Position Estimation Using UWB TDOA Measurements

Jun Xu, Maode Ma, Choi Look Law
Positioning and Wireless Technology Centre
Nanyang Technological University
50 Nanyang Drive, Research TechnoPlaza
Level 4, BorderX Block, Singapore 637553
xujun2005@pmail.ntu.edu.sg

Abstract- **UWB signal has high ranging precision due to its large bandwidth. In this paper, TDOA ranges measured by UWB signals are used in the position estimation process. The ranges are modeled for both LOS and NLOS environment. A TDOA error-minimizing localization method has been proposed to estimate the locations of blind nodes and its performance is investigated in both LOS and NLOS propagations. Cramer-Rao Lower Bound (CRLB) on the variance of the blind node positions is also derived for LOS situation. Furthermore, we demonstrate that TDOA error-minimizing method is more robust to range errors than Chan's method.**

I. Introduction

Positioning or localization is the process to determine the positions of the nodes. Knowing the positions of nodes in a wireless network enables a variety of functionalities such as emergency services, target identification and tracking, location dependent computing, health monitoring and geographic routing [1-5]. Global Positioning System (GPS) is the most recognized and widely used positioning system. However, it has physical limitations. GPS is not reliable for indoor applications because of severe degradation in ranging accuracy due to multi-paths. Furthermore, GPS receivers are expensive and have high power requirement. Many positioning systems have been developed. In [6], Hightower and Borriello describe a spectrum of positioning systems, e.g. Active Badge and Active Bat location systems developed by AT&T Cambridge, RADAR by Microsoft Research group.

Position estimation involves two main steps: range measuring and positioning based on the measured ranges. Angle of arrival (AOA), time of arrival (TOA) and time difference of arrival (TDOA) are the three commonly used ranging techniques. Here we are more interested in TDOA. TDOA is based on estimating the difference in the arrival times of the signal between the synchronized reference nodes. TDOA ranging does not require knowledge of the absolute time of the transmission.

UWB (Ultra Wideband) signals are well suited for range measurement for its large bandwidth. Ultra Wideband (UWB) is defined by Federal Communications Commission (FCC) to have fractional bandwidth larger than 20% or absolute bandwidth of more than 500MHz.Large bandwidth implies high temporal resolution, and hence UWB can achieve high ranging accuracy [7]. Furthermore UWB signals are robust to multipath interference and fading. Therefore good positioning precision can be achieved by using UWB [8]. Many ranging techniques based on UWB have been proposed. Lee [9] suggests a UWB TOA ranging scheme which implements a search algorithm for the detection of a direct path signal in the presence of dense multipath. Chung and Ha [10] proposed a method based on the estimation of time of arrival of the first multipath of UWB signals under the existence of the LOS path in a multipath environment. UWB ranging for Non-line-of-sight (NLOS) propagations is also investigated in [11].

Once the TDOA ranges have been estimated using UWB signals, we can obtain the range difference measurements by multiplying estimated time delay with radio speed c. These measurements can be converted into nonlinear hyperbolic equations. The equation of TDOA is:

$$R_{j,k} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} - \sqrt{(x_i - x_k)^2 + (y_i - y_k)^2} \quad (1)$$

Where $R_{j,k}$ is the range difference obtained from the estimated TDOA. (x_i, y_i) is the coordinate of the blind node and (x_j, y_j), (x_k, y_k) are the coordinates of the reference nodes.

In 2-D plane, the position of the blind node (node that has no knowledge of its location) can be estimated if there are at least 3 reference nodes (nodes with knowledge of their locations and are time synchronized)). Several algorithms have been proposed to solve the TDOA problem. Chan's method is one of the well-known algorithms [12]. It takes advantage of redundant measurements and can achieve high accuracy. However, it cannot work efficiently if the measurement has large errors, e.g. NLOS error. In [13] a relative localization algorithm is proposed and its accuracy increases with the number of blind nodes. However this algorithm is not applicable to the case where only TDOA measurements are available because synchronization of the blind nodes is required to obtain the TDOA ranges among the blind nodes.

