In the name of allah

Linear Control Systems

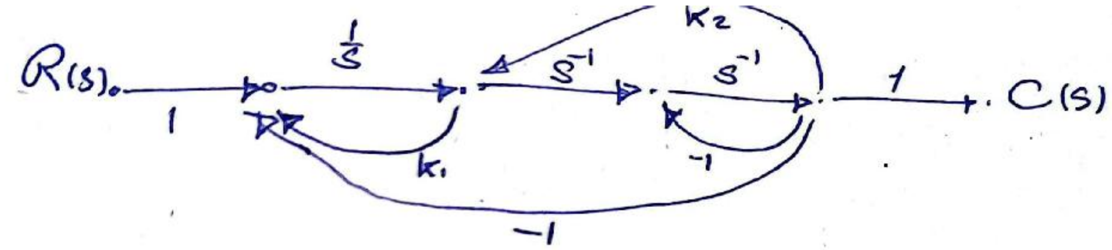
Hw 03

ali sadeghian 400101464

Fall 1402

Theory Assignments

تمرین



حل (1

$H(s)$:

$$P = s^{-3}$$

$$\Delta = 1 - \left(\frac{k_1}{s} - s^{-1} - s^{-3} + \frac{k_2}{s^2}\right) + \left(\frac{k_1}{s}\,\frac{-1}{s}\right)$$

$$H(s) = \frac{1}{s^3 - k_1 s^2 + s^2\,1 - k_2 s + k_1 s} = \frac{1}{s^3 + (1-k_1)s^2 + (k_2 - k_1)s + 1}$$

$$\begin{array}{lll} s^3 & 1 & -K_2 - K_1 \\ s^2 & 1-K_1 & 1 \\ s^1 & \frac{K_1^2 + (K_2-1)K_1 - K_2 + 1}{1-K_1} & 0 \\ s^0 & 0 & \end{array}$$

$\longrightarrow$ $K_1^2 + (K_2-1)K_1 - (K_2+1) = 0$

$$K_1 = \frac{-(K_2-1) \pm \sqrt{(K_2-1)^2 + 4(K_2+1)}}{2}$$

$$= \frac{-(K_2-1) \pm \sqrt{(K_2+1)^2 + 4}}{2}$$

$(1-K_1)s^2 + 1 = 0$

$K_2 = -1$

$$= -1 \pm 1 = -2, 0$$

$\rightarrow K_1 = -2 \,;\, K_2 = -1 \rightarrow \omega = \pm\frac{1}{\sqrt{3}}$

$ K_1 = 0 \,,\, K_2 = -1 \rightarrow \omega = \pm 1$

(20ج

1 $\dfrac{20}{(S+3)(S+30)} \rightarrow \dfrac{K}{S+P}$ $\boxed{\dfrac{K}{P} = \dfrac{2}{9}}$

$\dfrac{20\,(S+P)}{K(S+3)\,(S+30)} = \dfrac{20P + 20S}{90K + 33KS + KS^2}$ $\dfrac{20P}{90K}$ $\dfrac{1+\frac{1}{P}S}{1+\frac{11}{30}S+\frac{1}{90}S^2}$

$\Delta_0 = 1 \quad \Delta_1 = \frac{11}{30} \quad \Delta_2 = \frac{1}{45}$

$m_0 = 1 \quad m_1 = \frac{1}{P}$

$m_2 = -\dfrac{0 \quad 1}{2} + \dfrac{(1/P)^2}{1} - \dfrac{0 \quad 1}{2}$

$\Delta_2 = -\dfrac{1/45}{2} + \dfrac{(11/30)^2}{1} - \dfrac{1/45}{2}$

$\left(\dfrac{1}{P}\right)^2 = \left(\dfrac{11}{30}\right)^2 - \dfrac{1}{45} = \dfrac{121-20}{900},\ \dfrac{101}{900}$

$\rightarrow P = \dfrac{30}{\sqrt{101}}$

$\dfrac{K}{P} = \dfrac{2\cancel{0}}{9\cancel{0}} \rightarrow K = \dfrac{2}{9}P = \dfrac{20}{3\sqrt{101}}$

$P = K = \dfrac{30}{\sqrt{101}}$

$\Rightarrow H = \dfrac{\frac{20}{3\sqrt{101}}}{S + \frac{30}{\sqrt{101}}}$

2. $\dfrac{-0{,}2S+1}{(0{,}5S+1)(0{,}25S+1)(0{,}2S+1)} \rightarrow \dfrac{K}{(S+P_1)(S+P_2)}$ $\boxed{K = P_1P_2}$

$S^2+(P_1+P_2)S+P_1P_2$

$-0{,}2(P_1+P_2)\,/\,-0{,}2\,P_1P_2$

$1 \qquad P_1+P_2$

$\dfrac{(-0{,}2S+1)(S+P_1)(S+P_2)}{K(0{,}5S+1)(0{,}25S+1)(0{,}2S+1)} = \dfrac{-0{,}2S^3+(1-0{,}2(P_1+P_2))S^2+(P_1+P_2-0{,}2P_1P_2)S+P_1P_2}{0{,}025KS^3+0{,}275KS^2+0{,}95KS+K}$

$0{,}05S^2 + 0{,}45S \quad 1$

$1.5 \qquad 0{,}45$

$0{,}025 \qquad 0{,}225 \qquad 0{,}5$

$= \dfrac{\frac{-0{,}2}{P_1P_2}S^3 + \frac{(1-0{,}2(P_1+P_2))}{P_1P_2}S^2 + \frac{P_1+P_2-0{,}2P_1P_2}{P_1P_2}S + 1}{0{,}025S^3 + 0{,}275S^2 + 0{,}95S + 1}$

$\Delta^0 = 1 \quad \Delta^1 = \dfrac{P_1+P_2}{P_1P_2} - 0{,}2$

$m^0 = 1 \quad m^1 = 0{,}95$

$m^2 = \dfrac{2 - 0{,}4(P_1+P_2)}{P_1P_2}$

$\Delta^2 = 0{,}550$

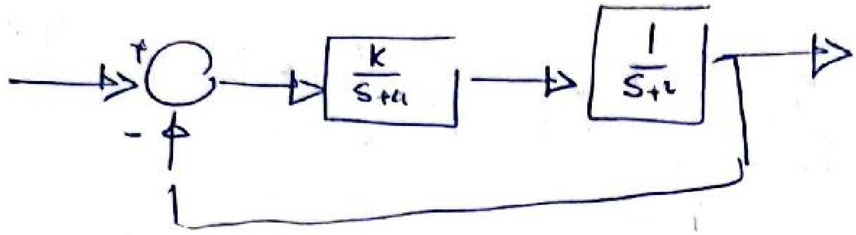
$m^3 = \dfrac{-0{,}2}{P_1P_2}$

$\Delta^3 = 0{,}15$

$M_2 = -\dfrac{\frac{2-0{,}4(P_1+P_2)}{P_1P_2}}{2} + \dfrac{(0{,}95)^2}{1} - \dfrac{\frac{2-0{,}4(P_1+P_2)}{P_1P_2}}{2}$

$\Delta_2 = -\dfrac{0{,}55}{2} + \dfrac{\left(\frac{P_1+P_2}{P_1P_2} - 0{,}2\right)^2}{1} - \dfrac{0{,}55}{2}$

(3 سوال



$$= \frac{k}{s^2+(a+2)s+2a+k} = \alpha \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

T_s: $\boxed{2\%}$ P.O. $\boxed{5\%}$

$$2 \geq T_s = \frac{4}{\xi\omega_n} , \quad \xi \geq 0.7$$

$$\Rightarrow \frac{4}{\xi\omega_n} = 1 \Rightarrow \frac{4}{\xi} = \omega_n , \quad \xi = 0.8 \Rightarrow \omega_n = 5$$

$$\Rightarrow \alpha \frac{25}{s^2+8s+25} = \frac{k}{s^2+(a+2)s+2a+k}$$

$$\left.\begin{array}{l} a+2 = 8 \\ k+2a = 25 \end{array}\right\} \Rightarrow \begin{array}{l} a = 6 \\ k = 13 \end{array}$$

$$\alpha = \frac{13}{25}$$

گرد کردم [illegible]

(4 سوال

	type	u(t)	t r(t)	P(t)	
L_1	1	0	$\frac{3}{2}$	∞	$K_v = \frac{20}{3}$
L_2	1	0	$\frac{24}{5}$	∞	$K_v = \frac{25}{12}$
L_3	0	$\frac{1}{1+35}$	∞	∞	$K_p = 35$
L_4	2	0	0	$\frac{6}{5}$	$K_a = \frac{5}{3}$


```
num = [13];
den = [1 8 25];
```

