In the name of allah

Linear Control Systems

Hw 01

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Fall 1402

Theory Assignments

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سوال 1

1) $T_1(s) = \dfrac{s+6}{s(s^2+4s+3)} = \dfrac{2}{s} + \dfrac{\frac{1}{2}}{s+3} + \dfrac{-\frac{5}{2}}{s+1}$

$(s+1)(s+3)$

$\Rightarrow 2u(t) + \frac{1}{2}u(t)e^{-3t} - \frac{5}{2}u(t)e^{-t}$

2) $T_2(s) = \dfrac{5}{s(s^2+4s+5)} = \dfrac{5}{s((s+2)^2+1)} = \dfrac{a}{s} + \dfrac{bs+c}{(s+2)^2+1}$

$as^2 + 4as + 5a + bs^2 + cs = 0$

$a = 1$

$a = -b \Rightarrow b = -1$

$c = -4a \Rightarrow c = -4$

$\dfrac{1}{s} + \dfrac{-s-4}{(s+2)^2+1} = \dfrac{1}{s} - \dfrac{s+2}{(s+2)^2+1} - \dfrac{2}{(s+2)^2+1} \Rightarrow u(t) - e^{-2t}(\cos(t) + 2\sin(t))$

3) $T_3(s) = \dfrac{s^2+2s+3}{s^3+6s^2+12s+8} = \dfrac{s^2+2s+3}{(s+2)^3} = \dfrac{a}{s+2} + \dfrac{b}{(s+2)^2} + \dfrac{c}{(s+2)^3}$

$u(t)(e^{-2t} - te^{-2t})$

$a = 1$

$b = -2$

$c = 3$

$a(s^2+4s+4) + b(s+2) + c$

$\rightarrow u(t)e^{-2t}(1 - 2t + \frac{3}{2}t^2)$

تمرین 6

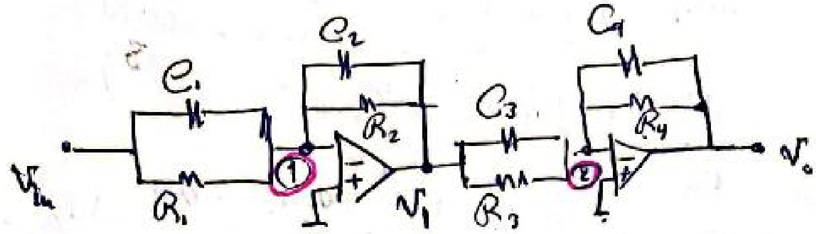
2 سوال

$F(s) = \dfrac{2s+1}{s(s+1)(s+2)}$ $\quad f(0^+) = \lim_{s\to\infty} sF(s) = 0$ $\left(\dfrac{2s^2}{s^3}\right)$

$\mathcal{L}\, f'(t) = sF(s) - f(0) = \dfrac{2s+1}{(s+1)(s+2)}$ $\quad f'(0) = \lim_{s\to\infty} sF(s) = 2$ $\left(\dfrac{2s^2}{s^2}\right)$

$\mathcal{L}\, f''(t) = s^2F(s) - sf(0) - f'(0) = \dfrac{2s^2+s}{(s+1)(s+2)} - 2$ $\quad f''(0) = \lim_{s\to\infty} sF(s) = -5$ $\left(\dfrac{-5s^2}{s^2}\right)$

3 سؤال)



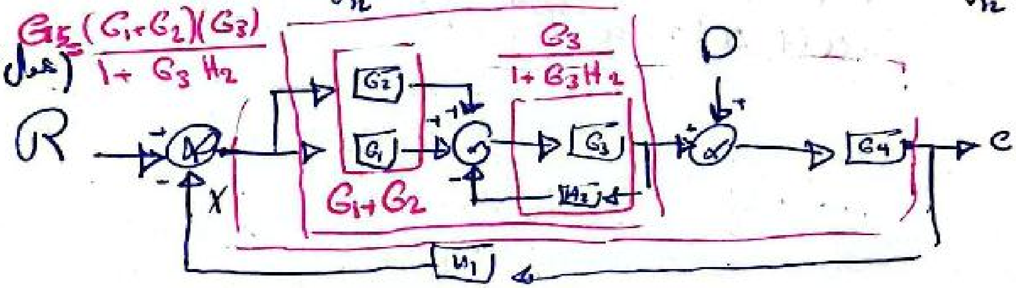
KCL 1) $V_1\left(\frac{1}{R_2}+C_2 s\right) = -V_{in}\left(\frac{1}{R_1}+C_1 s\right)$

