Linear Control Systems
25411

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Fall 1402 **Due Date: 2/11/1402**

Note 1: ***purple*** *problems are bonus ones.*

1 Plotting Nyquist Diagram by Hand

Plot hand-drawn Nyquist diagrams for the given systems and ascertain their stability or instability assuming both positive and negative feedback.

a) $G_1(s) = \dfrac{1}{s^2}$

b) $G_2(s) = \dfrac{1}{s^3}$

c) $G_3(s) = \dfrac{1}{s^2 - 1}$

d) $G_4(s) = \dfrac{-1}{s^2 + 1}$

e) $G_5(s) = \dfrac{2}{(s+1)(s+2)(s+3)}$

2 Closed-loop Stability vs. Open-loop Stability Margins

Nearly all electronic and power system designers believe that having a loop gain with positive gain and phase margins is equivalent to stability of closed-loop system (with unity negative feedback). So, electronic circuits and power control systems are daily designed (using Bode plot) with the goal of reaching satisfying positive margins.

a) Can you provide an example of loop gain with positive margins which closing the loop with unity feedback would result in an unstable closed-loop system.

b) Can you provide an example of minimum-phase loop gain with at least one negative margin which closing the loop with unity feedback would result in a stable closed-loop system.

c) According to obtained results in previous parts, one can conclude positive margins are not necessary nor sufficient to guarantee the closed-loop stability. So, why people are interested to guarantee closed-loop stability of system by designing controller with positive margins (in spite of using Nyquist stability criteria)? Why this method typically works?

d) In essence, does Bode plot contain all equivalent information of Nyquist plot? Justify your answer.

Hint: you might need to use MATLAB to do part a and part b.

3 Passivity

For a stable LTI system with input $u(t)$, output $y(t)$ and transfer function $G(s)$, consider the following definitions:

Passive:	$\int_0^\infty y(t)u(t)dt \geq 0$	$\equiv$	$\mathrm{Re}\{G(j\omega)\} \geq 0$
Input Strictly Passive (ISP):	$\int_0^\infty y(t)u(t)dt \geq \epsilon \int_0^\infty u^2(t)dt$	$\equiv$	$\mathrm{Re}\{G(j\omega)\} \geq \epsilon > 0$
Output Strictly Passive (OSP):	$\int_0^\infty y(t)u(t)dt \geq \epsilon \int_0^\infty y^2(t)dt$	$\equiv$	$\mathrm{Re}\{G(j\omega)\} \geq \epsilon \lvert G(j\omega)\rvert^2 > 0$

a) Can the following transfer function be implemented using circuit comprising solely with passive elements? Devise an electronic circuit with the following transfer function.

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

b) Ascertain the given systems passivity and determine their passivity type.

1) $G_1(s) = k$
2) $G_2(s) = \frac{1}{s}$
3) $G_3(s) = s$
4) $G_4(s) = \frac{k}{\tau s + 1}$
5) $G_5(s) = \frac{s^2 + 1}{(s+1)^2}$

c) Consider a system formed by connecting passive elements. Can this composite system also be classified as passive? For example consider a series or parallel connection of two $G(s) = \frac{1}{s+1}$ systems. Are all parallel combinations of passive systems passive?

d) Prove the following theorems ($\deg D(s) - \deg N(s) = r$ is the relative degree):

1) $G(s) = \frac{N(s)}{D(s)}$ is passive $\Rightarrow r \in \{-1, 0, 1\}$
2) $G(s) = \frac{N(s)}{D(s)}$ is ISP $\Rightarrow r \in \{-1, 0\}$
3) $G(s) = \frac{N(s)}{D(s)}$ is OSP $\Rightarrow r \in \{0, 1\}$

4 Time Delay Approximations

Approximating the delay using only zeros or only poles is problematic and useless. There is a suitable compromise based on approximation of the time delay transfer function using a rational function. One approach is dividing the exponential function into two parts of numerator and denominator as follows (it's not the Padé Approximations):

$$e^{-sT} = \frac{e^{-s\frac{T}{2}}}{e^{s\frac{T}{2}}}$$

Let's substitute the numerator and denominator with their Taylor series in order to approximate the delay:

$$e^{-sT} = \frac{1 + \frac{-s\frac{T}{2}}{1!} + \frac{(-s\frac{T}{2})^2}{2!} + \cdots}{1 + \frac{s\frac{T}{2}}{1!} + \frac{(s\frac{T}{2})^2}{2!} + \cdots}$$

a) Assume $T = 2$. Compare the Bode diagram of the Taylor series approximations with degree $n \in \{1, 2, 3, 4, 5\}$.

b) Compare step responses of the approximations in part a.

c) Is the Taylor series approximation of time delay with degree $n = 5$ stable?