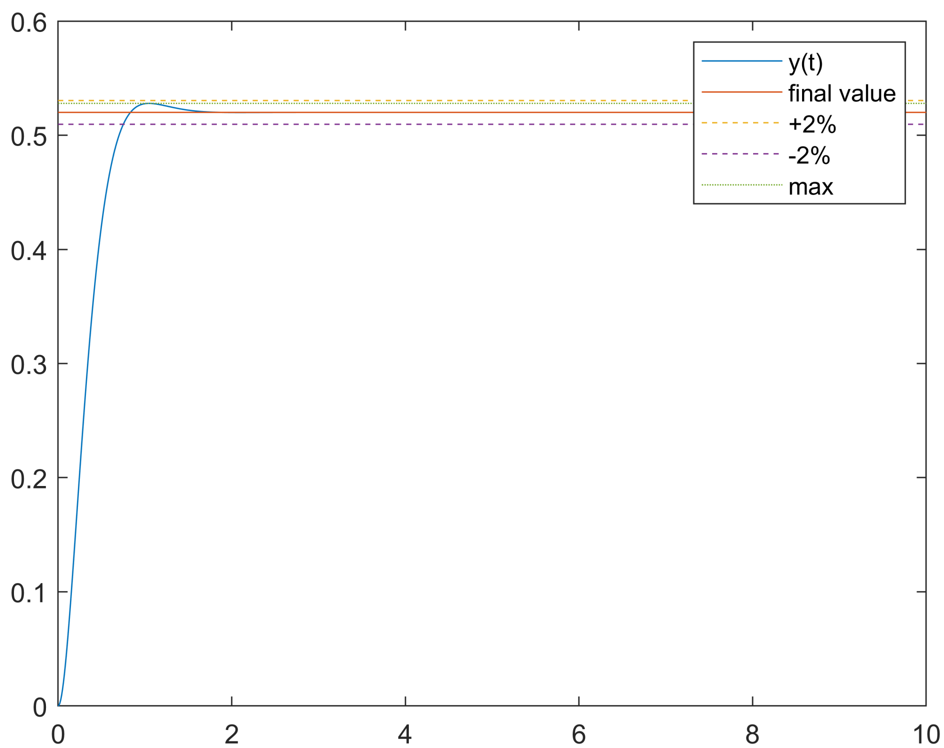
```
sys = tf(num, den)
```

```
sys =

        13
  --------------
  s^2 + 8 s + 25

Continuous-time transfer function.
```

```
t = 0:0.01:10;
u =  Step01(t);
y = lsim(sys, u, t);
Max = max(y);
PO = ((Max-13/25)/(13/25)) *100 % P.O.
```

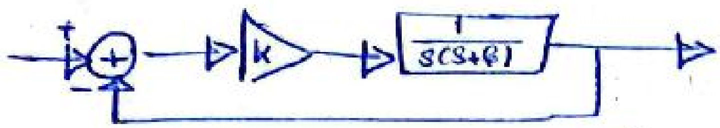
```
PO = 1.5163
```

```
figure
plot(t,y ,t,13/25.*u ,t,13/25.*u.*1.02, '--',t,13/25.*u.*0.98,'--',t,Max.*u,':')
legend("y(t)","final value","+2%","-2%","max")
```



سوال 5)

1



$$H(s) = \frac{K}{s^2+6s+K} \qquad \left.\begin{array}{l}\omega_n^2 = K \\ \xi\omega_n = 6\end{array}\right\}$$

1. min Setling time

$$y(t) \begin{cases} \xi \neq 0 \rightarrow T_s = \infty \\ 0<\xi<1 \quad 1-\frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(-) \\ \xi=1 \quad 1-e^{-\omega_n t}-\omega_n t e^{-\omega_n t} \\ \xi>1 \quad 1-K_1 e^{-\lambda_1 t}-K_2 e^{-\lambda_2 t} \qquad \lambda_1, \lambda_2 = \xi\omega_n \pm \omega_n\sqrt{\xi^2-1} \end{cases}$$

$0<\xi<1$:
$$e = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} = \frac{e^{-3t}}{\sqrt{1-\xi^2}} = \frac{2}{100}$$

$$e^{-3t} = \frac{2}{100}\sqrt{1-\xi^2}$$

$$-3t = \ln\frac{2}{100} + \frac{1}{2}\ln 1-\xi^2$$

$$t = -\frac{1}{3}\ln\frac{2}{100} - \frac{1}{6}\ln 1-\xi^2 \xrightarrow{\min} -\frac{1}{3}\ln\frac{2}{100} = 1{,}304$$

$\xi=1$
$$e = e^{-\omega_n t} + \omega_n t e^{-\omega_n t} \xrightarrow{\omega_n=3} T_s \simeq \frac{6}{\omega_n} = 2s$$
$$= 1{,}945s$$

$\xi>1 \quad \omega_n = \frac{3}{\xi}$
$$e = \frac{\lambda_2}{\lambda_2-\lambda_1} e^{+\lambda_1 t} + \frac{\lambda_1}{\lambda_1-\lambda_2} e^{+\lambda_2 t} \qquad \lambda_1, \lambda_2 = -\xi\omega_n \mp \omega_n\sqrt{\xi^2-1}$$

$\Rightarrow$ حل با desmos (همراه نمودار، مقدار تقریبی

$\Rightarrow T_{s\,\min} = 1{,}304$

2. min Setling time , $\forall t: y(t) \leq r(t)$

$\rightarrow \xi<1 \;\times \qquad (0<\xi<1 \quad e \rightarrow$ می‌تواند منفی شود ← از $r(t)$ بیشتر شود.)

$\Rightarrow \xi=1 \rightarrow T_s = 1{,}945$

Attached is a video that confirms this.

③ minimize $\int_0^\infty e^2(t)\,dt$

فقط در حالتی که ~~...~~ باشد بهترین پاسخ میدهد T_s، کمترین‌ها و e سریع تر میرا می‌شود.

$$e(t) = \sqrt{\frac{k}{k-9}}\, e^{-3t} \sin\left(\sqrt{k-9}\,t + \cos^{-1}\left(\frac{3}{\sqrt{k}}\right)\right)$$

$$|J(k)| = \left|\int_0^\infty e_k(t)^2\,dt\right| \le \frac{k}{k-9}\int_0^\infty e^{-6t}\,dt = \frac{k}{6(k-9)}$$

$$\left|\frac{k}{6(k-9)}\right| \ge \frac{1}{6} \Rightarrow$$ همین زمانی اتفاق می‌افتد که k به سمت بی‌نهایت برود.

در این صورت یک ~~...~~ کران پایین ~~...~~ برای خطا میرا می‌شود

$$\Rightarrow \int_0^\infty \left(e^{-3t} + 3te^{-3t}\right)^2 dt = \int_0^\infty e^{-6t} + ext\ dt = \frac{1}{6} + ext.$$

همان طور که دیده می‌شود خطا برای میرایی بحرانی از کران پایه برای زیرمیرا بالاتر است
و خطا برای میرایی ضعیف از میرایی بحرانی بیشتر است.

MATLAB Assignments

```
clc
close all
clear all
```

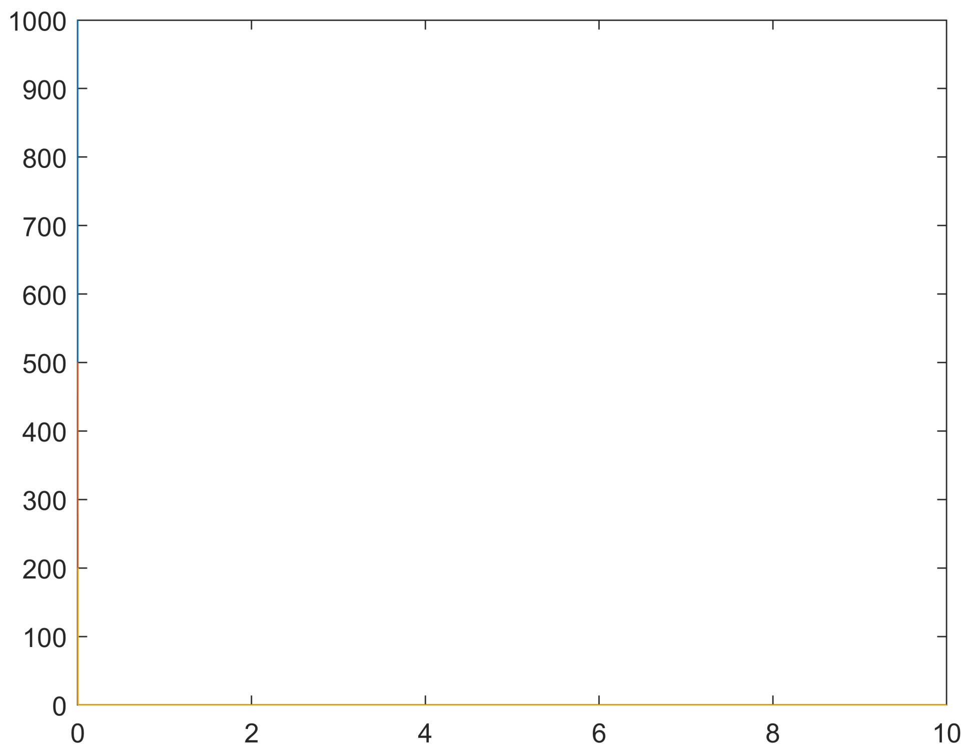
6 Symbolic e^{At}

At first, we define an imprecise definition for impulse as a causal signal, then we try to improve the answer with different widths.