$$\frac{V_1}{V_{in}} = \frac{\frac{1}{R_1}+C_1 s}{\frac{1}{R_2}+C_2 s} \Rightarrow \text{KCL 2)}\ \frac{V_o}{V_1} = \frac{\frac{1}{R_3}+C_3 s}{\frac{1}{R_4}+C_4 s}$$

$$\Rightarrow G(s) = \frac{V_o}{V_{in}} = \frac{\frac{1}{R_1}+C_1 s}{\frac{1}{R_2}+C_1 s} \cdot \frac{\frac{1}{R_3}+C_3 s}{\frac{1}{R_4}+C_4 s} = \frac{C_1C_3 s^2+\left(\frac{C_1}{R_3}+\frac{C_3}{R_1}\right)s+\frac{1}{R_1R_3}}{C_2C_4 s^2+\left(\frac{C_4}{R_2}+\frac{C_2}{R_4}\right)s+\frac{1}{R_2R_4}}$$

$G_5 = \frac{(G_1+G_2)(G_3)}{1+G_3H_2}$

4 سؤال)



$(G_5X+D)\,G_4 = C$

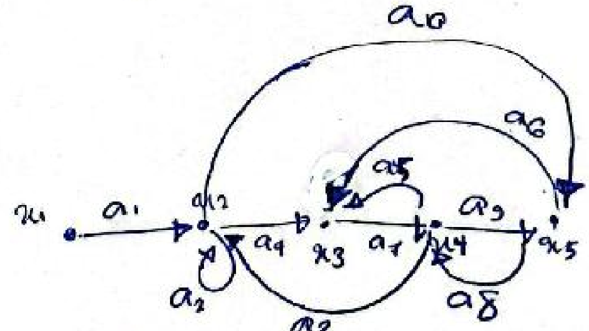
$X = R - CH_1$

$$\Rightarrow G_5G_4R + G_4D = (1+G_5G_4H_1)C$$

$$G_5 = \frac{(G_1+G_2)\,G_3}{1+G_3H_2}$$

$$C(s) = \frac{G_5G_4}{1+G_5G_4H_1}R + \frac{G_4}{1+G_5G_4H_1}D$$

5 سؤال)



Path: P_1 a_1a_{10}

P_2 $a_1a_4a_7a_9$

Loop: L_1 a_2

L_2 $a_4a_7a_3$

L_3 a_7a_5

L_4 $a_7a_9a_6$ L_7 $a_{10}a_7a_6a_3$

L_5 a_9a_8, L_6 $a_{10}a_8a_3$

non-touching L_{13} a_2, a_5a_7

L_{15} a_2, a_9a_8

L_{14} $a_2, a_7a_9a_6$

$$\Delta = 1 - \sum_1^7 L_i + \sum_3^5 L_{1i}$$

$\Delta_1 = 1-(L_3)$

$\Delta_2 = 1$

$$\Rightarrow \frac{x_5}{x_1} = \frac{\sum_1^2 \Delta_i P_i}{\Delta}$$

6 $U(s)$) $T(s) = \dfrac{1}{(\frac{s}{5}+1)(\frac{s}{10}+1)}$, $u(t) = \begin{cases} 1 & 2 \le t \le 4 \\ -1 & 6 \le t \le 8 \\ 0 & O.W. \end{cases} = u(t-2) - u(t-4)$
$- u(t-6) + u(t-8)$

$$U(s) = \frac{1}{s} \underbrace{\left(e^{-2s} - e^{-4s} - e^{-6s} + e^{-8s}\right)}_{f(s)}$$

$$\Rightarrow Y(s) = T(s)\,U(s) = \frac{f(s)}{s(\frac{s}{5}+1)(\frac{s}{10}+1)} = \underbrace{\left(\frac{1}{s} + \frac{-0.4}{\frac{s}{5}+1} + \frac{-0.2}{\frac{s}{10}+1}\right)}_{u(t)(1-2e^{-5t}-2e^{-10t})} f(s)$$

