

# Fairness

Christos Dimitrakakis

October 7, 2020

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- ▶ Meritocracy.

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- ▶ Proportionality and representation.

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- ▶ Proportionality and representation.
- ▶ Equal treatment.

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- ▶ **Meritocracy.**
- ▶ Proportionality and representation.
- ▶ Equal treatment.
- ▶ **Non-discrimination.**

# Meritocracy

# Meritocracy

## Example 1 (College admissions)

- ▶ Student  $A$  has a grade  $4/5$  from Gota Highschool.
- ▶ Student  $B$  has a grade  $5/5$  from Vasa Highschool.



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## Example 2 (Additional information)

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- ▶ 50% of admitted Vasa graduates with 5 get their degree.

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## Solutions

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- ▶ Admit **randomly**?

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## Solutions

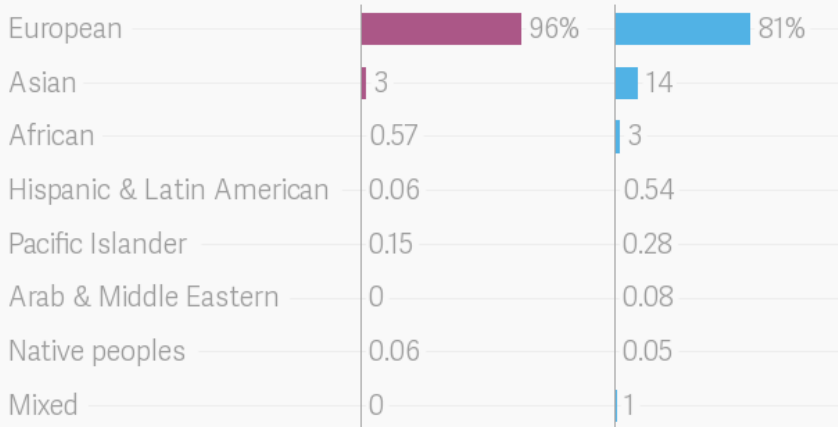
- ▶ Admit **everybody**?
- ▶ Admit **randomly**?
- ▶ Use **prediction** of individual academic performance?

# Proportional representation

Little progress is being made to improve diversity in genomics

Share of samples in genetic studies, by ancestry

■ 373 studies, up to 2009 ■ 2,511 studies, up to 2016



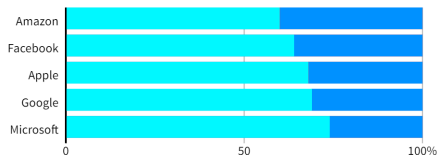
# Hiring decisions

## Dominated by men

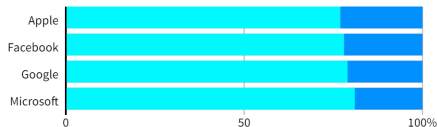
Top U.S. tech companies have yet to close the gender gap in hiring, a disparity most pronounced among technical staff such as software developers where men far outnumber women. Amazon's experimental recruiting engine followed the same pattern, learning to penalize resumes including the word "women's" until the company discovered the problem.

### GLOBAL HEADCOUNT

Male Female



### EMPLOYEES IN TECHNICAL ROLES



C. Dimitrakakis



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# Fairness and information

## Example 3 (College admissions data)

School	Male	Female
A	62%	82%
B	63%	68%
C	37%	34%
D	33%	35%
E	28%	24%
F	6%	7%
Average	45%	38%



