Neural Network with Two Layers

Welcome to your week three programming assignment. You are ready to build a neural network with two layers and train it to solve a classification problem.

After this assignment, you will be able to:

- Implement a neural network with two layers to a classification problem
- Implement forward propagation using matrix multiplication
- Perform backward propagation

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Packages

First, import all the packages you will need during this assignment.

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import colors
```

```
# A function to create a dataset.
from sklearn.datasets import make_blobs

# Output of plotting commands is displayed inline within the Jupyter 
notebook.
%matplotlib inline

# Set a seed so that the results are consistent.
np.random.seed(3)
```

Load the unit tests defined for this notebook.

```
import w3_unittest
```

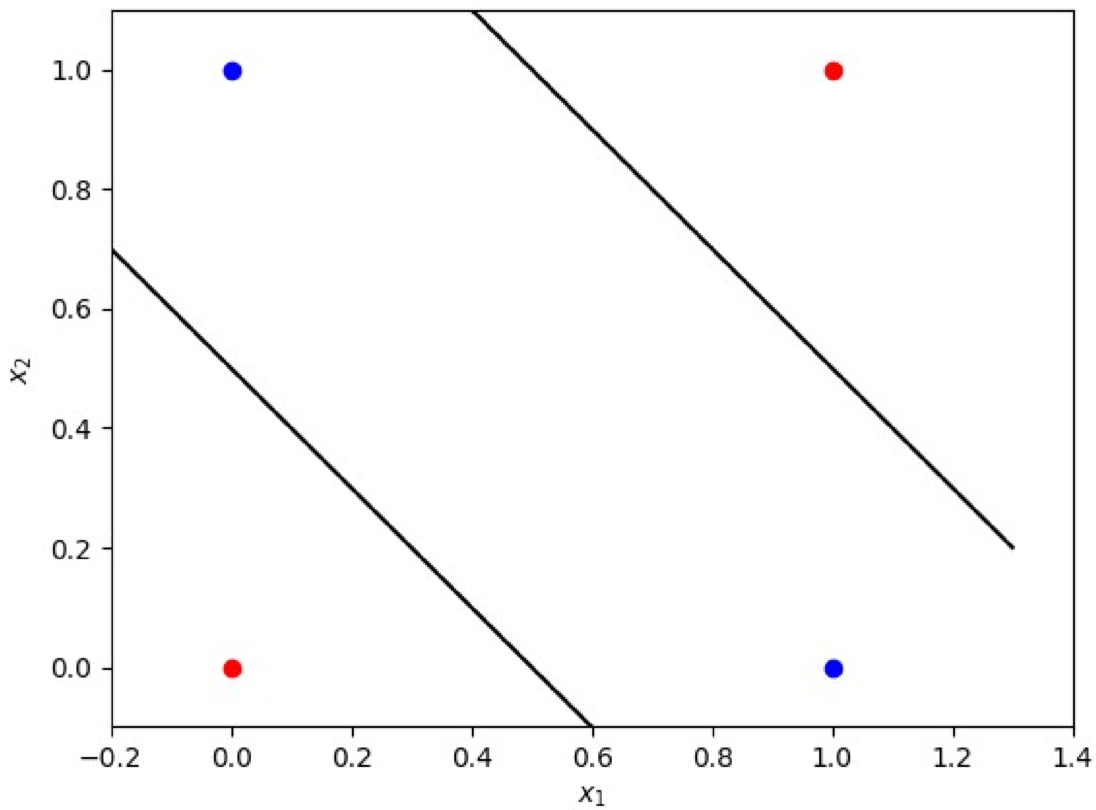
1 - Classification Problem

In one of the labs this week, you trained a neural network with a single perceptron, performing forward and backward propagation. That simple structure was enough to solve a "linear" classification problem - finding a straight line in a plane that would serve as a decision boundary to separate two classes.

Imagine that now you have a more complicated problem: you still have two classes, but one line will not be enough to separate them.

```
fig, ax = plt.subplots()
xmin, xmax = -0.2, 1.4
x_line = np.arange(xmin, xmax, 0.1)
# Data points (observations) from two classes.
ax.scatter(0, 0, color="r")
ax.scatter(0, 1, color="b")
ax.scatter(1, 0, color="b")
ax.scatter(1, 1, color="r")
ax.set_xlim([xmin, xmax])
ax.set_ylim([-0.1, 1.1])
ax.set_xlabel('$x_1$')
ax.set_ylabel('$x_2$')
# Example of the lines which can be used as a decision boundary to 
separate two classes.
ax.plot(x_line, -1 * x_line + 1.5, color="black")
ax.plot(x_line, -1 * x_line + 0.5, color="black")
plt.plot()