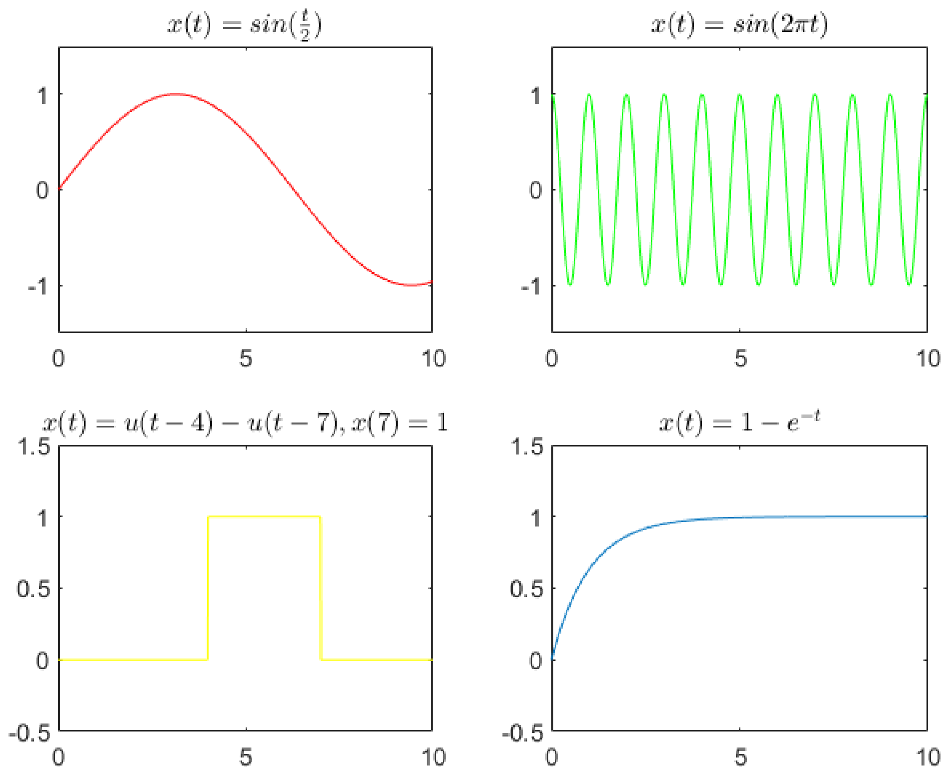
$$\xrightarrow[\text{شیفت زمانی}]{f(s)} y(t) = u(t-2)\left(1 - 2e^{-5t-10} - 2e^{-10t-20}\right)$$
$$- u(t-4)\left(1 - 2e^{-5t-20} - 2e^{-10t-40}\right)$$
$$- u(t-6)\left(1 - 2e^{-5t-30} - 2e^{-10t-60}\right)$$
$$+ u(t-8)\left(1 - 2e^{-5t-40} - 2e^{-10t-80}\right)$$