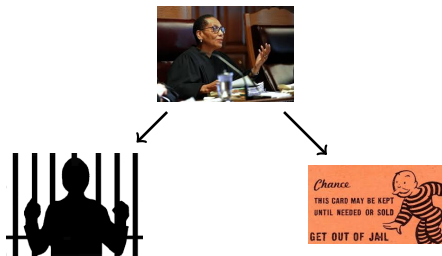
# Bail decisions



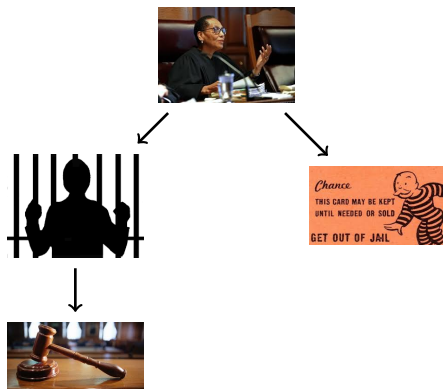
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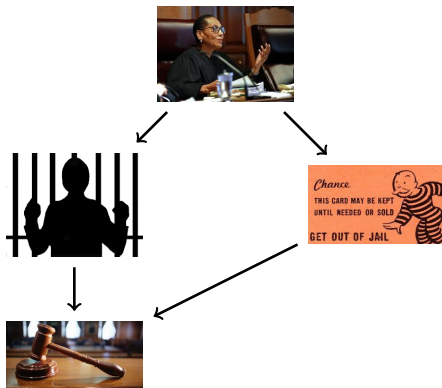
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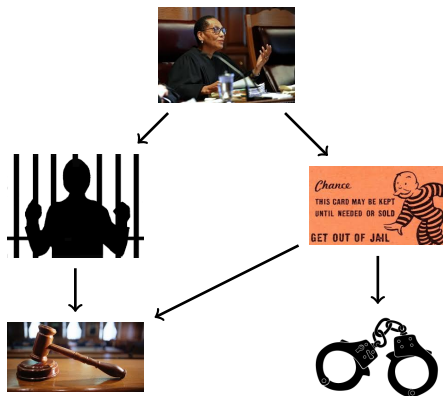
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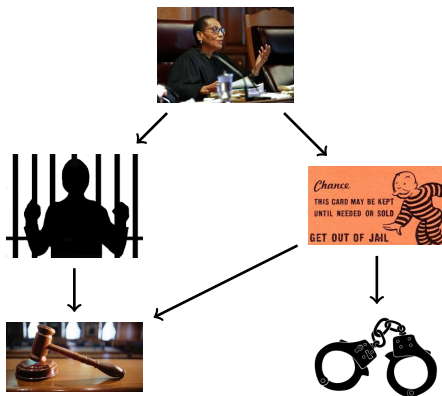
# Bail decisions



# Bail decisions



# Bail decisions



## His honour the machine

Prisoners released on bail\*  
%

Chosen by  
judges

18.6

of which: re-offend<sup>†</sup>

Suggested  
by algorithm

14.9

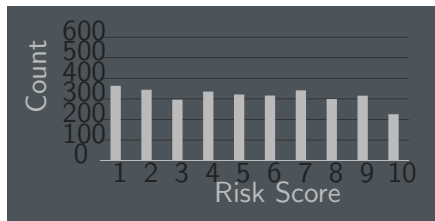
\*From a representative sample of the US Department of Justice database 1990-2009

Source: Jens Ludwig,  
University of Chicago

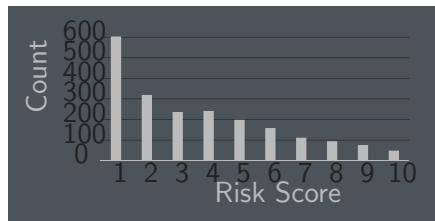
<sup>†</sup>Failure to appear in court and  
re-arrest before trial

Economist.com

# Whites get lower scores than blacks<sup>1</sup>



Black



White

**Figure:** Apparent bias in risk scores towards black versus white defendants.

<sup>1</sup>Pro-publica, 2016



But scores equally accurately predict recidivism<sup>2</sup>



Figure: Recidivism rates by risk score.

## But non-offending blacks get higher scores

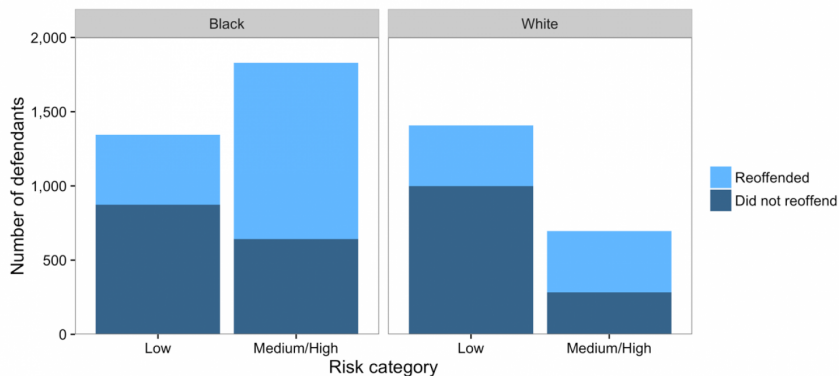


Figure: Score breakdown based on recidivism rates.

# Graphical models and independence

- ▶ Why is it not possible to be fair in all respects?
- ▶ Different notions of **conditional independence**.
- ▶ Can only be satisfied rarely simultaneously.

# Graphical models

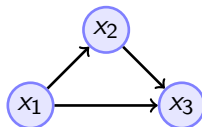


Figure: Graphical model (directed acyclic graph) for three variables.