In this paper, we describe an error-minimizing localization method which uses TDOA ranges measured using UWB to derive the positions of the blind nodes. All the blind nodes in the network are seen as a group and their locations are

1-4244-0102-X/06/$20.00 ©2006 IEEE

estimated simultaneously. This method compares the measured TDOA ranges with those computed from the node coordinates, and the blind node positions are estimated by minimizing the error sum. Cramer-Rao Lower Bound (CRLB) is derived for Gaussian distributed TDOA range measurements. We investigate the performance of the error-minimizing method for both LOS and NLOS propagations. For the same number of reference nodes and the same propagation environment, TDOA error-minimizing achieves higher accuracy than Chan's method.

The remainder of the paper is organized as follows. In section II we described how we model the measured ranges for both LOS and NLOS. Section III presents our localization scheme and simulation results. Section IV presents our conclusion.

II. RANGE MODELING

Line-of-sight (LOS) propagation is necessary for accurate range estimates. If there is no LOS between two nodes, the UWB pulse will travel an extra distance and hence extra time of propagation [7]. NLOS error is the dominant error in range estimation. Range errors for TOA measurements are modeled in [14, 15]. It is known that TDOA is actually the difference of TOAs which are estimated relative to a common reference time but independent on the actual transmission time [16].

In LOS propagation, the range errors are normally modeled as zero-mean Gaussian, which is caused by the measurement system itself. Therefore we can model the measured range between two nodes in LOS propagation as:

$$(R_{m,n})_{los} = (\| p_i - p_m \| - \| p_i - p_n \|) + n_1 \tag{2}$$

where $R_{m,n}$ is the range difference from node m and n to the querying blind node i, n_1 is Gaussian measurement noise, p_m and p_n are coordinates of two reference nodes, and p_i is the coordinate of the blind node.

For NLOS, some obstacles may exist between the nodes (Fig. 1), and LOS is no longer present.

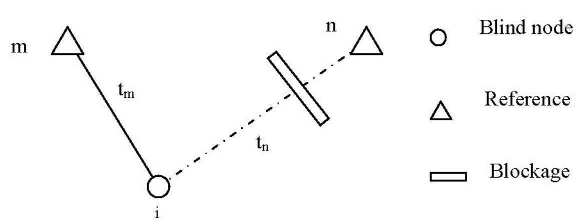


Fig. 1 illustration of NLOS propagation: solid line and dashed line represent LOS and NLOS respectively

For TOA in NLOS environment, a positive bias is introduced in the measurement due to the delay of received signal because the LOS between two nodes is blocked. Its magnitude of the positive bias depends on the propagation environment. The major errors also include Gaussian measurement error. The TOA range for NLOS can be modeled as:

$$(R_n)_{nlos} = \| p_i - p_n \| + n_2 + n_3 \tag{3}$$

where R_n is the TOA range between node n and i, n_2 is Gaussian measurement noise, and n_3 is a positive bias present when LOS is blocked.

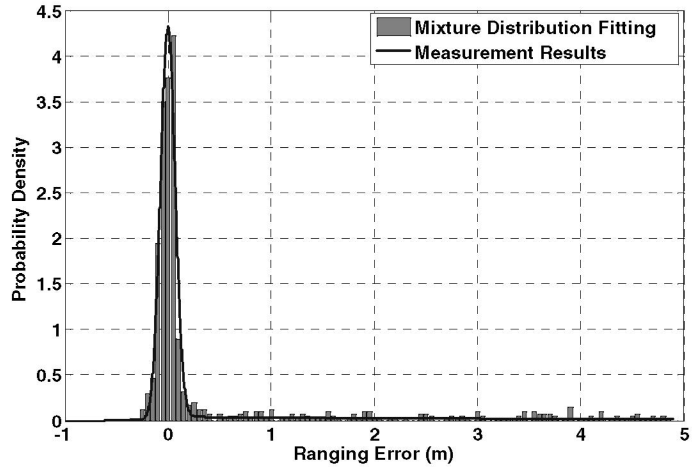


Fig. 2 Histogram of NLOS ranging errors for TOA

The histogram in Fig.2 is obtained from our TOA range measurements, which shows the probability density of the range errors in NLOS environment. The measurements were performed at the Positioning and Wireless Technology Center (PWTC), Nanyang Technological University, Singapore. UWB signals are generated by an in-house designed pulse generator at a pulse repetition frequency of 2 MHz, and the ranges are obtained by sending UWB signals from one transmitter to multiple receivers which are located at different locations [17]. The probability of the ranging errors can be expressed in a mathematical form as a combination of Gaussian distribution function and exponential distribution function:

$$f(x) = \alpha \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right) + (1-\alpha)\frac{1}{\beta}\exp\left(-\frac{x}{\beta}\right), \tag{4}$$

where

$\sigma = 0.071$ m, standard deviation of the Gaussian distribution function,

$\beta = 5.235$ m, mean of exponential distribution function

$\alpha = 0.767$ m, weighting factor

The values of above parameters are obtained by fitting the measured range error PDF shown in Fig. 2.