[]
```



This logic can appear in many applications. For example, if you train a model to predict whether you should buy a house knowing its size and the year it was built. A big new house will not be affordable, while a small old house will not be worth buying. So, you might be interested in either a big old house, or a small new house.

The one perceptron neural network is not enough to solve such classification problem. Let's look at how you can adjust that model to find the solution.

In the plot above, two lines can serve as a decision boundary. Your intuition might tell you that you should also increase the number of perceptrons. And that is absolutely right! You need to feed your data points (coordinates x_1, x_2) into two nodes separately and then unify them somehow with another one to make a decision.

Now let's figure out the details, build and train your first multi-layer neural network!

2 - Neural Network Model with Two Layers

2.1 - Neural Network Model with Two Layers for a Single Training Example

The input and output layers of the neural network are the same as for one perceptron model, but there is a **hidden layer** now in between them. The training examples $x^{(i)}=\begin{bmatrix} x_1^{(i)} \\ x_2^{(i)} \end{bmatrix}$ from the input layer of size $n_x=2$ are first fed into the hidden layer of size $n_h=2$. They are simultaneously fed into the first perceptron with weights $W_1^{[1]}=\begin{bmatrix} w_{1,1}^{[1]} & w_{2,1}^{[1]} \end{bmatrix}$, bias $b_1^{[1]}$; and into the second perceptron with weights $W_2^{[1]}=\begin{bmatrix} w_{1,2}^{[1]} & w_{2,2}^{[1]} \end{bmatrix}$, bias $b_2^{[1]}$. The integer in the square brackets $\square^{[1]}$ denotes the layer number, because there are two layers now with their own parameters and outputs, which need to be distinguished.

\begin{align} z_1^{1} &= w_{1,1}^{[1]} x_1^{(i)} + w_{2,1}^{[1]} x_2^{(i)} + b_1^{[1]} = W_1^{[1]}x^{(i)} + b_1^{[1]},\ z_2^{1} &= w_{1,2}^{[1]} x_1^{(i)} + w_{2,2}^{[1]} x_2^{(i)} + b_2^{[1]} = W_2^{[1]}x^{(i)} + b_2^{[1]}.\tag{1} \end{align}

These expressions for one training example $x^{(i)}$ can be rewritten in a matrix form :

$$z^{[1](i)}=W^{[1]}x^{(i)}+b^{[1]},$$

where

$z^{[1](i)}=\begin{bmatrix} z_1^{[1](i)} \\ z_2^{[1](i)} \end{bmatrix}$ is vector of size $(n_h\times 1)=(2\times 1)$;

$W^{[1]}=\begin{bmatrix} W_1^{[1]} \\ W_2^{[1]} \end{bmatrix}=\begin{bmatrix} w_{1,1}^{[1]} & w_{2,1}^{[1]} \\ w_{1,2}^{[1]} & w_{2,2}^{[1]} \end{bmatrix}$ is matrix of size $(n_h\times n_x)=(2\times 2)$;

$b^{[1]}=\begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \end{bmatrix}$ is vector of size $(n_h\times 1)=(2\times 1)$.

Next, the hidden layer activation function needs to be applied for each of the elements in the vector $z^{[1](i)}$. Various activation functions can be used here and in this model you will take the sigmoid function $\sigma(x)=\frac{1}{1+e^{-x}}$. Remember that its derivative is $\frac{d\sigma}{dx}=\sigma(x)(1-\sigma(x))$. The output of the hidden layer is a vector of size $(n_h\times 1)=(2\times 1)$:

$$a^{[1](i)}=\sigma\left(z^{[1](i)}\right)=\begin{bmatrix} \sigma\left(z_1^{[1](i)}\right) \\ \sigma\left(z_2^{[1](i)}\right) \end{bmatrix}.$$

Then the hidden layer output gets fed into the output layer of size $n_y=1$. This was covered in the previous lab, the only difference are: $a^{[1](i)}$ is taken instead of $x^{(i)}$ and layer notation $\square^{[2]}$ appears to identify all parameters and outputs:

$$z^{[2](i)}=w_1^{[2]}a_1^{[1](i)}+w_2^{[2]}a_2^{[1](i)}+b^{[2]}=W^{[2]}a^{[1](i)}+b^{[2]},$$

$z^{[2](i)}$ and $b^{[2]}$ are scalars for this model, as $(n_y\times 1)=(1\times 1)$;

$W^{[2]} = \begin{bmatrix} w_1^{[2]} & w_2^{[2]} \end{bmatrix}$ is vector of size $\left(n_y \times n_h\right) = \left(1 \times 2\right)$.

Finally, the same sigmoid function is used as the output layer activation function:

$$a^{[2](i)} = \sigma\left(z^{[2](i)}\right).$$

Mathematically the two layer neural network model for each training example $x^{(i)}$ can be written with the expressions $\left(2\right) - \left(5\right)$. Let's rewrite them next to each other for convenience:

\begin{align} z^{1} &= W^{[1]} x^{(i)} + b^{[1]},\ a^{1} &= \sigma\left(z^{1}\right),\ z^{2} &= W^{[2]} a^{1} + b^{[2]},\ a^{2} &= \sigma\left(z^{2}\right).\ \tag{6} \end{align}

Note, that all of the parameters to be trained in the model are without $\square^{(i)}$ index - they are independent on the input data.

Finally, the predictions for some example $x^{(i)}$ can be made taking the output $a^{[2](i)}$ and calculating $\hat{y}$ as: $\hat{y} = \begin{cases} 1 & \text{if } a^{[2](i)} > 0.5, \\ 0 & \text{otherwise}. \end{cases}$.

2.2 - Neural Network Model with Two Layers for Multiple Training Examples

Similarly to the single perceptron model, m training examples can be organised in a matrix X of a shape ($2 \times m$), putting $x^{(i)}$ into columns. Then the model $\left(6\right)$ can be rewritten in terms of matrix multiplications:

\begin{align} Z^{[1]} &= W^{[1]} X + b^{[1]},\ A^{[1]} &= \sigma\left(Z^{[1]}\right),\ Z^{[2]} &= W^{[2]} A^{[1]} + b^{[2]},\ A^{[2]} &= \sigma\left(Z^{[2]}\right),\ \tag{7} \end{align}