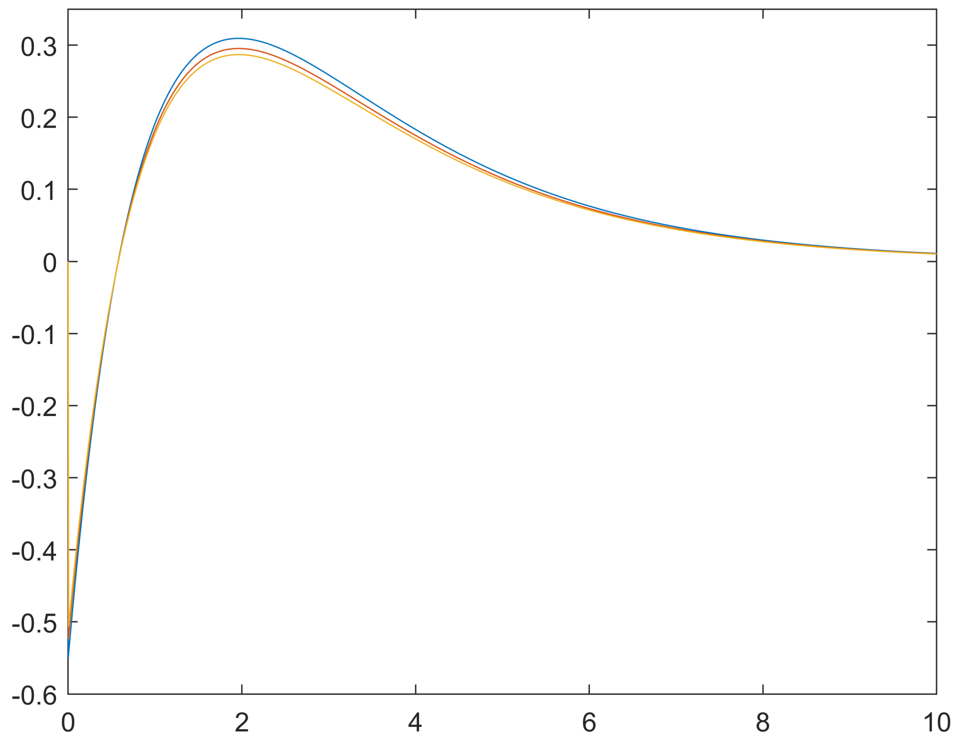
1)lsim

```
num = [-1 1];
```

```
den = [2 3 1];
sys = tf(num, den);

t_1 = 0:0.0001:10;
u  =  impulse_me( 1000 , t_1);
u1 =  impulse_me( 500  , t_1);
u2 =  impulse_me( 200  , t_1);
y_11 = lsim(sys, u, t_1);
y_12 = lsim(sys, u1, t_1);
y_13 = lsim(sys, u2, t_1);

figure
plot(t_1,u,t_1,u1,t_1,u2)
xlim([0 10])
```




```
figure
plot(t_1,y_11,t_1,y_12,t_1,y_13)
xlim([0 10])
ylim([-0.6 0.35])
```



2)ilaplace

We will use the `ilaplace()` function in MATLAB to obtain the inverse Laplace transform of the transfer function, which is equivalent to

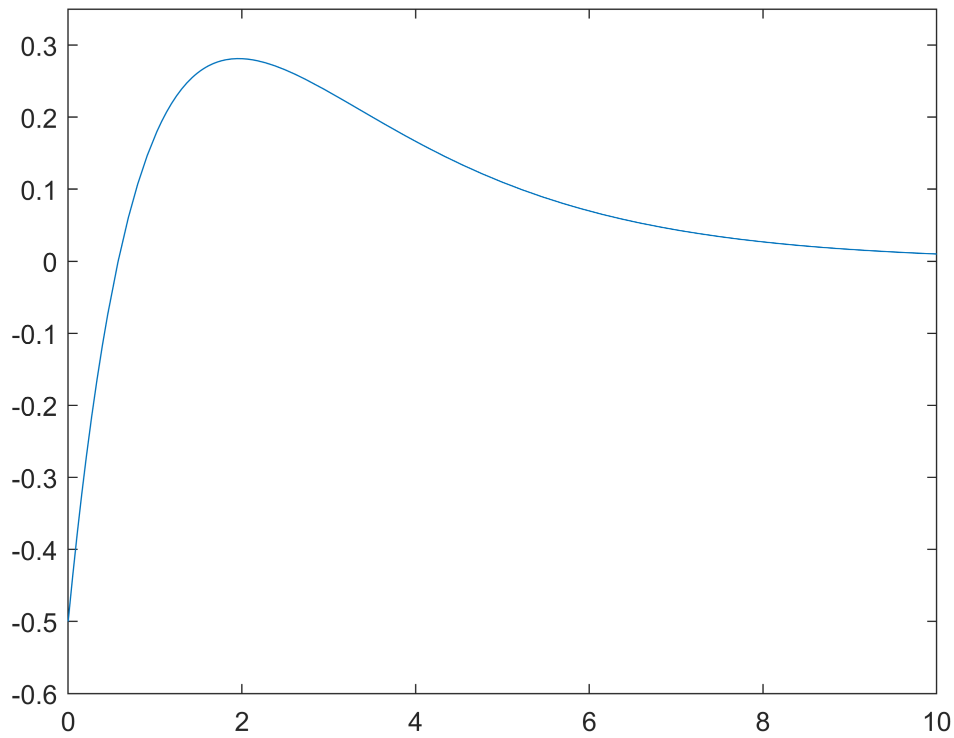
its time-domain impulse response. We will then compare the results to verify their accuracy.

```
syms s
sys = (1-s)/(2*s^2+3*s+1);

y_2 = ilaplace(sys)
```

```
y_2 =
```

$$\frac{3\,e^{-\frac{t}{2}}}{2} - 2\,e^{-t}$$

```
figure
fplot(y_2)
xlim([0 10])
ylim([-0.6 0.35])
```



3)expm function

We can use the `expm()` function in MATLAB to obtain `eAt`. Then, we can use equation $y = \mathrm{C}\mathrm{e}^{\mathrm{A}t}B$ to calculate the impulse response.

```
num = [-1 1];
den = [2 3 1];
sys = tf(num, den);
sys = ss(sys);

syms t
A = sys.A*t;
eAt = expm(A)
```

```
eAt =
```

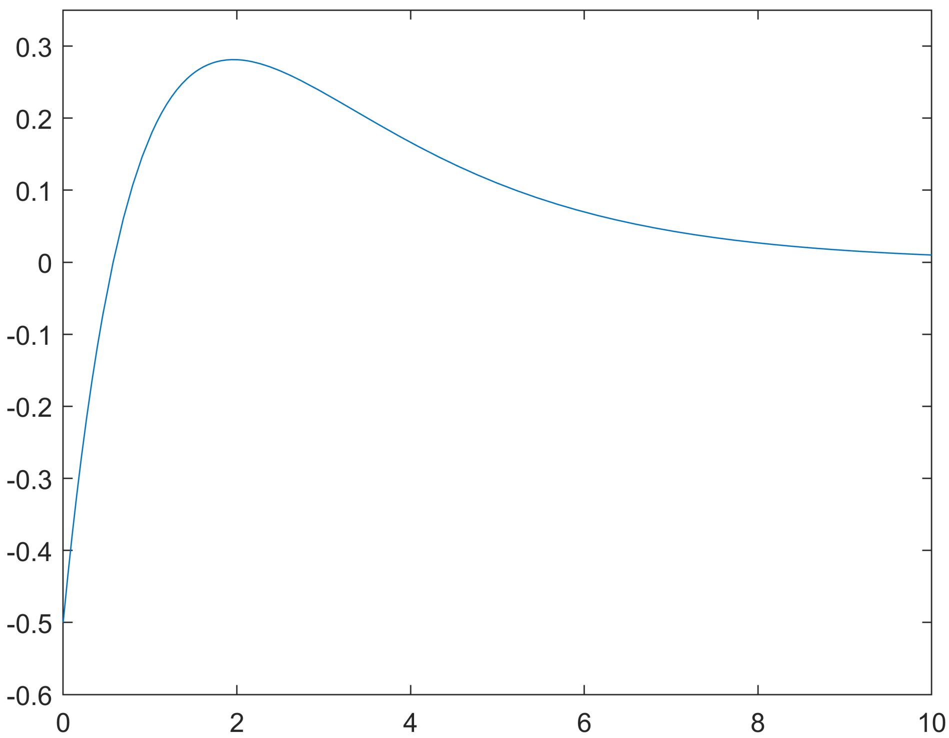
$$\begin{pmatrix} 2\,\mathrm{e}^{-t} - \mathrm{e}^{-\frac{t}{2}} & \mathrm{e}^{-t} - \mathrm{e}^{-\frac{t}{2}} \\ 2\,\mathrm{e}^{-\frac{t}{2}} - 2\,\mathrm{e}^{-t} & 2\,\mathrm{e}^{-\frac{t}{2}} - \mathrm{e}^{-t} \end{pmatrix}$$

```
eAt3 = eAt;
B = sys.B;
C = sys.C;
impulse_response = C * eAt * B
```

```
impulse_response =
```

