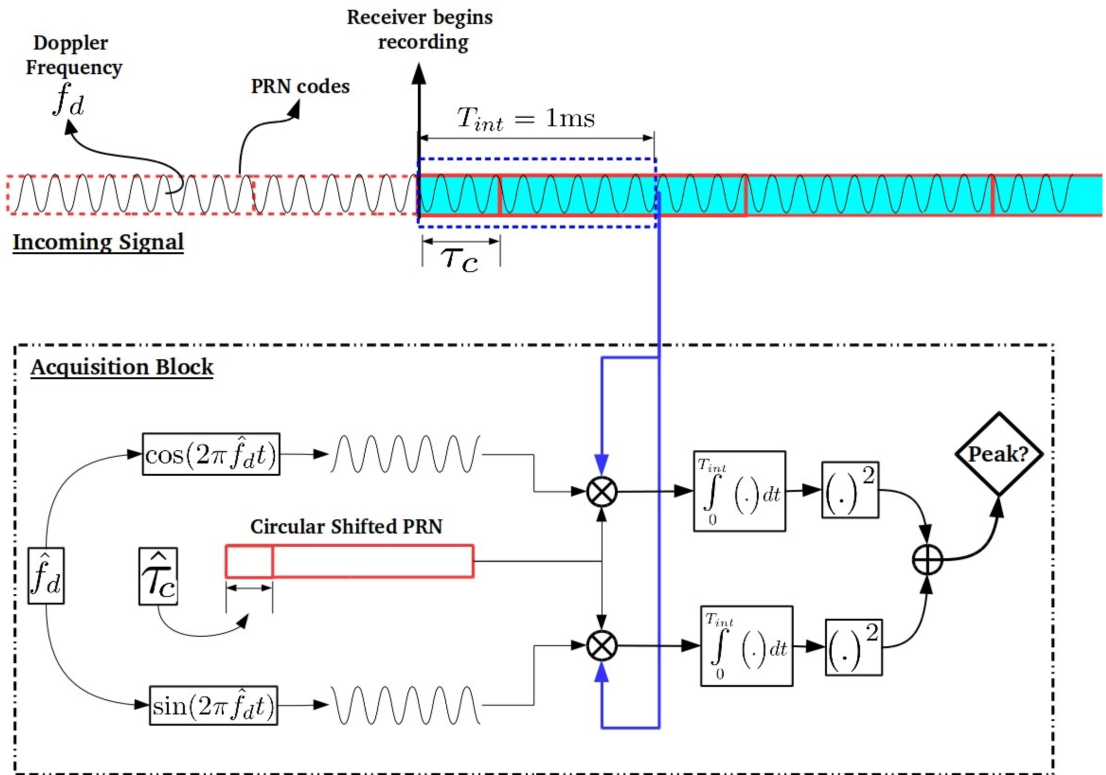

Figure 2.11: Acquisition with $T_{int} = 1\text{ms}$ (Note that f_{IF} has been taken to be 0)

2.3.2 Acquisition

The process of finding the satellites signals present and getting a coarse estimate of the Doppler frequency shift and the 'code-phase' for each of them is referred to as 'Acquisition'. When cold started, the receiver starts recording the incoming signal from an unknown point of the PRN sequence and it differs from satellite to satellite since the different signals arrive out of sync depending on the distance of satellites from the receiver. To be able to de-spread the signal from the j^{th} satellite, the receiver must align with the PRN sequence $x_j(t)$ of that satellite and then decode the data bits. This requires knowledge of τ_j modulo 1ms as described below. But this operation requires the baseband signal i.e $D_j(t - \tau_j)\, x_j(t - \tau_j)$. So the carrier must be stripped off the signal, but it is necessary to know the Doppler shift $f_{d,j}$ to do so.. Thus if the signal from a particular satellite is present, the first step is to get a coarse estimate of the f_d and (τ mod 1), without knowing θ. Knowing f_d will eliminate the carrier completely and give the baseband signal. Knowing (τ mod 1) then will align the receiver code generator with incoming signal by introducing a code-offset of (τ mod 1) as described below. However given the low SNR of the CDMA signal, it requires a brute-force search over frequency and code-phase space to get the coarse estimates. Since each satellite needs to be acquired independently, in subsequent analysis, we will consider the signal only from one satellite and treat signals from other satellites as noise. Thus