If we measure TDOA ranges in the same environment, the same obstacles exist. Therefore the same amount of bias will be present for a UWB pulse traveling between the nodes. Thus we can reasonably approximate the TDOA range value of blind node i in Fig.1 as:

$$(R_{m,n})_{nlos} = (\| p_i - p_m \| + n_m) - (\| p_i - p_n \| + n_n), \tag{5}$$

Where $R_{m,n}$ is the measured TDOA range between node m and n; n_m and n_n are range noise for UWB signal traveling from node m and n respectively.

III. LOCALIZATION

Our goal is to estimate the positions of the blind nodes in the network, given a set of TDOA measurements obtained using UWB and the positions of the reference nodes. For simplicity, we restrict ourselves to two dimensions. In our UWB localization system, it is assumed that the radio range of the reference nodes covers the whole area and the reference nodes are all synchronized

A. TDOA Error-minimizing Localization

Consider a network with N nodes *(1, 2, ..., r,r+1, ..., N)* as depicted in Fig. 3, where the first r nodes are reference nodes, and the remaining (*N-r*) nodes are blind nodes. $R_{m,n}$ is the range difference from node m and n to the querying blind node i, which equals radio speed times measured TDOA, $R_{m,n}= c\cdot t_{tdoa}$. p_i, p_m and p_n are positions of blind node i, reference nodes m, n, respectively.

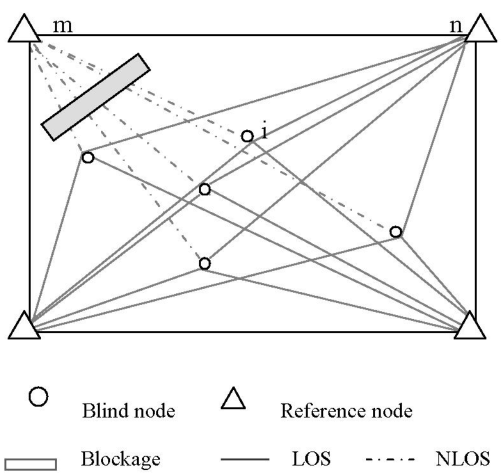


Fig. 3 A Network example with 4 reference nodes (time synchronized) and 5 blind nodes. Reference node m has NLOS to all the blind nodes

Then the Euclidean distances between node i, m and node i, n are $\| p_i - p_m \|$ and $\| p_i - p_n \|$, and hence their difference is $(\| p_i - p_m \| - \| p_i - p_n \|)$. If $R_{m,n}$ is accurate, without error, and p_i, p_m *and* p_n are the true coordinates of the three nodes, then $R_{m,n} = (\| p_i - p_m \| - \| p_i - p_n \|)$. However, $R_{m,n}$ is always with noise, as mentioned in Section II, and among the three nodes *i, m, and n*, at least the coordinate of the querying node i is unknown. Therefore $R_{m,n}$ and $(\| p_i - p_m \| - \| p_i - p_n \|)$ are most likely not equal. Then we have to minimize the difference between $R_{m,n}$ and $(\| p_i - p_m \| - \| p_i - p_n \|)$ in order to find the positions of the blind nodes. All the blind nodes are seen as a group, and a cost function Eq. (6) is formed. Conjugate gradient method can be used to do the minimization [18].

$$f(p)=\sum_{i=r+1}^{N}\sum_{m=1}^{r-1}\sum_{n=m+1}^{r}(\| p_i - p_m \| - \| p_i - p_n \| - R_{m,n})^2 \qquad (6)$$

We measure the performance of the positioning method with root mean square error (RMSE), which is obtained by comparing estimated positions with actual positions. Lower RMSE means better performance. RMSE is expressed as Eq. (7).

$$RMSE=\sqrt{\frac{\sum_{i=1}^{N-r}\|(p_{est})_i-(p_{actual})_i\|^2}{N-r}} \qquad (7)$$

where p_{est} and p_{actual} are estimated and actual coordinates, respectively.

