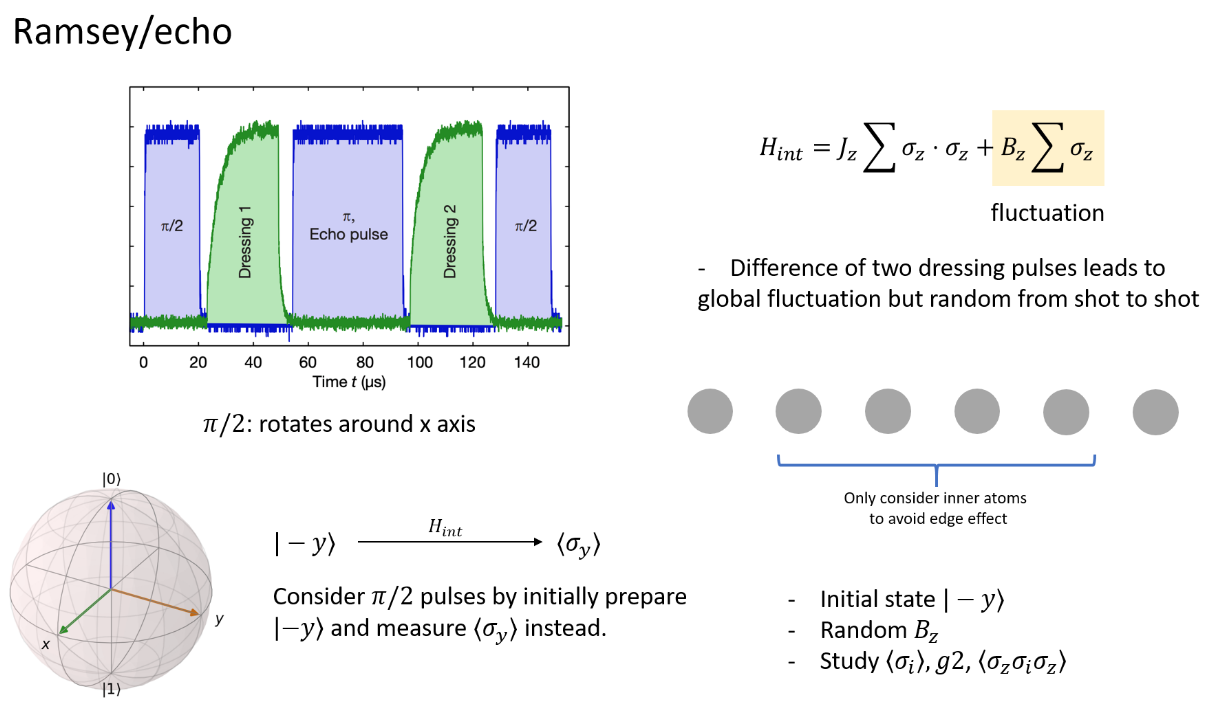
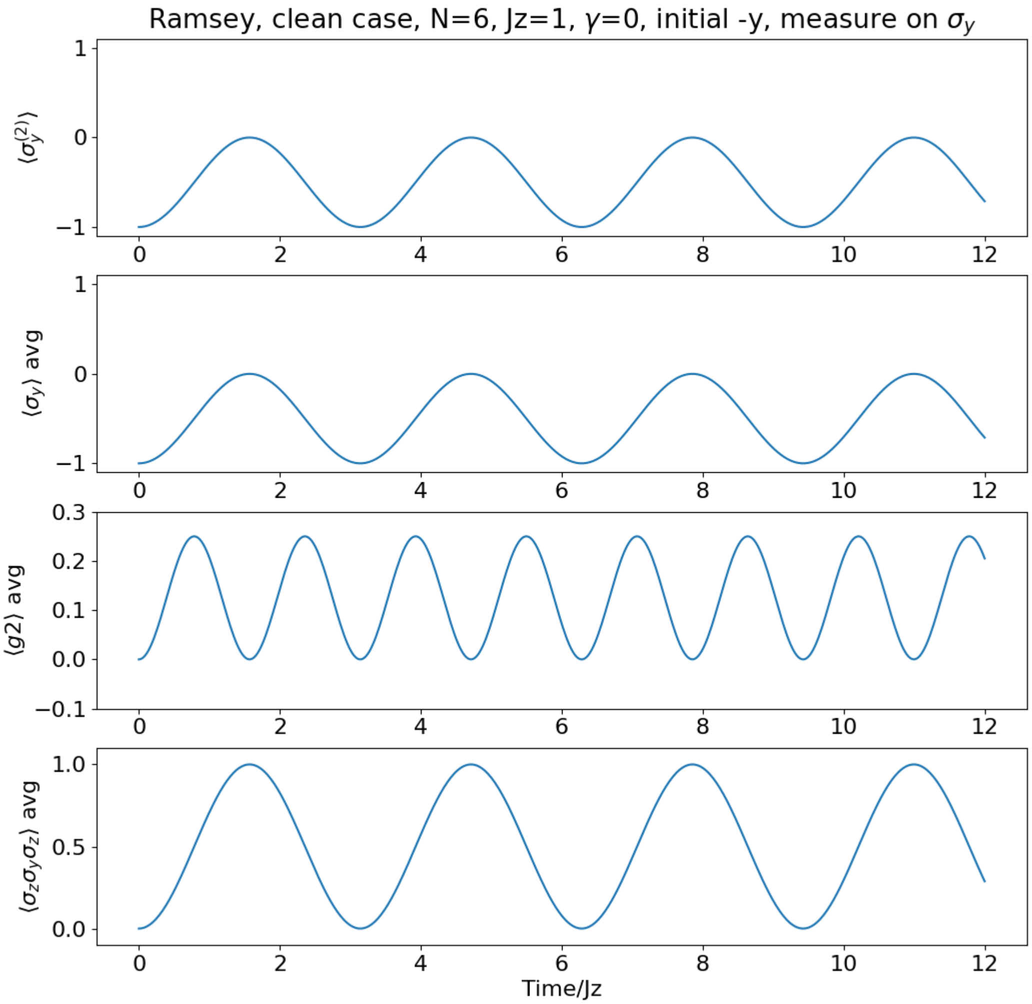
Results from Ising 1D chain simulation

