In the name of allah

Linear Control Systems

Hw 05

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Fall 1402

Theory Assignments

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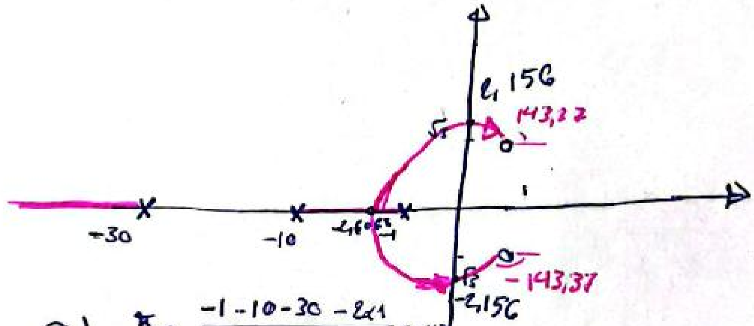
حل 1)

$$G_1 = \frac{(S-1+j\sqrt{3})(S-1-j\sqrt{3})}{(S+1)(S+10)(S+30)}$$



a) $\sigma = \frac{-1-10-30-2\times1}{1} = -43$

b) $$\frac{dG_1}{dS} = \frac{(2S-2)(S^3+41S^2+340S+300) - (S^2-2S+4)(3S^2+82S+340)}{(S+1)^2(S+10)^2(S+30)^2}$$

$-2S^3 - 82S^2 - 680S - 600$ $\quad -12S^2 - 528S - 1360$

$2S^4 + 82S^3 + 680S^2 + 600S$ $\quad 6S^3 + 164S^2 + 680S$

$-3S^4 - 82S^3 - 340S^2$

$= -S^4 + 4S^3 + 410S^2 + 272S - 1960$

$S = 22{,}54,\ -17{,}813,\ \boxed{-2{,}6068},\ 1{,}872$

خط‌ها

نقطه شکست

c) $S^3 + 41S^2 + 340S + 300 + kS^2 - 2kS + 4k$

S^3	1	$340-2k$
S^2	$k+41$	$300+4k$
S^1	α	
S^0	$300+4k$	

$k > -41$ $\quad -41 > k$ $\quad -49{,}674 < k < 167{,}674$

$k > -75$

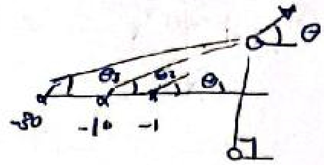
$\Rightarrow -49{,}674 < k < 167{,}674$

$$\alpha = \frac{-2k^2 + 254k + 13640}{k+41}$$

$(41+k)S^2 + 300 + 4k = 0$

$k = 167{,}674 \rightarrow S = \pm 2{,}156j$

d) زوایای خروج از قطب‌ها مشخص است



$\theta + 90 - \theta_1 - \theta_2 - \theta_3 = 180$

$\theta + 90 - (40{,}9 + 8{,}94 + 3{,}2) = 180 \Rightarrow \theta_{1+\sqrt{3}i} = 143{,}37$

$\hookrightarrow \theta_{1-\sqrt{3}i} = -143{,}37$

e) breaking Point

$S = -2{,}6068$

$\Rightarrow k = 20{,}326$

$$G_2 = \frac{S+4}{S(S+6)(S+8)(S^2+3S+41)}$$

a) $k>0$: $N-m = 4$ $\theta_a = 45, 135, -45, -135$

$$\sigma_A = \frac{0-6-8-3+4}{4} = \frac{-13}{4} = -3{,}25$$

b) $\frac{d}{ds}\left(\frac{S+4}{S(S+6)(S+8)(S^2+3S+41)}\right) = 0 \Rightarrow S = -7{,}0775$, قابل قبول

break aut point

c) $S^5 + 17S^4 + 94S^3 + 200S^2 + 192S + kS + 4k$

S^5	1	94	$192+k$
S^4	17	200	$4k$
S^3	82,235	a	
S^2	b	4k	
S^1	c		
S^0	4k		

$k < 1014$

$-2080 < k < 122$, $1014 < k$

$k > 0$

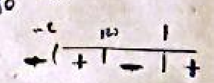
$a = \frac{13k + 3264}{17} = \frac{13}{17}k + 192$

$b = \frac{-13k^2 - 3264 + 16447}{82{,}35}$

$c = \frac{ba - 4k \times 82{,}235}{b}$

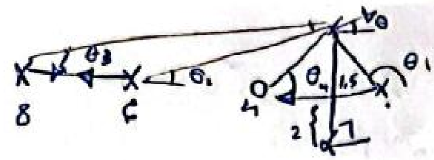
$c = 0 \Rightarrow$ marginal unstable $k = 122$, -2080