where $b^{[1]}$ is broadcasted to the matrix of size $\left(n_h \times m\right) = \left(2 \times m\right)$ and $b^{[2]}$ to the vector of size $\left(n_y \times m\right) = \left(1 \times m\right)$. It would be a good exercise for you to have a look at the expressions $\left(7\right)$ and check that sizes of the matrices will actually match to perform required multiplications.

You have derived expressions to perform forward propagation. Time to evaluate your model and train it.

2.3 - Cost Function and Training

For the evaluation of this simple neural network you can use the same cost function as for the single perceptron case - log loss function. Originally initialized weights were just some random values, now you need to perform training of the model: find such set of parameters $W^{[1]}$, $b^{[1]}$, $W^{[2]}$, $b^{[2]}$, that will minimize the cost function.

Like in the previous example of a single perceptron neural network, the cost function can be written as:

$$\mathcal{L}\left(W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]}\right) = \frac{1}{m}\sum_{i=1}^{m} L\left(W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]}\right) = \frac{1}{m}\sum_{i=1}^{m} \large\left(\small -y^{(i)}\log\left(a^{[2](i)}\right) - (1-y^{(i)})\log\left(1- a^{[2](i)}\right) \large \right), \small\tag{8}$$

where $y^{(i)} \in \{0,1\}$ are the original labels and $a^{[2](i)}$ are the continuous output values of the forward propagation step (elements of array $A^{[2]}$).

To minimize it, you can use gradient descent, updating the parameters with the following expressions:

\begin{align} W^{[1]} &= W^{[1]} - \alpha \frac{\partial \mathcal{L} }{ \partial W^{[1]} },\ b^{[1]} &= b^{[1]} - \alpha \frac{\partial \mathcal{L} }{ \partial b^{[1]} },\ W^{[2]} &= W^{[2]} - \alpha \frac{\partial \mathcal{L} }{ \partial W^{[2]} },\ b^{[2]} &= b^{[2]} - \alpha \frac{\partial \mathcal{L} }{ \partial b^{[2]} },\ \tag{9} \end{align}

where α is the learning rate.

To perform training of the model you need to calculate now $\frac{\partial L}{\partial W^{[1]}}, \frac{\partial L}{\partial b^{[1]}}, \frac{\partial L}{\partial W^{[2]}}, \frac{\partial L}{\partial b^{[2]}}$.

Let's start from the end of the neural network. You can rewrite here the corresponding expressions for $\frac{\partial L}{\partial W}$ and $\frac{\partial L}{\partial b}$ from the single perceptron neural network:

\begin{align} \frac{\partial \mathcal{L} }{ \partial W } &= \frac{1}{m}\left(A-Y\right)X^T,\ \frac{\partial \mathcal{L} }{ \partial b } &= \frac{1}{m}\left(A-Y\right)\mathbf{1},\ \end{align}

where 1 is just a $(m \times 1)$ vector of ones. Your one perceptron is in the second layer now, so W will be exchanged with $W^{[2]}$, b with $b^{[2]}$, A with $A^{[2]}$, X with $A^{[1]}$:

\begin{align} \frac{\partial \mathcal{L} }{ \partial W^{[2]} } &= \frac{1}{m}\left(A^{[2]}-Y\right)\left(A^{[1]}\right)^T,\ \frac{\partial \mathcal{L} }{ \partial b^{[2]} } &= \frac{1}{m}\left(A^{[2]}-Y\right)\mathbf{1}.\ \tag{10} \end{align}

Let's now find $\frac{\partial L}{\partial W^{[1]}} = \begin{bmatrix} \frac{\partial L}{\partial w_{1,1}^{[1]}} & \frac{\partial L}{\partial w_{2,1}^{[1]}} \\ \frac{\partial L}{\partial w_{1,2}^{[1]}} & \frac{\partial L}{\partial w_{2,2}^{[1]}} \end{bmatrix}$. It was shown in the videos that

$$\frac{\partial L}{\partial w_{1,1}^{[1]}} = \frac{1}{m} \sum_{i=1}^{m} \left(\left(a^{[2](i)} - y^{(i)} \right) w_1^{[2]} \left(a_1^{[1](i)} \left(1 - a_1^{[1](i)} \right) \right) x_1^{(i)} \right)$$

If you do this accurately for each of the elements $\frac{\partial L}{\partial W^{[1]}}$, you will get the following matrix:

$$\frac{\partial L}{\partial W^{[1]}} = \begin{bmatrix} \frac{\partial L}{\partial w_{1,1}^{[1]}} & \frac{\partial L}{\partial w_{2,1}^{[1]}} \\ \frac{\partial L}{\partial w_{1,2}^{[1]}} & \frac{\partial L}{\partial w_{2,2}^{[1]}} \end{bmatrix}$$

$$¿\frac{1}{m} \begin{bmatrix} \sum_{i=1}^{m} \left(\left(a^{[2](i)} - y^{(i)} \right) w_1^{[2]} \left(a_1^{[1](i)} \left(1 - a_1^{[1](i)} \right) \right) x_1^{(i)} \right) & \sum_{i=1}^{m} \left(\left(a^{[2](i)} - y^{(i)} \right) w_1^{[2]} \left(a_1^{[1](i)} \left(1 - a_1^{[1](i)} \right) \right) x_2^{(i)} \right) \\ \sum_{i=1}^{m} \left(\left(a^{[2](i)} - y^{(i)} \right) w_2^{[2]} \left(a_2^{[1](i)} \left(1 - a_2^{[1](i)} \right) \right) x_1^{(i)} \right) & \sum_{i=1}^{m} \left(\left(a^{[2](i)} - y^{(i)} \right) w_2^{[2]} \left(a_2^{[1](i)} \left(1 - a_2^{[1](i)} \right) \right) x_2^{(i)} \right) \end{bmatrix}$$

Looking at this, you can notice that all terms and indices somehow are very consistent, so it all can be unified into a matrix form. And that's true! $\left(W^{[2]}\right)^T = \begin{bmatrix} w_1^{[2]} \\ w_2^{[2]} \end{bmatrix}$ of size $(n_h \times n_y) = (2 \times 1)$ can be multiplied with the vector $A^{[2]} - Y$ of size $(n_y \times m) = (1 \times m)$, resulting in a matrix of size $(n_h \times m) = (2 \times m)$:

$$\left(W^{[2]}\right)^T \left(A^{[2]} - Y\right) = \begin{bmatrix} w_1^{[2]} \\ w_2^{[2]} \end{bmatrix} \begin{bmatrix} \left(a^{[2](1)} - y^{(1)}\right) & \cdots & \left(a^{[2](m)} - y^{(m)}\right) \end{bmatrix} = \begin{bmatrix} \left(a^{[2](1)} - y^{(1)}\right) w_1^{[2]} & \cdots & \left(a^{[2](m)} - y^{(m)}\right) w_1^{[2]} \\ \left(a^{[2](1)} - y^{(1)}\right) w_2^{[2]} & \cdots & \left(a^{[2](m)} - y^{(m)}\right) w_2^{[2]} \end{bmatrix}$$

.

Now taking matrix $A^{[1]}$ of the same size $(n_h \times m) = (2 \times m)$,

$$A^{[1]} = \begin{bmatrix} a_1^{[1](1)} & \cdots & a_1^{[1](m)} \\ a_2^{[1](1)} & \cdots & a_2^{[1](m)} \end{bmatrix},$$

you can calculate:

$$A^{[1]} \cdot \left(1 - A^{[1]}\right) = \begin{bmatrix} a_1^{[1](1)} \left(1 - a_1^{[1](1)}\right) & \cdots & a_1^{[1](m)} \left(1 - a_1^{[1](m)}\right) \\ a_2^{[1](1)} \left(1 - a_2^{[1](1)}\right) & \cdots & a_2^{[1](m)} \left(1 - a_2^{[1](m)}\right) \end{bmatrix},$$

where "$\cdot$" denotes **element by element** multiplication.

With the element by element multiplication,

$$\left(W^{[2]}\right)^T \left(A^{[2]} - Y\right) \cdot \left(A^{[1]} \cdot \left(1 - A^{[1]}\right)\right) = \begin{bmatrix} \left(a^{[2](1)} - y^{(1)}\right) w_1^{[2]} \left(a_1^{[1](1)} \left(1 - a_1^{[1](1)}\right)\right) & \cdots & \left(a^{[2](m)} - y^{(m)}\right) w_1^{[2]} \left(a_1^{[1](m)} \left(1 - a_1^{[1](m)}\right)\right) \\ \left(a^{[2](1)} - y^{(1)}\right) w_2^{[2]} \left(a_2^{[1](1)} \left(1 - a_2^{[1](1)}\right)\right) & \cdots & \left(a^{[2](m)} - y^{(m)}\right) w_2^{[2]} \left(a_2^{[1](m)} \left(1 - a_2^{[1](m)}\right)\right) \end{bmatrix}.$$

If you perform matrix multiplication with X^T of size $(m \times n_x) = (m \times 2)$, you will get matrix of size $(n_h \times n_x) = (2 \times 2)$:

$$\left(\left(W^{[2]}\right)^T\left(A^{[2]}-Y\right)\cdot\left(A^{[1]}\cdot\left(1-A^{[1]}\right)\right)\right)X^T=\begin{bmatrix}\left(a^{[2](1)}-y^{(1)}\right)w_1^{[2]}\left(a_1^{[1](1)}\left(1-a_1^{[1](1)}\right)\right) & \cdots & \left(a^{[2](m)}-y^{(m)}\right)w_1^{[2]}\left(a_1^{[1](m)}\left(1-a_1^{[1](m)}\right)\right)\\ \left(a^{[2](1)}-y^{(1)}\right)w_2^{[2]}\left(a_2^{[1](1)}\left(1-a_2^{[1](1)}\right)\right) & \cdots & \left(a^{[2](m)}-y^{(m)}\right)w_2^{[2]}\left(a_2^{[1](m)}\left(1-a_2^{[1](m)}\right)\right)\end{bmatrix}\begin{bmatrix}x_1^{(1)}\\ \cdots\\ x_1^{(m)}\end{bmatrix}$$

$$¿\begin{bmatrix}\sum_{i=1}^{m}\left(\left(a^{[2](i)}-y^{(i)}\right)w_1^{[2]}\left(a_1^{[1](i)}\left(1-a_1^{[1](i)}\right)\right)x_1^{(i)}\right) & \sum_{i=1}^{m}\left(\left(a^{[2](i)}-y^{(i)}\right)w_1^{[2]}\left(a_1^{[1](i)}\left(1-a_1^{[1](i)}\right)\right)x_2^{(i)}\right)\\ \sum_{i=1}^{m}\left(\left(a^{[2](i)}-y^{(i)}\right)w_2^{[2]}\left(a_2^{[1](i)}\left(1-a_2^{[1](i)}\right)\right)x_1^{(i)}\right) & \sum_{i=1}^{m}\left(\left(a^{[2](i)}-y^{(i)}\right)w_2^{[2]}\left(a_2^{[1](i)}\left(1-a_2^{[1](i)}\right)\right)x_2^{(i)}\right)\end{bmatrix}$$

This is exactly like in the expression (12)! So, $\frac{\partial L}{\partial W^{[1]}}$ can be written as a mixture of multiplications:

$$\frac{\partial L}{\partial W^{[1]}}=\frac{1}{m}\left(\left(W^{[2]}\right)^T\left(A^{[2]}-Y\right)\cdot\left(A^{[1]}\cdot\left(1-A^{[1]}\right)\right)\right)X^T,$$

where "$\cdot$" denotes element by element multiplications.

Vector $\frac{\partial L}{\partial b^{[1]}}$ can be found very similarly, but the last terms in the chain rule will be equal to 1, i.e. $\frac{\partial z_1^{[1](i)}}{\partial b_1^{[1]}}=1$. Thus,

$$\frac{\partial L}{\partial b^{[1]}}=\frac{1}{m}\left(\left(W^{[2]}\right)^T\left(A^{[2]}-Y\right)\cdot\left(A^{[1]}\cdot\left(1-A^{[1]}\right)\right)\right)\mathbf{1},$$

where $\mathbf{1}$ is a $(m\times 1)$ vector of ones.

Expressions (10), (13) and (14) can be used for the parameters update (9) performing backward propagation:

\begin{align} \frac{\partial \mathcal{L} }{ \partial W^{[2]} } &= \frac{1}{m}\left(A^{[2]}-Y\right)\left(A^{[1]}\right)^T,\ \frac{\partial \mathcal{L} }{ \partial b^{[2]} } &= \frac{1}{m}\left(A^{[2]}-Y\right)\mathbf{1},\ \frac{\partial \mathcal{L} }{ \partial W^{[1]}} &= \frac{1}{m}\left(\left(W^{[2]}\right)^T \left(A^{[2]} - Y\right)\cdot \left(A^{[1]}\cdot\left(1-A^{[1]}\right)\right)\right)X^T,\ \ \frac{\partial \mathcal{L} }{ \partial b^{[1]}} &= \frac{1}{m}\left(\left(W^{[2]}\right)^T \left(A^{[2]} - Y\right)\cdot \left(A^{[1]}\cdot\left(1-A^{[1]}\right)\right)\right)\mathbf{1},\ \tag{15} \end{align}

where $\mathbf{1}$ is a $(m\times 1)$ vector of ones.

So, to understand deeply and properly how neural networks perform and get trained, **you do need knowledge of linear algebra and calculus joined together**! But do not worry! All together it is not that scary if you do it step by step accurately with understanding of maths.

Time to implement this all in the code!

2.2 - Dataset

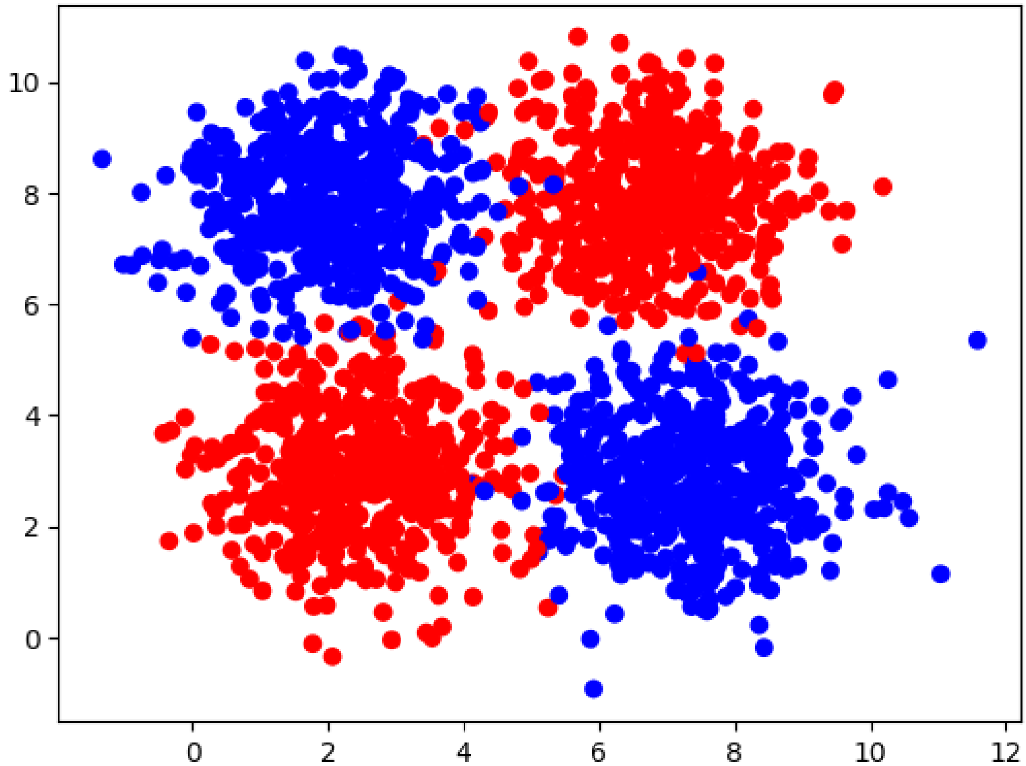
First, let's get the dataset you will work on. The following code will create $m=2000$ data points (x_1, x_2) and save them in the `NumPy` array `X` of a shape $(2 \times m)$ (in the columns of the array). The labels (0: blue, 1: red) will be saved in the `NumPy` array `Y` of a shape $(1 \times m)$.