## Joint probability

Let  $P$  is a probability measure on  $(\Omega, \Sigma)$ . Then let the random variable  $\mathbf{x} = (x_1, \dots, x_n)$  so that  $\mathbf{x} : \Omega \rightarrow X$ ,  $X = \prod_i X_i$ . The joint probability of  $\mathbf{x}$  can be written in terms of the underlying probability measure  $P$ :

$$\mathbb{P}(\mathbf{x} \in A) = P(\{\omega \in \Omega \mid \mathbf{x}(\omega) \in A\}).$$

## Factorisation

$$\mathbb{P}(\mathbf{x}) = \mathbb{P}(\mathbf{x}_B \mid \mathbf{x}_C) \mathbb{P}(\mathbf{x}_C), \quad B, C \subseteq [n]$$

# Graphical models

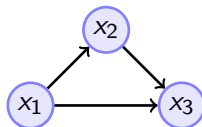


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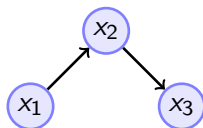
$$\mathbb{P}(\mathbf{x} \in A) = P(\{\omega \in \Omega \mid \mathbf{x}(\omega) \in A\}).$$

## Factorisation

So we can write any joint distribution as

$$P(x_1)P(x_2 \mid x_1)P(x_3 \mid x_1, x_2) \cdots P(x_n \mid x_1, \dots, x_{n-1}).$$

# Directed graphical models



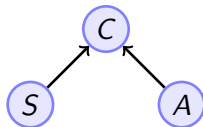
**Figure:** Graphical model for the factorisation  $\mathbb{P}(x_3 \mid x_2) \mathbb{P}(x_2 \mid x_1) \mathbb{P}(x_1)$ .

## Conditional independence

We say  $x_i$  is conditionally independent of  $x_B$  given  $x_D$  and write  $x_i \mid x_D \perp\!\!\!\perp x_B$  iff

$$\mathbb{P}(x_i, x_B \mid x_D) = \mathbb{P}(x_i \mid x_D) \mathbb{P}(x_B \mid x_D).$$

## Example 4 (Smoking and lung cancer)

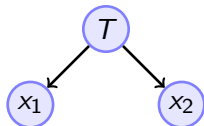


**Figure:** Smoking and lung cancer graphical model, where  $S$ : Smoking,  $C$ : cancer,  $A$ : asbestos exposure.

### Explaining away

Even though  $S, A$  are independent, they become dependent once you know  $C$ .

## Example 5 (Time of arrival at work)



**Figure:** Time of arrival at work graphical model where  $T$  is a traffic jam and  $x_1$  is the time John arrives at the office and  $x_2$  is the time Jane arrives at the office.

## Conditional independence

Even though  $x_1, x_2$  are correlated, they become independent once you know  $T$ .



## Example 6 (Treatment effects)

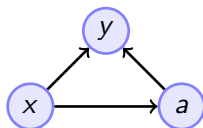


Figure: Kidney treatment model, where  $x$ : severity,  $y$ : result,  $a$ : treatment applied

	Treatment A	Treatment B
Small stones	87	270
Large stones	263	80
Severity	Treatment A	Treatment B
Small stones )	93%	87%
Large stones	73%	69%
Average	78%	83%

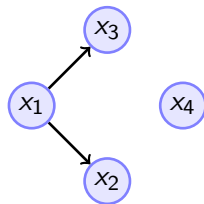
## Example 7 (School admission)



**Figure:** School admission graphical model, where  $z$ : gender,  $s$ : school applied to,  $a$ : whether you were admitted.

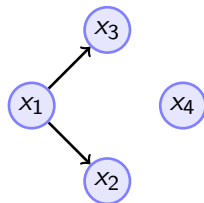
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<i>Average</i>	<i>45%</i>	<i>38%</i>

## Exercise 1



Factorise the following graphical model.

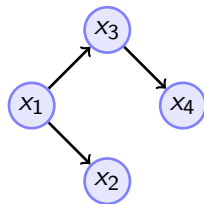
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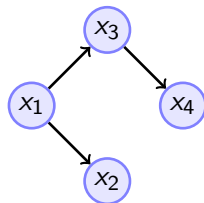
$$\mathbb{P}(\mathbf{x}) = \mathbb{P}(x_1) \mathbb{P}(x_2 \mid x_1) \mathbb{P}(x_3 \mid x_1) \mathbb{P}(x_4)$$

## Exercise 2



Factorise the following graphical model.

## Exercise 2



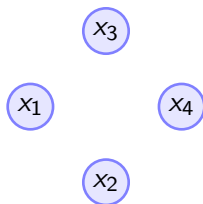
Factorise the following graphical model.

$$\mathbb{P}(\mathbf{x}) = \mathbb{P}(x_1) \mathbb{P}(x_2 \mid x_1) \mathbb{P}(x_3 \mid x_1) \mathbb{P}(x_4 \mid x_3)$$

### Exercise 3

What dependencies does the following factorisation imply?

