

Evolution of Anime Co-Viewership Networks (2006–2018)

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ABSTRACT

This study analyzes the structural evolution of the MyAnimeList platform (2006–2018). Utilizing time-sliced networks, we examine the dynamics of taste communities and user navigation. We employ the Leiden algorithm to trace the genealogy of clusters from a cohesive core to a fragmented landscape. Regarding user modeling, we find that stochastic random walker agents diverge from empirical trajectories, indicating the limitations of purely topological simulation. In contrast, machine learning experiments successfully predict user migration patterns. Crucially, we demonstrate that the inclusion of community-based clustering metrics significantly improves model accuracy, highlighting the predictive value of mesoscale network features.

1 INTRODUCTION

1.1 The MyAnimeList Ecosystem as a Complex Network

The digital aggregation of cultural preferences has transformed the study of computational sociology. MyAnimeList (MAL), established in 2006, serves as a primary registry for anime consumption. Unlike generic social networks where edges represent declared friendships, the primary structure of MAL is a bipartite interest graph connecting users to titles. This structure provides an optimal environment for analyzing "affinity networks," where community formation is driven by taste homophily rather than geographic proximity.

The period between 2006 and 2018 represents a critical epoch in the globalization of Japanese animation, characterized by the transition from a niche subculture to a mainstream entertainment force. Analyzing this period allows for the observation of a complete evolutionary cycle of a complex network—from a cohesive core to a fragmented, multi-polar topology.

1.2 Objectives and Research Questions

The primary objective of this study is to model the MAL community as a dynamic, evolving system. Traditional static graph analysis fails to capture the temporal fluidity of user interests. To address this, we employ a multi-stage analytical framework combining dynamic community detection, stochastic simulation, and predictive modeling.

A key focus of our investigation is the evaluation of topological agents. We test the hypothesis that standard Random Walker models, which rely solely on structural connectivity, are sufficient to simulate user navigation in affinity networks. Furthermore, we explore whether the predictive power of machine learning models can be enhanced by incorporating mesoscale network features derived from community detection.

The research is guided by the following key questions:

- **Topological Evolution:** How did the structural properties of the MAL graph change during the community's expansion (2006–2018)?
- **Community Dynamics:** How do taste clusters evolve over time, and can we identify distinct phases of fragmentation using dynamic community detection (Leiden)?
- **Limits of Topological Simulation:** To what extent do empirical user trajectories diverge from theoretical Random Walk models?
- **Predictive Modeling:** Can we predict user migration between communities using supervised learning, and does the inclusion of clustering metrics improve model performance?

2 DATA AND METHODOLOGY

2.1 Dataset Acquisition and Preprocessing

The data originates from a publicly available kaggle datasets aggregated from MAL profiles [1, 5], covering the period from 2006 to 2018. Given the platform's predominantly international user base, the dataset reflects global consumption patterns rather than domestic Japanese trends.

To ensure data quality, we applied a multi-stage filtering pipeline:

- **Bot Detection:** Removal of inactive accounts and profiles exhibiting automated behavior.
- **Percentile-based Truncation:** We excluded users falling into the extreme tails of the activity distribution. This removes users with too few votes (insufficient signal for clustering) and those with implausibly high vote counts, ensuring the analysis focuses on human-scale consumption patterns.

The final processed dataset comprises approximately **85,000 unique users** and **6,500 anime titles**.

2.2 Graph Projection and Topology Construction

We model the system as a bipartite graph which is subsequently projected into two distinct monopartite networks. The detailed construction pipelines are described in Sections ?? and ??, respectively.

2.2.1 Anime-Anime Network. In this projection, an edge exists if two titles share a common voter. To account for varying audience sizes, we utilized the **Jaccard Similarity** index as the edge weight.

$$J(A, B) = \frac{|U_A \cap U_B|}{|U_A \cup U_B|}$$

Given the extreme density of the raw projection (where a single popular anime could fully connect thousands of users), we applied a hard threshold of $J > 0.05$. This effectively prunes weak links formed by random coincidences while preserving significant genre or fandom connections.

2.2.2 User-User Network. Here, an edge connects two users if they have rated the same anime. The edge weight is defined as the raw count of shared titles (co-votes). A major challenge in this projection is the variance in edge weights, which range from negligible values (2–3 shared items) to tens of thousands (10^4). To address this, we implemented a cutoff threshold: edges were retained only if users shared more than **3 titles**.