B. Cramer-Rao Lower Bound (CRLB)

Before we go to the simulation, we first look at the CRLB on the variance of the positions of the blind node for LOS propagation. CRLB is a theoretical lower bound of the variance of the position estimates and it is defined as the inverse of the Fisher information matrix (FIM) [19]. We assume the measured TDOA ranges are Gaussian with mean $d_{m,n}$ and variance σ^2:

$$R_{m,n} \sim N\,(d_{m,n}, \sigma^2) \qquad (8)$$

Where

$$d_{m,n}=\sqrt{(x-x_m)^2+(y-y_m)^2}-\sqrt{(x-x_n)^2+(y-y_n)^2}$$

Then the jointly conditional PDF function is:

$$p(d\,|\,x,y)=\prod_{m=1}^{r-1}\prod_{n=m+1}^{r}\frac{1}{2\pi\sigma^2}\exp\!\left(-\frac{(d_{m,n}-R_{m,n})^2}{2\sigma^2}\right) \qquad (9)$$

We express the FIM as

$$I(x,y)=\begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix} \qquad (10)$$

where

$$I_{11}=-E\left(\frac{\partial^2 \ln p(d\,|\,x,y)}{\partial x^2}^{2}\right) \qquad I_{22}=-E\left(\frac{\partial^2 \ln p(d\,|\,x,y)}{\partial y^2}^{2}\right)$$

$$I_{12}=I_{21}=-E\left(\frac{\partial^2 \ln p(d\,|\,x,y)}{\partial x \partial y}^{2}\right)$$

The elements of the above FIM can be derived:

$$I_{11}=\frac{1}{\sigma^2}\sum_{m=1}^{r-1}\sum_{n=m+1}^{r}(A_m-A_n)^2 \qquad I_{22}=\frac{1}{\sigma^2}\sum_{m=1}^{r-1}\sum_{n=m+1}^{r}(B_m-B_n)^2$$

$$I_{12}=I_{21}=\frac{1}{\sigma^2}\sum_{m=1}^{r-1}\sum_{n=m+1}^{r}((A_m-A_n)(B_m-B_n))$$

where

$$A_i=\frac{(x-x_i)}{\sqrt{(x-x_i)^2+(y-y_i)^2}}\quad B_i=\frac{(y-y_i)}{\sqrt{(x-x_i)^2+(y-y_i)^2}}$$

The bound of the variance of the blind node position can be obtained:

$$\sigma_{blind}^2=\mathrm{var}(x)+\mathrm{var}(y)\geq\frac{I_{22}}{I_{11}I_{22}-I_{12}^2}+\frac{I_{11}}{I_{11}I_{22}-I_{12}^2}$$

$$\sigma_{blind}^2\geq\frac{\sigma^2(\sum_{m=1}^{r-1}\sum_{n=m+1}^{r}(A_m-A_n)^2+\sum_{m=1}^{r-1}\sum_{n=m+1}^{r}(B_m-B_n)^2)}{\sum_{m=1}^{r-1}\sum_{n=m+1}^{r}(A_m-A_n)^2\sum_{m=1}^{r-1}\sum_{n=m+1}^{r}(B_m-B_n)^2-(\sum_{m=1}^{r-1}\sum_{n=m+1}^{r}(A_m-A_n)(B_m-B_n))^2} \tag{11}$$