$$\frac{3\,e^{-\frac{t}{2}}}{2} - 2\,e^{-t}$$

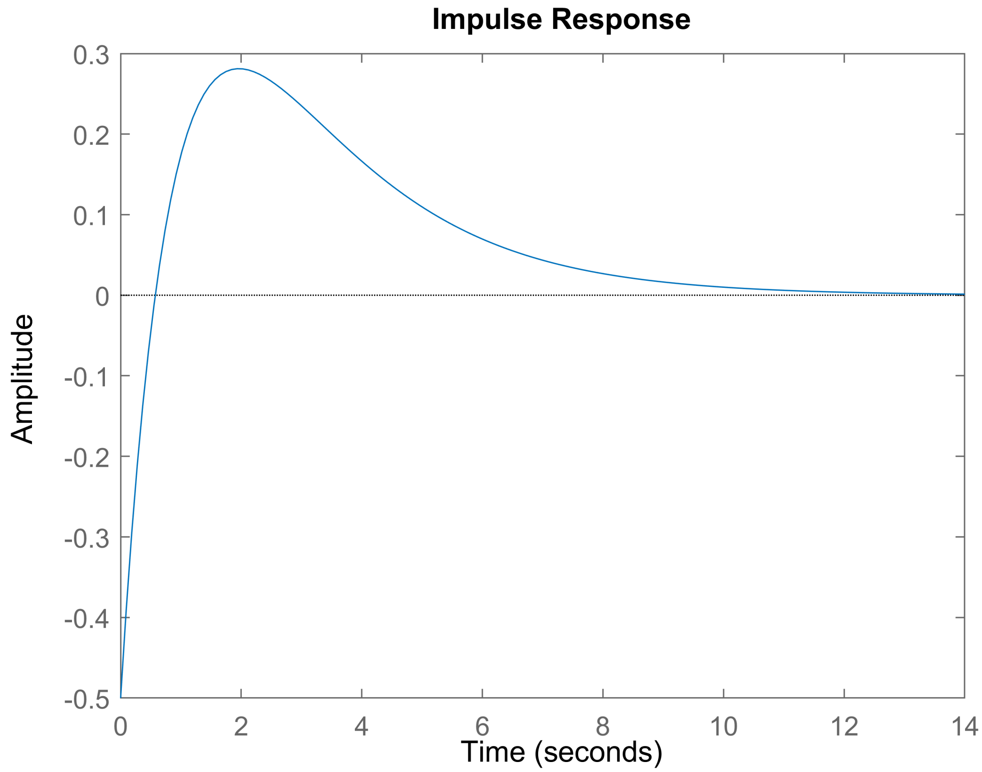
```
figure
fplot(impulse_response)
xlim([0 10])
ylim([-0.6 0.35])
```



4)impulse

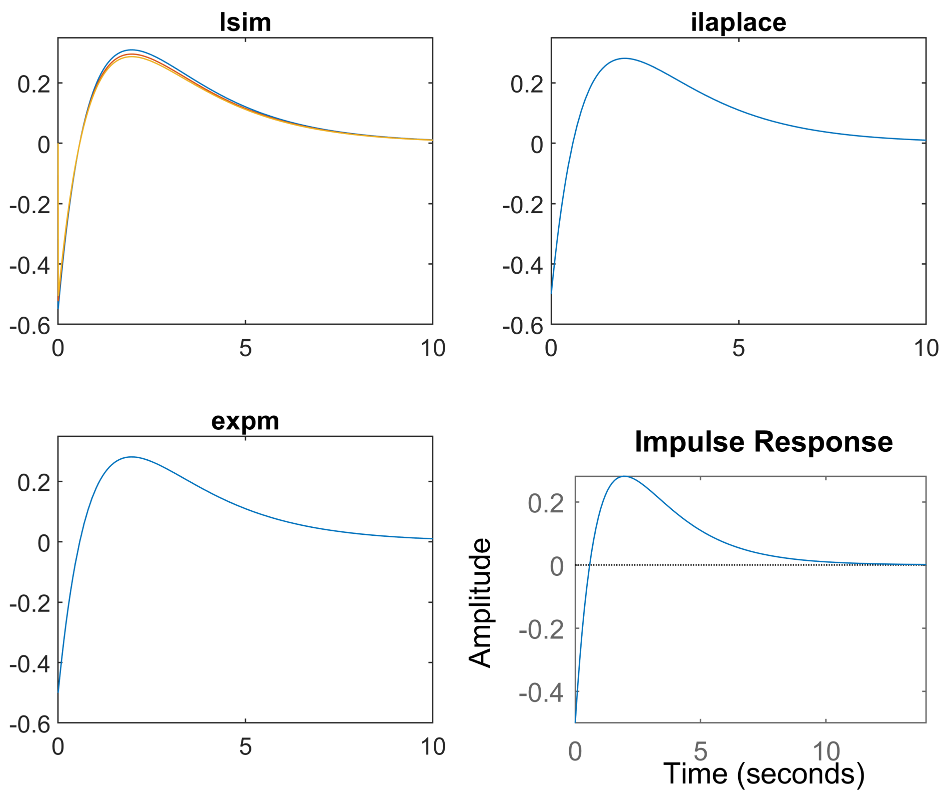
This is the result of getting the impulse response with impulse() function.

```
num = [-1 1];
den = [2 3 1];
sys = tf(num, den);

impulse(sys)
```



5) In this section we compare previews part :

```
figure %1
subplot(2,2,1)
plot(t_1,y_11,t_1,y_12,t_1,y_13)
title("lsim")
xlim([0 10])
ylim([-0.6 0.35])
subplot(2,2,2) %2
fplot(y_2)
title("ilaplace")
xlim([0 10])
ylim([-0.6 0.35])
subplot(2,2,3) %3
fplot(impulse_response)
title("expm")
xlim([0 10])
ylim([-0.6 0.35])
subplot(2,2,4)
impulse(sys) %4
```



It's crystal clear that all these plots are showing the impulse response of G(S)

6) As expected e^{At}, matrices calculated from those ways are identical!

```
num = [-1 1];
den = [2 3 1];
sys = tf(num, den);
sys = ss(sys);
A = sys.A
```

```
A = 2×2
   -1.5000   -0.5000
    1.0000         0
```

```
eAt = ilaplace((s*eye(size(A,1)) - A )^-1)
```

eAt =

$$\begin{pmatrix} 2\,e^{-t} - e^{-\frac{t}{2}} & e^{-t} - e^{-\frac{t}{2}} \\ 2\,e^{-\frac{t}{2}} - 2\,e^{-t} & 2\,e^{-\frac{t}{2}} - e^{-t} \end{pmatrix}$$

```
eAt3
```

eAt3 =

$$\begin{pmatrix} 2e^{-t}-e^{\frac{-t}{2}} & e^{-t}-e^{\frac{-t}{2}} \\ 2e^{\frac{-t}{2}}-2e^{-t} & 2e^{\frac{-t}{2}}-e^{-t} \end{pmatrix}$$

as we see ,they are equal

7 System 1

سوال 7)

$$x_1' = x_2$$

$$x_2' = \frac{u(t) - kx_1 - x_2}{m}$$

$$y = x_1$$

روش 2

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ \frac{-k}{m} & \frac{-1}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

روش 1

$$x' = Ax + Bu$$

$$\underline{x' - Ax} = Bu$$

ضرب طرفین e^{At}

$$\rightarrow e^{-At}x' - e^{-At}Ax = e^{-At}Bu$$

$$(e^{-At}x)' = e^{-At}Bu$$

t_s: زمانی که u شروع به مقدار گیری می‌کند

$$e^{-At}x = x(0) + \int_0^t e^{-A\tau}Bu\,d\tau$$

$$x = e^{At}x(0) + e^{At}\int_0^t e^{-A\tau}Bu\,d\tau \Rightarrow e^{At}\underline{x(t_s)} + \int_{t_s}^{t} e^{-A(\tau-t)}Bu\,d\tau + \underline{K}e^{At} = x$$

$$\xrightarrow{x(t_s)=0 \Rightarrow K=0} \int_{t_s}^{t} e^{-A(\tau-t)}Bu\,d\tau = x$$

$\boxed{x(t_s)=0}$ نباشد LTI نیست $\Leftarrow$. ببینیم

$$u_1+u_2 = y_1+y_2$$

$$u(t-T) = y(t-T)$$

3) finding G(s)

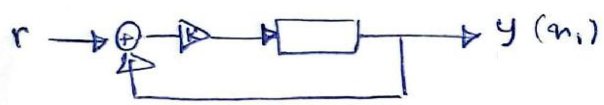
```
syms k m
A = [0 1;-k/m -1/m];
B = [0 ;1/m];
C = [1 0];
D = 0;

syms s
G = C*(s*eye(size(A))-A)^(-1)*B
```

G =

$$\frac{1}{m\,s^2+s+k}$$

سوال 4

$$u(t) = K_p\left(r(t) - x_1(t)\right)$$



سوال 5

$$x' = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{1}{m} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \cancel{u}\ \left(K_p\left(r(t) - x_1(t)\right)\right)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

$$x' = \begin{bmatrix} 0 & 1 \\ \frac{-k-K_p}{m} & -\frac{1}{m} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{K_p}{m} \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

6) finding T(s)

```
syms k m K
A = [0 1;(-(k+K)/m) -1/m];
B = [0 ;K/m];
C = [1 0];
D = 0;

syms s
T = C*(s*eye(size(A))-A)^(-1)*B
```

T =

$$\frac{K}{m\,s^2+s+K+k}$$

پارت 7

$$T = \frac{K}{ms^2 + s + k + K}$$

$$ms^2 + s + k + K = 0$$

$$\frac{s^2}{4} + s + \frac{1}{4} + K = 0$$

$$s^2 + 4s + 1 + 4K$$

1	$1+4K$
4	0
$1+4K$	

$\Rightarrow K < -1/4$

$\Rightarrow K > -1/4$ پایدار است.