Anyway, even after thresholding, the user-user raw network projection suffered from extreme density saturation. Popular "blockbuster" titles (e.g., *Death Note*, *Attack On Titan*) act as super-hubs; a single vote for such a title effectively connects a user to thousands of others, creating a near-clique structure that obscures genuine taste communities. So, these titles had to be deleted from the dataset prior to projection.

2.2.3 Further Sparsification Attempts. More aggressive sparsification techniques (e.g., Backbone extraction, k-NN) were tested but did not reveal significantly distinct structural patterns. Consequently, we retained the simpler approach to avoid unnecessary information loss while maintaining structural clarity.

2.3 Resulting Topology

These thresholding strategies proved effective in mitigating the "hairball" phenomenon common in social graphs. The resulting networks exhibited a graph density in the range of 0.2–0.3, striking a balance between sparsity (for efficient clustering) and connectivity (preserving the Giant Connected Component).

2.4 Sparsification and Topology Correction

To extract the meaningful "backbone" of the network, we applied a multi-stage sparsification pipeline:

- (1) **k-Nearest Neighbors (k-NN):** Restricting node connections to their k strongest peers to preserve local structure.
- (2) **Disparity Filter (Backbone):** Extracting statistically significant edges to retain multiscale structure while discarding random noise.

Only after these topological corrections is the graph subjected to the Leiden community detection algorithm and Random Walk simulations.

3 TOPOLOGICAL EVOLUTION OF PROJECTED NETWORKS

In this section, we analyze the structural evolution of the projections derived from the bipartite graph. We first examine the Anime-Anime network to understand how content relationships shifted over time, followed by an analysis of the User-User network.

3.1 Anime-Anime Network

The projected anime-anime network experienced explosive growth over the analyzed period (2006–2018), transforming from a compact, niche community into a sprawling, heterogeneous ecosystem. This transformation is defined by three primary phenomena: the densification-sparsification paradox, increasing taste divergence, and the crystallization of a "rich-club" core.

3.1.1 The Densification-Sparsification Paradox. The network underwent a dramatic scale expansion: the number of nodes (anime titles) increased from 732 in 2006 to 6,129 in 2018, while the volume of connections (edges) surged from ~64,000 to ~819,000. However, this volumetric growth reveals a fundamental structural shift.

While the absolute number of connections increased by an order of magnitude, the potential number of connections grew quadratically (N^2). Consequently, the global Graph Density declined precipitously from 0.2387 (2006) to 0.0436 (2018).

This indicates a transition from a "Village" topology—where the community is small enough for high interconnectedness—to a "Metropolis" structure. In the modern era, the ecosystem has become highly specialized; while the total volume of interactions is higher, individual anime titles connect to a significantly smaller fraction of the total population. The network has shifted from a monolithic block to a spread-out, sparse landscape.

3.1.2 Increasing Social Distance and Taste Divergence. To quantify the "cost" of traversing this expanding network, we analyzed weighted path metrics. Since edge weights represent similarity (Jaccard), the weighted distance can be interpreted as "social distance" or taste divergence.

The evolution of these metrics is presented in **Figure 1**. As shown in the *upper-left panel*, the average weighted path length rose sharply from 7.1 in 2006 to 44.4 in 2018. This metric represents the "resistance" to navigation: connecting a fan of a niche genre to a mainstream hit now requires passing through significantly more intermediaries.

Simultaneously, the network diameter (*upper-right panel*) expanded from 29 to 188.5 weighted units. This confirms that the "taste universe" is expanding. Distinct clusters (e.g., modern idols vs. vintage mecha) are moving mathematically further apart, creating deep topological fissures.

3.1.3 Local Cohesion and the "Fandom" Effect. Despite the global sparsification, the network maintains robust local connectivity. The *bottom-left panel* of **Figure 1** illustrates the Average Clustering Coefficient. After an initial adjustment, the metric stabilized at a remarkably high value of ≈ 0.59 . This indicates that the "Small-World" property is preserved locally. If Anime A is connected to B and C, there is a consistent $\sim 60\%$ probability that B and C are also connected. This proves that the sparsification did not destroy community cohesion; instead, the landscape fractured into tight, self-reinforcing "genre bubbles" (fandoms).