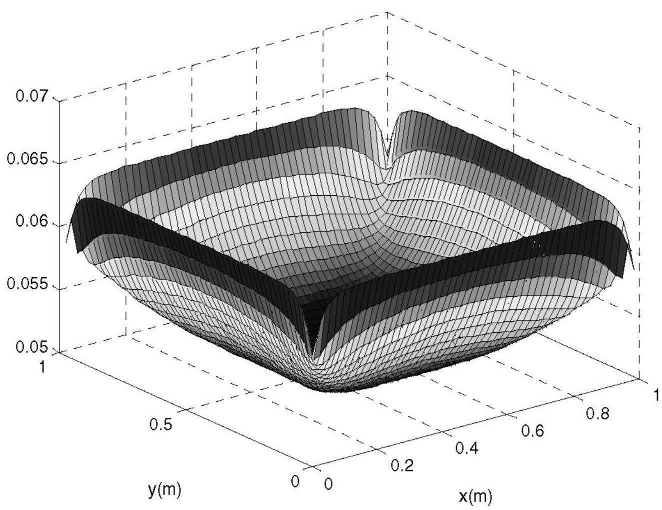
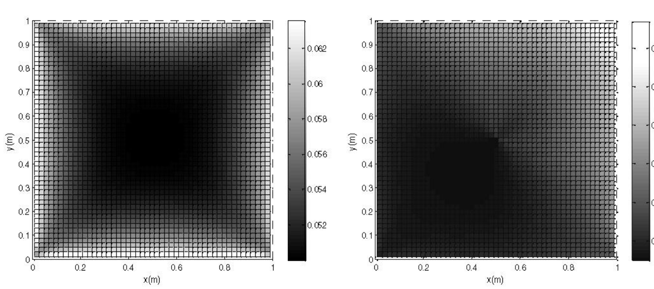


Fig. 4 Three dimensional plot of CRLB, r=4, σ=0.1m



(a) (b)

Fig.5 (a) CRLB for the network with 4 reference nodes at four corners
(b) CRLB for the network with 3 reference nodes at three corners and one reference node at the center

It is clear that the right part of the inequality (Eq. (11)) is proportional to the variance of the TDOA measurements, which means that the lower bound on the positioning error of the blind node increases with standard deviation of the TDOA range errors. We also observe that A_i *and* B_i have the relation as $A_i^2+B_i^2=1$, and thus we can represent A and B as $cos\beta_i$ and $sin\beta_i$, respectively, where β_i is the angle of the blind node to the reference node i with respect to x axis. Therefore we can conclude that the CRLB of the variance of the blind node position is closely related to the geometry of the network, i.e. CRLB varies with the positions of the reference nodes and blind nodes.

Fig. 4 shows the CRLB for standard deviation σ=0.1m in a network with four reference nodes placed at the four corners of a *1mx1m* area. Blind node at the central area has lower position error and the minimum value is 0.05m. Fig. 5 compares the CRLBs for two networks with different reference node placement. Fig.5 (a) is two-dimensional view of Fig.4, while Fig. 5(b) has three reference nodes at three corners and one at the center. It verifies that CRLB varies with the geometry (Darker area has lower CRLB values).

C. Simulation Results

We simulated the above method using Microsoft Visual C++ and MatLab. Reference nodes are placed at the edges of a 50mx50m region, and blind nodes are randomly placed in this region. It is assumed that the radio range of the reference nodes covers the whole area and the reference nodes are all time synchronized. The initial positions of the blind nodes are guessed as long as they are within the rectangular area. For LOS propagation, we assume the range errors are Gaussian with zero mean and standard deviation of 0.1m, $R_{m,n}\sim N(d_{m,n},\sigma^2)$, where $d_{m,n}$ is the actual range difference. For NLOS case, we assume that the UWB signals received at one of the reference nodes located at the corners have NLOS to all the blind nodes due to an obstacle which is close to that reference node (Fig.3), while the remaining reference nodes have clear LOS to the blind nodes. Then TDOA ranges can be represented by Eq. (5) and range noise due to link blockage is modeled with the PDF function given in Eq. (4).

a. RMSE vs. Number of Nodes

For the simulation, the number of reference nodes is increased from 4 to 8; the number of blind nodes increases from 5 to 150. Fig. 6 shows the resulted RMSE for both LOS and NLOS. It is obvious that RMSE for LOS is much smaller compared to NLOS RMSE. This is because for a given positioning method, the only source of error is derived from the accuracy of range measurements and NLOS error is the major error in TDOA measurements. For both cases, the accuracy of location estimation increases as the number of reference nodes increases. We know that, in a network with total r reference nodes, the maximum number of TDOA range measurements (node pairs in the network between reference nodes) we can obtain for each blind node is r*(r-1)/2, increasing with the reference node number.

