

Homework Assignment 2

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This will keep you off the street for a while so please start early!

Consider the so called advection diffusion ODE

$$\frac{d}{dx} \left(\sigma(x) \frac{dp}{dx} \right) + \mu \frac{dp}{dx} = q$$

with boundary conditions

$$p(0) = 0 \quad p(1) = 0$$

Part One - Pen and paper

- 1 Use the flux $J = \sigma p_x$ and rewrite the equation as a system of first order equations
- 2 Use the following discretization to solve the problem. Put p on the **nodes** of your mesh and J in **cell centers**.

Write a discrete system of equations that approximate the continuous system to second order accuracy. Use index notation.

- 3 Discuss your approximation of boundary conditions.
- 4 Write your system using matrix notation.
- 5 Eliminate J from the equations to obtain a system on p

Part two - Computer

1. Code your discretization into a function
function[p] = convAdvec(h,sigma,mu,q)
See further instructions in the starter code file `convAdvec.m` on GitHub.
2. Use the following functions to test your code

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- $p = \sin(2\pi x)$, $\sigma = 1 + x^2$, $\mu = 0.1$.
- $p = \sin(2\pi x)$, $\sigma = 1 + x^2$, $\mu = 10$.

Use $h = 10^{-i}$, $i = 2, \dots, 5$. Note that you will have to compute the corresponding source function q for each test function and write your own testing script. Discuss the results of your tests.