> [20210316]
>
> Github



Clean case Ising interaction

- $H_{int} = J_z \sum_{<i,j>} \sigma_z \cdot \sigma_z$
- N = 6 particles and consider only inner particles to avoid edge effect.
- Initialy prepare atoms in $|-y\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$.
- Only $\langle\sigma_y\rangle$ shows the dynamic of H_{int}.
- $\langle\sigma_z\sigma_y\sigma_z\rangle$ is multibody correlation for Cluster state. It oscillates at the same period as magnitization.



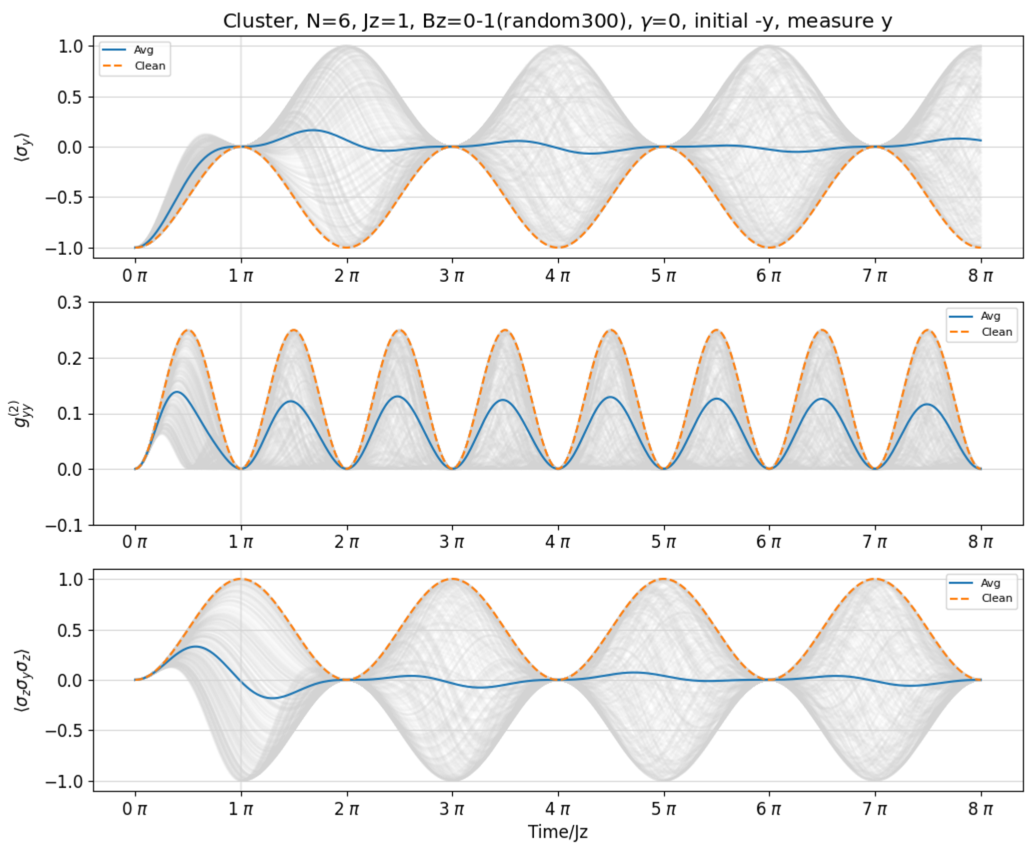
Put random fluctuation on Bz

- $H_{int} = J_z \sum_{<i,j>} \sigma_z \cdot \sigma_z + B_z\sigma_z$
- This is a global fluctuation phase but different between shot to shot leads to random vector on equatorial plane of bloch sphere.
- B_z is random between 0 to $0.1J_z, 1J_z, 10J_z$.
- Both $\langle\sigma_x\rangle$ and $\langle\sigma_y\rangle$ have a correlation dynamic but $\langle\sigma_z\rangle$ is not.