```
m = 2000
samples, labels = make_blobs(n_samples=m,
                             centers=([2.5, 3], [6.7, 7.9], [2.1, 
7.9], [7.4, 2.8]),
                             cluster_std=1.1,
                             random_state=0)
labels[(labels == 0) | (labels == 1)] = 1
labels[(labels == 2) | (labels == 3)] = 0
X = np.transpose(samples)
Y = labels.reshape((1, m))

plt.scatter(X[0, :], X[1, :], c=Y, cmap=colors.ListedColormap(['blue', 
'red']));

print ('The shape of X is: ' + str(X.shape))
print ('The shape of Y is: ' + str(Y.shape))
print ('I have m = %d training examples!' % (m))
```

```
The shape of X is: (2, 2000)
The shape of Y is: (1, 2000)
I have m = 2000 training examples!
```



2.3 - Define Activation Function

Exercise 1

Define sigmoid activation function $\sigma\left(z\right) =\frac{1}{1+e^{-z}} $.

```
def sigmoid(z):
    ### START CODE HERE ### (~ 1 line of code)
    res = 1/(1 + np.exp(-z))
    ### END CODE HERE ###

    return res

print("sigmoid(-2) = " + str(sigmoid(-2)))
print("sigmoid(0) = " + str(sigmoid(0)))
print("sigmoid(3.5) = " + str(sigmoid(3.5)))
```

```
sigmoid(-2) = 0.11920292202211755
sigmoid(0) = 0.5
sigmoid(3.5) = 0.9706877692486436
```

Expected Output

Note: the values may vary in the last decimal places.

```
sigmoid(-2) = 0.11920292202211755
sigmoid(0) = 0.5
sigmoid(3.5) = 0.9706877692486436
```

```
w3_unittest.test_sigmoid(sigmoid)
```

```
 All tests passed
```

3 - Implementation of the Neural Network Model with Two Layers

3.1 - Defining the Neural Network Structure

Exercise 2

Define three variables:

- `n_x`: the size of the input layer
- `n_h`: the size of the hidden layer (set it equal to 2 for now)
- `n_y`: the size of the output layer

```
# GRADED FUNCTION: layer_sizes

def layer_sizes(X, Y):
    """
    Arguments:
    X -- input dataset of shape (input size, number of examples)
    Y -- labels of shape (output size, number of examples)

    Returns:
    n_x -- the size of the input layer
    n_h -- the size of the hidden layer
    n_y -- the size of the output layer
    """
    ### START CODE HERE ### (~ 3 lines of code)
    # Size of input layer.
    n_x = X.shape[0]
    # Size of hidden layer.
    n_h = 2
    # Size of output layer.
```

```
    n_y = Y.shape[0]
    ### END CODE HERE ###
    return (n_x, n_h, n_y)

(n_x, n_h, n_y) = layer_sizes(X, Y)
print("The size of the input layer is: n_x = " + str(n_x))
print("The size of the hidden layer is: n_h = " + str(n_h))
print("The size of the output layer is: n_y = " + str(n_y))

The size of the input layer is: n_x = 2
The size of the hidden layer is: n_h = 2
The size of the output layer is: n_y = 1
```

Expected Output

```
The size of the input layer is: n_x = 2
The size of the hidden layer is: n_h = 2
The size of the output layer is: n_y = 1

w3_unittest.test_layer_sizes(layer_sizes)

 All tests passed
```

3.2 - Initialize the Model's Parameters

Exercise 3

Implement the function `initialize_parameters()`.

Instructions:

- Make sure your parameters' sizes are right. Refer to the neural network figure above if needed.
- You will initialize the weights matrix with random values.
  - Use: `np.random.randn(a,b) * 0.01` to randomly initialize a matrix of shape (a,b).
- You will initialize the bias vector as zeros.
  - Use: `np.zeros((a,b))` to initialize a matrix of shape (a,b) with zeros.