$122 > k > -2080$



$bS^2 + 4k$ .. $k = 122 \Rightarrow S = \pm i\, 1{,}86$

d)



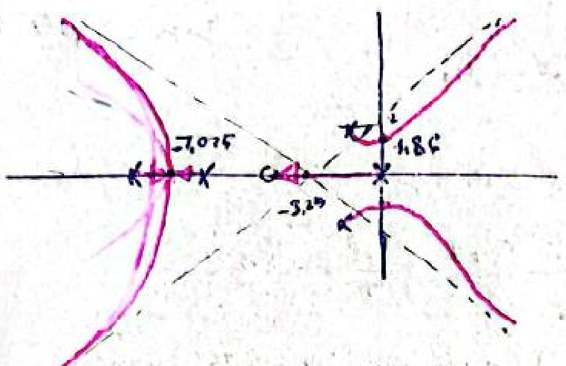
$$\Rightarrow \theta_4 - (\theta_1 + \theta_2 + \theta_3 + 90 + \theta) = 180$$

$$38.66 - (27 + 23.76 + 90 + 17\theta) = 180$$

$$\Rightarrow \theta = -39.4$$

e) $s = (-7.0775)$ breaking out Point

$\Rightarrow k = 75.112$



(2Obs

$$1 + K G(s) = 1 + k \frac{\prod^{m} (s + z_i)}{\prod^{n} (s + P_i)} = 1 + K \frac{s^m + \sum^m z_i s^{m-1} - -}{s^n + \sum^n P_i s^{n-1} + -}$$

$$s_o^n + \left(\sum^n P_i\right) s_o^{n-1} + k s_o^m + k \left(\sum^m z_i\right) s_o^{m-1} \simeq 0$$

$$-k = \frac{s_o^n + \sum^n P_i s_o^{n-1}}{s_o^m + \sum^m z_i s_o^{m-1}} = s_o^{n-m} \left(1 + \frac{\sum^n P_i - \sum^m z_i}{s_o} + -\right)$$

$$\Rightarrow (-k)^{\frac{1}{n-m}} = s_o \left(1 + \frac{\sum P_i - \sum z_i}{s_o}\right)^{\frac{1}{n-m}} \simeq s_o \left(1 + \frac{\sum^n P_i - \sum^m z_i}{s_o (n-m)}\right)$$

$$s_o \simeq (-k)^{\frac{1}{n-m}} - \frac{\sum^n P_i - \sum^m z_i}{n-m}$$

$$s_o = k^{\frac{1}{n-m}} e^{\frac{j(2l+1)}{n-m}} - \frac{\sum P_i - \sum z_i}{n-m} \Rightarrow \sigma_A = \frac{+\sum^n -P_i - \sum^m -z_i}{n-m}$$

$\boxed{k > 0}$

سوال ۳)

$$\frac{s+2}{(s+1)^2} \xrightarrow[\text{شیفت می‌دهیم}]{\text{جابه‌جا به راست}} \frac{s}{(s-1)^2} \xrightarrow{\text{قطب‌ها}} s^2-(2-k)s+1$$

$$\Rightarrow \frac{-k+2 \pm \sqrt{(2-k)^2-4}}{2} \xRightarrow{0<k<4} |P| = \frac{(-k+2)^2 + (2-k)^2 - 4}{4} = 1$$

پس قطب‌ها روی دایره به شعاع ۱ و مرکز ‎-2‎ هستند.

سوال 4)

$$G(s) = \frac{s+0{,}1}{s(s-0{,}2)(s^2+s+0{,}6)} \qquad \begin{matrix} -0{,}1 \\ 0,\ 0{,}2,\ -0{,}5 \pm \sqrt{\frac{k1}{40}} \end{matrix}$$

a) $$\sigma_A = \frac{0+0{,}2-1+0{,}1}{3} = \frac{-7}{30}$$

$$\theta_A^{k>0} = \frac{\pi}{3}, \pi, -\frac{\pi}{3} \quad , \quad \theta_A^{k<0} = 0, \frac{2}{3}\pi, -\frac{2}{3}\pi$$

b)

$$\Rightarrow \frac{d}{ds}\, \frac{s+0{,}1}{s(s-0{,}2)(s^2+s+0{,}6)} = 0$$

$$(0{,}1)\, s(s-0{,}2)(s^2+s+0{,}6) - (s+0{,}1)\left(s(s-0{,}2)(s^2+s+0{,}6)\right)' = 0$$

$$s = \boxed{\begin{matrix}-0{,}3647 \\ 0{,}0815603\end{matrix}} \text{ و } -0{,}191778 \pm 0{,}312583i$$