8)

```
k = 0.25;
m = 0.25;
K = 200;

A = [0 1;(-(k+K)/m) -1/m];
B = [0 ;K/m];
C = [1 0];
D = 0;

syms s
T = C*(s*eye(size(A))-A)^(-1)*B
```

T =

$$\frac{800}{s^2 + 4\,s + 801}$$

```
poles(T,s)
```

ans =

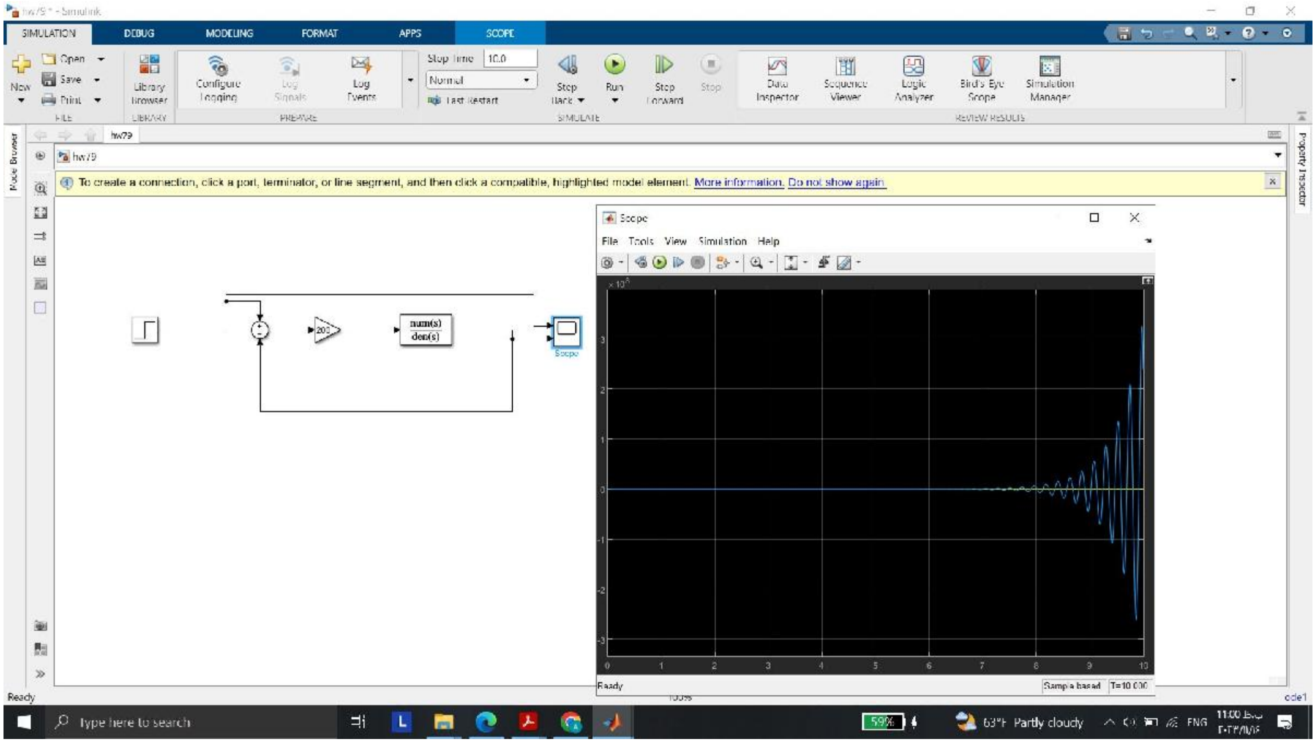
$$\begin{pmatrix} -2 + \sqrt{797}\ \mathrm{i} \\ -2 - \sqrt{797}\ \mathrm{i} \end{pmatrix}$$

Considering that the poles and zeros are on the left side of the imaginary axis, then the system is stable

9)The system per se is unstable as we can see in the figure the output starts to rise monotonically. But using the feedback

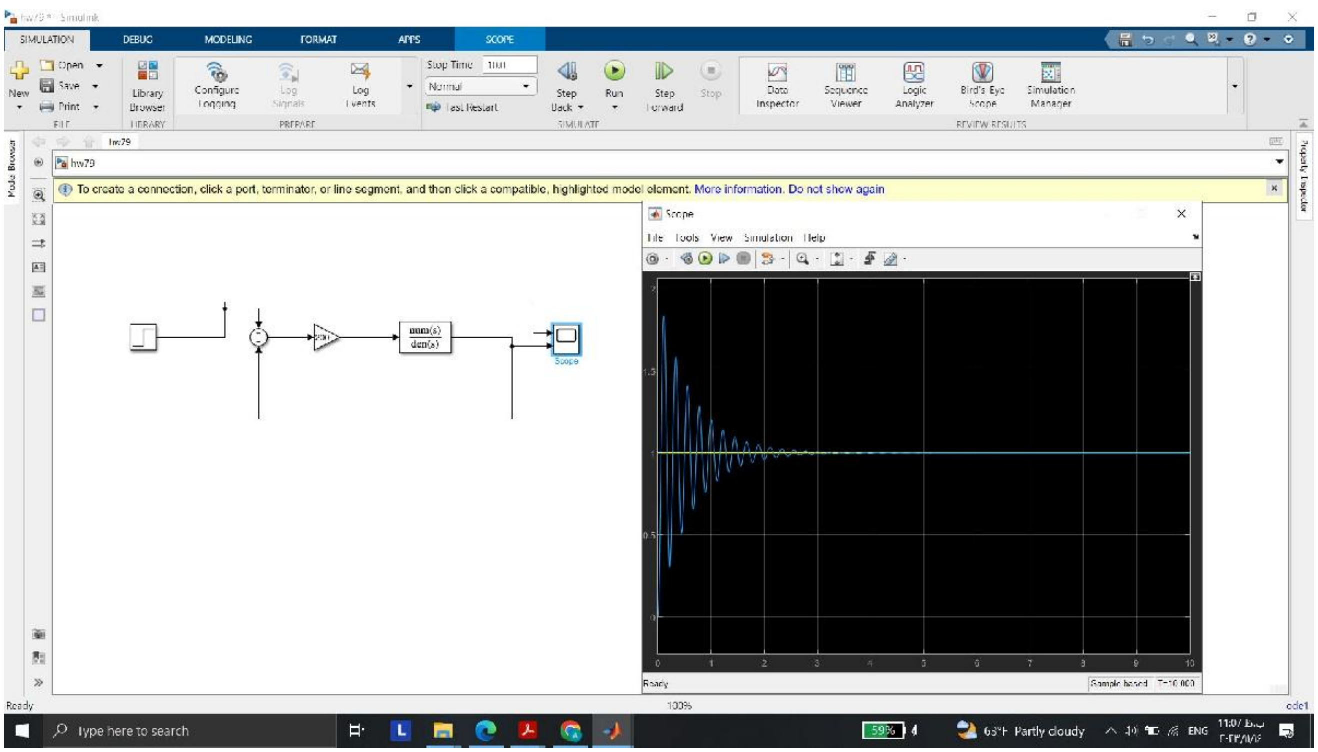
in next part we will stabilize the system.

ode1 Ts=0.01



10)

ode1 Ts=0.001



Using the given time step and the solver in the instruction and the results, we can deduce that the system is stable, since by getting bounded input we got bunded output and furthermore the output converges to zero as time passes. The feedback indeed stabilized the system!