MATLAB

```
clear all
close all
```

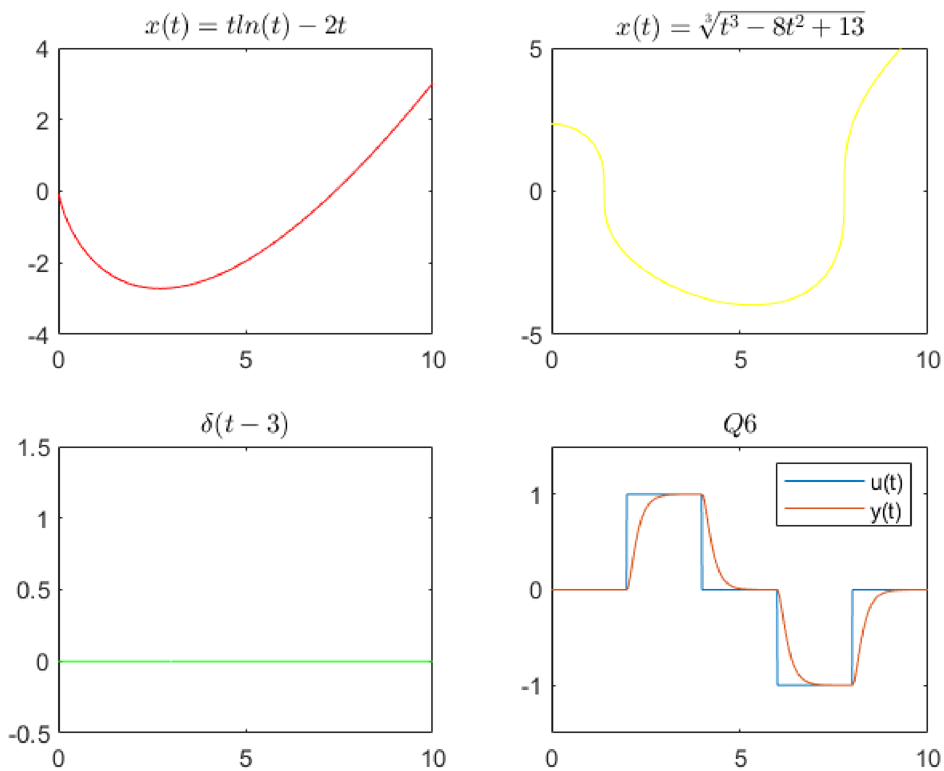
7 Continues-Time Signals

```
t_start = 0;
t_end = 10;

figure
subplot(2,2,1)
num = 1001;
t = linspace(t_start,t_end,num);
x1 =  sin(t./2);
plot(t,x1,'Color','r')
title('$x(t)=sin(\frac{t}{2}) $','Interpreter','latex','FontSize',10)
xlim([0 10])
ylim([-1.5 1.5])


subplot(2,2,2)
num = 1001;
t = linspace(t_start,t_end,num);
x2 =  cos(2*pi*t);
plot(t,x2,'Color','g')
title('$x(t)=sin(2\pi t) $','Interpreter','latex','FontSize',10)
xlim([0 10])
ylim([-1.5 1.5])


subplot(2,2,3)
num = 1001 ;
t = linspace(t_start,t_end,num);
x3 =  Step01(t-4)-Step00(t-7);
plot(t,x3,'Color','y')
title('$x(t)=u(t-4)-u(t-7),x(7)=1 $','Interpreter','latex','FontSize',10)
xlim([0 10])
ylim([-0.5 1.5])



subplot(2,2,4)
num = 1001;
t = linspace(t_start,t_end,num);
x4 =  1 -exp(-t);
plot(t,x4)
title('$x(t)=1-e^{-t}$','Interpreter','latex','FontSize',10)
xlim([0 10])
ylim([-0.5 1.5])
```




```
figure
subplot(2,2,1)
num = 1001 ;
t = linspace(t_start,t_end,num);
x5 =  t.*log(t)-2*t;
plot(t,x5,'Color','r')
title('$x(t)=tln(t)-2t$','Interpreter','latex','FontSize',10)
xlim([0 10])
ylim([-4 4])



subplot(2,2,2)
num = 1001;
t = linspace(t_start,t_end,num);
x6 =  nthroot(t.^3-8*t.^2+13,3);
plot(t,x6,'Color','y')
title('$x(t)=\sqrt[3]{t^3-8t^2+13}$','Interpreter','latex','FontSize',10)
xlim([0 10])
ylim([-5 5])


subplot(2,2,3)
num = 1001;
t = linspace(t_start,t_end,num);
x7 = dirac(t-3);
plot(t,x7,'Color','g')
title('$ \delta (t-3)$','Interpreter','latex','FontSize',10)
```

