i=1:s
    if C(i)>0
        u(i)=1;
    else
        u(i)=0;
    end
end
end
```