```
# GRADED FUNCTION: initialize_parameters

def initialize_parameters(n_x, n_h, n_y):
    """
    Argument:
    n_x -- size of the input layer
    n_h -- size of the hidden layer
    n_y -- size of the output layer
```

```
    Returns:
    params -- python dictionary containing your parameters:
                    W1 -- weight matrix of shape (n_h, n_x)
                    b1 -- bias vector of shape (n_h, 1)
                    W2 -- weight matrix of shape (n_y, n_h)
                    b2 -- bias vector of shape (n_y, 1)
    """

    ### START CODE HERE ### (~ 4 lines of code)
    W1 = np.random.randn(n_h, n_x) * 0.01
    b1 = np.zeros((n_h, 1))
    W2 = np.random.randn(n_y, n_h) * 0.01
    b2 = np.zeros((n_y, 1))
    ### END CODE HERE ###

    assert (W1.shape == (n_h, n_x))
    assert (b1.shape == (n_h, 1))
    assert (W2.shape == (n_y, n_h))
    assert (b2.shape == (n_y, 1))

    parameters = {"W1": W1,
                  "b1": b1,
                  "W2": W2,
                  "b2": b2}

    return parameters

parameters = initialize_parameters(n_x, n_h, n_y)

print("W1 = " + str(parameters["W1"]))
print("b1 = " + str(parameters["b1"]))
print("W2 = " + str(parameters["W2"]))
print("b2 = " + str(parameters["b2"]))
```

```
W1 = [[ 0.01788628  0.0043651 ]
 [ 0.00096497 -0.01863493]]
b1 = [[0.]
 [0.]]
W2 = [[-0.00277388 -0.00354759]]
b2 = [[0.]]
```

Expected Output

Note: the elements of the arrays W1 and W2 maybe be different due to random initialization. You can try to restart the kernel to get the same values.

```
W1 = [[ 0.01788628  0.0043651 ]
 [ 0.00096497 -0.01863493]]
b1 = [[0.]
```

```
 [0.]]
W2 = [[-0.00277388 -0.00354759]]
b2 = [[0.]]

# Note:
# Actual values are not checked here in the unit tests (due to random initialization).
w3_unittest.test_initialize_parameters(initialize_parameters)
```

```
 All tests passed
```

3.3 - The Loop

Exercise 4

Implement `forward_propagation()`.

Instructions:

- Look above at the mathematical representation (7) of your classifier (section 2.2): \ begin{align} Z^{[1]} &= W^{[1]} X + b^{[1]},\ A^{[1]} &= \sigma\left(Z^{[1]}\right),\ Z^{[2]} &= W^{[2]} A^{[1]} + b^{[2]},\ A^{[2]} &= \sigma\left(Z^{[2]}\right).\ \end{align}
- The steps you have to implement are:
    a. Retrieve each parameter from the dictionary "parameters" (which is the output of `initialize_parameters()`) by using `parameters[".."]`.
    b. Implement Forward Propagation. Compute `Z1` multiplying matrices `W1`, `X` and adding vector `b1`. Then find `A1` using the `sigmoid` activation function. Perform similar computations for `Z2` and `A2`.

```
# GRADED FUNCTION: forward_propagation

def forward_propagation(X, parameters):
    """
    Argument:
    X -- input data of size (n_x, m)
    parameters -- python dictionary containing your parameters (output of initialization function)

    Returns:
    A2 -- the sigmoid output of the second activation
    cache -- python dictionary containing Z1, A1, Z2, A2
    (that simplifies the calculations in the back propagation step)
    """
    # Retrieve each parameter from the dictionary "parameters".
    ### START CODE HERE ### (~ 4 lines of code)
    W1 = parameters["W1"]
```

```
    b1 = parameters["b1"]
    W2 = parameters["W2"]
    b2 = parameters["b2"]
    ### END CODE HERE ###

    # Implement forward propagation to calculate A2.
    ### START CODE HERE ### (~ 4 lines of code)
    Z1 = np.matmul(W1, X) + b1
    A1 = sigmoid(Z1)
    Z2 = np.matmul(W2, A1) + b2
    A2 = sigmoid(Z2)
    ### END CODE HERE ###

    assert(A2.shape == (n_y, X.shape[1]))

    cache = {"Z1": Z1,
             "A1": A1,
             "Z2": Z2,
             "A2": A2}

    return A2, cache

A2, cache = forward_propagation(X, parameters)

print(A2)
```

```
[[0.49920157 0.49922234 0.49921223 ... 0.49921215 0.49921043
0.49920665]]
```

Expected Output

Note: the elements of the array A2 maybe be different depending on the initial parameters. If you would like to get exactly the same output, try to restart the Kernel and rerun the notebook.

```
[[0.49920157 0.49922234 0.49921223 ... 0.49921215 0.49921043
0.49920665]]

# Note:
# Actual values are not checked here in the unit tests (due to random
initialization).
w3_unittest.test_forward_propagation(forward_propagation)
```

```
 All tests passed
```

Remember, that your weights were just initialized with some random values, so the model has not been trained yet.

Exercise 5

Define a cost function (8) which will be used to train the model:

$$\mathcal{L}\left(W, b\right) = \frac{1}{m}\sum_{i=1}^{m} \large\left(\small - y^{(i)}\log\left(a^{(i)}\right) - (1-y^{(i)})\log\left(1- a^{(i)}\right) \large \right) \small.$$

```
def compute_cost(A2, Y):
    """
    Computes the cost function as a log loss

    Arguments:
    A2 -- The output of the neural network of shape (1, number of examples)
    Y -- "true" labels vector of shape (1, number of examples)

    Returns:
    cost -- log loss

    """
    # Number of examples.
    m = Y.shape[1]

    ### START CODE HERE ### (~ 2 lines of code)
    logloss = - np.multiply(np.log(A2), Y) - np.multiply(np.log(1 - A2), 1 - Y)
    cost = 1/m * np.sum(logloss)
    ### END CODE HERE ###

    assert(isinstance(cost, float))

    return cost

print("cost = " + str(compute_cost(A2, Y)))
```

```
cost = 0.6931477703826823
```

Expected Output

Note: the elements of the arrays W1 and W2 maybe be different!