3.1.4 Structural Phase Shift: The 2006 Anomaly. The year 2006 represents a distinct topological phase. In this nascent period, the network exhibited disassortative mixing (Degree Assortativity ≈ -0.11), suggesting a star-like structure where popular titles served as hubs connecting primarily to niche nodes. From 2007 onward, the network flipped to positive assortativity (≈ 0.50), signaling the emergence of the "Rich-Club" phenomenon.

3.1.5 Intensity vs. Topology. Finally, we examine the distribution of influence using Node Strength. The log-log plot in the *bottom-right panel* of **Figure 1** confirms the scale-free nature of the modern network, following a clear Power Law distribution. The network is dominated by a few "mega-hubs," validating the "preferential attachment" growth model.

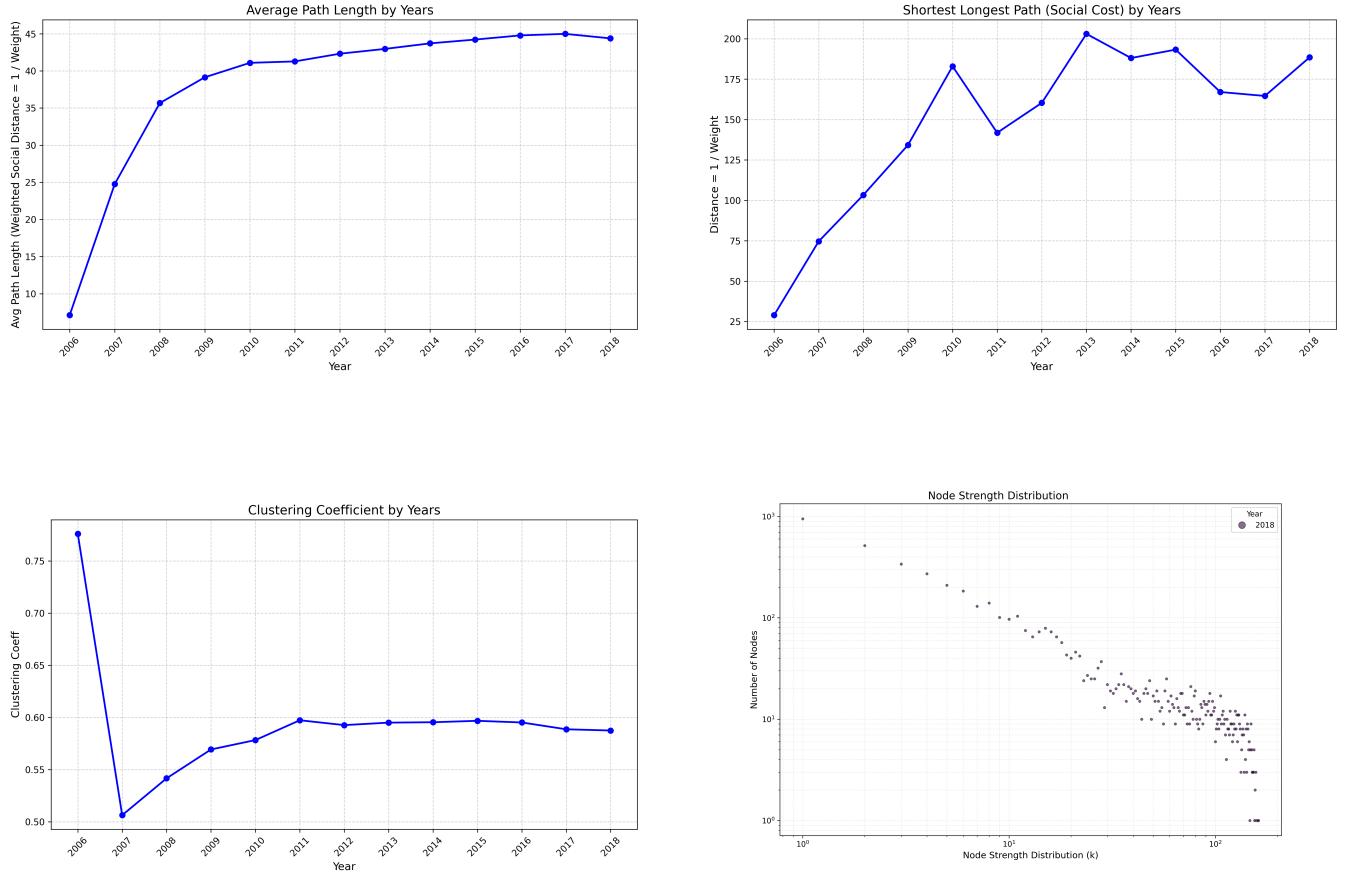


Figure 1: Evolution of Anime Network Topology (2006–2018). Upper-left: Average Weighted Path Length showing increased navigation difficulty. Upper-right: Network Diameter indicating the expansion of the "taste universe". Bottom-left: Average Clustering Coefficient stabilizing around 0.59, suggesting persistent local cohesion. Bottom-right: Node Strength Distribution (2018) confirming the scale-free ($P(k) \sim k^{-\gamma}$) nature of the modern network.

However, the Assortativity of Strength (≈ 0.18) is consistently lower than the Assortativity of Degree (≈ 0.50). This implies that while popular shows are structurally connected, the strongest taste affinities are located in the niche clusters, not the mainstream core.

3.2 User-User Network

In stark contrast to the Anime content network—which became "sparse" and harder to traverse as it grew—the User interaction network exhibits the classic properties of Network Densification. As the community expanded, the social distance between users collapsed, making the network significantly more interconnected.

While the "universe" of users grew, the social structure did not fragment into isolated islands. Instead, it evolved into a tight, integrated "global village," where new users actively connected to existing hubs rather than the periphery.

3.2.1 Global Integration and the "Shrinking World". The analysis of weighted path metrics reveals a community that is becoming functionally smaller and easier to traverse, despite growing in physical size.

The evolution of these metrics is presented in the *upper panels* of **Figure 2**. As shown in the *upper-left panel*, the average weighted path length dropped sharply from ~ 0.45 in 2006 to ~ 0.31 by 2009, maintaining this lower baseline through 2018. This reduction is a hallmark of the "Small World" effect: as the platform matured, users formed bridging connections, accelerating the flow of information across the graph.

Similarly, the network diameter (*upper-right panel*) contracted from ~ 1.08 to < 0.99 . Unlike the Anime graph, where taste divergence created massive gaps, the social graph's diameter is shrinking. This indicates that even socially distant groups (e.g., distinct language communities) are becoming more connected to the mainstream core.