$$s_r(t) = \sqrt{P}\, D(t-\tau)\, x(t-\tau) \cos(2\pi(f_{IF} + f_d)t + \theta) + \widetilde{n}(t)$$

where $\widetilde{n}(t) = n(t) + \sum^{K-1} \sqrt{P_i}\, D_i(t-\tau_i)\, x_i(t-\tau_i) \cos(2\pi(f_{IF} + f_{d,i})t + \theta_i)$

Now, refer to Figure 2.11. Here, the receiver begins recording from some arbitrary point in the signal. The nearest beginning of the PRN sequence is say a τ_c ms away. We will refer to this as the acquisition code-phase. Note that actually τ_c = (τ modulo 1ms). Consider a time interval of length T_{int}, which is a multiple of 1ms, the length of PRN codes. $s_r(t)$ is co-related with a local signal generated according to receiver's estimate, say $\{\hat{f}_d, \hat{\tau_c}\}$. Co-relation is computed as:

$$\mathfrak{R}_I(\hat{f}_d, \hat{\tau_c}) = \int_0^{T_{int}} s_r(t)\, x(t-\hat{\tau_c})\, \cos(2\pi(f_{IF} + \hat{f}_d)t)\, dt$$

$$\mathfrak{R}_Q(\hat{f}_d, \hat{\tau_c}) = \int_0^{T_{int}} s_r(t)\, x(t-\hat{\tau_c})\, \sin(2\pi(f_{IF} + \hat{f}_d)t)\, dt$$

Let, $\mathfrak{R}(\hat{f}_d, \hat{\tau_c}) = \mathfrak{R}_I + j\mathfrak{R}_Q$, where $j = \sqrt{-1}$,

$$\|\mathfrak{R}(\hat{f}_d, \hat{\tau_c})\|^2 = \mathfrak{R}_I^2 + \mathfrak{R}_Q^2$$

$\|\mathfrak{R}\|^2$ is known as the ambiguity function. The aim of acquisition is to search over the space of $\hat{f}_d, \hat{\tau_c}$ to find a distinctly high value of $\|\mathfrak{R}\|^2$. Since PRN code of a satellite correlates strongly only with a correctly aligned PRN code of the same satellite, presence of a peak confirms presence of signal from that satellite and also identifies the correct alignment. If such a peak occurs, the satellite is declared present, the bin $\hat{f}_d, \hat{\tau_c}$ at which it occurs is chosen as the coarse estimate and the receiver proceeds to tracking. Let $\{\tilde{f}_d, \tilde{\tau_c}\}$ be the estimates concluded from the acquisition process. Thus the output of acquisition is,

$$\{\tilde{f}_d, \tilde{\tau_c}\} = \underset{\{\hat{f}_d, \hat{\tau_c}\}}{\operatorname{argmax}} \quad \|\mathfrak{R}(\hat{f}_d, \hat{\tau_c})\|^2$$

Alternatively, $\mathfrak{R}$ can be written in complex notation.

$$\mathfrak{R}(\hat{f}_d, \hat{\tau_c}) = \int_0^{T_{int}} s_r(t)\, x(t-\hat{\tau})\, e^{(j2\pi\hat{f}_d t)}\, dt$$