11)First, we find the permanent error theoretically, then we confirm it by placing a large time in the response equation.

steady state error:

$$K_p = \lim_{s\longrightarrow 0} \frac{K}{\mathrm{ms}^2 + s + k} = \frac{K}{k}$$

$$e = \frac{A}{1 + K_p} = \frac{\mathrm{Ak}}{k + K}$$

```
k = 0.25;
m = 0.25;
K = 200;

A = [0 1;(-(k+K)/m) -1/m];
B = [0 ;K/m];
C = [1 0];
D = 0;

syms s t
T =  C*(s*eye(size(A))-A)^(-1)*B;
T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s;
y_1 = ilaplace(T1);
e_T1=k/(k+K);

k = 5;
m = 0.125;
K = 100;

A = [0 1;(-(k+K)/m) -1/m];
B = [0 ;K/m];
C = [1 0];
D = 0;
e_T2=k/(k+K);
syms s
T =  C*(s*eye(size(A))-A)^(-1)*B;
T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s;
y_2 = ilaplace(T1);


k = 10;
m = 3;
K = 50;

A = [0 1;(-(k+K)/m) -1/m];
B = [0 ;K/m];
C = [1 0];
D = 0;
```

```
e_T3=k/(k+K);
syms s
T =  C*(s*eye(size(A))-A)^(-1)*B;
T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s;
y_3 = ilaplace(T1);

eR1=1-eval(subs(y_1, t, 1000))
```

```
eR1 = 0.0012
```

```
e_T1
```

```
e_T1 = 0.0012
```

```
eR2=1-eval(subs(y_2, t, 1000))
```

```
eR2 = 0.0476
```

```
e_T2
```

```
e_T2 = 0.0476
```

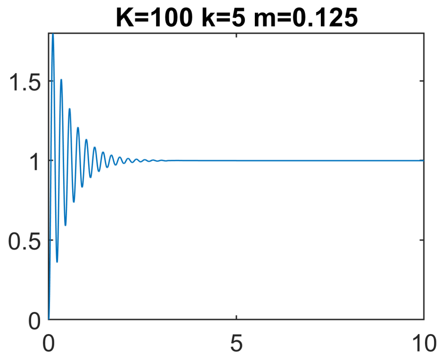
```
eR3=1-eval(subs(y_3, t, 1000))
```

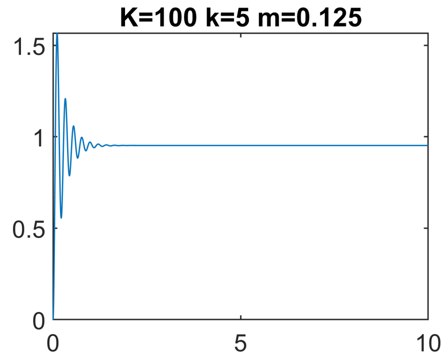
```
eR3 = 0.1667
```

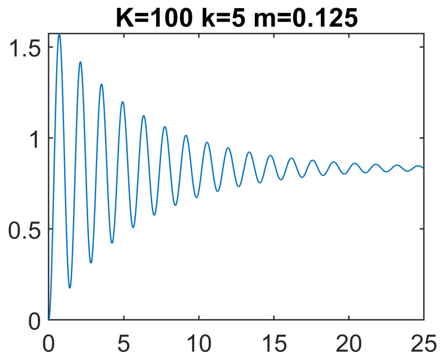
```
e_T3
```

```
e_T3 = 0.1667
```

```
figure
subplot(2,2,1)
fplot(y_1)
title("K=100 k=5 m=0.125")
xlim([0 10])
subplot(2,2,2)
fplot(y_2)
title("K=100 k=5 m=0.125")
xlim([0 10])
subplot(2,2,3)
fplot(y_3)
title("K=100 k=5 m=0.125")
xlim([0 25])
```







12)We know that $H(s) = \dfrac{4k_p}{s^2 + 4s + (1 + 4k_p)} = \dfrac{4k_p}{s^2 + 2\zeta\omega s + \omega^2}$. If we want the system to show critically damped behavior,

then we must have =1which results in $\xi = \dfrac{2}{\sqrt{1 + 4k_p}} = 1 \longrightarrow k_p = 0.75$.

```
k = 0.25;
m = 0.25;
K = 0.75;

A = [0 1;(-(k+K)/m) -1/m];
B = [0 ;K/m];
C = [1 0];
D = 0;

syms s
T =  C*(s*eye(size(A))-A)^(-1)*B
```

T =

$$\frac{3}{s^2 + 4\,s + 4}$$

```
T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s;
y = ilaplace(T1)
```

y =

$$\frac{3}{4} - \frac{3\,t\,e^{-2\,t}}{2} - \frac{3\,e^{-2\,t}}{4}$$

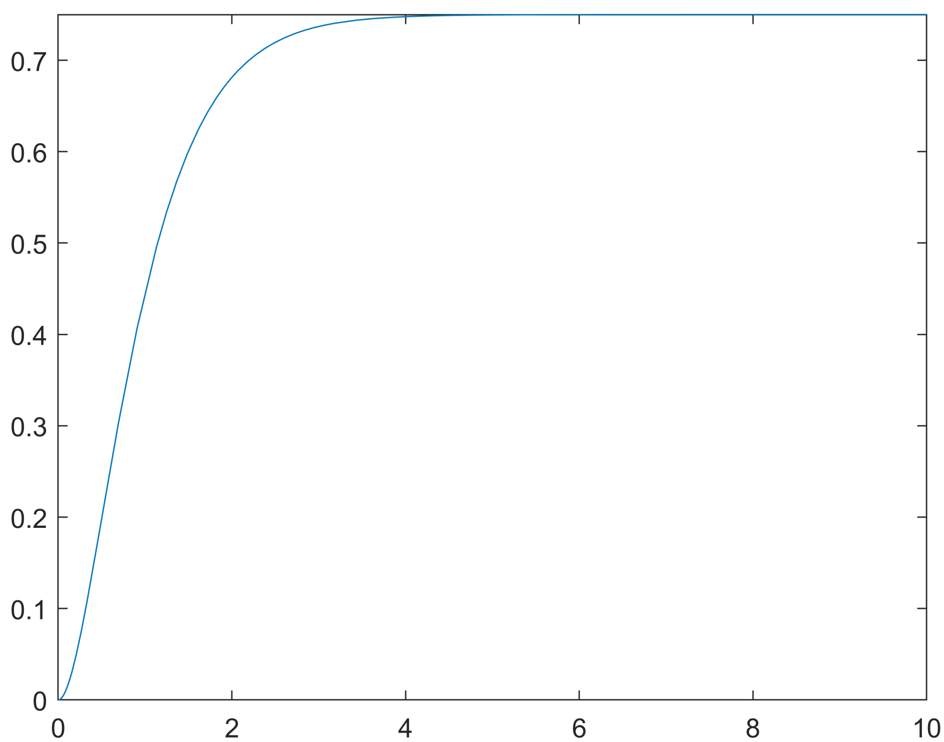
```
eR=1-eval(subs(y, t, 1000))
```

eR = 0.2500

```
e_T=k/(k+K)
```

e_T = 0.2500

```
figure
fplot(y)
xlim([0 10])
```



13)

finding G(s):

```
clear all

k = 0.25;
m = 0.25;
syms K


A = [0 1;(-(k+K)/m) -1/m];
```

```
B = [0 ;K/m];
C = [1 0];
D = 0;

syms s
T =  C*(s*eye(size(A))-A)^(-1)*B
```

T =

$$\frac{4\,K}{s^2+4\,s+4\,K+1}$$

$\omega_n = \sqrt{4K+1}\,, \zeta\omega_n = 2$

$\Rightarrow T_s \min : 0 < \zeta < 1 \;\Rightarrow K > 0.75$

$e = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} = \frac{e^{-2t}}{\sqrt{1-\xi^2}} < 0.02 \Rightarrow \xi = \varepsilon \Rightarrow T_s \approx 2$