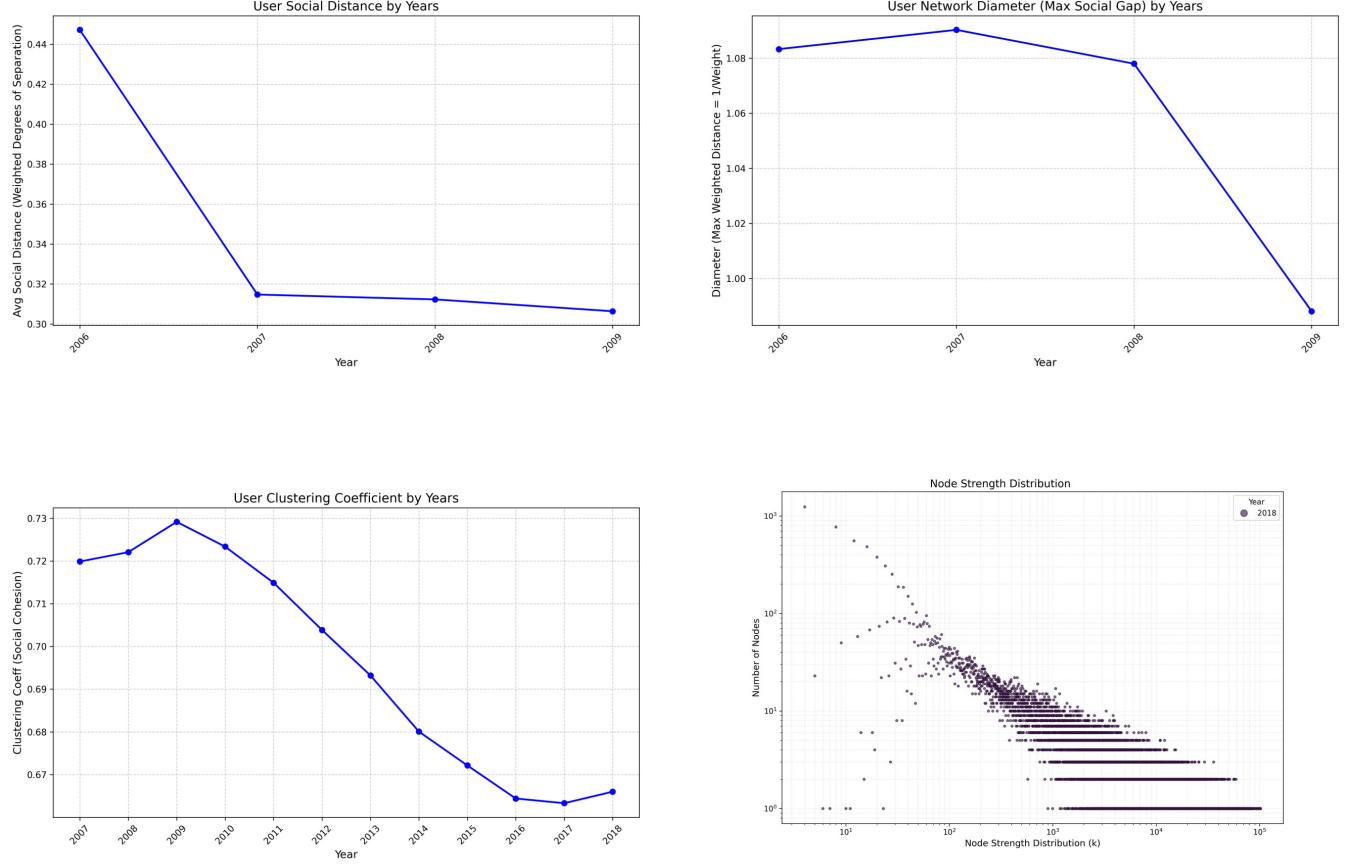


Figure 2: Evolution of User Network Topology (2006–2018). Upper-left: Avg. Weighted Path Length decreasing, indicating higher integration. Upper-right: Network Diameter contracting, showing the "shrinking world" phenomenon. Bottom-left: Clustering Coefficient peaking in 2009 and slowly stabilizing, reflecting the balance between clique formation and expansion. Bottom-right: Node Strength Distribution (2018) following a strict Power Law, highlighting the dominance of "Super-Users".

3.2.2 Local Cohesion and Community Structure. While the network became globally smaller, the local structure evolved to balance rapid growth with intimate social circles. The *bottom-left panel* of **Figure 2** shows the Average Clustering Coefficient. It peaked in 2009 (~ 0.73) during the platform’s initial boom, followed by a gentle decline to ~ 0.66 .

A score of 0.66 remains exceptionally high for a large social network. The slight decline suggests a natural dilution of cliques as users added diverse friends, but the high retention proves the community is fundamentally built on strong, overlapping friend groups rather than loose acquaintances.

3.2.3 Influence and Inequality. The distribution of influence confirms a highly stratified social hierarchy. The log-log plot in the *bottom-right panel* demonstrates a strict linear descent, characteristic of a Scale-Free Network ($P(k) \sim k^{-\gamma}$).

The graph is dominated by a tiny fraction of "Super-Users" (hubs) who possess nearly 1,000 \times the connectivity of the average user.

These hubs act as the structural "glue" that holds the giant component together, enabling the short path lengths observed in Section 3.2.

4 DATASET OVERVIEW

MyAnimeList (MAL) is one of the largest online platforms dedicated to anime and manga tracking. Users maintain personal lists of watched titles, assign numerical ratings, write reviews, and participate in community discussions.

Although MAL does not represent the entire global audience, it functions as a *large-scale, organically formed social network* structured around media consumption. Its importance for data analysis stems from several factors:

- it has millions of active users with voluntarily provided preference data;
- users form clusters and communities around genres, studios, eras, and specific titles;
- the rating patterns reflect collective taste dynamics, hype cycles, long-tail phenomena, and cross-cultural differences;

- the anime graph (users × titles) behaves like a sparse bipartite network with hubs, where highly popular titles act as central connectivity points.

For these reasons, MAL is a valuable source for studying recommendation systems, preference modeling, popularity prediction, and structural patterns of entertainment consumption.