- Looking from measurement on both $\langle\sigma_x\rangle$ and $\langle\sigma_y\rangle$, the random phase makes the $\sigma_z \cdot \sigma_z$ interaction term cross-talk to both x and y basises. This also consistance with the decreasing of g2 on $\langle\sigma_y\rangle$ and appears on $\langle\sigma_x\rangle$.

1. $\langle\sigma_y\rangle$ measurement

- Random phase makes total magnitization $\sum_i \langle\sigma_y^i\rangle$ go to zero.

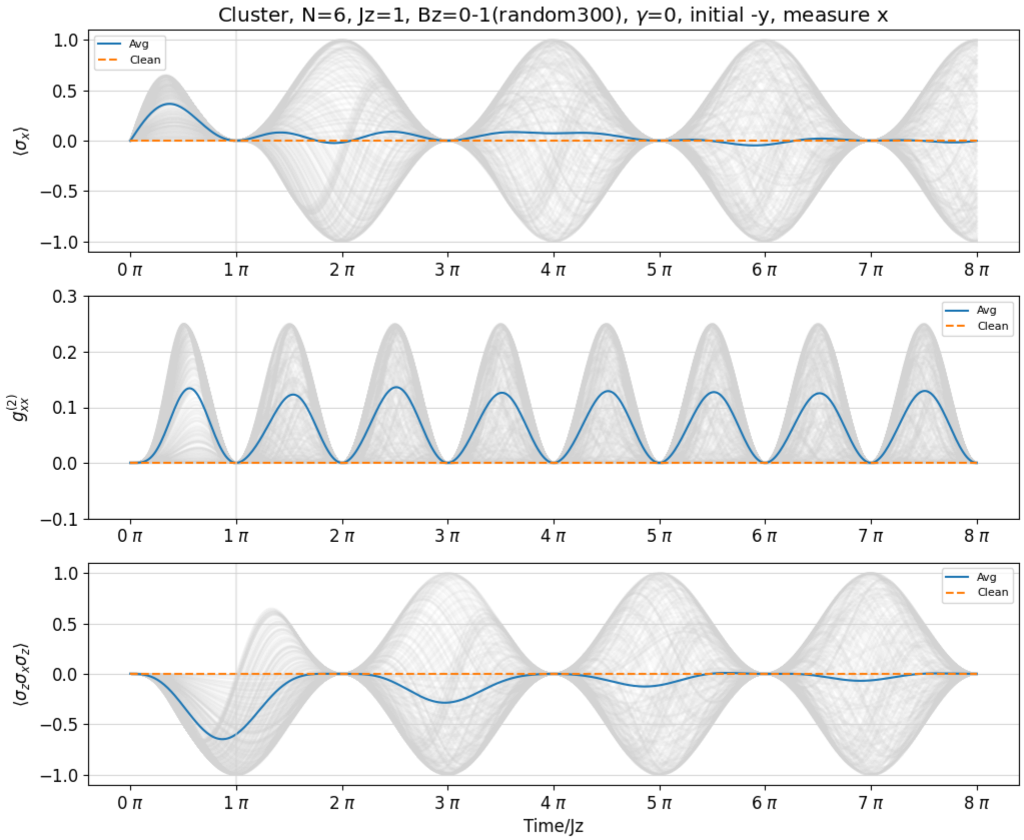
- The g2 correlation due to $\sigma_z \cdot \sigma_z$ still survives but the mplitude reduces by factor of 0.5 and stays constant at fully random phase.
- $\langle\sigma_y^i\rangle$ plot has a signature loops where the envelope is governed by clean case (No B_z fluctuation). The minimum uncertainty are presented at the same position as zero magnitization for clean case (Can we use this to clame for entanglement state of the system in random phase case?).
- Multibody correlation is also disappear when the is fluctuation.
- Compare with Random phase where $J_z = 0$, $\langle\sigma_y^i\rangle$ plot shows fully uncertainty.