$K = \infty$

To prove this, we find settlingtime for different k:

```
N = 100;
for i=1:N
K = exp(exp(i/20) - 120)
num = [4*K];
dem = [1 4 4*K+1];
sys = tf(num,dem);
settlingTime= stepinfo(sys).SettlingTime
end
```

```
K = 2.1939e-52
settlingTime = 14.8789
K = 2.3154e-52
settlingTime = 14.8789
K = 2.4504e-52
settlingTime = 14.8789
K = 2.6008e-52
settlingTime = 14.8789
K = 2.7689e-52
settlingTime = 14.8789
K = 2.9573e-52
settlingTime = 14.8789
K = 3.1692e-52
settlingTime = 14.8789
K = 3.4084e-52
settlingTime = 14.8789
K = 3.6794e-52
settlingTime = 14.8789
K = 3.9874e-52
settlingTime = 14.8789
K = 4.3391e-52
settlingTime = 14.8789
K = 4.7424e-52
settlingTime = 14.8789
K = 5.2068e-52
settlingTime = 14.8789
K = 5.7441e-52
settlingTime = 14.8789
K = 6.3689e-52
```

```
settlingTime = 14.8789
K = 7.0991e-52
settlingTime = 14.8789
K = 7.9572e-52
settlingTime = 14.8789
K = 8.9713e-52
settlingTime = 14.8789
K = 1.0177e-51
settlingTime = 14.8789
K = 1.1620e-51
settlingTime = 14.8789
K = 1.3357e-51
settlingTime = 14.8789
K = 1.5465e-51
settlingTime = 14.8789
K = 1.8040e-51
settlingTime = 14.8789
K = 2.1211e-51
settlingTime = 14.8789
K = 2.5148e-51
settlingTime = 14.8789
K = 3.0076e-51
settlingTime = 14.8789
K = 3.6301e-51
settlingTime = 14.8789
K = 4.4240e-51
settlingTime = 14.8789
K = 5.4464e-51
settlingTime = 14.8789
K = 6.7770e-51
settlingTime = 14.8789
K = 8.5276e-51
settlingTime = 14.8789
K = 1.0858e-50
settlingTime = 14.8789
K = 1.3997e-50
settlingTime = 14.8789
K = 1.8280e-50
settlingTime = 14.8789
K = 2.4202e-50
settlingTime = 14.8789
K = 3.2508e-50
settlingTime = 14.8789
K = 4.4330e-50
settlingTime = 14.8789
K = 6.1420e-50
settlingTime = 14.8789
K = 8.6533e-50
settlingTime = 14.8789
K = 1.2408e-49
settlingTime = 14.8789
K = 1.8123e-49
settlingTime = 14.8789
K = 2.6989e-49
settlingTime = 14.8789
K = 4.1022e-49
settlingTime = 14.8789
K = 6.3705e-49
settlingTime = 14.8789
K = 1.0119e-48
settlingTime = 14.8789
K = 1.6459e-48
settlingTime = 14.8789
K = 2.7447e-48
```

```
settlingTime = 14.8789
K = 4.6986e-48
settlingTime = 14.8789
K = 8.2683e-48
settlingTime = 14.8789
K = 1.4978e-47
settlingTime = 14.8789
K = 2.7972e-47
settlingTime = 14.8789
K = 5.3937e-47
settlingTime = 14.8789
K = 1.0757e-46
settlingTime = 14.8789
K = 2.2225e-46
settlingTime = 14.8789
K = 4.7662e-46
settlingTime = 14.8789
K = 1.0629e-45
settlingTime = 14.8789
K = 2.4697e-45
settlingTime = 14.8789
K = 5.9923e-45
settlingTime = 14.8789
K = 1.5215e-44
settlingTime = 14.8789
K = 4.0523e-44
settlingTime = 14.8789
K = 1.1349e-43
settlingTime = 14.8789
K = 3.3505e-43
settlingTime = 14.8789
K = 1.0456e-42
settlingTime = 14.8789
K = 3.4594e-42
settlingTime = 14.8789
K = 1.2169e-41
settlingTime = 14.8789
K = 4.5660e-41
settlingTime = 14.8789
K = 1.8333e-40
settlingTime = 14.8789
K = 7.9051e-40
settlingTime = 14.8789
K = 3.6737e-39
settlingTime = 14.8789
K = 1.8472e-38
settlingTime = 14.8789
K = 1.0090e-37
settlingTime = 14.8789
K = 6.0128e-37
settlingTime = 14.8789
K = 3.9264e-36
settlingTime = 14.8789
K = 2.8230e-35
settlingTime = 14.8789
K = 2.2456e-34
settlingTime = 14.8789
K = 1.9867e-33
settlingTime = 14.8789
K = 1.9656e-32
settlingTime = 14.8789
K = 2.1871e-31
settlingTime = 14.8789
K = 2.7536e-30
```

```
settlingTime = 14.8789
K = 3.9476e-29
settlingTime = 14.8789
K = 6.4873e-28
settlingTime = 14.8789
K = 1.2306e-26
settlingTime = 14.8789
K = 2.7146e-25
settlingTime = 14.8789
K = 7.0173e-24
settlingTime = 14.8789
K = 2.1432e-22
settlingTime = 14.8789
K = 7.7997e-21
settlingTime = 14.8789
K = 3.4130e-19
settlingTime = 14.8789
K = 1.8127e-17
settlingTime = 14.8789
K = 1.1802e-15
settlingTime = 14.8789
K = 9.5193e-14
settlingTime = 14.8789
K = 9.6160e-12
settlingTime = 14.8789
K = 1.2307e-09
settlingTime = 14.8789
K = 2.0200e-07
settlingTime = 14.8789
K = 4.3064e-05
settlingTime = 14.8762
K = 0.0121
settlingTime = 14.1549
K = 4.5286
settlingTime = 1.9036
K = 2.2994e+03
settlingTime = 1.9361
K = 1.6068e+06
settlingTime = 1.9554
K = 1.5710e+09
settlingTime = 1.9559
K = 2.1861e+12
settlingTime = 1.9549
```

14

Since $H(s) = \frac{G(s)}{G(s)+1} = \frac{1}{\mathrm{ms}^2 + s + (k+1)}$,in order to achive a pure oscillation, we should have $H(s) = \frac{f_v \omega_0^2}{s^2 + \omega_0^2}$

which clearly cnnnot happan!

8 System 2

Here are the state-space representation of the open-loop system:

$$x' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{2k}{m} & -\frac{1}{m} & \frac{2k}{m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{2k}{M} & 0 & -\frac{2k}{M} & -\frac{1}{M} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x + [0]u$$

2) finding G1 and G2

```
syms k m M
A =[0 1 0 0;-2*k/m -1/m 2*k/m 0;
    0 0 0 1; 2*k/M 0 -2*k/M -1/M];
B = [0;0;0;1/M];
C = [1 0 0 0;0 1 0 0];
D = 0;

syms s
G =  C*(s*eye(size(A))-A)^(-1)*B +D
```

G =

$$\begin{pmatrix} \frac{2\,k}{4\,k\,s + M\,s^3 + m\,s^3 + s^2 + 2\,k\,m\,s^2 + 2\,M\,k\,s^2 + M\,m\,s^4} \\ \frac{2\,k}{4\,k + s + M\,s^2 + m\,s^2 + 2\,M\,k\,s + 2\,k\,m\,s + M\,m\,s^3} \end{pmatrix}$$

3) replace u(t) with $K_p(r(t) - y(t))$:

$$x' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{2k}{m} & -\frac{1}{m} & \frac{2k}{m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{2k - K_p}{M} & 0 & -\frac{2k}{M} & -\frac{1}{M} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_p}{M} \end{bmatrix} u$$

$$y_1 = [1 \quad 0 \quad 0 \quad 0]x + [0]u$$

$$x' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{2k}{m} & -\frac{1}{m} & \frac{2k}{m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{2k}{M} & -\frac{K_p}{M} & -\frac{2k}{M} & -\frac{1}{M} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_p}{M} \end{bmatrix} u$$

$$y_2 = [1 \quad 0 \quad 0 \quad 0]x + [0]u$$

$$T_1(s) = \frac{\mathrm{KpG}_1(s)}{1 + G_1(s)}, T_2(s) = \frac{\mathrm{KpG}_2(s)}{1 + G_2(s)}$$

```
syms k m M Kp
A =[0 1 0 0;-2*k/m -1/m 2*k/m 0;
    0 0 0 1; (2*k-Kp)/M 0 -2*k/M -1/M];
B = [0;0;0;Kp/M];
C = [1 0 0 0];
D = 0;

syms s
T1 =  C*(s*eye(size(A))-A)^(-1)*B +D
```

T1 =

$$\frac{2\,\mathrm{Kp}\,k}{2\,\mathrm{Kp}\,k + 4\,k\,s + M\,s^3 + m\,s^3 + s^2 + 2\,k\,m\,s^2 + 2\,M\,k\,s^2 + M\,m\,s^4}$$

```
syms k m M Kp
A =[0 1 0 0;-2*k/m -1/m 2*k/m 0;
    0 0 0 1; 2*k/M -Kp/M -2*k/M -1/M];
B = [0;0;0;Kp/M];
C = [0 1 0 0];
D = 0;

syms s
T2 =  C*(s*eye(size(A))-A)^(-1)*B +D
```

T2 =

$$\frac{2\,\mathrm{Kp}\,k}{4\,k + s + 2\,\mathrm{Kp}\,k + M\,s^2 + m\,s^2 + 2\,M\,k\,s + 2\,k\,m\,s + M\,m\,s^3}$$

4)

$$\lim_{s\longrightarrow 0}\begin{pmatrix}\dfrac{2\,k}{4\,k\,s + M\,s^3 + m\,s^3 + s^2 + 2\,k\,m\,s^2 + 2\,M\,k\,s^2 + M\,m\,s^4}\\[2ex] \dfrac{2\,k}{4\,k + s + M\,s^2 + m\,s^2 + 2\,M\,k\,s + 2\,k\,m\,s + M\,m\,s^3}\end{pmatrix} =$$

$$\lim_{s\longrightarrow 0}\begin{pmatrix}\dfrac{2\,k}{4\,k\,s + M\,s^3 + m\,s^3 + s^2 + 2\,k\,m\,s^2 + 2\,M\,k\,s^2 + M\,m\,s^4}\\[2ex] \dfrac{2\,k}{4\,k + s + M\,s^2 + m\,s^2 + 2\,M\,k\,s + 2\,k\,m\,s + M\,m\,s^3}\end{pmatrix} = \begin{bmatrix}\infty\\ 0.5\end{bmatrix} = k_p$$

$$\lim_{s\longrightarrow 0} s\begin{pmatrix}\dfrac{2\,k}{4\,k\,s + M\,s^3 + m\,s^3 + s^2 + 2\,k\,m\,s^2 + 2\,M\,k\,s^2 + M\,m\,s^4}\\[2ex] \dfrac{2\,k}{4\,k + s + M\,s^2 + m\,s^2 + 2\,M\,k\,s + 2\,k\,m\,s + M\,m\,s^3}\end{pmatrix} =$$

$$\lim_{s\longrightarrow 0} s\begin{pmatrix}\dfrac{2\,k}{4\,k\,s + M\,s^3 + m\,s^3 + s^2 + 2\,k\,m\,s^2 + 2\,M\,k\,s^2 + M\,m\,s^4}\\[2ex] \dfrac{2\,k}{4\,k + s + M\,s^2 + m\,s^2 + 2\,M\,k\,s + 2\,k\,m\,s + M\,m\,s^3}\end{pmatrix} = \begin{bmatrix}0.5\\ 0\end{bmatrix} = k_V$$