4.1 Dataset Structure

We use the 2020 “Anime Recommendation Database” from Kaggle, which combines several processed dumps of MyAnimeList:¹

anime.csv. Metadata for approximately 17,000 titles, with columns including:

- identifiers: MAL_ID, Name, English name, Japanese name;
- quality and popularity metrics: Score, Ranked, Popularity, Members, Favorites;
- categorical descriptors: Genres, Type, Source, Studios, Producers;
- structural info: Episodes, Duration, Rating (age restriction);
- engagement counters: Watching, Completed, Dropped, etc.;
- per-score vote counts: Score-1 ... Score-10.

anime_withSynopsis.csv. A reduced version containing MAL_ID, Name, Score, Genres, Synopsis. Useful for NLP tasks such as clustering by textual description.

animelist.csv. Contains approximately 300 million user-anime interactions, with columns: user_id, anime_id, rating, watching_status, watched_episodes. This is the core behavioral dataset representing the user-item matrix.

rating_complete.csv. A filtered version of *animelist.csv* containing only rows with watching_status = 2 (“Completed”), columns: anime_id, user_id, rating. Commonly used for training recommender systems.

watching_status.csv. Lookup table pairing each integer code with a textual description (“Currently Watching”, “Completed”, etc.).

4.2 Strengths of the Dataset

Despite being collected from an entertainment platform, the dataset has several significant advantages for data science practice:

- **Large scale:** Tens of millions of ratings across thousands of titles enable analysis of long-tail distributions, user segmentation, genre-level statistics, and popularity dynamics.
- **Natural heterogeneity of users:** No incentive to game the system; tastes are diverse and clusters form organically.
- **Multiple complementary tables:** Metadata, textual features, and behavioral interactions allow content-based, collaborative, graph-based, and hybrid recommender analyses.
- **Excellent for methodological demonstration:** Useful for data cleaning, exploratory data analysis (EDA), imputation, outlier detection, recommendation algorithms, and metadata fusion with NLP.

¹Where external statistics are used (industry reports, demographic data), and where it is possible, we use sources closest to 2020 to maintain temporal consistency.

4.3 Limitations and Potential Biases

The dataset reflects behaviors of a specific community and inherits biases from the platform:

- **Geographical bias:** MAL is most popular in North America, parts of Europe, and Southeast Asia. For example, China is one of the global leaders in anime licensing [4], but Chinese users are absent on MAL.
- **Cultural and language bias:** English-speaking communities dominate the dataset.
- **Sparse user-item matrix:** Most users have watched only a small fraction of all anime; this affects collaborative filtering model performance and cold-start dynamics.
- **Artifacts and inconsistency in dataset:** Example for rating_complete.csv: filtering by watching_status = Completed is not sufficient. Some users marked anime as completed but watched fewer episodes than the total and gave non-zero ratings. Since MAL does not allow rating = 0, zero ratings indicate *no vote* rather than dislike. These cases are rare (~0.1%) but must be considered during cleaning.
- **No timestamps:** Limits temporal modeling of tastes and popularity.
- **Caution in interpreting popularity:** The dataset reflects preferences of *dedicated fans* rather than the general population. So, MAL ratings and engagement metrics do not directly translate to global popularity. MAL users are biased towards committed anime fans; casual viewers are underrepresented.
- **Licensing bias:** In many regions, anime consumption occurs primarily through piracy, so official licensing or platform metrics severely underestimate actual viewership².
- **MAL - only one of the biggest anime hubs:** Country-level popularity is not reliably inferable without additional corrections or external data sources; such adjustments are beyond the scope of this project.

5 SYNTHETIC USER GENERATION

To simulate global user interactions while preserving privacy, we generate synthetic user profiles based on demographic and traffic data.

5.1 Data Sources

- **World cities and populations:** Dataset [3]. Used columns ASCII Name, Country Name EN, Population, Latitude, Longitude.
- **MyAnimeList traffic per country:** Scrapped manually from semrush.com for October 2025, with columns Country, Number of Visitors.
- **Age and sex distributions:** From demographic studies[2] providing mean and standard deviation for age, and male/female proportions.

Additionally, the demographic survey[2] collected country-level data, but it was conducted at a large anime convention, so the sample is a *convenience sample* and not representative of the general

²Even in the USA, the leader in licensed anime, nearly half of users watch primarily via unofficial services [2]

population. Nevertheless, the observed distributions roughly align with the traffic patterns obtained from semrush.com.

The survey also collected additional information, such as self-reported life satisfaction, hobbies, gender, preferred decades of favorite titles, and other personal attributes. These variables are not directly used in this project, as our analysis relies on synthetic profiles generated from traffic and demographic distributions.