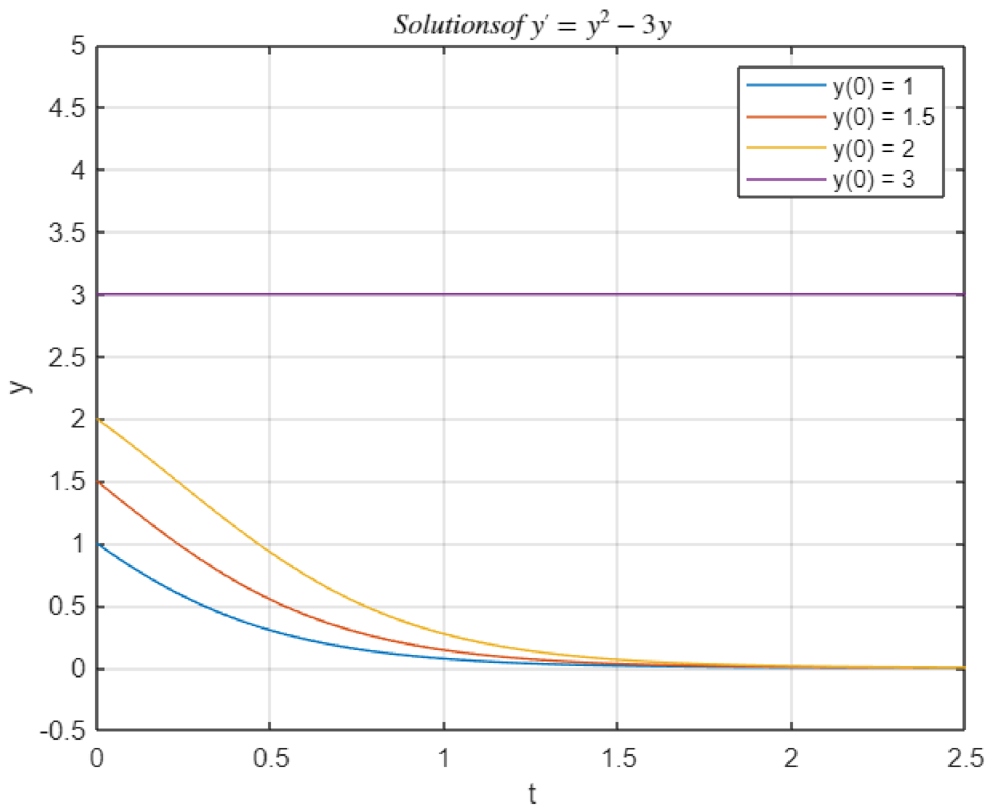
```
xlim([0 10])
ylim([-0.5 1.5])



num = 1001;
t = linspace(t_start,t_end,num);
u =  Step01(t-2)-Step01(t-4)-Step01(t-6)+Step01(t-8);
y=Step01(t-2).*(1-2*exp(-5*(t-2))+exp(-10*(t-2)))-Step01(t-4).*(1-2*exp(-5*(t-4))+ ...
  exp(-10*(t-4)))-Step01(t-6).*(1-2*exp(-5*(t-6))+exp(-10*(t-6)))+ ...
  Step01(t-8).*(1-2*exp(-5*(t-8))+exp(-10*(t-8)));
subplot(2,2,4)
plot(t,u,t,y)
title('$Q6$','Interpreter','latex','FontSize',10)
legend("u(t)","y(t)")
xlim([0 10])
ylim([-1.5 1.5])
```