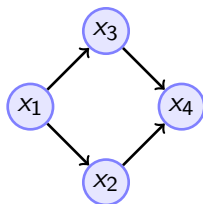
$$\mathbb{P}(\mathbf{x}) = \mathbb{P}(x_1) \mathbb{P}(x_2 \mid x_1) \mathbb{P}(x_3 \mid x_1) \mathbb{P}(x_4 \mid x_2, x_3)$$



### Exercise 3

What dependencies does the following factorisation imply?

$$\mathbb{P}(\mathbf{x}) = \mathbb{P}(x_1) \mathbb{P}(x_2 \mid x_1) \mathbb{P}(x_3 \mid x_1) \mathbb{P}(x_4 \mid x_2, x_3)$$





## Deciding conditional independence

There is an algorithm for deciding conditional independence of any two variables in a graphical model.

# Inference and prediction in graphical models

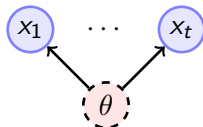


Figure: Inference and prediction in a graphical model

## Inference of latent variables

$$\mathbb{P}(\theta \mid x_1, \dots, x_t)$$

- Model parameters.
- System states.

# Inference and prediction in graphical models

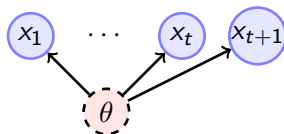


Figure: Inference and prediction in a graphical model

## Prediction

$$\mathbb{P}(x_{t+1} \mid x_1, \dots, x_t) = \int_{\theta} \mathbb{P}(x_{t+1} \mid \theta) d\mathbb{P}(\theta \mid x_1, \dots, x_t)$$

Predictions are **testable**.

# Coin tossing, revisited

## Example 8

The Beta-Bernoulli prior



Figure: Graphical model for a Beta-Bernoulli prior

$$\theta \sim \text{Beta}(\xi_1, \xi_2), \quad \text{i.e. } \xi \text{ are Beta distribution parameters} \quad (3.1)$$

$$x \mid \theta \sim \text{Bernoulli}(\theta), \quad \text{i.e. } P_\theta(x) \text{ is a Bernoulli} \quad (3.2)$$

## Example 9

The  $n$ -meteorologists problem (continuation of Exercise ??)

- Meteorological models  $\mathcal{M} = \{\mu_1, \dots, \mu_n\}$
- Rain predictions at time  $t$ :  $p_{t,\mu} \triangleq P_\mu(x_t = \text{rain})$ .
- Prior probability  $\xi(\mu) = 1/n$  for each model.
- Decision  $a$ , resulting in utility  $U(a, x_{t+1})$

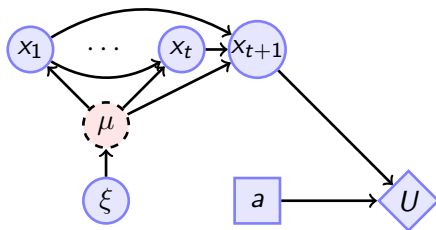


Figure: Inference, prediction and decisions in a graphical model.

# Measuring independence

## Theorem 10

If  $x_i \mid x_D \perp\!\!\!\perp x_B$  then

$$\mathbb{P}(x_i \mid x_B, x_D) = \mathbb{P}(x_i \mid x_D)$$

## Example 11

$$\|\mathbb{P}(a \mid y, z) - \mathbb{P}(a \mid y)\|_1$$

which for discrete  $a, y, z$  is:

$$\max_{i,j} \|\mathbb{P}(a \mid y = i, z = j) - \mathbb{P}(a \mid y = i)\|_1 = \max_{i,j} \left\| \sum_k \mathbb{P}(a = k \mid y = i, z = j) - \mathbb{P}(a = k \mid y = i) \right\|_1$$

# Measuring independence

## Theorem 10

If  $x_i \mid x_D \perp\!\!\!\perp x_B$  then

$$\mathbb{P}(x_i \mid x_B, x_D) = \mathbb{P}(x_i \mid x_D)$$

This implies

$$\mathbb{P}(x_i \mid x_B = b, x_D) = \mathbb{P}(x_i \mid x_B = b', x_D)$$

so we can measure independence by seeing how the distribution of  $x_i$  changes when we vary  $x_B$ , keeping  $x_D$  fixed.

## Example 11

$$\|\mathbb{P}(a \mid y, z) - \mathbb{P}(a \mid y)\|_1$$

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## Example 12

An alternative model for coin-tossing This is an elaboration of Example ?? for hypothesis testing.