قابل‌قبول

c) $$s^4 + 0{,}8s^3 + 0{,}4s^2 + (0{,}1k - 0{,}12)s + 0{,}1k$$

$k = 1{,}71 \Rightarrow s = \pm j0{,}713$

$k = 3{,}09 \Rightarrow s = \pm j1{,}374$

$$\begin{array}{l|lll} s^4 & 1 & 0{,}4 & 0{,}1k \\ s^3 & 0{,}8 & 0{,}1k-0{,}12 & \\ s^2 & a & 0{,}1k & \\ s^1 & b & & \\ s^0 & 0{,}1k & & \end{array}$$

$k < 4{,}4$

$1{,}71 < k < 3{,}09$ یا $k > 4{,}4$

$k > 0$

$$a = \frac{0{,}1k - 0{,}12 - 0{,}32}{-0{,}8} = \frac{4{,}4-k}{8}$$

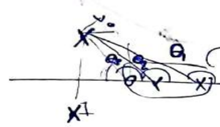
$$b = \frac{-0{,}08k + a(0{,}1k-0{,}12)}{a}$$

$1{,}7 \quad 3 \quad 4{,}4$

$- \quad + \quad - \quad +$

$\Rightarrow 1{,}71 < k < 3{,}09 \Leftarrow$ پایدار است

d)

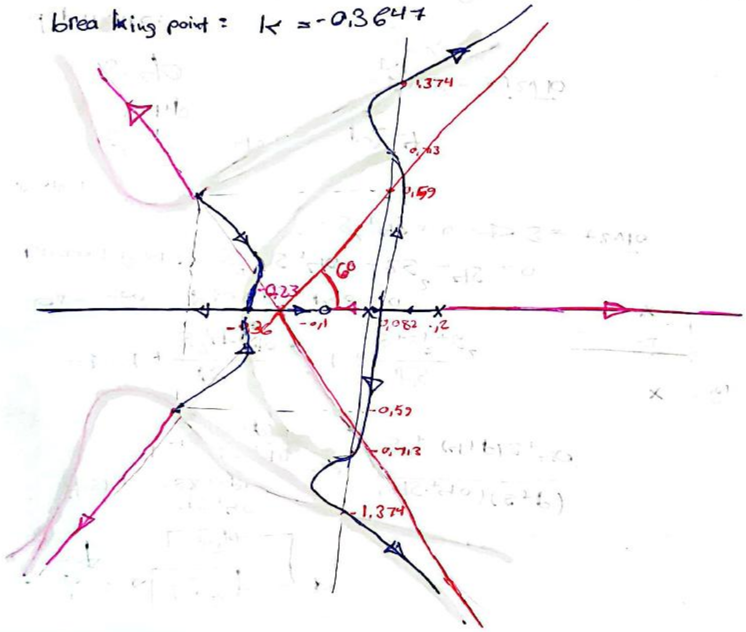


زاویه‌های ورود و خروج مشخص است.

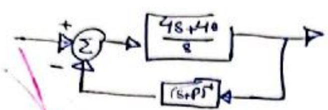
$K>0$ $\quad 180 = \overset{124}{\theta_3} - (90 + \overset{139}{\theta_1} + \overset{130}{\theta_2} + \theta) \Rightarrow \theta = -55$

$K<0$ $\quad 360 = \quad \Rightarrow \theta = 125$

e) breaking point: $K = -0.3647$

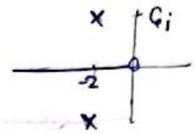


(5 حل



$$H(s) = \frac{\frac{4s+40}{s}}{1+\frac{4s+40}{(s+p)s}} = \frac{(4s+40)(s+p)}{s^2+(4+p)s+40}$$

$$\Rightarrow 1+\frac{ps}{s^2+4s+40} = 1+\frac{ps}{(s+2)^2+6^2}$$



σ_A = نیست $\quad p>0:\ \pi,\quad p<0:\ 0$

Breaking Point: $s^2+4s+40-2s^2-4s=0$

$$-s^2+40=0 \Rightarrow s=\pm 2\sqrt{10}$$

Roth:

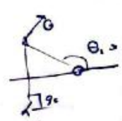
s^2	1	40
s^1	$4+p$	
s^0	40	

$p>-4$

$\Rightarrow s=\pm j2\sqrt{10}$

$\Rightarrow p<0$

departure:

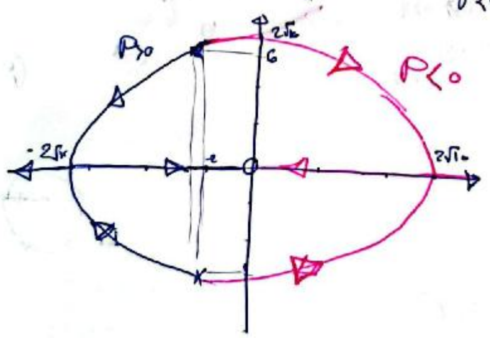


$\theta_1 = 108.43$

$p>0$: $-\theta-90+108.43=180$

$p<0$: $\theta = -161.5$

$\theta = 18.43$



CS Scanned with CamScanner

سوال 6

1) با توجه به اینکه نویز ورودی از فیلتر رد خواهد شد محتوای فرکانس بالای خود را از دست خواهد داد و به صورت نرم تغییر خواهد کرد D
2) از آنجایی که نویز خروجی دارای محتوای فرکانسی بالاست H
3) چون مقدار K به عنوان گین dc است C
4) با کاهش مقدار این قطب به محور موهومی شاهد سریعتر شدن و با دور شدن آن شاهد کندشدن پاسخ پله می شویم A
5) چون این قطب از سایر قطب ها دورتر است،تاثیر کمی روی پاسخ دارد E
6) تغییر مقدار ζ مقدار اورشوت و اندر شوت تغییر می کند F
7) تغییرات ω موجب تغییرات شدید روی فرکانس میرایی می شود I
8) این صفر ،صفر سمت چپ بوده بنابرین روی دامنه پاسخ گذرا را اثر می گذارد G
9) صفر سمت راست اندر شوت را به شدت تقویت می کند B

```
clc
close all
clear all
```

MATLAB Assignments

7 Breakaway Points

part a)

```
syms  s
G1(s) =  (s^2+2*s+2)/(s*(s^2+0.25))
```

```
G1(s) =
```

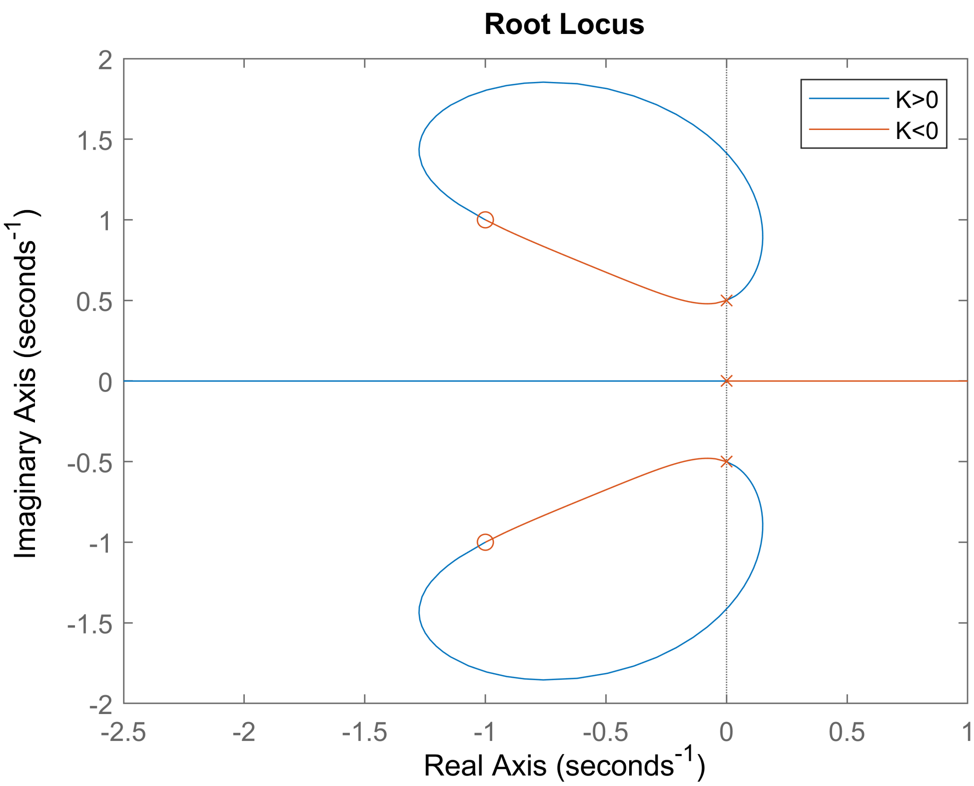
$$\frac{s^2+2\,s+2}{s\,\left(s^2+\frac{1}{4}\right)}$$

```
anser = partA(G1,s)
```

```
anser =

Empty sym: 0-by-1
```

```
sys1 = tf([1 2 2],[1 0 0.25 0]);
sys2 = -sys1;
rlocus(sys1,sys2)
legend("K>0","K<0")
```




```
G2(s) = (s^2 / (s^2-1)^2)
```