```
cost = 0.6931477703826823
```

```
# Note:
# Actual values are not checked here in the unit tests (due to random initialization).
w3_unittest.test_compute_cost(compute_cost, A2)
```

```
 All tests passed
```

Calculate partial derivatives as shown in (15):

$$\begin{align} \frac{\partial \mathcal{L} }{ \partial W^{[2]} } &= \frac{1}{m}\left(A^{[2]}-Y\right)\left(A^{[1]}\right)^T,\ \frac{\partial \mathcal{L} }{ \partial b^{[2]} } &= \frac{1}{m}\left(A^{[2]}-Y\right)\mathbf{1},\ \frac{\partial \mathcal{L} }{ \partial W^{[1]}} &= \frac{1}{m}\left(\left(W^{[2]}\right)^T \left(A^{[2]} - Y\right)\cdot \left(A^{[1]}\cdot\left(1-A^{[1]}\right)\right)\right)X^T,\ \frac{\partial \mathcal{L} }{ \partial b^{[1]}} &= \frac{1}{m}\left(\left(W^{[2]}\right)^T \left(A^{[2]} - Y\right)\cdot \left(A^{[1]}\cdot\left(1-A^{[1]}\right)\right)\right)\mathbf{1}.\ \end{align}$$

```python
def backward_propagation(parameters, cache, X, Y):
    """
    Implements the backward propagation, calculating gradients

    Arguments:
    parameters -- python dictionary containing our parameters
    cache -- python dictionary containing Z1, A1, Z2, A2
    X -- input data of shape (n_x, number of examples)
    Y -- "true" labels vector of shape (n_y, number of examples)

    Returns:
    grads -- python dictionary containing gradients with respect to 
different parameters
    """
    m = X.shape[1]

    # First, retrieve W from the dictionary "parameters".
    W1 = parameters["W1"]
    W2 = parameters["W2"]

    # Retrieve also A1 and A2 from dictionary "cache".
    A1 = cache["A1"]
    A2 = cache["A2"]

    # Backward propagation: calculate partial derivatives denoted as 
dW1, db1, dW2, db2 for simplicity.
    dZ2 = A2 - Y
    dW2 = 1/m * np.dot(dZ2, A1.T)
    db2 = 1/m * np.sum(dZ2, axis = 1, keepdims = True)
    dZ1 = np.dot(W2.T, dZ2) * A1 * (1 - A1)
    dW1 = 1/m * np.dot(dZ1, X.T)
    db1 = 1/m * np.sum(dZ1, axis = 1, keepdims = True)

    grads = {"dW1": dW1,
             "db1": db1,
             "dW2": dW2,
             "db2": db2}

    return grads

grads = backward_propagation(parameters, cache, X, Y)

print("dW1 = " + str(grads["dW1"]))
```

```
print("db1 = " + str(grads["db1"]))
print("dW2 = " + str(grads["dW2"]))
print("db2 = " + str(grads["db2"]))
```

```
dW1 = [[-1.49856632e-05  1.67791519e-05]
 [-2.12394543e-05  2.43895135e-05]]
db1 = [[5.11207671e-07]
 [7.06236219e-07]]
dW2 = [[-0.00032641 -0.0002606 ]]
db2 = [[-0.00078732]]
```

Exercise 6

Implement `update_parameters()`.

Instructions:

- Update parameters as shown in (9) (section 2.3): \begin{align} W^{[1]} &= W^{[1]} - \alpha \frac{\partial \mathcal{L} }{ \partial W^{[1]} },\ b^{[1]} &= b^{[1]} - \alpha \frac{\partial \mathcal{L} }{ \partial b^{[1]} },\ W^{[2]} &= W^{[2]} - \alpha \frac{\partial \mathcal{L} }{ \partial W^{[2]} },\ b^{[2]} &= b^{[2]} - \alpha \frac{\partial \mathcal{L} }{ \partial b^{[2]} }.\ \end{align}
- The steps you have to implement are:
    a. Retrieve each parameter from the dictionary "parameters" (which is the output of `initialize_parameters()`) by using `parameters[".."]`.
    b. Retrieve each derivative from the dictionary "grads" (which is the output of `backward_propagation()`) by using `grads[".."]`.
    c. Update parameters.

```
def update_parameters(parameters, grads, learning_rate=1.2):
    """
    Updates parameters using the gradient descent update rule

    Arguments:
    parameters -- python dictionary containing parameters
    grads -- python dictionary containing gradients
    learning_rate -- learning rate for gradient descent

    Returns:
    parameters -- python dictionary containing updated parameters
    """
    # Retrieve each parameter from the dictionary "parameters".
    ### START CODE HERE ### (~ 4 lines of code)
    W1 = parameters["W1"]
    b1 = parameters["b1"]
    W2 = parameters["W2"]
    b2 = parameters["b2"]
```

```
    ### END CODE HERE ###

    # Retrieve each gradient from the dictionary "grads".
    ### START CODE HERE ### (~ 4 lines of code)
    dW1 = grads["dW1"]
    db1 = grads["db1"]
    dW2 = grads["dW2"]
    db2 = grads["db2"]
    ### END CODE HERE ###

    # Update rule for each parameter.
    ### START CODE HERE ### (~ 4 lines of code)
    W1 = W1 - learning_rate * dW1
    b1 = b1 - learning_rate * db1
    W2 = W2 - learning_rate * dW2
    b2 = b2 - learning_rate * db2
    ### END CODE HERE ###

    parameters = {"W1": W1,
                  "b1": b1,
                  "W2": W2,
                  "b2": b2}

    return parameters

parameters_updated = update_parameters(parameters, grads)

print("W1 updated = " + str(parameters_updated["W1"]))
print("b1 updated = " + str(parameters_updated["b1"]))
print("W2 updated = " + str(parameters_updated["W2"]))
print("b2 updated = " + str(parameters_updated["b2"]))
```

```
W1 updated = [[ 0.01790427  0.00434496]
 [ 0.00099046 -0.01866419]]
b1 updated = [[-6.13449205e-07]
 [-8.47483463e-07]]
W2 updated = [[-0.00238219 -0.00323487]]
b2 updated = [[0.00094478]]
```

Expected Output

Note: the actual values can be different!

```
W1 updated = [[ 0.01790427  0.00434496]
 [ 0.00099046 -0.01866419]]
b1 updated = [[-6.13449205e-07]
 [-8.47483463e-07]]
W2 updated = [[-0.00238219 -0.00323487]]
b2 updated = [[0.00094478]]
```

```
w3_unittest.test_update_parameters(update_parameters)
```

```
 All tests passed
```

3.4 - Integrate parts 3.1, 3.2 and 3.3 in nn_model()

Exercise 7

Build your neural network model in `nn_model()`.

Instructions: The neural network model has to use the previous functions in the right order.