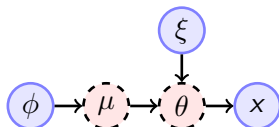


Figure: Graphical model for a hierarchical prior

- $\mu_1$ : A Beta-Bernoulli model with  $\text{Beta}(\xi_1, \xi_2)$
- $\mu_0$ : The coin is fair.

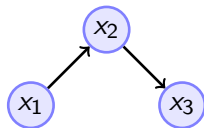
$$\theta \mid \mu = \mu_0 \sim \mathcal{D}(0.5), \quad \text{i.e. } \theta \text{ is always } 0.5 \quad (3.3)$$

$$\theta \mid \mu = \mu_1 \sim \text{Beta}(\xi_1, \xi_2), \quad \text{i.e. } \theta \text{ has a Beta distribution} \quad (3.4)$$

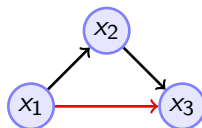
$$x \mid \theta \sim \text{Bernoulli}(\theta), \quad \text{i.e. } P_\theta(x) \text{ is Bernoulli} \quad (3.5)$$



# Bayesian testing of independence



(a)  $\Theta_0$  assumes independence



(b)  $\Theta_1$  does **not** assume independence

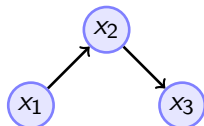
## Example 13

Assume data  $D = \{x_1^t, x_2^t, x_3^t \mid t = 1, \dots, T\}$  with  $x_i^t \in \{0, 1\}$ .

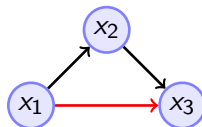
$$P_{\theta}(D) = \prod_t P_{\theta}(x_3^t \mid x_2^t) P_{\theta}(x_2^t \mid x_1^t) P_{\theta}(x_1^t), \quad \theta \in \Theta_0 \quad (3.6)$$

$$P_{\theta}(D) = \prod_t P_{\theta}(x_3^t \mid x_2^t, x_1^t) P_{\theta}(x_2^t \mid x_1^t) P_{\theta}(x_1^t), \quad \theta \in \Theta_1 \quad (3.7)$$

# Bayesian testing of independence



(a)  $\theta_0$  assumes independence



(b)  $\theta_1$  does **not** assume independence

## Example 13

$$\theta_1 \triangleq P_{\theta}(x_1^t = 1) \quad (\mu_0, \mu_1)$$

$$\theta_{2|1}^i \triangleq P_{\theta}(x_2^t = 1 \mid x_1^t = i) \quad (\mu_0, \mu_1)$$

$$\theta_{3|2}^j \triangleq P_{\theta}(x_3^t = 1 \mid x_2^t = j) \quad (\mu_0)$$

$$\theta_{3|2,1}^{i,j} \triangleq P_{\theta}(x_3^t = 1 \mid x_2^t = j, x_1^t = i) \quad (\mu_1)$$



Figure: Hierarchical model.

$$\mu_i \sim \phi \quad (3.6)$$

$$\theta \mid \mu = \mu_i \sim \xi_i \quad (3.7)$$

## Marginal likelihood

$$\mathbb{P}_\phi(D) = \phi(\mu_0) \mathbb{P}_{\mu_0}(D) + \phi(\mu_1) \mathbb{P}_{\mu_1}(D) \quad (3.8)$$

$$\mathbb{P}_{\mu_i}(D) = \int_{\Theta_i} P_\theta(D) \mathrm{d}\xi_i(\theta). \quad (3.9)$$



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## Model posterior

$$\phi(\mu \mid D) = \frac{\mathbb{P}_\mu(D)\phi(\mu)}{\sum_i \mathbb{P}_{\mu_i}(D)\phi(\mu_i)} \quad (3.8)$$

# Calculating the marginal likelihood

## Monte-Carlo approximation

$$\int_{\Theta} P_{\theta}(D) d\xi(\theta) \approx \frac{1}{N} \sum_{n=1}^N P_{\theta_n}(D) + O(1/\sqrt{N}), \quad \theta_n \sim \xi \quad (3.9)$$

## Importance sampling

$$\int_{\Theta} P_{\theta}(D) d\xi(\theta) \quad (3.10)$$

# Calculating the marginal likelihood

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$$\int_{\Theta} P_{\theta}(D) d\xi(\theta) = \int_{\Theta} P_{\theta}(D) \frac{d\psi(\theta)}{d\xi(\theta)} d\xi(\theta) \quad (3.10)$$

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$$\int_{\Theta} P_{\theta}(D) d\xi(\theta) \approx \sum_{n=1}^N P_{\theta}(D) \frac{d\xi(\theta_n)}{d\psi(\theta_n)}, \quad \theta_n \sim \psi \quad (3.10)$$