Substituting for $s_r(t)$ from 2.3, we get,

$$
\begin{aligned}
\mathfrak{R}(\hat{f}_d, \hat{\tau_c}) &= \int_0^{T_{int}} \left(\sqrt{P_r} D(t-\tau) x(t-\tau) \cos(2\pi(f_{IF} + f_d)t + \theta) + \widetilde{n}(t) \right) x(t - \hat{\tau_c})\, e^{(j2\pi(f_{IF} + \hat{f}_d))t)}\, dt \\
&= \left(\sqrt{P_r} \int_0^{T_{int}} D(t-\tau) x(t-\tau) x(t-\hat{\tau_c}) \cos(2\pi(f_{IF} + f_d)t + \theta) e^{(j2\pi(f_{IF} + \hat{f}_d)t)}\, dt \right) + \widetilde{\mathfrak{n}}
\end{aligned}
$$

where $\widetilde{\mathfrak{n}}$ is a term resulting from noise. If T_{int} is taken much less than 20ms, like 1 or 2ms, then $D(t)$ would be constant in the interval. So take it to be $D = \pm 1$ (neglecting the possibility a data bit boundary in the T_{int}-long segment). When $\hat{\tau_c} = \tau_c (= \tau \text{ modulo } 1)$, $x(t-\tau)x(t-\hat{\tau_c}) = x(t-\tau)^2 = 1$. Then,

$$
\begin{aligned}
\mathfrak{R}(\hat{f}_d, \tau_c) &= \sqrt{P_r} D \int_0^{T_{int}} \left(\frac{e^{j(2\pi(f_{IF} + f_d)t + \theta)} + e^{-j(2\pi(f_{IF} + f_d)t + \theta)}}{2} \right) e^{j2\pi(f_{IF} + \hat{f}_d)t)}\, dt + \widetilde{\mathfrak{n}} \\
&= \sqrt{P_r} D \int_0^{T_{int}} \left(\frac{e^{j(2\pi(2f_{IF} + f_d + \hat{f}_d)t + \theta)} + e^{j(2\pi(\hat{f}_d - f_d)t + \theta)}}{2} \right) dt + \widetilde{\mathfrak{n}}
\end{aligned}
$$

Since $2f_{IF} \gg \frac{1}{T_{int}}$, we have $\int_0^{T_{int}} e^{j(2\pi(2f_{IF} + f_d + \hat{f}_d)t + \theta)} dt \approx 0$

$$
\begin{aligned}
\therefore \mathfrak{R}(\hat{f}_d, \tau_c) &\approx \sqrt{P_r} D \int_0^{T_{int}} \left(\frac{e^{j(2\pi(\hat{f}_d - f_d)t + \theta)}}{2} \right) dt + \widetilde{\mathfrak{n}} \\
&= \frac{\sqrt{P_r} e^{j\theta}}{2} D \int_0^{T_{int}} \left(e^{j2\pi\delta_{f_d} t} \right) dt + \widetilde{\mathfrak{n}} \qquad (\delta_{f_d} = \hat{f}_d - f_d) \\
&= (D e^{j\theta} \sqrt{P_r}) \frac{\sin(\pi \delta_{f_d} T_{int})}{2\pi \delta_{f_d}} e^{\pi \delta_{f_d} T_{int}} + \widetilde{\mathfrak{n}} \qquad (2.5)
\end{aligned}
$$

Neglecting the effect of $\widetilde{\mathfrak{n}}$, observe that:

- $\|\mathfrak{R}\|^2$ is independent of θ and D. Thus acquisition process will function the same without modification irrespective of the carrier phase on reception and the navigation data in the chosen signal segment
- $\lim_{\delta_{f_d} \to 0} \|\mathfrak{R}\|^2 = P_r$. For values of $\{\hat{f}_d, \hat{\tau_c}\}$ other than $\{f_d, \tau_c\}$, $\|\mathfrak{R}\|^2 < P_r$. Thus ideally, under low noise conditions, output of acquisition are the actual Doppler and code shifts.