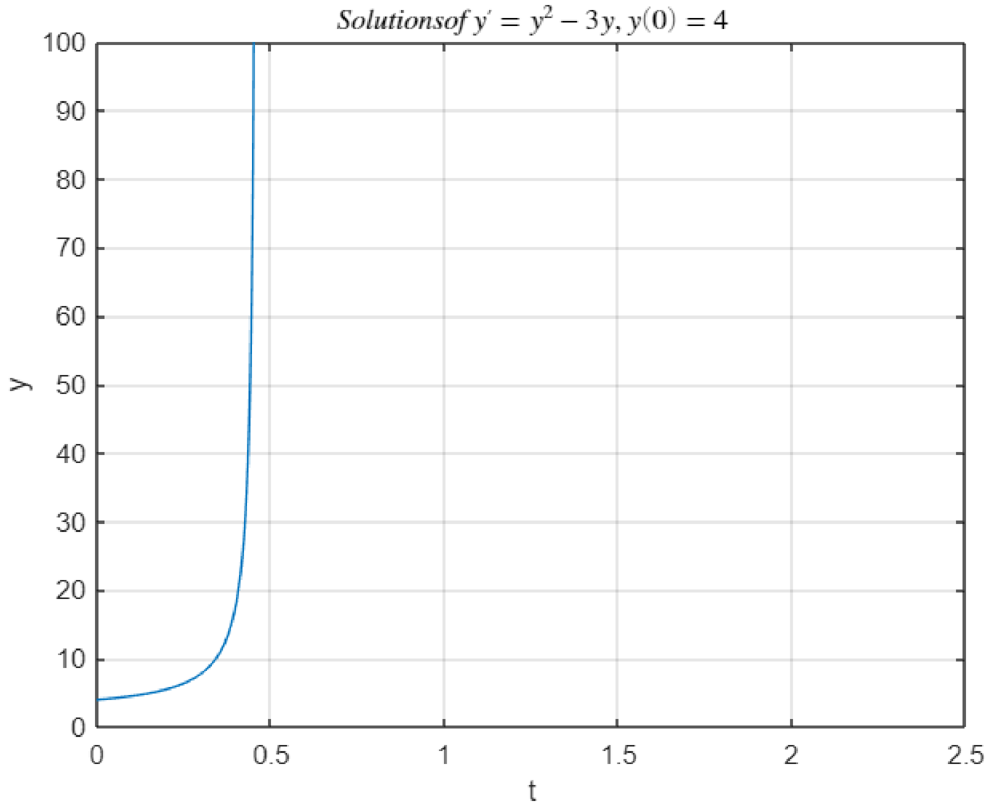
8 Numerical Solution of Ordinary Differential Equations

```
tspan = [0 2.5];
y0 = [1 1.5 2 3];

figure

[t, y] = ode45(@odefun, tspan, y0);
```

```
plot(t, y);
xlabel('Time');
ylabel('y(t)');
grid on
xlim([0 2.5])
ylim([-0.5 5])
xlabel('t')
ylabel('y')
legend("y(0) = 1","y(0) = 1.5","y(0) = 2","y(0) = 3")
title('$Solutions of y'' = y^{2}-3y$','interpreter','latex')
```




```
tspan = [0 0.46];
y0 = 4;


[t, y] = ode45(@odefun, tspan, y0);

plot(t, y);
xlabel('Time');
ylabel('y(t)');
grid on
xlim([0 2.5])
ylim([0 100])
xlabel('t')
ylabel('y')
title('$Solutions of y'' = y^{2}-3y, y(0) = 4$','interpreter','latex')
```

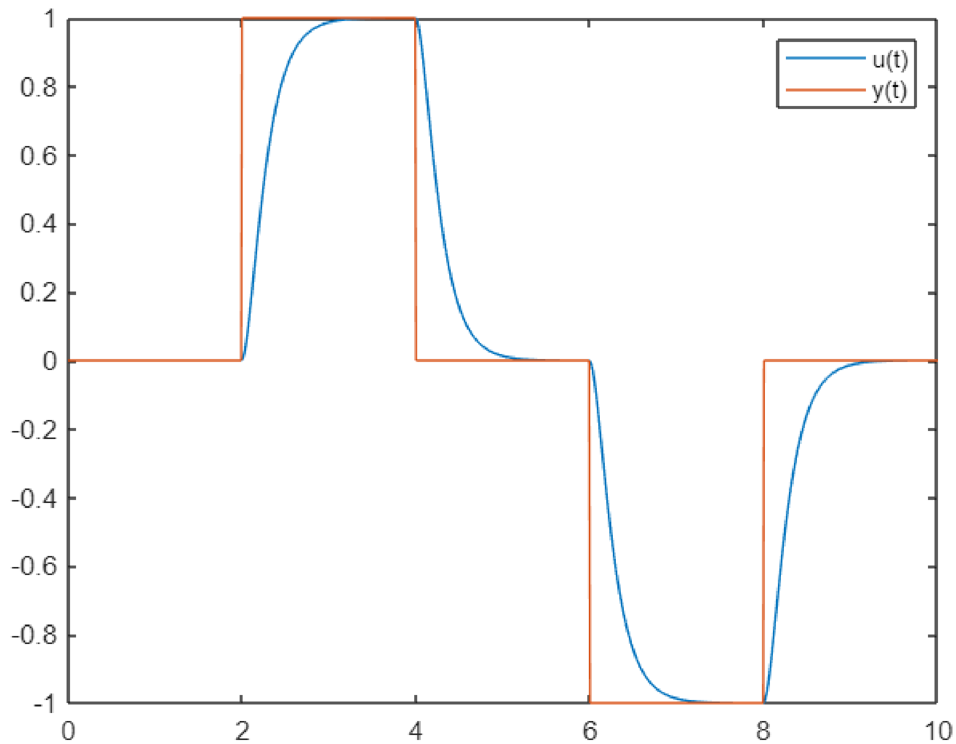


9 Numerical Methods to Find the Response of a Linear System

lsim:

