Linear Control Systems
25411

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Fall 1402 **Due Date: 1402/8/30**

Note 1: ***purple*** *problems are bonus ones.*

1 Routh-Hurwitz Criterion

For each of the following transfer functions, use the Routh-Hurwitz criterion to determine whether the system is stable, marginally stable or unstable. In unstable cases determine how many poles are in the right-half plane.

1. $G_1(s) = \dfrac{5s+5}{s^4+3s^3+3s^2+3s+2}$
2. $G_2(s) = \dfrac{8s+1}{s^4+3s^3+3s^2+9s+1}$
3. $G_3(s) = \dfrac{4}{s^4+4s^3+3s^2-4s-4}$
4. $G_4(s) = \dfrac{2s-1}{s^5+2s^4+2s^3+4s^2+11s+10}$
5. $G_5(s) = \dfrac{2}{s^6+7s^5+20s^4+30s^3+25s^2+11s+2}$

2 Range of a Gain for Closed-Loop Stability

Given a closed-loop system with unity feedback, where the open-loop transfer function is $G(s)$ and the proportional controller is K, determine the interval of values for K that ensures the stability of the closed-loop system.

$$G(s) = \frac{2s^3+s^2+s-1}{s^3-5s^2+2s+8}$$

Generate some step response plots of the closed-loop system with varying values of K to verify your solution.

3 Evaluating Stability Condition Using Impulse Response

Consider the set of bounded continuous-time signals. The l^∞ norm for this set is defined as follows:

$$||x||_\infty = \sup_{t \geq 0} |x(t)|$$

System S is an arbitrary LTI system with impulse response $h(t)$, and $||S||$ is defined as follows:

$$||S|| = \sup_{||u||_\infty \neq 0} \frac{||y||_\infty}{||u||_\infty}$$

a) Prove the following equation:

$$||y||_\infty \leq ||u||_\infty \int_0^\infty |h(\tau)| d\tau$$

b) Prove the following equation:

$$||S|| = \int_0^\infty |h(\tau)| d\tau$$

Hint: find a signal that satisfies the equality (=) in part a.

c) By calculating the corresponding integral for each of the following systems, determine whether the system is stable or not (For a stable system S_i we have $||S_i|| < \infty$).

1) $S_1 : G_1(s) = \dfrac{1}{s+1}$
2) $S_2 : G_2(s) = \dfrac{1}{s-1}$
3) $S_3 : G_3(s) = \dfrac{1}{s^2+1}$
4) $S_4 : G_4(s) = \dfrac{1}{(s+1)^2}$
5) $S_5 : h_5(t) = \dfrac{1}{t+1}$
6) $S_6 : h_6(t) = \dfrac{1}{t^2+1}$

For each of the systems in part c, generate the step response plots to confirm your solution.

4 Step Response Characteristics

Second-order underdamped systems transfer function can be represented as below:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \quad \omega_n > 0, \quad 0 < \zeta < 1$$

So, the Laplace transform of the output to the unit step input $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$ is:

$$Y(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

and

$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \phi)$$

where $\omega_d = \omega_n\sqrt{1-\zeta^2}$ and $\cos(\phi) = \zeta$.

a) Prove the following equation:

$$T_s \approx \frac{4}{\zeta\omega_n}$$

where settling time T_s is time to go from zero to within 2% of steady state. For which interval of ζ the equation is valid? How can we rectify the equation for $\zeta = 1$?

b) Prove the following equation:

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

where overshoot $M_p = \max(y(t)) - 1$. For which interval of ζ the equation is valid? For a fixed ω_n, how can we get the fastest response without overshoot?

MATLAB Assignments

5 Curve Fitting Toolbox

a) Determine the step response of the following transfer function in terms of K and τ:

$$G(s) = \frac{K}{\tau s + 1}$$

b) Using given *simulation.slx* file, generate the step response data ($u(t)$ and $y(t)$) of the unknown plant.

c) Open Curve Fitting Toolbox (type *cftool* in command line) and consider the obtained results in part b as inputs. Using obtained results in part a, estimate the values of K and τ parameters.

d) Generate and compare the step response plots of the unknown and the identified system from the previous part.

e) Using the step response of unknown system along with visual inspection, estimate the values of the time constant τ and DC gain K of the unknown system.

f) Modify the amplitude of the input signal and repeat parts c and d. Note that input signal must satisfy condition $-5 < u(t) < 5$ to ensure linear behavior.

g) Attempt to estimate the values of K and τ parameters using a sinusoidal input signal and repeat parts c and d.

h) Examine the effect of increasing length of data on the accuracy of the identified system.

i) Use lowpass filter $F(s) = \dfrac{1}{as+1}$ (with appropriate value of a) to reduce the effect of noise and apply the curve fitting toolbox to estimate the values of K and τ parameters (Determine the step response of transfer function $G(s)F(s)$ in time domain and identify the entire system but remember that the final result should be $G(s)$).

j) Compare all of the results and suggest the best method.

k) Use the Curve Fitting Toolbox to estimate the values of K, τ_1 and τ_2 parameters from the transfer function $G_2(s) = \dfrac{K}{(\tau_1 s+1)(\tau_1 s+1)}$ (by calculating the structure of step response in time domain) and try to make it as accurate as you can (by changing the input data set).

6 System Identification Toolbox

a) Using *simulation.slx* file, once again generate the step response data ($u(t)$ and $y(t)$) of the unknown plant.

b) Apply the System Identification toolbox (type *ident* in command line) to identify first order transfer function $G(s) = \frac{k}{s+a}$ using the data set obtained from part a.

c) Modify the amplitude of the input signal and repeat part b. Note that input signal must satisfy condition $-5 < u(t) < 5$ to ensure linear behavior.

d) Use an arbitrary sinusoidal input signal and repeat part b.

e) Use white noise input signal and repeat part b.

f) Use lowpass filter $F(s) = \frac{1}{as+1}$ to reduce the effect of noise and apply the System Identification toolbox to identify first order transfer function $G(s) = \frac{k}{s+a}$.

g) Compare all of the results and suggest the best method.

h) Use the best method to identify second-order transfer function $G_2(s) = \frac{1}{s^2 + a_1 s + a_0}$.