** gray lines show global dynamic where B_z different from shot to shot.

2. $\langle\sigma_x\rangle$ measurement

- Random phase makes total magnitization $\sum_i\langle\sigma_x^i\rangle$ stay at zero.
- The g2 correlation of $\langle\sigma_x\rangle$ is zero for clean case. In contrust, the g2 correlation appears and increases to 0.13(50% of clean case in $\langle\sigma_y\rangle$ measurement) on random phase.
- Multibody correlation is also disappear when the is fluctuation.
- The $\langle\sigma_x^i\rangle$ plot also shows the same signature loops as measured in $\langle\sigma_y^i\rangle$.



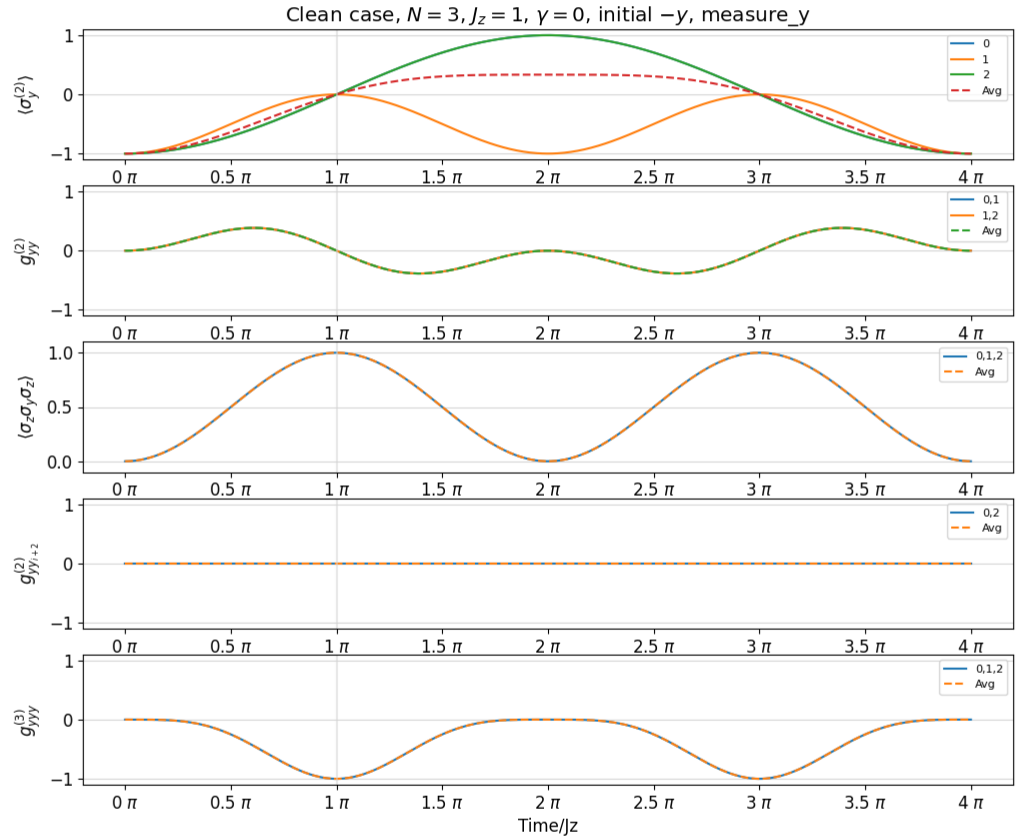
** gray lines show global dynamic where B_z different from shot to shot.