G2(s) =

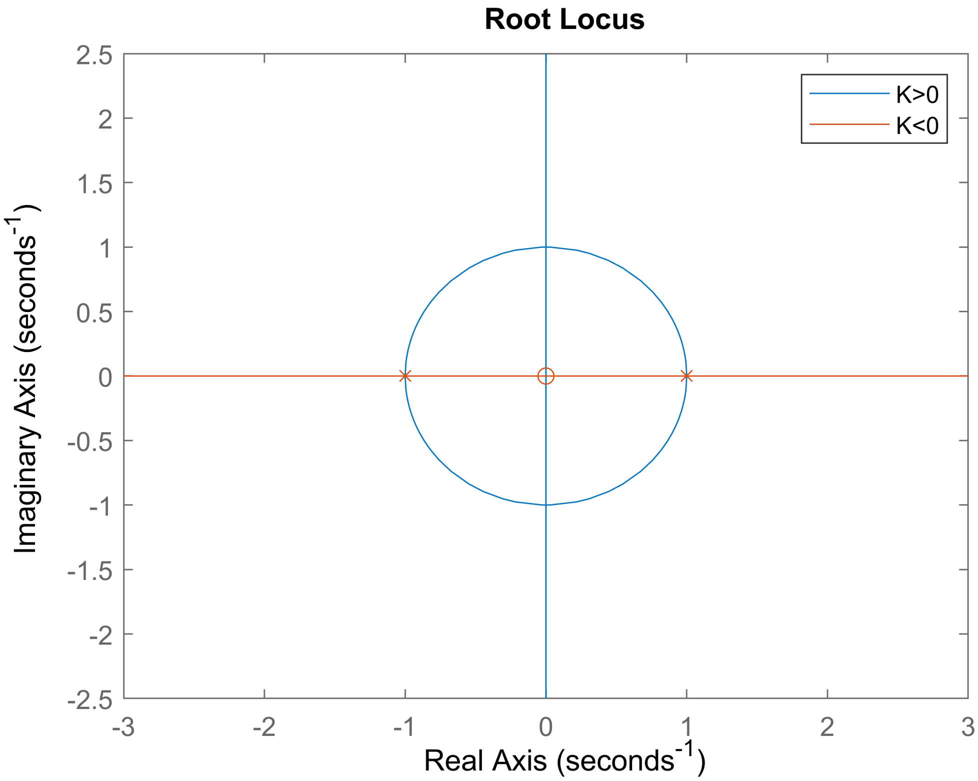
$$\frac{s^2}{(s^2-1)^2}$$

```
anser = partA(G2,s)
```

anser =

$$\begin{pmatrix} 0 \\ -1.0\,\mathrm{i} \\ 1.0\,\mathrm{i} \end{pmatrix}$$

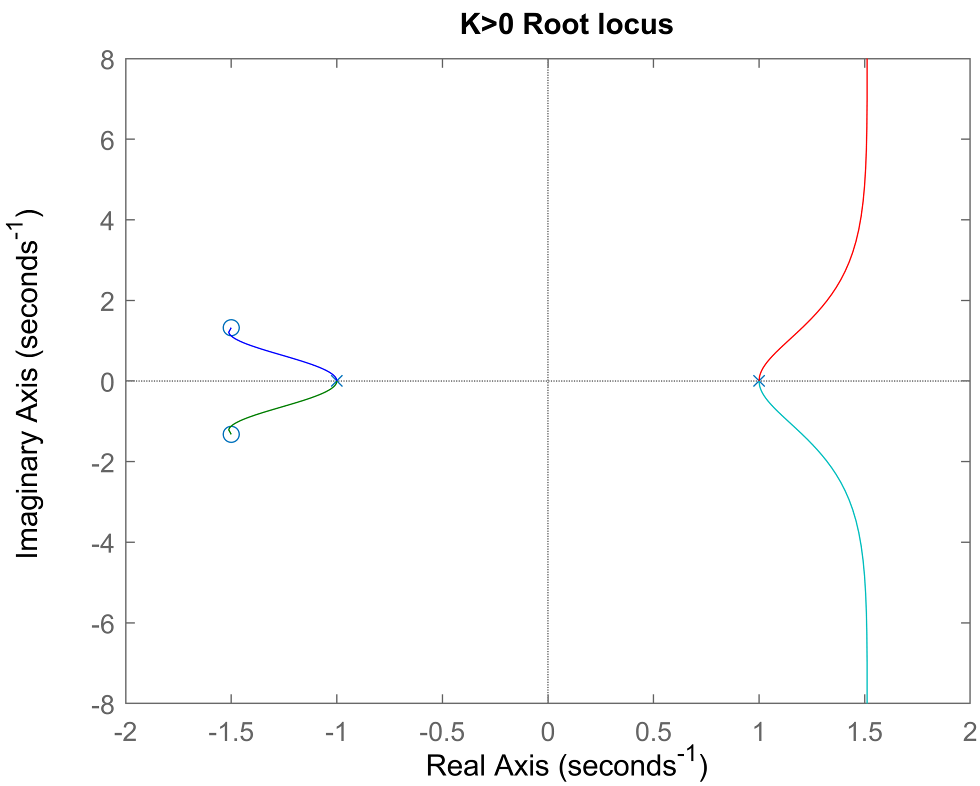
```
sys1 = tf([1 0 0],[1 0 -2 0 1]);
sys2 = -sys1;
rlocus(sys1,sys2)
legend("K>0","K<0")
```