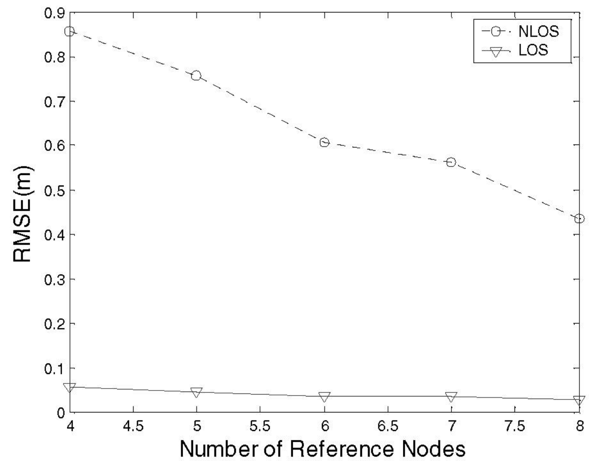


Fig. 6 RMSE versus the number of reference nodes for both LOS and NLOS

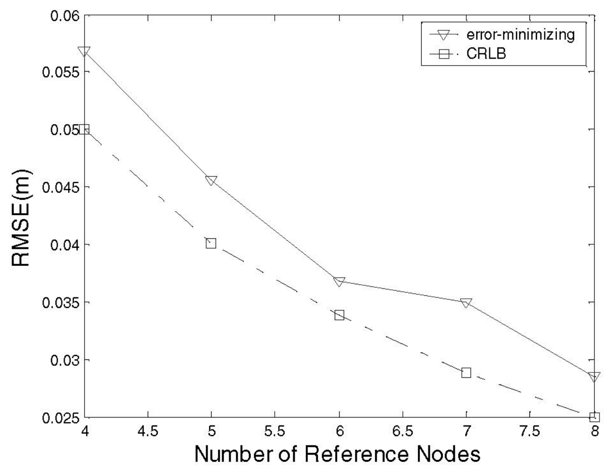


Fig. 7 Comparison of RMSE of TDOA error-minimizing method with CRLB

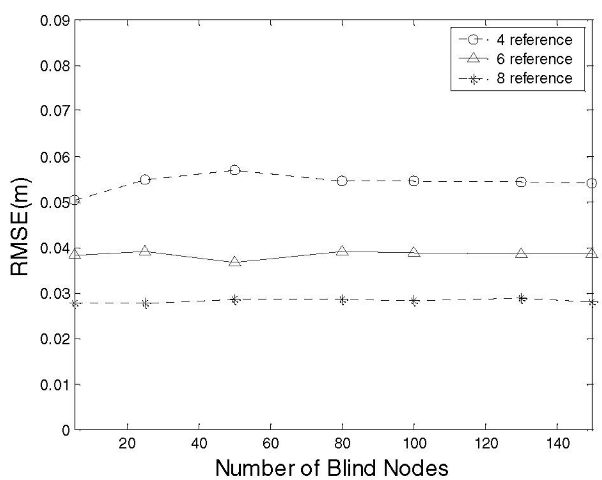


Fig.8 LOS: RMSE versus number of blind nodes based on different number of reference nodes.

Fig. 7 compares the RMSE of error minimizing method with CRLB values. The positioning error is very close to CRLB using TDOA error-minimizing. In Fig. 7, the CRLB values are the lowest values for a given number of reference nodes. Fig. 8 illustrates the relationship between the RMSE and the number of blind nodes. It is observed that RMSE almost keeps constant, and is independent of the blind node number for the same number of reference nodes. Therefore we can obtain the positions of a group of blind nodes simultaneously using error-minimizing method without reducing the positioning accuracy.

b. TDOA Error-minimizing Localization vs. Chan's Method

Chan's method takes advantage of redundant measurements and can achieve high accuracy. It rearranges a set of nonlinear equations that are formed based on TDOA measurements, produces an approximate of maximum likelihood estimator, and gives a closed-form, non-iterative solution. It consists of two Least Squares steps: initial position estimate of the blind node (*x', y'*) and R_1' are solved from Eq. (12) using Least Squares. The *x', y'* and R_1' are then substituted into Eq. (13). Final position estimate (*x, y*) can be obtained by solving Eq. (13) [12].