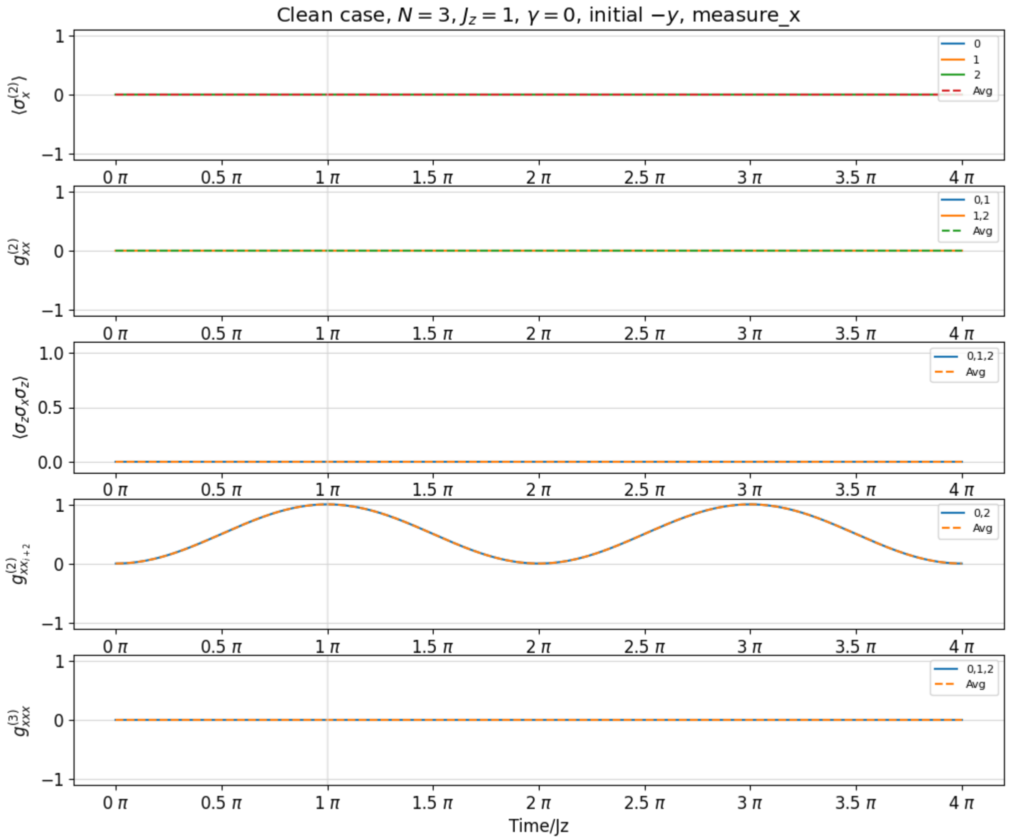
Correlation

We looking more in the details of correlation depending on the edge effect. Correlation $K = \langle \sigma_z^i \sigma^{i+1} \sigma_z^{i+2} \rangle$, $g^{(2)}_{i,i+1}$, $g^{(2)}_{i,i+2}$, $g^{(3)}_{i,i+1,i+2}$ on basis x, y are compared with $N = 3$ and $N = 5$ atoms.

N=3

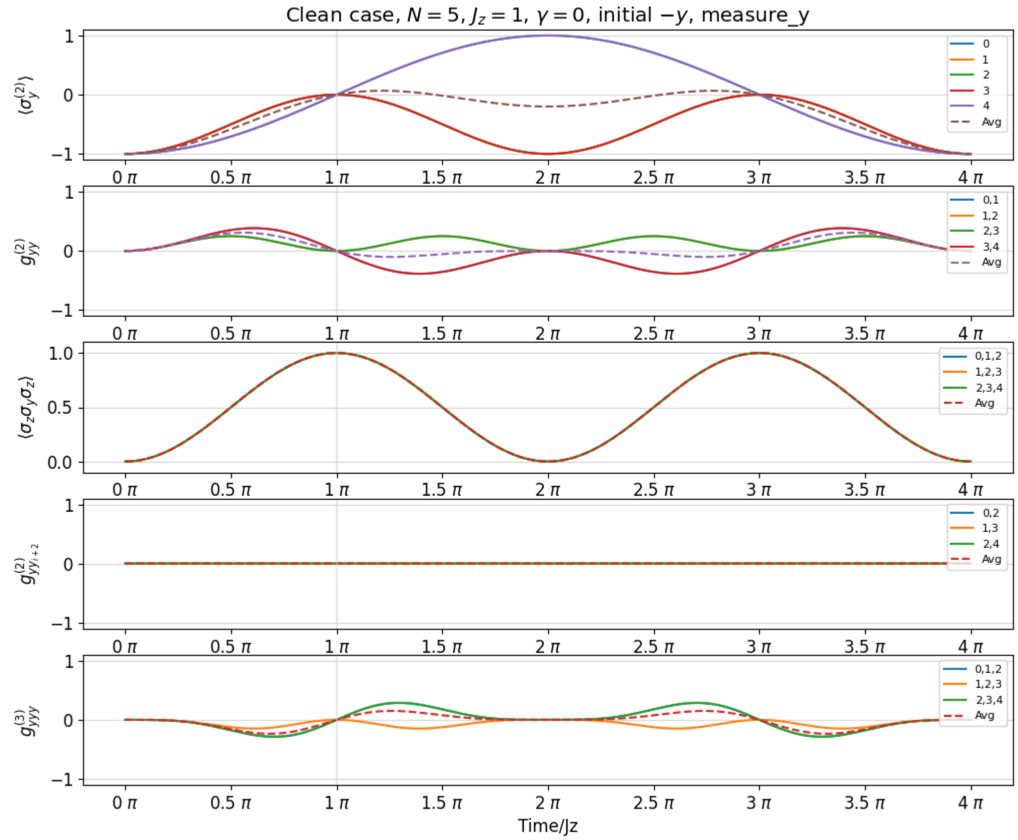
- All correlation on z are zero.
- Correlation on x are zero except $g^{(2)}_{x_i,x_{i+2}}$.
- $g^{(2)}_{y_i,y_{i+2}}$ (atoms at the edge) is zero but $g^{(2)}_{x_i,x_{i+2}}$ go from 0 to 1 at $T_c = \pi/J_z$.
- $g^{(3)}_{y_i,y_{i+1},y_{i+2}}$ (all atoms) go from 0 to -1 at $T_c/J_z = \pi$.

