ROFORMER-ENHANCED TRANSFORMER WITH ROTARY POSITION EMBEDDING

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2.3 Relative position embedding

The authors of Shaw et al. [2018] applied different settings of Equation (1) as following:

$$
\begin{aligned}
f_q(\boldsymbol{x}_m) &:= \boldsymbol{W}_q(\boldsymbol{x}_m + p_m) \quad \text{— Query.} \quad \text{Query has posi} \\
f_k(\boldsymbol{x}_n, n) &:= \boldsymbol{W}_k(\boldsymbol{x}_n + \tilde{\boldsymbol{p}}_r^k) \\
f_v(\boldsymbol{x}_n, n) &:= \boldsymbol{W}_v(\boldsymbol{x}_n + \tilde{\boldsymbol{p}}_r^v)
\end{aligned} \tag{5}
$$

where $\tilde{\boldsymbol{p}}_r^k, \tilde{\boldsymbol{p}}_r^v \in \mathbb{R}^d$ are trainable relative position embeddings. Note that $r = \text{clip}(m - n, r_{\min}, r_{\max})$ represents the relative distance between position m and n. They clipped the relative distance with the hypothesis that precise relative position information is not useful beyond a certain distance. Keeping the form of Equation (3), the authors Dai et al. [2019] have proposed to decompose $\boldsymbol{q}_m^\intercal \boldsymbol{k}_n$ of Equation (2) as

$$f_k(Y_{n,r}) = \quad \boldsymbol{q}_m^\intercal \boldsymbol{k}_n = \boldsymbol{x}_m^\intercal \boldsymbol{W}_q^\intercal \boldsymbol{W}_k \boldsymbol{x}_n + \boldsymbol{x}_m^\intercal \boldsymbol{W}_q^\intercal \boldsymbol{W}_k \boldsymbol{p}_n + \boldsymbol{p}_m^\intercal \boldsymbol{W}_q^\intercal \boldsymbol{W}_k \boldsymbol{x}_n + \boldsymbol{p}_m^\intercal \boldsymbol{W}_q^\intercal \boldsymbol{W}_k \boldsymbol{p}_n, \tag{6}$$

the key idea is to replace the absolute position embedding $\boldsymbol{p}_n$ with its sinusoid-encoded relative counterpart $\tilde{\boldsymbol{p}}_{m-n}$, while the absolute position $\boldsymbol{p}_m$ in the third and fourth term with two trainable vectors $\mathbf{u}$ and $\mathbf{v}$ independent of the query positions. Further, $\boldsymbol{W}_k$ is distinguished for the content-based and location-based key vectors $\boldsymbol{x}_n$ and $\boldsymbol{p}_n$, denoted as $\boldsymbol{W}_k$ and $\widetilde{\boldsymbol{W}}_k$, resulting in:

$$\boldsymbol{q}_m^\intercal \boldsymbol{k}_n = \boldsymbol{x}_m^\intercal \boldsymbol{W}_q^\intercal \boldsymbol{W}_k \boldsymbol{x}_n + \boldsymbol{x}_m^\intercal \boldsymbol{W}_q^\intercal \widetilde{\boldsymbol{W}}_k \tilde{\boldsymbol{p}}_{m-n} + \mathbf{u}^\intercal \boldsymbol{W}_q^\intercal \boldsymbol{W}_k \boldsymbol{x}_n + \mathbf{v}^\intercal \boldsymbol{W}_q^\intercal \widetilde{\boldsymbol{W}}_k \tilde{\boldsymbol{p}}_{m-n} \tag{7}$$

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The authors of He et al. [2020] argued that the relative positions of two tokens could only be fully modeled using the middle two terms of Equation (6). As a consequence, the absolute position embeddings $\boldsymbol{p}_m$ and $\boldsymbol{p}_n$ were simply replaced with the relative position embeddings $\tilde{\boldsymbol{p}}_{m-n}$:

$$\boldsymbol{q}_m^\intercal \boldsymbol{k}_n = \boldsymbol{x}_m^\intercal \boldsymbol{W}_q^\intercal \boldsymbol{W}_k \boldsymbol{x}_n + \boldsymbol{x}_m^\intercal \boldsymbol{W}_q^\intercal \boldsymbol{W}_k \tilde{\boldsymbol{p}}_{m-n} + \tilde{\boldsymbol{p}}_{m-n}^\intercal \boldsymbol{W}_q^\intercal \boldsymbol{W}_k \boldsymbol{x}_n \tag{10}$$

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$$\langle f_q(\boldsymbol{x}_m, m), f_k(\boldsymbol{x}_n, n)\rangle = g(\boldsymbol{x}_m, \boldsymbol{x}_n, m - n). \tag{11}$$

The ultimate goal is to find an equivalent encoding mechanism to solve the functions $f_q(\boldsymbol{x}_m, m)$ and $f_k(\boldsymbol{x}_n, n)$ to conform the aforementioned relation.

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RoFormer

representation.