## Sequential updating of the marginal likelihood

$$\mathbb{P}_{\xi}(D)$$

(3.14)

### Example 14 (Beta-Bernoulli)

$$\mathbb{P}_{\xi}(x_t = 1 \mid x_1, \dots, x_{t-1}) = \frac{\alpha_t}{\alpha_t + \beta_t},$$

with  $\alpha_t = \alpha_0 + \sum_{n=1}^{t-1} x_n$ ,  $\beta_t = \beta_0 + \sum_{n=1}^{t-1} (1 - x_n)$

## Sequential updating of the marginal likelihood

$$\mathbb{P}_{\xi}(D) = \mathbb{P}_{\xi}(x_1, \dots, x_T)$$

(3.14)

### Example 14 (Beta-Bernoulli)

$$\mathbb{P}_{\xi}(x_t = 1 \mid x_1, \dots, x_{t-1}) = \frac{\alpha_t}{\alpha_t + \beta_t},$$

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## Sequential updating of the marginal likelihood

$$\begin{aligned}\mathbb{P}_\xi(D) &= \mathbb{P}_\xi(x_1, \dots, x_T) \\ &= \mathbb{P}_\xi(x_2, \dots, x_T \mid x_1) \mathbb{P}_\xi(x_1)\end{aligned}\tag{3.11}$$

(3.14)

### Example 14 (Beta-Bernoulli)

$$\mathbb{P}_\xi(x_t = 1 \mid x_1, \dots, x_{t-1}) = \frac{\alpha_t}{\alpha_t + \beta_t},$$

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## Sequential updating of the marginal likelihood

$$\mathbb{P}_\xi(D) = \mathbb{P}_\xi(x_1, \dots, x_T) \quad (3.11)$$

$$= \mathbb{P}_\xi(x_2, \dots, x_T \mid x_1) \mathbb{P}_\xi(x_1) \quad (3.12)$$

$$= \prod_{t=1}^T \mathbb{P}_\xi(x_t \mid x_1, \dots, x_{t-1})$$

$$(3.14)$$

### Example 14 (Beta-Bernoulli)

$$\mathbb{P}_\xi(x_t = 1 \mid x_1, \dots, x_{t-1}) = \frac{\alpha_t}{\alpha_t + \beta_t},$$

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## Sequential updating of the marginal likelihood

$$\mathbb{P}_\xi(D) = \mathbb{P}_\xi(x_1, \dots, x_T) \quad (3.11)$$

$$= \mathbb{P}_\xi(x_2, \dots, x_T \mid x_1) \mathbb{P}_\xi(x_1) \quad (3.12)$$

$$= \prod_{t=1}^T \mathbb{P}_\xi(x_t \mid x_1, \dots, x_{t-1}) \quad (3.13)$$

$$= \prod_{t=1}^T \int_{\Theta} P_{\theta_n}(x_t) \underbrace{d\xi(\theta \mid x_1, \dots, x_{t-1})}_{\text{posterior at time } t} \quad (3.14)$$

### Example 14 (Beta-Bernoulli)

$$\mathbb{P}_\xi(x_t = 1 \mid x_1, \dots, x_{t-1}) = \frac{\alpha_t}{\alpha_t + \beta_t},$$

with  $\alpha_t = \alpha_0 + \sum_{n=1}^{t-1} x_n$ ,  $\beta_t = \beta_0 + \sum_{n=1}^{t-1} (1 - x_n)$

# Further reading

## Python sources

- ▶ A simple python measure of conditional independence  
`src/fairness/ci_test.py`
- ▶ A simple test for discrete Bayesian network  
`src/fairness/DirichletTest.py`
- ▶ Using the PyMC package  
[https://docs.pymc.io/notebooks/Bayes\\_factor.html](https://docs.pymc.io/notebooks/Bayes_factor.html)

# Bail decisions, revisited

 $X$  $\downarrow \pi$ 

# Bail decisions, revisited

 $x$ 

 $\pi$ 

 $a_1$ 


$$\pi(a \mid x)$$

(policy)



# Bail decisions, revisited

 $x$ 

 $\pi$ 

 $a_1$ 

 $a_2$ 

 $\pi(a | x)$ 

(policy)

# Bail decisions, revisited

 $x$ 

 $\downarrow \pi$ 

 $\pi(a \mid x)$  (policy)

 $\mathbb{P}(y \mid a, x)$  (outcome)

 $a_1$ 

 $a_2$ 

 $\downarrow y_1$ 


# Bail decisions, revisited

 $x$ 

 $\downarrow \pi$ 

 $\pi(a \mid x)$  (policy)