The Doppler shift in frequency is in most cases known to be in within $\pm$6kHz. Hence the frequency search space is $-6000 \leq \hat{f}_d \leq 6000$. And since the PRN sequence lasts 1ms, code-

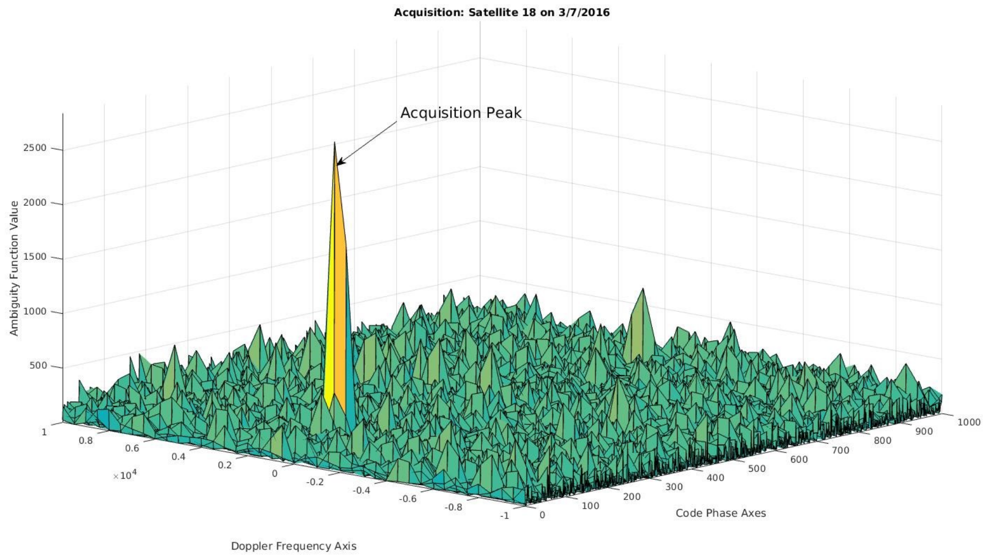

Figure 2.12: Result of Acquisition procedure as a 2D plot on frequency code-phase plane

phase search space is 0ms $\leq \hat{\tau_c} <$ 1ms. If we could search over all possible values in this search space, we can obtain the the exact Doppler shift and align the local code generator exactly with the incoming signal. This routine could be repeated every once in a while when Doppler frequency or the code-phase has changed and realign the receiver. However, it is not possible to search over the entire search space, so we discretize the search space with a frequency bin size of say Δf_d and code-phase bin of $\Delta\tau_c$. That is, the ambiguity function is only computed for every Δf_d^{th} frequency and every $\Delta\tau_c^{\text{th}}$ code-phase shift. And hence the estimates $\{\tilde{f_d}, \tilde{\tau_c}\}$ thus obtained are 'coarse'. To check all code shifts in the PRN sequence of length 1023, we need $\Delta\tau_c \leq \frac{1}{1023}$ms and for reasons detailed in Chapter 3, we need $\Delta f_d \leq$ 500Hz if $T_{int} = 1ms$. If we take them as $\frac{1}{1023}$ms and 500Hz, the output $\tilde{f_d}$ and $\tilde{\tau_c}$ will be among $\{-6000, -5500, \cdots - 500, 0, 500, \ldots, 6000\}$Hz and $\{0, \frac{1}{1023}, \frac{2}{1023} \cdots \frac{1022}{1023}\}$ms respectively. The number of ambiguity functions to be computed (or size of search space) is $\frac{6000-(-6000)}{\Delta f_d} \times \frac{1}{\Delta\tau_c} \approx 24,000$. Performing 24,000 correlations is a computationally expensive task, thus even after discrete-izing the search. Besides, f_d and τ_c change only slowly and smoothly with time. So it is wasteful to perform acquisition often. In practice, acquisition is thus performed only once while the receiver is cold-started. The resulting coarse estimate is used as the initial condition for the tracking block which then settles to the a finer estimate and tracks f_d and τ_c to keep the receiver aligned with incoming signal thereafter. An example of acquisition output is shown in Figure 2.12. As long as the SNR is high enough (typically $\geq$ 40 dB-Hz), it is unlikely that noise generates a peak which is higher than at the nearest estimates for $\{f_d, \tau_c\}$. Let us assume so and proceed to tracking.