Q1 $\sqrt{d}$.,

$$
\begin{aligned}
a_{m,n} &= \frac{\exp\left(\frac{\boldsymbol{q}_m^\intercal \boldsymbol{k}_n}{\sqrt{d}}\right)}{\sum_{j=1}^{N} \exp\left(\frac{\boldsymbol{q}_m^\intercal \boldsymbol{k}_j}{\sqrt{d}}\right)} \\
\mathbf{o}_m &= \sum_{n=1}^{N} a_{m,n} \boldsymbol{v}_n \quad \Rightarrow ?\ \text{word embedding?}
\end{aligned} \tag{2}
$$

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$$
\begin{cases}
\boldsymbol{p}_{i,2t} &= \sin(k/10000^{2t/d}) \\
\boldsymbol{p}_{i,2t+1} &= \cos(k/10000^{2t/d})
\end{cases}
$$

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$e^{\theta I}$. $\quad e^{i\theta} = \cos\theta + i\sin\theta$.

$$
\begin{aligned}
f_q(\boldsymbol{x}_m, m) &= (\boldsymbol{W}_q \boldsymbol{x}_m) e^{im\theta} \\
f_k(\boldsymbol{x}_n, n) &= (\boldsymbol{W}_k \boldsymbol{x}_n) e^{in\theta} \\
g(\boldsymbol{x}_m, \boldsymbol{x}_n, m - n) &= \text{Re}[(\boldsymbol{W}_q \boldsymbol{x}_m)(\boldsymbol{W}_k \boldsymbol{x}_n)^* e^{i(m-n)\theta}]
\end{aligned} \tag{12}
$$

where $\text{Re}[\cdot]$ is the real part of a complex number and $(\boldsymbol{W}_k \boldsymbol{x}_n)^*$ represents the conjugate complex number of $(\boldsymbol{W}_k \boldsymbol{x}_n)$. $\theta \in \mathbb{R}$ is a preset non-zero constant. We can further write $f_{\{q,k\}}$ in a multiplication matrix:

$$f_{\{q,k\}}(\boldsymbol{x}_m, m) = \begin{pmatrix} \cos m\theta & -\sin m\theta \\ \sin m\theta & \cos m\theta \end{pmatrix} \begin{pmatrix} W_{\{q,k\}}^{(11)} & W_{\{q,k\}}^{(12)} \\ W_{\{q,k\}}^{(21)} & W_{\{q,k\}}^{(22)} \end{pmatrix} \begin{pmatrix} x_m^{(1)} \\ x_m^{(2)} \end{pmatrix} \tag{13}$$

$g \Rightarrow$. matrix. in 2D case

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$$f_{\{q,k\}}(\boldsymbol{x}_m, m) = \boldsymbol{R}_{\Theta,m}^d \boldsymbol{W}_{\{q,k\}} \boldsymbol{x}_m \tag{14}$$

where

$\begin{pmatrix} \cos n\theta_1 & \sin n\theta_1 \\ -\sin n\theta_1 & \cos n\theta_1 \end{pmatrix}$

$\cos(n-m)\theta_1$, $-\sin(n-m)\theta_1$
$\sin(n-m)\theta_1$, $\cos(n-m)\theta_1$

$$\boldsymbol{R}_{\Theta,m}^d = \begin{pmatrix}
\cos m\theta_1 & -\sin m\theta_1 & 0 & 0 & \cdots & 0 & 0 \\
\sin m\theta_1 & \cos m\theta_1 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cos m\theta_2 & -\sin m\theta_2 & \cdots & 0 & 0 \\
0 & 0 & \sin m\theta_2 & \cos m\theta_2 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & \cos m\theta_{d/2} & -\sin m\theta_{d/2} \\
0 & 0 & 0 & 0 & \cdots & \sin m\theta_{d/2} & \cos m\theta_{d/2}
\end{pmatrix} \tag{15}$$

is the rotary matrix with pre-defined parameters $\Theta = \{\theta_i = 10000^{-2(i-1)/d}, i \in [1, 2, ..., d/2]\}$. A graphic illustration of RoPE is shown in Figure (1). Applying our RoPE to self-attention in Equation (2), we obtain:

$$\boldsymbol{q}_m^\intercal \boldsymbol{k}_n = (\boldsymbol{R}_{\Theta,m}^d \boldsymbol{W}_q \boldsymbol{x}_m)^\intercal (\boldsymbol{R}_{\Theta,n}^d \boldsymbol{W}_k \boldsymbol{x}_n) = \boldsymbol{x}^\intercal \boldsymbol{W}_q R_{\Theta,n-m}^d \boldsymbol{W}_k \boldsymbol{x}_n \tag{16}$$

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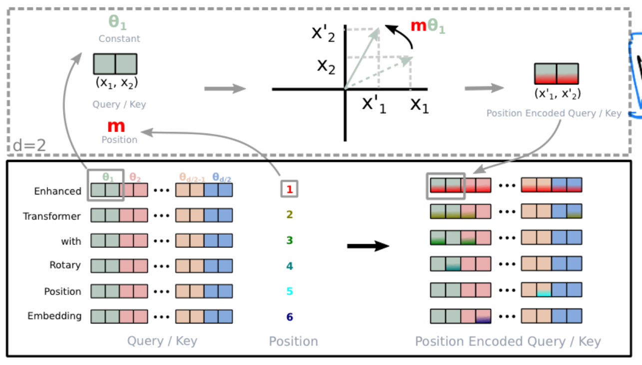
tion.



why?

Figure 1: Implementation of Rotary Position Embedding(RoPE).

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$$\boldsymbol{R}_{\Theta,m}^d \boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \vdots \\ x_{d-1} \\ x_d \end{pmatrix} \otimes \begin{pmatrix} \cos m\theta_1 \\ \cos m\theta_1 \\ \cos m\theta_2 \\ \cos m\theta_2 \\ \vdots \\ \cos m\theta_{d/2} \\ \cos m\theta_{d/2} \end{pmatrix} + \begin{pmatrix} -x_2 \\ x_1 \\ -x_4 \\ x_3 \\ \vdots \\ -x_{d-1} \\ x_d \end{pmatrix} \otimes \begin{pmatrix} \sin m\theta_1 \\ \sin m\theta_1 \\ \sin m\theta_2 \\ \sin m\theta_2 \\ \vdots \\ \sin m\theta_{d/2} \\ \sin m\theta_{d/2} \end{pmatrix}$$