 $\mathbb{P}(y \mid a, x)$  (outcome)

 $a_1$ 

 $a_2$ 

 $\downarrow y_1$ 


# Bail decisions, revisited

 $x$ 

 $\pi$ 

 $\pi(a | x)$  (policy)

 $\mathbb{P}(y | a, x)$  (outcome)

 $a_1$ 

 $y_1$ 

 $a_2$ 

 $y_2$ 


# Bail decisions, revisited

 $x$ 

 $\downarrow \pi$ 

 $a_1$ 

 $y_1$ 

 $a_2$ 

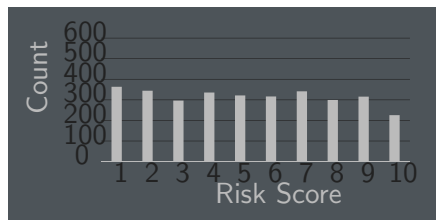
 $y_2$ 

 $\pi(a | x)$  (policy)

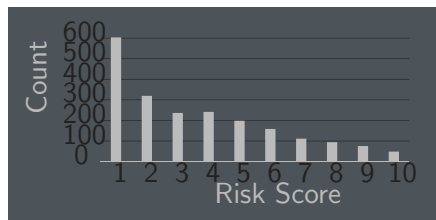
 $\mathbb{P}(y | a, x)$  (outcome)

 $U(a, y)$  (utility)

# Independence



Black



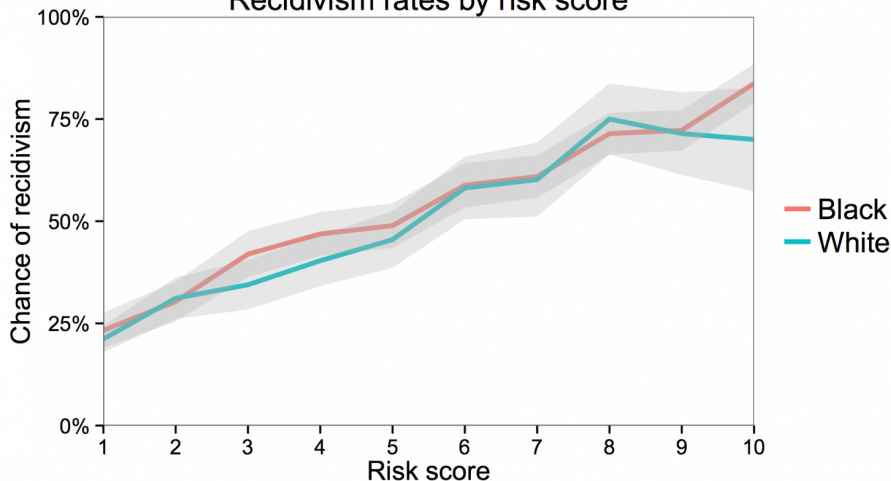
White

**Figure:** Apparent bias in risk scores towards black versus white defendants.

$$\mathbb{P}_{\theta}^{\pi}(a \mid z) = \mathbb{P}_{\theta}^{\pi}(a)$$

(non-discrimination)

# Recidivism rates by risk score



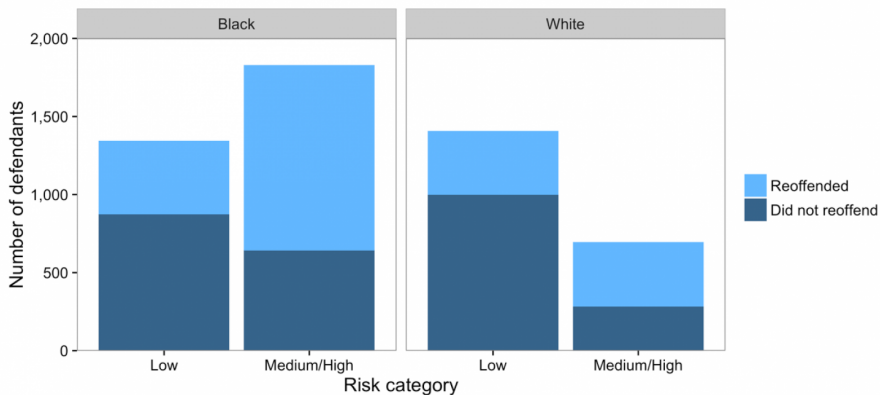
$y$  Result.

$a$  Assigned score.

$z$  Race.

$$\mathbb{P}^{\pi}(y \mid a, z) = \mathbb{P}^{\pi}(y \mid a) \quad (\text{calibration})$$

$$\mathbb{P}^{\pi}(a \mid y, z) = \mathbb{P}^{\pi}(a \mid y) \quad (\text{balance})$$



$y$  Result.