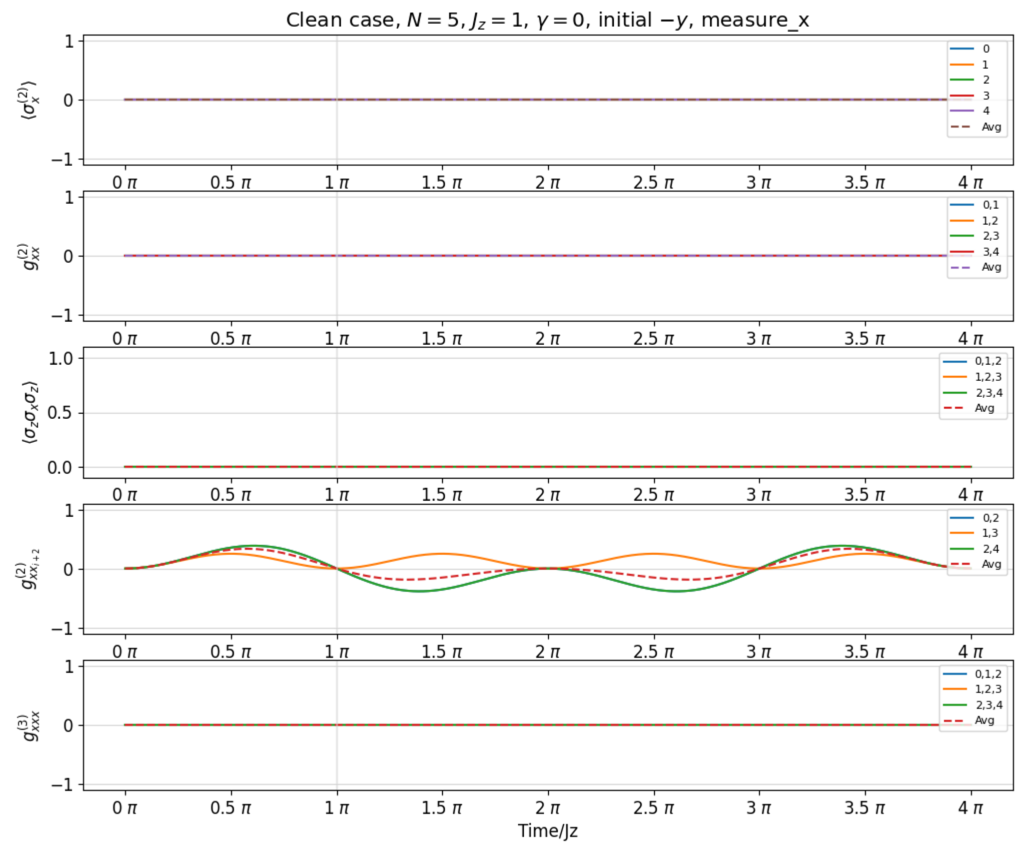

Clean case, $N = 3, J_z = 1, \gamma = 0$, initial $-y$, measure_y
$\langle\sigma_y^{(2)}\rangle$
0
1
2
Avg
$g_{yy}^{(2)}$
0,1
1,2
Avg
$\langle\sigma_z\sigma_y\sigma_z\rangle$
0,1,2
Avg
$g_{yy_{i+2}}^{(2)}$
0,2
Avg
$g_{yyy}^{(3)}$
0,1,2
Avg
0 π
0.5 π
1 π
1.5 π
2 π
2.5 π
3 π
3.5 π
4 π
Time/Jz



N=5

- When N=5, $g^{(3)}_{y_i,y_{i+1},y_{i+2}}$ (all atoms) go from 0 to -0.25 at $t = t_c/2 = \pi/2J_z$ and back to zero at $t = t_c = \pi/J_z$
- $g^{(2)}_{2_i,2_{i+1}}$ go from 0 to 0.25 at $t = t_c/2 = \pi/2J_z$ and back to zero at $t = t_c = \pi/J_z$


Clean case, $N = 5$, $J_z = 1$, $\gamma = 0$, initial $-y$, measure_y
$\langle\sigma_y^{(2)}\rangle$
$g_{yy}^{(2)}$
$\langle\sigma_z\sigma_y\sigma_z\rangle$
$g_{yy_{i+2}}^{(2)}$
$g_{yyy}^{(3)}$
0
1
2
3
4
Avg
0,1
1,2
2,3
3,4
0,1,2
1,2,3
2,3,4
0,2
1,3
2,4
0 π
0.5 π
1 π
1.5 π
2 π
2.5 π
3 π
3.5 π
4 π
Time/Jz



Pair state at t_c

We check states and entanglement of middle pair atoms depending on neighboring pair basis projection.