5.2 Generation Procedure

- (1) Compute country-level user proportions based on MAL traffic:

$$P_{\text{country}} = \frac{\text{MAL users in country } X}{\text{Total MAL users}}.$$

- (2) Distribute users to cities within each country proportionally to city population:

$$P_{\text{city}} = P_{\text{country}} \cdot \frac{\text{City population}}{\sum \text{City populations in country}}.$$

- (3) Sample user attributes:

- age ~ Normal distribution (mean, std from [2]),
- sex ~ Bernoulli(p) (male/female proportion from [2]),
- latitude, longitude assigned according to the selected city.

- (4) Assign user_id sequentially and compile all attributes into profiles.csv with columns:

user_id, country, city, age, sex, latitude, longitude.

5.3 Notes

- This synthetic population allows testing recommendation algorithms and demographic analyses without revealing actual user data. - The proportions reflect December 2025 traffic, but are applied to simulate the 2020 dataset contextually.

6 DATASET AND INITIAL EXPLORATION

For the analytical part of this project, we use the *Anime Recommendation Database 2020* from Kaggle, which combines metadata about anime titles with large-scale user rating data. The dataset contains over 12,000 anime entries and more than 73,000 users, resulting in approximately 7.8 million individual rating interactions. This structure naturally splits the data into two main tables:

- **Anime information:** title, genres, type (TV, movie, OVA), number of episodes, release year, studio, popularity metrics, and short descriptions.
- **User ratings:** user identifiers and their numeric ratings for specific anime.

At first glance, the data appears rich and diverse, but it also reflects several typical characteristics of real-world datasets:

- **Incomplete records:** many anime lack a release year, genre tags, or episode counts.
- **Inconsistent categorical data:** genre lists differ in formatting and ordering; some entries use non-standard tags.

- **Long-tailed distributions:** a small number of highly popular anime dominate ratings, while the majority receive very few.
- **Sparse user behavior:** most users rate only a tiny fraction of available titles.

These properties are not defects; they represent the typical landscape of large, user-generated media datasets. They also motivate several of the analytical directions in this project, such as identifying rating patterns, studying genre clusters, and modeling the structure of user-anime interactions.

Synthetic Users and Locations

The original dataset does not contain geographic or demographic information about users. To explore cross-cultural and regional patterns, we generate synthetic user metadata. Each user is assigned a plausible location (country and optionally city), following real-world population distributions. This augmented dataset allows us to ask new types of questions, such as whether certain genres correlate with specific geographic regions, or whether user communities cluster differently across countries.

The synthetic data is clearly separated from the original records and is used only for exploratory purposes, without affecting the underlying rating matrix.

Data Cleaning

Before we can meaningfully analyze the data, we must resolve the inconsistencies and structural issues inherited from the raw dataset. This involves:

- normalizing genre representations and splitting multi-genre fields;
- removing or correcting obviously invalid entries (e.g. anime with zero episodes released in the 1800s);
- handling missing values through imputation or category-specific defaults;
- joining anime metadata with synthetic user information to form a unified analytical table;
- reducing noise in the user-anime interaction graph by filtering out extremely sparse users or entries.

These preprocessing steps create a clean, analyzable foundation for the exploratory data analysis that follows and ensure that all subsequent insights reflect meaningful patterns rather than artifacts of data collection.

7 RANDOM-WALK MODEL FOR USER TRAJECTORY SIMULATION

7.1 Motivation

User activity on a large interaction graph can be interpreted as a sequence of transitions between nodes (e.g. items, topics, or communities). Given such a sequence for each user, our goal is to construct a probabilistic model that captures the *structural tendencies* of user navigation. This model is later used to generate synthetic trajectories—“random walkers”—that approximate the observed behavior of the real user. The ensemble of walkers provides a natural way to measure how typical or atypical a given user trajectory is, relative to the structure of the graph.

Because the underlying graph is large (on the order of thousands of nodes and millions of edges), all computations must be local and efficient. We exploit the fact that the graph evolves year by year, so a user trajectory is implicitly aligned with a sequence of yearly graphs.

7.2 Definition of the Random Walker

Let $G_t = (V_t, E_t)$ denote the interaction graph in year t . For a user u we observe a trajectory

$$\mathbf{x}^{(u)} = (x_0, x_1, \dots, x_T),$$

where $x_t \in V_t$ is the node visited by the user in year t . We construct a *random walker* whose behavior in year t is governed only by the local structure of G_t and the user's starting point x_0 .