$a$  Assigned score.

$z$  Race.

$$\mathbb{P}^{\pi}(y \mid a, z) = \mathbb{P}^{\pi}(y \mid a) \quad (\text{calibration})$$

$$\mathbb{P}^{\pi}(a \mid y, z) = \mathbb{P}^{\pi}(a \mid y) \quad (\text{balance})$$



## Meritocratic decision

$$a_t(\theta, x_t) \in \arg \max_a \mathbb{E}_\theta(U \mid a, x_t) = \int_{\mathcal{Y}} U(a_t, y) \mathbb{P}_\theta(y \mid a_t, x_t) \quad (4.1)$$

# Smooth fairness

$$D[\pi(a \mid x), \pi(a \mid x')] \leq \rho(x, x'). \quad (4.2)$$

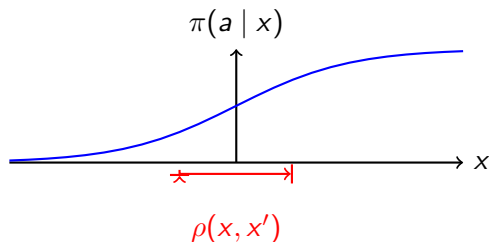


Figure: A Lipschitz function

## The constrained maximisation problem

$$\max_{\pi} \{ U(\pi) \mid \rho(x, x') \leq \epsilon \} \quad (4.3)$$

# The value of a policy

## Fairness metrics: balance

$$F_{\text{balance}}(\theta, \pi) \triangleq \sum_{y,z,a} |\mathbb{P}_{\theta}^{\pi}(a \mid y, z) - \mathbb{P}_{\theta}^{\pi}(a \mid y)|^2 \quad (4.4)$$

# The value of a policy

## Fairness metrics: balance

$$F_{\text{balance}}(\theta, \pi) \triangleq \sum_{y,z,a} |\mathbb{P}_{\theta}^{\pi}(a \mid y, z) - \mathbb{P}_{\theta}^{\pi}(a \mid y)|^2 \quad (4.4)$$

## Utility: Classification accuracy

$$U(\theta, \pi) = \mathbb{P}_{\theta}^{\pi}(y_t = a_t)$$

# The value of a policy

## Fairness metrics: balance

$$F_{\text{balance}}(\theta, \pi) \triangleq \sum_{y, z, a} |\mathbb{P}_{\theta}^{\pi}(a \mid y, z) - \mathbb{P}_{\theta}^{\pi}(a \mid y)|^2 \quad (4.4)$$

## Utility: Classification accuracy

$$U(\theta, \pi) = \mathbb{P}_{\theta}^{\pi}(y_t = a_t)$$

## Use $\lambda$ to trade-off utility and fairness

$$V(\lambda, \theta, \pi) = (1 - \lambda) \overbrace{U(\theta, \pi)}^{\text{utility}} - \lambda \underbrace{F(\theta, \pi)}_{\text{unfairness}} \quad (4.5)$$

# Model uncertainty

$\theta$  is unknown

## Theorem 15

*A decision rule in the form of a lottery, i.e.*

$$\pi(a \mid x) = p_a$$

*can be the only way to satisfy balance for all possible  $\theta$ .*

## Possible solutions

- ▶ Marginalize over  $\theta$  ("expected" model)
- ▶ Use Bayesian reasoning

## The value of a policy

Let  $\lambda$  represent the trade-off between utility and fairness.

$$V(\lambda, \theta, \pi) = \lambda \overbrace{U(\theta, \pi)}^{\text{utility}} - \underbrace{(1 - \lambda)F(\theta, \pi)}_{\text{fairness violation}} \quad (4.6)$$

# The Bayesian decision problem

## The Bayesian value of a policy

$$V(\lambda, \xi, \pi) = \int_{\Theta} V(\lambda, \theta, \pi) d\xi(\theta). \quad (4.7)$$



## Online resources

- ▶ COMPAS analysis by propublica  
<https://github.com/propublica/compas-analysis>
- ▶ Open policing database <https://openpolicing.stanford.edu/>

# Learning outcomes

## Understanding

- ▶ Graphical models and conditional independence.
- ▶ Fairness as independence and meritocracy.

## Skills

- ▶ Specify a graphical model capturing dependencies between variables.
- ▶ Testing for conditional independence.
- ▶ Verify if a policy satisfies a fairness condition.

## Reflection

- ▶ How should we be fair with respect to sensitive attributes?
- ▶ Balancing the needs of individuals, the decision maker and society?
- ▶ Does having more data available make it easier to achieve fairness?
- ▶ What is the relation to game theory and welfare economics?