In GPS, on cold start, the receiver has no information about the visible satellites. Thus it performs acquisition with all satellite PRN codes till it finds 4 or more satellites. In case of IRNSS however all the 7 satellites (as of June 2017) are visible all the time over Indian

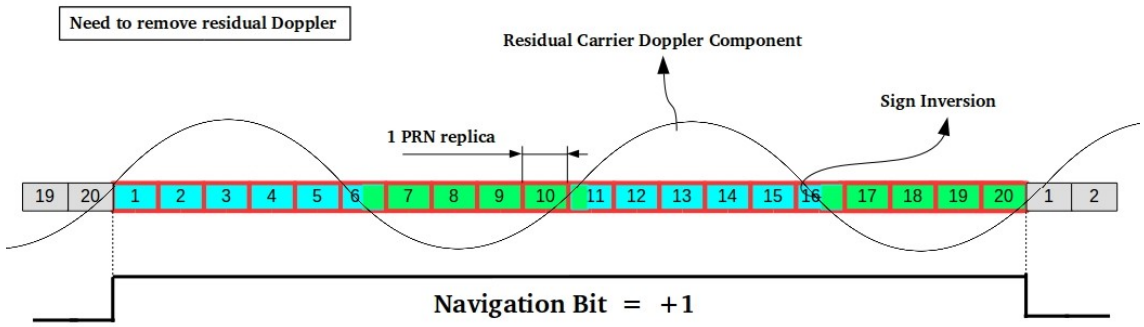

Figure 2.13: Effect of Residual Doppler after acquisition. Green implies inverted code-chips

subcontinent. The estimates $\{\tilde{f}_{di}, \tilde{\tau}_{ci}\}$ for each of the satellites are used to initializes that many separate tracking loops. Each such loop is called a 'channel' and the specification, 'number of channels' in a receiver refers to the maximum number of satellites that the receiver can track simultaneously. But sometimes it also simply refers to the number of correlators in the receiver. The number of channels can be as large as 22 to track multi-constellation satellites for better accuracy or simply to reduce time-to-first-fix by computing many correlations parallel.

2.3.3 Tracking

We consider the tracking block for one satellite. If acquisition detects the correct peak, the receiver concludes the estimates $\{\tilde{f}_d, \tilde{\tau}_c\}$, where we have,

$$|\tilde{f}_d - f_d| \leq \frac{\Delta f_d}{2} \tag{2.6}$$

$$|\tilde{\tau}_c - \tau_c| \leq \frac{\Delta \tau_c}{2} \tag{2.7}$$

The aim now is to not only decode the Navigation bits, but also to maintain the code-phase synchronization thereby precisely detecting bit and PRN sequence boundaries, used in pseudo-range computation. We now motivate the need and functioning of the two main components that make up the tracking block.

PLL: Remove Residual Doppler and Phase

Observe from Equation 2.5 that, if $\delta_{f_d} = 0$, $\mathfrak{R} = De^{j\theta}$ (ignoring P_r and noise). Thus, ignoring the effects of noise, if $\delta_{f_d} = 0$ is maintained, the data bit sequence can be obtained from the sign of real (or imaginary) part of $\mathfrak{R}$ alone, it is $D\cos\theta$ (or $D\cos\theta$). Note that the bit sequence so obtained will either by D or $-D$ depending on what θ is, which can then be distinguished using NAV frame elements, like the preamble. Moreover, we can find θ from the phase of complex number $\mathfrak{R}$, and get $D = e^{-j\theta}\mathfrak{R}$. This is like making θ zero, in which case all the power in the signal is in the In-phase component. This is helpful because, when considering the effect of noise term $\tilde{\mathfrak{n}} = \tilde{\mathfrak{n}}_I + j\tilde{\mathfrak{n}}_Q$, taking the sign without correcting for θ actually gives $\text{sign}[D\cos(\theta) + \tilde{\mathfrak{n}}_I]$.