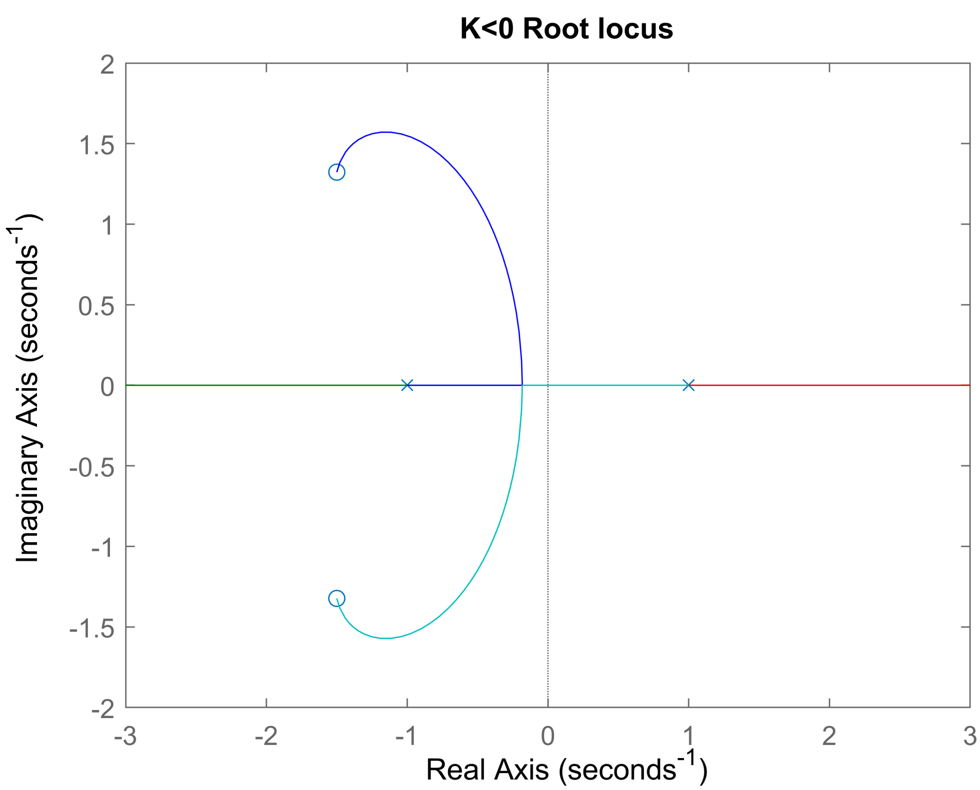
part (b)

We can plot complementary root locus with By multiplying -1 in the transfer function