$$e_p = \frac{A}{1+K_p}, e_p = \begin{bmatrix} 0 \\ \frac{2}{3} \end{bmatrix}$$

$$e_v = \frac{A}{K_v}, e_v = \begin{bmatrix} 2 \\ \infty \end{bmatrix}$$

```
k = 10;
m = 1;
M = 1;
Kp = 5;

% case 1
syms s t
A =[0 1 0 0;-2*k/m -1/m 2*k/m 0;
    0 0 0 1; (2*k-Kp)/M 0 -2*k/M -1/M];
B = [0;0;0;Kp/M];
C = [1 0 0 0];

T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s;
eval(poles(T1))
```

```
ans = 5×1 complex
  -0.5000 + 6.0886i
  -0.5000 - 6.0886i
  -0.5000 + 1.5587i
  -0.5000 - 1.5587i
   0.0000 + 0.0000i
```

stable

```
ys_1 = ilaplace(T1);
T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s^2;
yr_1 = ilaplace(T1);

A =[0 1 0 0;-2*k/m -1/m 2*k/m 0;
    0 0 0 1; 2*k/M -Kp/M -2*k/M -1/M];
B = [0;0;0;Kp/M];
C = [0 1 0 0];

T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s;
ys_11 = ilaplace(T1);
eval(poles(T1))
```

```
ans = 4×1 complex
   0.5702 + 6.6525i
   0.5702 - 6.6525i
  -3.1403 + 0.0000i
   0.0000 + 0.0000i
```

unstable

```
T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s^2;
yr_11 = ilaplace(T1);
eval(poles(T1))
```

```
ans = 4×1 complex
   0.5702 + 6.6525i
   0.5702 - 6.6525i
  -3.1403 + 0.0000i
   0.0000 + 0.0000i
```

```
% case 2
k = 5;
m = 1;
M = 2;
Kp = 1;

syms s t
A =[0 1 0 0;-2*k/m -1/m 2*k/m 0;
    0 0 0 1; (2*k-Kp)/M 0 -2*k/M -1/M];
B = [0;0;0;Kp/M];
C = [1 0 0 0];

T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s;
ys_2 = ilaplace(T1);
eval(poles(T1))
```

```
ans = 5×1 complex
  -0.3326 - 0.4814i
  -0.3326 + 0.4814i
  -0.4174 + 3.7984i
  -0.4174 - 3.7984i
   0.0000 + 0.0000i
```

stable

```
T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s^2;
yr_2 = ilaplace(T1);
eval(poles(T1))
```

```
ans = 5×1 complex
  -0.3326 - 0.4814i
  -0.3326 + 0.4814i
  -0.4174 + 3.7984i
  -0.4174 - 3.7984i
   0.0000 + 0.0000i
```

```
A =[0 1 0 0;-2*k/m -1/m 2*k/m 0;
    0 0 0 1; 2*k/M -Kp/M -2*k/M -1/M];
B = [0;0;0;Kp/M];
C = [0 1 0 0];

T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s;
ys_21 = ilaplace(T1);
eval(poles(T1))
```

```
ans = 4×1 complex
  -0.2500 + 3.8649i
  -0.2500 - 3.8649i
  -1.0000 + 0.0000i
   0.0000 + 0.0000i
```

stable

```
T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s^2;
yr_21 = ilaplace(T1);
eval(poles(T1))
```

```
ans = 4×1 complex
  -0.2500 + 3.8649i
  -0.2500 - 3.8649i
  -1.0000 + 0.0000i
   0.0000 + 0.0000i
```

```
% case 3
k = 10;
m = 1;
M = 10;
Kp = 100;


syms s t
A =[0 1 0 0;-2*k/m -1/m 2*k/m 0;
    0 0 0 1; (2*k-Kp)/M 0 -2*k/M -1/M];
B = [0;0;0;Kp/M];
C = [1 0 0 0];

T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s;
ys_3 = ilaplace(T1);
eval(poles(T1))
```

```
ans = 5×1 complex
   1.0328 + 3.3176i
   1.0328 - 3.3176i
  -1.5828 + 3.7497i
  -1.5828 - 3.7497i
   0.0000 + 0.0000i
```

unsatble

```
T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s^2;
yr_3 = ilaplace(T1);
eval(poles(T1))
```

```
ans = 5×1 complex
   1.0328 + 3.3176i
   1.0328 - 3.3176i
  -1.5828 + 3.7497i
  -1.5828 - 3.7497i
   0.0000 + 0.0000i
```

```
A =[0 1 0 0;-2*k/m -1/m 2*k/m 0;
    0 0 0 1; 2*k/M -Kp/M -2*k/M -1/M];
B = [0;0;0;Kp/M];
C = [0 1 0 0];

T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s;
ys_31 = ilaplace(T1);
eval(poles(T1))
```

```
ans = 4×1 complex
   1.9266 - 6.1216i
   1.9266 + 6.1216i
  -4.9532 + 0.0000i
   0.0000 + 0.0000i
```

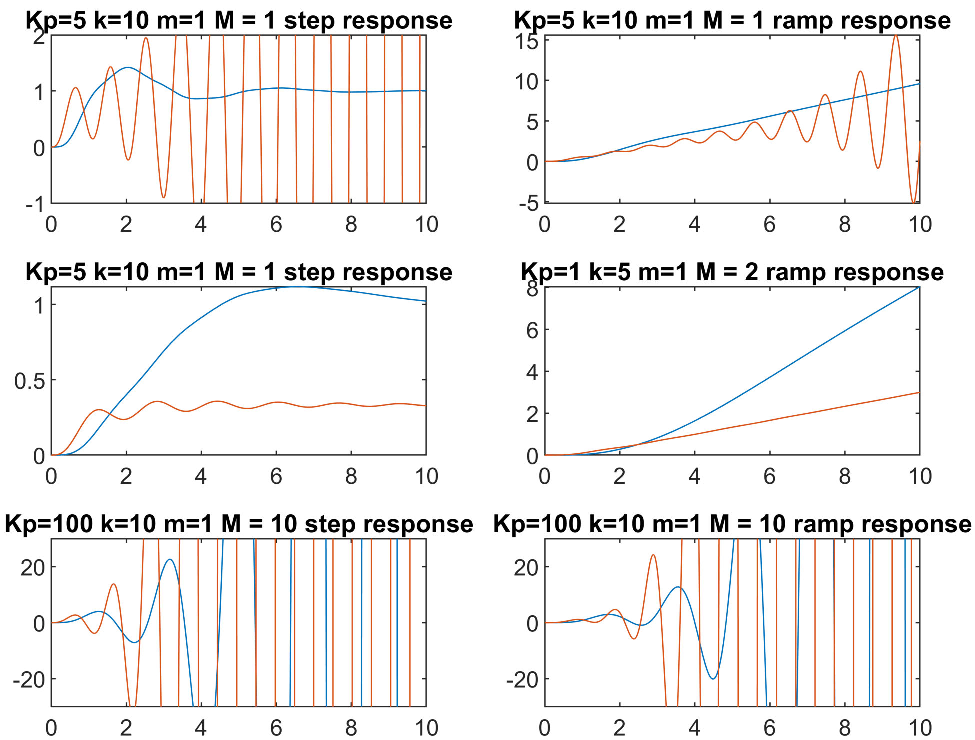
unsatble

```
T1 = C*(s*eye(size(A))-A)^(-1)*B * 1/s^2;
yr_31 = ilaplace(T1);
eval(poles(T1))
```

```
ans = 4×1 complex
   1.9266 - 6.1216i
   1.9266 + 6.1216i
  -4.9532 + 0.0000i
   0.0000 + 0.0000i
```

```
figure
syms t
subplot(3,2,1)
fplot(ys_1)
hold on
fplot(ys_11)
title("Kp=5 k=10 m=1 M = 1 step response")
xlim([0 10])
ylim([-1 2])

subplot(3,2,2)
fplot(yr_1)
hold on
fplot(yr_11)
title("Kp=5 k=10 m=1 M = 1 ramp response")
xlim([0 10])

subplot(3,2,3)
fplot(ys_2)
hold on
fplot(ys_21)
title("Kp=5 k=10 m=1 M = 1 step response")
xlim([0 10])

subplot(3,2,4)
fplot(yr_2)
hold on
fplot(yr_21)
title("Kp=1 k=5 m=1 M = 2 ramp response")
xlim([0 10])

subplot(3,2,5)
fplot(ys_3)
hold on
fplot(ys_31)
title("Kp=100 k=10 m=1 M = 10 step response")
xlim([0 10])
```

```
ylim([-30 30])

subplot(3,2,6)
fplot(yr_3)
hold on
fplot(yr_31)
title("Kp=100 k=10 m=1 M = 10 ramp response")
ylim([-30 30])
xlim([0 10])
```



2end is stable ,but 1st and 3th is unstable.

The steady error for the second case is the same as in theory.

In firt case , state 1 is stable but second isn't.

```
function [u] = impulse_me(e,t)

u = e* (Step01(t) - Step00( t-(1/(e)) ) );
end
function [u] = Step01(C)
u = C;
s = length(C);
for i=1:s
    if C(i)>=0
        u(i)=1;
    else
        u(i)=0;
```

```
    end
end
end
function [u] = Step00(C)
u = C;
s = length(C);
for i=1:s
    if C(i)>0
        u(i)=1;
    else
        u(i)=0;
    end
end
end
```