Middle pair states

All entanglement states of the middle pair are just the super position of Bell's states with combination of phase as showed in the table.

- This table shows N=4 atoms and project atom 0 and 3 to $yy, y(-y), (-y)y, (-y)(-y)$. The middle pair states at $t=t_c$ are super position of $|00\rangle, |01\rangle, |10\rangle, |11\rangle$ with the same amplitude 0.5.

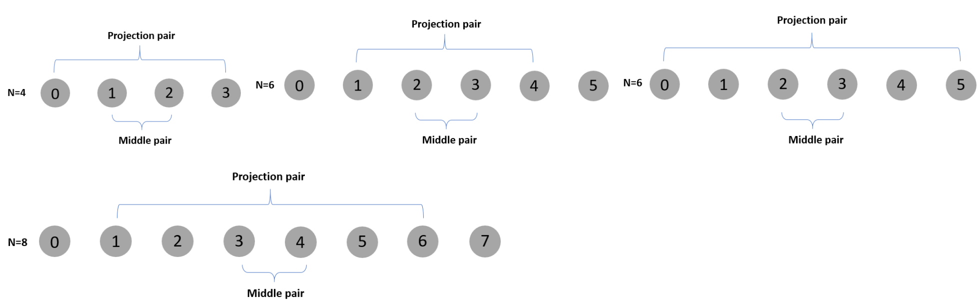
Projects (1,4) ($\rho_{init=-y}$)	Projects (1,4) ($\rho_{init=+y}$)	State of middle pair ($\rho^{3,4}$)	Expand	Concurrent C
$Y^{(1)}\rho Y^{(4)}$	$Y^{(1)}\rho Y^{(4)}$	$\frac{\Psi^- + \Phi^+}{\sqrt{2}}$	$0.5(\|00\rangle + \|01\rangle + \|10\rangle - \|11\rangle)$	1
$(-Y^{(1)})\rho(-Y^{(4)})$	$(-Y^{(1)})\rho(-Y^{(4)})$	$\frac{\Psi^- - \Phi^+}{\sqrt{2}}$	$0.5(\|00\rangle - \|01\rangle - \|10\rangle - \|11\rangle)$	1
$Y^{(1)}\rho(-Y^{(4)})$	$Y^{(1)}\rho(-Y^{(4)})$	$\frac{\Psi^- + \Phi^+}{\sqrt{2}}$	$0.5(\|00\rangle + \|01\rangle + \|10\rangle - \|11\rangle)$	1
$(-Y^{(1)})\rho Y^{(4)}$	$(-Y^{(1)})\rho Y^{(4)}$	$\frac{\Psi^- + \Phi^+}{\sqrt{2}}$	$0.5(\|00\rangle + \|01\rangle + \|10\rangle - \|11\rangle)$	1

- $Tr_{0,3}\{\rho\}$ => $\rho_{1,2}$ =>mix state
- $Tr_{0,3}\{|y_0\rangle\langle y_0|\rho|y_3\rangle\langle y_3|\}$ => $\rho^y_{1,2}$ => maximum entangle state
- $Tr_{0,3}\{|z_0\rangle\langle z_0|\rho|z_3\rangle\langle z_3|\}$ => $\rho^z_{1,2}$ => maximum entangle state
- $Tr_{0,3}\{|x_0\rangle\langle x_0|\rho|x_3\rangle\langle x_3|\}$ => $\rho^x_{1,2}$ => pure state