Formally, for each year t the walker occupies a state $X_t \in V_t$. Conditioned on $X_t = v$, the walker chooses its position in year $t+1$ according to a probability distribution over the neighbors of v in G_t :

$$\mathbb{P}(X_{t+1} = w \mid X_t = v) = \frac{1}{\deg_{G_t}(v)} \quad \text{for all } (v, w) \in E_t.$$

That is, the walker performs a uniformly random step along the edges that exist in the corresponding yearly graph.

A single random walker generates one synthetic trajectory

$$\mathbf{Y} = (Y_0, Y_1, \dots, Y_T), \quad Y_0 = x_0.$$

To model uncertainty and to obtain stable statistical estimates, we simulate an ensemble of K independent walkers for each user.

7.3 Asynchrony and Year-Level Dynamics

A key detail is that the walkers evolve *asynchronously*. Each walker only moves when the global simulation clock advances to a year in which that walker still has remaining steps. This design is necessary because real user trajectories can have different lengths, and the yearly graphs G_t may differ substantially in size and connectivity.

Thus the simulation proceeds by iterating over years $t = 0, 1, \dots, T$ and, for each walker whose trajectory length is greater than t , performing exactly one step in G_t . Walkers whose length is shorter than the current year simply remain inactive.

7.4 Ensemble-Based Evaluation

Given a user u with observed trajectory $\mathbf{x}^{(u)}$ and an ensemble of simulated trajectories $\{\mathbf{Y}^{(k)}\}_{k=1}^K$, we can quantify how well the random-walk model explains the user's behavior.

Let $d(\cdot, \cdot)$ be a similarity or distance measure between two trajectories. In this work we primarily use a weighted node-overlap metric that penalizes long-distance mismatches. The average similarity of the user to the ensemble is

$$\bar{s}^{(u)} = \frac{1}{K} \sum_{k=1}^K s(\mathbf{x}^{(u)}, \mathbf{Y}^{(k)}),$$

with an accompanying variance

$$\text{Var}^{(u)} = \frac{1}{K} \sum_{k=1}^K \left(s(\mathbf{x}^{(u)}, \mathbf{Y}^{(k)}) - \bar{s}^{(u)} \right)^2.$$

These metrics estimate how "typical" the user is relative to repeated realizations of the random-walk model.

Such quantities naturally extend to population-level statistics: distributions of similarities, identification of outliers, and hypothesis testing against the null model provided by random walks.

7.5 Consensus Models

To summarize the global behavior of the entire ensemble, we consider two forms of consensus:

Markov Consensus. All walker trajectories across all users define empirical transition counts

$$C_{vw} = \#\{\text{times a walker moves } v \rightarrow w\}.$$

Normalizing the rows yields an empirical transition matrix

$$P_{vw} = \frac{C_{vw}}{\sum_{w'} C_{vw'}}.$$

The matrix P defines a global Markov model that captures the average transition tendencies dictated by the graph structure and the distribution of starting points. This model can be used to compute likelihoods of real user trajectories, to generate new synthetic walkers, or to build a deterministic "most probable" consensus path by greedy selection.

Medoid Trajectory. As a complementary summary, we compute the *medoid* of a set of walker trajectories—the trajectory that minimizes the total distance to all others:

$$k^* = \arg \min_k \sum_{j=1}^K d(\mathbf{Y}^{(k)}, \mathbf{Y}^{(j)}).$$

The medoid offers an interpretable representative path that arises from an actual random walker, as opposed to the probabilistic object represented by P .

7.6 Interpretation

The random-walk construction provides an explicit null model driven solely by the graph's local connectivity. If a real user's trajectory significantly deviates from the ensemble predicted by the graph, the deviation may reveal hidden structure, atypical behavior, or external influences that are not captured by topology alone.

Conversely, if the random-walk ensemble closely matches the user, the graph alone is sufficient to explain the observed behavior.

This duality—graph-driven randomness versus user-specific structure—is the central object of analysis in the subsequent sections of the report.

8 MARS_1.0 PROJECT TREE

```
MARS_1.0/
  data/
    anime_ranks/
      anime.csv
      animelist.csv
      anime_withSynopsis.csv
      rating_complete.csv
      watching_status.csv
    anime_timestamps/
      anime_timestamps.csv
```

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