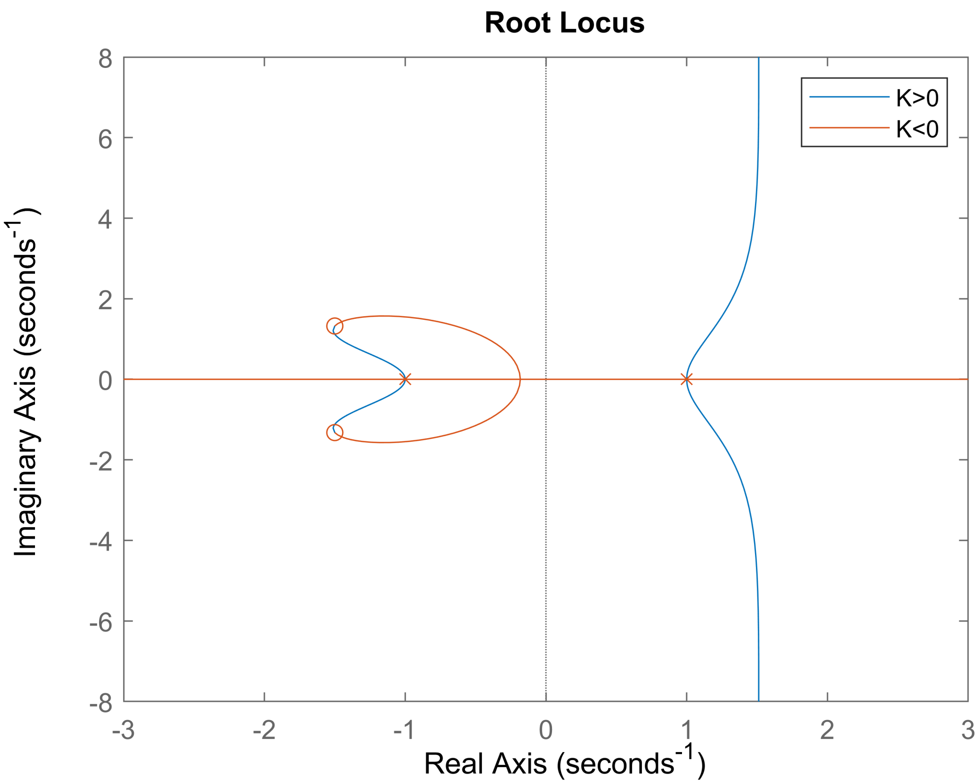
```
sys1 = tf([1 3 4],[1 0 -2 0 1]);
sys2 = -sys1;
rlocus(sys1)
title("K>0 Root locus")
```




```
rlocus(sys2)
title("K<0 Root locus")
```



```
rlocus(sys1,sys2)
legend("K>0","K<0")
```



part (c)

```
G1(s) =  (s^2+2*s+2)/(s*(s^2+0.25))
```

G1(s) =

$$\frac{s^2+2\,s+2}{s\,\left(s^2+\frac{1}{4}\right)}$$

```
[R,complementaryR,non] = partC(G1,s)
```

R =

Empty sym: 0-by-1

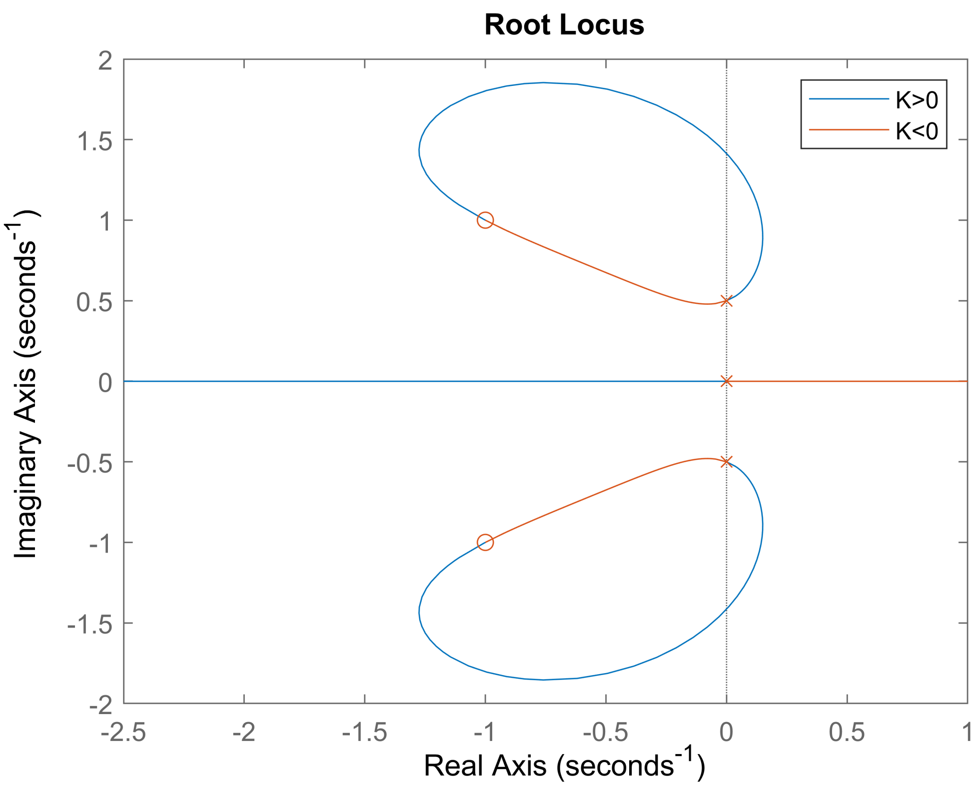
complementaryR =

Empty sym: 0-by-1

non =

$$\begin{pmatrix} -2.0291289456597803634180201034811 - 1.3357918919869738626969712269717\,i \\ -2.0291289456597803634180201034811 + 1.3357918919869738626969712269717\,i \\ 0.029128945659780363418020103481059 - 0.28960800211921472630369875126637\,i \\ 0.029128945659780363418020103481059 + 0.28960800211921472630369875126637\,i \end{pmatrix}$$

```
sys1 = tf([1 2 2],[1 0 0.25 0]);
sys2 = -sys1;
rlocus(sys1,sys2)
legend("K>0","K<0")
```




```
G2(s) = (s^2 / (s^2-1)^2)
```