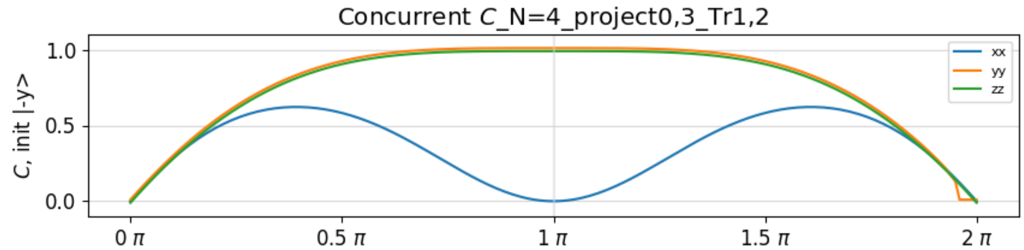
Entanglement

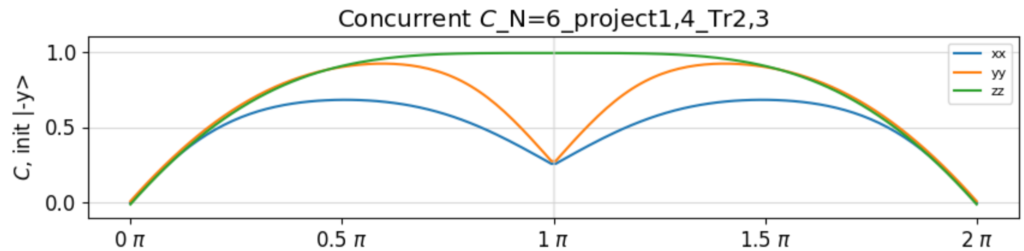
Concerrent C is used to certify entanglement of two particle. $C = 1$ is maximally entangled states while $C = 0$ is separable states. PRA 77 062330, Arxiv

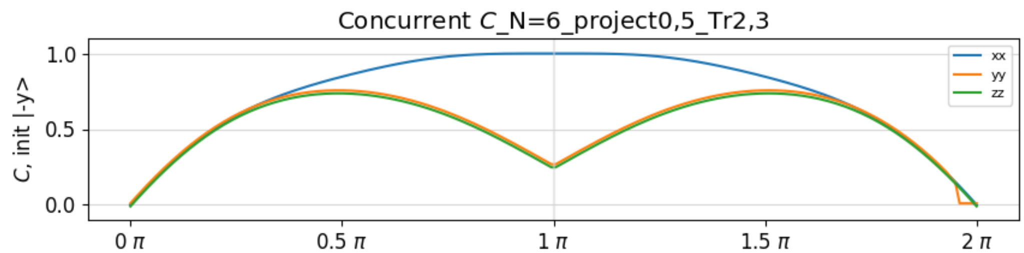
N=4,6,8 **(atom position is label from 0 to N-1)** are compared with different projection of neighbor/next neighbor pair. We start from initial state $|-y\rangle$ and let it evolves in $\sigma_z \cdot \sigma_z$ interaction. At $t = t_c$, spin i, j are project on $x/y/z$ and do a patial trace out leave only the middle pair state.

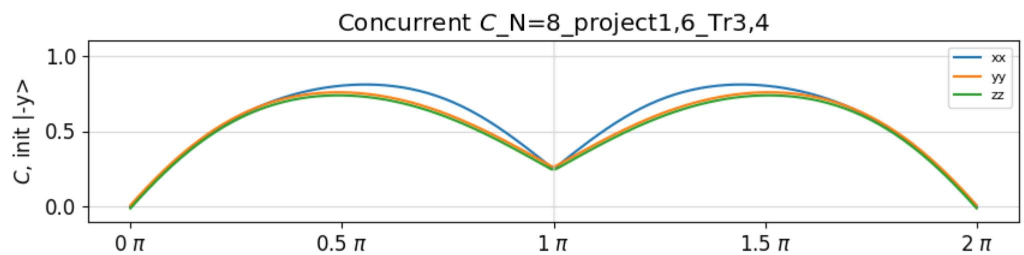


- N=4, projection on y_0y_3, z_0z_3 results an entanglement of middle pair atoms at $t = tc$ but not in x_0x_3.
- N=6, two additional atoms are add at the start and end of the chain. Only projection on z_1z_4 shows an entanglement.
- N=6, projection are shifted to next nearest neighbor atom (atoms at the edge). Only projection on x_0x_5 shows an entanglement.
- N=8, two additional atoms are add at the start and end of the chain. Any projection basis on (1,6) cannot lead to entanglement of middle pair atoms.


Concurrent C_N=4_project0,3_Tr1,2
C, init |-y>
1.0
0.5
0.0
0 π
0.5 π
1 π
1.5 π
2 π
xx
yy
zz


Concurrent C_N=6_project1,4_Tr2,3
C, init |-y>
1.0
0.5
0.0
0 π
0.5 π
1 π
1.5 π
2 π
xx
yy
zz


Concurrent C_N=6_project0,5_Tr2,3
C, init |-y>
1.0
0.5
0.0
0 π
0.5 π
1 π
1.5 π
2 π
xx
yy
zz


Concurrent C_N=8_project1,6_Tr3,4
C, init |-y>
1.0
0.5
0.0
0 π
0.5 π
1 π
1.5 π
2 π
xx
yy
zz