$$\frac{1}{(0.2s+1)(0.1s+1)} = \frac{1}{0.02s^2+0.3s+1}$$

```
Ts = 0.01;
sys = tf(1,[0.02 0.3 1]);
t = 0:Ts:10;
u =  Step01(t-2)-Step01(t-4)-Step01(t-6)+Step01(t-8);
y0 =[0 0];
y = lsim(sys,u,t);
figure
plot(t,y,t,u)
legend("u(t)","y(t)")
```



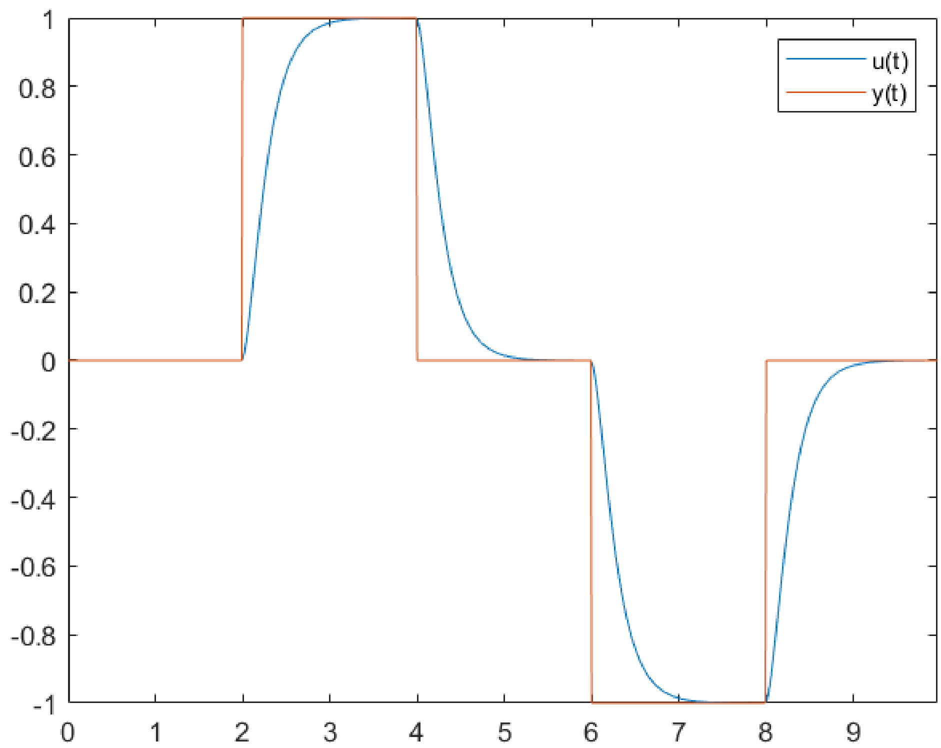
Euler method:

$$\Rightarrow 0.02y'' + 0.3y' + y = u \Rightarrow y(n) = u(n) - \frac{0.3}{h}(y(n) - y(n-1)) - \frac{0.02}{h^2}(y(n) - 2y(n-1) + y(n-2))$$