The Padé approximation for time delay in control systems provides a way to represent time delays as transfer functions, enabling control engineers to apply standard control system design techniques to systems with time delays:

$$e^{-sT} \approx \sum_{i=0}^{m+n} \frac{(-Ts)^i}{i!} \approx R_{m,n}(Ts) = \frac{\sum_{i=0}^{m} p_i (Ts)^i}{\sum_{i=0}^{n} q_i (Ts)^i}, \qquad p_0 = q_0 = 1$$

d) Use the given formula above to determine the p_1 and q_1 coefficients in the standard Padé approximation $R_{1,1}(Ts)$ which introduced in class.

e) Use the given formula above to determine the p_i and q_i coefficients in the standard Padé approximation $R_{2,2}(Ts)$. Are obtained coefficients similar to those obtained by Teylor method?

The standard Padé approximation recommends to pick equal numerator and denominator degree (i.e., $m = n$), however it exhibits a jump at time $t = 0$ in it's step response (prove it). This is highly undesirable in simulating time delays. To avoid this phenomena we shall reconsider the Padé approximation with different numerator degrees.

It is possible to show that you can calculate the p_i and q_i coefficients in the Padé approximation $R_{m,n}(Ts)$ using the following formulas

$$p_i = (-1)^i \frac{(m+n-i)!m!}{(m+n)!i!(m-i)!}, \qquad q_i = \frac{(m+n-i)!n!}{(m+n)!i!(n-i)!}$$

f) Assume $T = 2$. Compare the Bode diagram of the standard Padé approximations $R_{1,1}(Ts)$, $R_{2,2}(Ts)$, $R_{3,3}(Ts)$, $R_{4,4}(Ts)$ and $R_{5,5}(Ts)$.

g) Compare step responses of the approximations in part f.

h) Assume $T = 2$. Compare the Bode diagram of the Padé approximations $R_{1,2}(Ts)$, $R_{2,3}(Ts)$, $R_{3,4}(Ts)$ and $R_{4,5}(Ts)$.

i) Compare step responses of the approximations in part h.

j) Why do people use Padé approximations with an equal degree of numerator and denominator for frequency domain analysis and Padé approximations with a lower degree of numerator than denominator for time domain analysis?

MATLAB Assignments

5 Root-locus of Time Delayed Systems

In class, the Nyquist plot of time delayed systems was discussed. Assume that we wish to draw the root-locus of following simple time-delayed system, despite the fact that the MATLAB `rlocus` command does not support time-delayed systems.

$$L(s) = \frac{e^{-Ts}}{s+P}$$

a) Derive corresponding conditions that a desirable point in s-domain belongs to the root-locus plot of $L(s)$.

b) Unfortunately, equations in part a are nonlinear. Write a MATLAB code which for a known proportional gain K, calculates the associated closed-loop poles (assuming unity negative feedback). $T = 0.1$ and $P = 2$.

c) Use a `for` loop to compute closed-loop poles as K varies in range of $0 : 0.1 : 20$.

d) Sketch the root-locus. Note that for some values of K we might have more that one roots! Actually since the equations contains sine and cosine functions, in some cases we might have an infinite number of roots. This property of time-delayed systems is interesting.

e) Respecting the obtained root-locus, compute the value of K which causes instability.

f) For the value of K obtained in part e, what is your expectation from Nyquist plot? sketch it using `nyquist` command and validate your expectation.

g) Simulate step response of system regarding the obtained value of K in part e and show that system is on the verge of instability.

Hint: To do part b, it might be helpful to use `syms, solve` and `subs` commands.