$$0.5\begin{bmatrix} {R_{2,1}}^2 - K_2 + K_1 \\ {R_{3,1}}^2 - K_3 + K_1 \\ \cdots \\ {R_{r,1}}^2 - K_r + K_1 \end{bmatrix} = \begin{bmatrix} x_{2,1} & y_{2,1} & R_{2,1} \\ x_{3,1} & y_{3,1} & R_{3,1} \\ . & . & . \\ x_{r,1} & y_{r,1} & R_{r,1} \end{bmatrix} \begin{bmatrix} x' \\ y' \\ R_1' \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} (x'-x_1)^2 \\ (y'-y_1)^2 \\ R_1'^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} (x-x_1)^2 \\ (y-y_1)^2 \end{bmatrix} \tag{13}$$

where

$$K_i = x_i^2 + y_i^2, \quad x_{i,1} = x_i - x_1, \quad y_{i,1} = y_i - y_1$$

$$R_i = \sqrt{(x_i - x)^2 + (y_i - y)^2}, \quad R_{m,n} = R_m - R_n$$

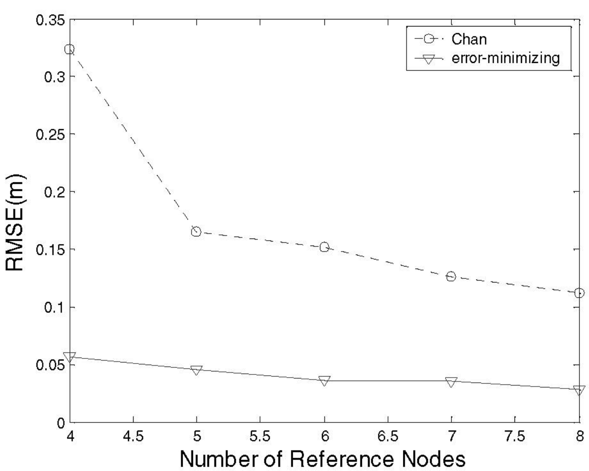
We compare TDOA error-minimizing localization method with Chan's method for both LOS and NLOS cases. 50 blind nodes are placed in the 50mx50m rectangular area, and the number of reference nodes is increased from 4 to 8. As shown in Fig.9, given the same number of reference nodes, TDOA error-minimizing localization method can achieve better accuracy compared to Chan's method. For NLOS, locations of up to 16 nodes cannot be determined with four reference nodes using Chan's method (RMSE>5m), while error-minimizing method can estimate all the blind node locations with RMSE less than 0.9m. Therefore, we can conclude that error-minimizing method is more robust to ranging errors compared to Chan's method.

IV. Conclusion

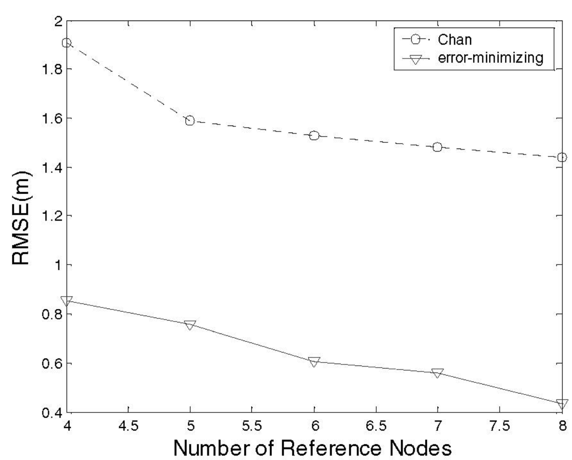
The problem of location positioning using UWB ranging measurements was addressed in the paper. We modeled the measured ranges for both LOS and NLOS propagations, and investigated the effect of range errors on the positioning estimation accuracy. The positions of the blind nodes were estimated by TDOA error-minimizing method. The simulation results demonstrated that the positions of a group of blind nodes can be estimated simultaneously without reducing the positioning accuracy. The Cramer-Rao Lower Bound for TDOA was also derived for LOS case, and it was observed that accuracy of TDOA error-minimizing method is close to the derived CRLB value. It was also found that TDOA error-minimizing method performed more efficiently and accurately than Chan's method for large range errors.

ACKNOWLEDGEMENT

The authors would like to thank Mr. Xu Chi (a PhD Student in PWTC center) for providing the measurement data and ASTAR UWB-SC TSRP for partial funding this project.



