(b) $q(n,k)$

1 $\quad$ k $\quad$ k+1 $\quad$ n

$n-1$

$x_k \le y$

$P(x_k < y) = y$

$\max\{y\}$

$$\int_0^1 ({}^kC_1) \cdot dy \cdot (y)^{n-1}$$

$$\Rightarrow ({}^kC_1) \cdot \left[\frac{y^n}{n}\right]_0^1 = \frac{k}{n}$$

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(c) $p(n,k)$

k $\quad$ 1 $\quad$ k+1 $\quad$ n

max $y$ $\qquad$ $y_1$ $\qquad$ $y_1 > y$

k $\quad$ k+1

$y_1$

k+1 $\quad$ n

$\Rightarrow$ $y_r \to$ $(k+r)^{th}$ $\qquad$ $r \in 1, n-k$

$y_{r+1}$ $\qquad$ $(k+r)!$ $\qquad$ $n-r-k$

~~n-r+1~~ $\quad$ ~~n-r-k~~

$x$ $\qquad$ ~~n-r+1~~ $\quad$ ~~r+k+x = n~~

$i - r + 1 + x = n - r$

$x = n - i - k$

$< y_0$ $\quad (k+i) \quad - \quad (n-i-k)$

$y_1 \qquad < y_1$

$$\int_{y}^{y_1} (y_1)^{(i-1)} (dy_1) \cdot (y_1)^{n-i-k}$$

$$\Rightarrow y^{(i-1)} \cdot \int_{y}^{1} y_1^{(n-i-k)} dy$$

$$y^{(i-1)} \left( \frac{y_1^{(n-i-k+1)}}{(n-i-k+1)} \right)_y^1$$

$$\frac{y^{i-1} - y^{n-i-k+1+i-1}}{n-i-k+1} = \frac{y^{i-1} - y^{n-k}}{n-i-k+1}$$

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$$\Rightarrow \int \left( \frac{y^{i-1} - y^{n-k}}{n-k-i+1} \right) \cdot k y^{k-1} \, dy$$

$$k \int_0^1 \frac{y^{i+k-2} - y^{n-1}}{(n-k-i+1)} \, dy$$

$$k \left( \frac{\frac{y^{i+k-1}}{i+k-1} - \frac{y^{n}}{n}}{n-i-k+1} \right)_0^1$$

$$\Rightarrow k \left( \frac{\cancel{n-i-k+1}}{n(i+k-1)\cancel{(n-i-k+1)}} \right)$$

$$\sum_{i=1}^{n-k} \left( \frac{k}{n(i+k-1)} \right) \rightarrow \text{winning}$$

losing

$$1 - \sum_{i=1}^{n-k} \frac{k}{n(i+k-1)} \rightarrow$$

$$\left( 1 - \sum_{i=1}^{n-k} \frac{k}{n(i+k-1)} - \frac{k}{n} \right)$$

$\hookrightarrow P(n, k)$