G2(s) =

$$\frac{s^2}{(s^2-1)^2}$$

```
[R,complementaryR,non] = partC(G2,s)
```

R =

$$\begin{pmatrix} -1.0\,i \\ 1.0\,i \end{pmatrix}$$

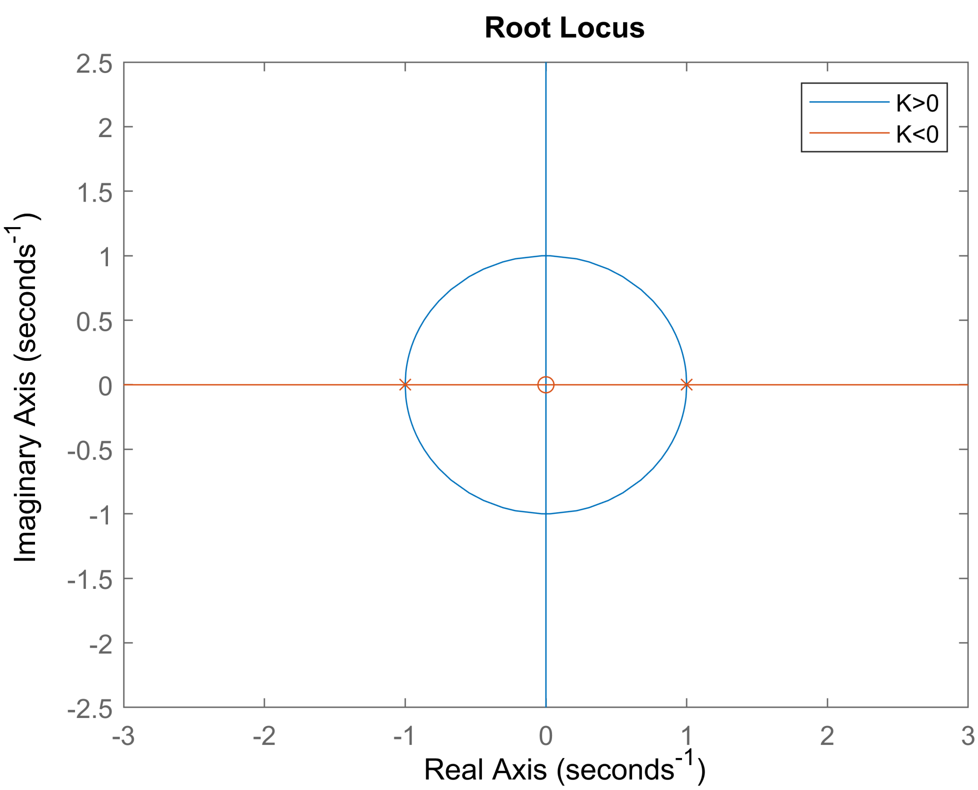
complementaryR =

Empty sym: 0-by-1

```
non =

Empty sym: 0-by-1
```

```
sys1 = tf([1 0 0],[1 0 -2 0 1]);
sys2 = -sys1;
rlocus(sys1,sys2)
legend("K>0","K<0")
```




```
function G = partA(Gi , s)
Gip(s) = diff(Gi , s);          % finding diff
s1 = vpasolve(Gip == 0, s);     % finding valid s
GG = Gi(s1);                    % value of G(s)
idg = imag(GG) == 0;            % find real G(s0)
G = s1(idg);                    % delete complex G(s0)
end

function [G,Gc,N] = partC(Gi , s)
Gip(s) = diff(Gi , s);          % finding diff
s1 = vpasolve(Gip == 0, s);     % finding valid s
GG = Gi(s1);                    % value of G(s)
idg = (imag(GG) == 0);          % find real G(s0)
N = s1(~idg);                   % delete all real values G(s0)
idgc = imag(GG) == 0 & real(GG) > 0;  % find real G(s0)>0 :K<0
idga = imag(GG) == 0 & real(GG) < 0;  % find real G(s0)<0 :K>0
```

```
G = s1(idga);                       % add just K>0
Gc = s1(idgc);                      % add just K<0

end
```