(a) LOS



(b) NLOS

Fig.9 TDOA Error-minimizing Localization vs. Chan's Method

References

[1] Barry Brumitt, John Krumm, B. Meyers, and S. Shafer, "Ubiquitous computing and the role of geometry," *In Special Issue on Smart Spaces and Environments*, volume 7-5, pages 41-43. IEEE Personal Communications, October 2000.

[2] D. Estrin, D. Culler, K. Pister and G. Sukhatme, "Connecting the physical world with pervasive networks," *IEEE Pervasive Computing*, Volume 1, Issue 1, Page(s): 59 – 69, Jan.-March 2002

[3] F.Zhao, J. Liu, J.J.Liu, L.Guibas, and J.Reich, "Collaborative signal and information processing: an information directed approach," *Proc.IEEE*, 91(8): 1199-1209,2003

[4] Mike Dempsey, " Indoor Positioning Systems in Healthcare: A Basic Overview of Technologies," http://www.cimit.org/pubs/ips_in_healthcare.pdf

[5] Y.-B. Ko and N. H. Vaidya, "Location-aided routing (LAR) in mobile ad hoc networks," *Proc. ACM/IEEE 4th Annu. Int. Conf. Mobile Computing and Networking (MobiCom'98)*, pp. 66–75, 1998

[6] J. Hightower and G. Borriello, "Location systems for ubiquitous computing," Computer, Volume 34, Issue 8, Page(s): 57 – 66, Aug. 2001

[7] S. Gezici, Zhi Tian, G.B. Giannakis, H. Kobayashi, A. F. Molisch, H.V. Poor and Z. Sainoglu, "Localization via ultra-wideband radios: a look at positioning aspects for future sensor networks," *IEEE Signal Processing Magazine*, Volume 22, Issue 4, Page(s):70 – 84, July 2005

[8] S.J.Ingram, D.Harmer and M.Quinlan, "UltraWideBand indoor positioning systems and their use in emergencies," *Position Location and Navigation Symposium*, Page(s):706 – 715, 26-29 April 2004

[9] J.Y. Lee, "Ultra-Wideband Ranging in Dense Multipath environments," Ph.D. Dissertation, Dept. of Electrical Engineering, University of Southern California, May 2002

[10] W. C. Chung and D. S. Ha, "An Accurate Ultra Wideband (UWB) ranging for precision asset location," in *Proc. IEEE Conf. Ultra WidebandSystems and Technologies*, Nov. 2003

[11] B.Denis, J.Keignart and N.Daniele, "Impact of NLOS propagation upon ranging precision in UWB systems," *IEEE Conference on Ultra Wideband Systems and Technologies*, Page(s):379 – 383,16-19 Nov. 2003

[12] Y. Chan and K.C. Ho, "A simple and efficient estimator for hyperbolic location," *IEEE Transactions on Signal Processing*, Volume 42, Issue 8, Page(s):1905 - 1915 Aug. 1994

[13] N. Patwari, A. O. Hero, M.Perkins, N.S. Correal, and R.J.O'Dea, "Relative location estimation in wireless sensor networks," *IEEE Transactions on Signal Processing*, Volume 51, Issue 8, pp.2137 – 2148, Aug. 2003

[14] B. Alavi and K. Pahlavan, "Modeling of the distance error for indoor geolocation," IEEE *Wireless Communications and Networking*, Volume 1, Page(s):668 – 672, 16-20 March 2003

[15] B. Alavi and K. Pahlavan, "Modeling of the TOA-based distance measurement error using UWB indoor radio measurements," *IEEE Communications Letters*, Volume 10, Issue 4, Page(s):275 – 277, Apr 2006

[16] Ranging Subcommittee Final Report, www.ieee802.org /15/pub /04/15-04-0581-07-004a-ranging-subcommittee-final-report.doc

[17] Z. N. Low, J. H. Cheong and C. L. Law, "Novel low cost higher derivative Gaussian pulse generator circuit,", *Proc-ICCS2004*, Singapore, pp. 30-34, 2004

[18] W. H. Press, B. P. Flannery, S. A. Teukolsky and W.T. Vetterling, *Numerical recipes in C : the art of scientific computing*, Cambridge University Press

[19] Steven M. Kay, *Fundamentals of statistical signal processing: Estimation Theory*, Prentice Hall, Englewood Cliff, New Jersey, USA