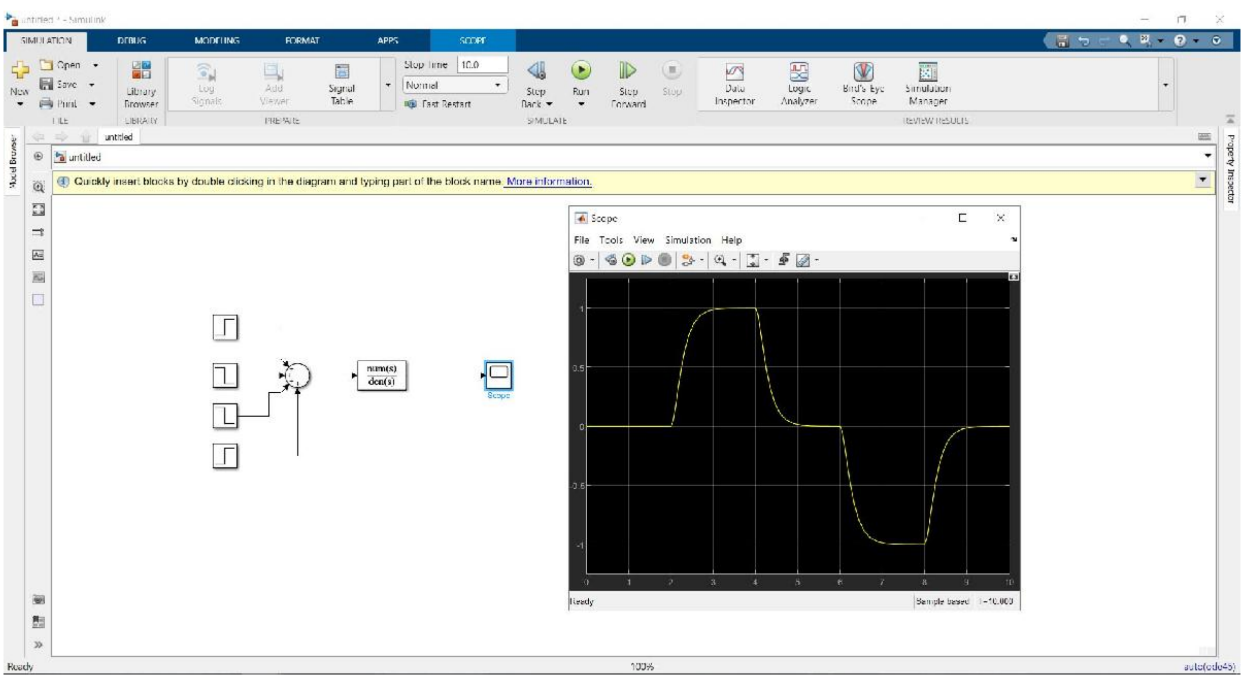
$$\Rightarrow \left(1 + \frac{0.3}{h} + \frac{0.02}{h^2}\right) y(n) = u(n) + \left(\frac{0.3}{h} + \frac{0.04}{h^2}\right) y(n-1) - \frac{0.02}{h^2} y(n-2)$$

$$\Rightarrow y(n) = \frac{1}{\left(1 + \frac{0.3}{h} + \frac{0.02}{h^2}\right)} u(n) + \frac{\left(\frac{0.3}{h} + \frac{0.04}{h^2}\right)}{\left(1 + \frac{0.3}{h} + \frac{0.02}{h^2}\right)} y(n-1) - \frac{\frac{0.02}{h^2}}{\left(1 + \frac{0.3}{h} + \frac{0.02}{h^2}\right)} y(n-2)$$

```
Ts = 0.01;
t = 0:Ts:10;
Y=zeros(size(t));
U=Step01(t-2)-Step01(t-4)-Step01(t-6)+Step01(t-8);

Y(1)=0;
Y(2)=0;

for i=3:length(Y)
    Y(i)=(1/(1+(0.3/(Ts))+(0.02/(Ts^2))))*(U(i)+((0.3/Ts) ...
        +(0.04/(Ts^2)))*Y(i-1)-(0.02/(Ts^2))*Y(i-2)); %f
end
figure
plot(t,Y,t,U)
legend("u(t)","y(t)")
```



10 Simulink



According to questions 8, 9, and 10, all outputs are equal to the answer in section 6 with proper accuracy

Functions

```
function dydt = odefun(t, y)
    dydt = y.^2 - 3*y;
end

function [u] = Step01(C)
u = C;
s = length(C);
for i=1:s
    if C(i)>=0
        u(i)=1;
    else
        u(i)=0;
    end
end
end
function [u] = Step00(C)
u = C;
s = length(C);
for i=1:s
    if C(i)>0
        u(i)=1;
    else
        u(i)=0;
    end
end
end
```