```
# GRADED FUNCTION: nn_model

def nn_model(X, Y, n_h, num_iterations=10, learning_rate=1.2,
print_cost=False):
    """
    Arguments:
    X -- dataset of shape (n_x, number of examples)
    Y -- labels of shape (n_y, number of examples)
    num_iterations -- number of iterations in the loop
    learning_rate -- learning rate parameter for gradient descent
    print_cost -- if True, print the cost every iteration

    Returns:
    parameters -- parameters learnt by the model. They can then be
used to predict.
    """

    n_x = layer_sizes(X, Y)[0]
    n_y = layer_sizes(X, Y)[2]

    # Initialize parameters.
    ### START CODE HERE ### (~ 1 line of code)
    parameters = initialize_parameters(n_x, n_h, n_y)
    ### END CODE HERE ###

    # Loop.
    for i in range(0, num_iterations):

        ### START CODE HERE ### (~ 4 lines of code)
        # Forward propagation. Inputs: "X, parameters". Outputs: "A2,
cache".
        A2, cache = forward_propagation(X, parameters)

        # Cost function. Inputs: "A2, Y". Outputs: "cost".
```

```
        cost = compute_cost(A2, Y)

        # Backpropagation. Inputs: "parameters, cache, X, Y". Outputs:
"grads".
        grads = backward_propagation(parameters, cache, X, Y)

        # Gradient descent parameter update. Inputs: "parameters,
grads, learning_rate". Outputs: "parameters".
        parameters = update_parameters(parameters, grads,
learning_rate)
        ### END CODE HERE ###

        # Print the cost every iteration.
        if print_cost:
            print ("Cost after iteration %i: %f" %(i, cost))

    return parameters

parameters = nn_model(X, Y, n_h=2, num_iterations=3000,
learning_rate=1.2, print_cost=True)
print("W1 = " + str(parameters["W1"]))
print("b1 = " + str(parameters["b1"]))
print("W2 = " + str(parameters["W2"]))
print("b2 = " + str(parameters["b2"]))

W1 = parameters["W1"]
b1 = parameters["b1"]
W2 = parameters["W2"]
b2 = parameters["b2"]
```

```
Cost after iteration 0: 0.693166
Cost after iteration 1: 0.693153
Cost after iteration 2: 0.693149
Cost after iteration 3: 0.693148
Cost after iteration 4: 0.693147
Cost after iteration 5: 0.693147
Cost after iteration 6: 0.693147
Cost after iteration 7: 0.693147
Cost after iteration 8: 0.693147
Cost after iteration 9: 0.693147
Cost after iteration 10: 0.693147
Cost after iteration 11: 0.693147
Cost after iteration 12: 0.693147
Cost after iteration 13: 0.693147
Cost after iteration 14: 0.693147
Cost after iteration 15: 0.693147
Cost after iteration 16: 0.693147
Cost after iteration 17: 0.693147
Cost after iteration 18: 0.693147
Cost after iteration 19: 0.693147
```

```
Cost after iteration 20: 0.693146
Cost after iteration 21: 0.693146
Cost after iteration 22: 0.693146
Cost after iteration 23: 0.693146
Cost after iteration 24: 0.693146
Cost after iteration 25: 0.693146
Cost after iteration 26: 0.693146
Cost after iteration 27: 0.693146
Cost after iteration 28: 0.693146
Cost after iteration 29: 0.693146
Cost after iteration 30: 0.693146
Cost after iteration 31: 0.693146
Cost after iteration 32: 0.693146
Cost after iteration 33: 0.693146
Cost after iteration 34: 0.693146
Cost after iteration 35: 0.693146
Cost after iteration 36: 0.693146
Cost after iteration 37: 0.693146
Cost after iteration 38: 0.693146
Cost after iteration 39: 0.693146
Cost after iteration 40: 0.693146
Cost after iteration 41: 0.693146
Cost after iteration 42: 0.693146
Cost after iteration 43: 0.693145
Cost after iteration 44: 0.693145
Cost after iteration 45: 0.693145
Cost after iteration 46: 0.693145
Cost after iteration 47: 0.693145
Cost after iteration 48: 0.693145
Cost after iteration 49: 0.693145
Cost after iteration 50: 0.693145
Cost after iteration 51: 0.693145
Cost after iteration 52: 0.693145
Cost after iteration 53: 0.693145
Cost after iteration 54: 0.693145
Cost after iteration 55: 0.693145
Cost after iteration 56: 0.693145
Cost after iteration 57: 0.693145
Cost after iteration 58: 0.693145
Cost after iteration 59: 0.693145
Cost after iteration 60: 0.693145
Cost after iteration 61: 0.693144
Cost after iteration 62: 0.693144
Cost after iteration 63: 0.693144
Cost after iteration 64: 0.693144
Cost after iteration 65: 0.693144
Cost after iteration 66: 0.693144
Cost after iteration 67: 0.693144
Cost after iteration 68: 0.693144
```

```
Cost after iteration 69: 0.693144
Cost after iteration 70: 0.693144
Cost after iteration 71: 0.693144
Cost after iteration 72: 0.693144
Cost after iteration 73: 0.693144
Cost after iteration 74: 0.693143
Cost after iteration 75: 0.693143
Cost after iteration 76: 0.693143
Cost after iteration 77: 0.693143
Cost after iteration 78: 0.693143
Cost after iteration 79: 0.693143
Cost after iteration 80: 0.693143
Cost after iteration 81: 0.693143
Cost after iteration 82: 0.693143
Cost after iteration 83: 0.693143
Cost after iteration 84: 0.693143
Cost after iteration 85: 0.693142
Cost after iteration 86: 0.693142
Cost after iteration 87: 0.693142
Cost after iteration 88: 0.693142
Cost after iteration 89: 0.693142
Cost after iteration 90: 0.693142
Cost after iteration 91: 0.693142
Cost after iteration 92: 0.693142
Cost after iteration 93: 0.693142
Cost after iteration 94: 0.693141
Cost after iteration 95: 0.693141
Cost after iteration 96: 0.693141
Cost after iteration 97: 0.693141
Cost after iteration 98: 0.693141
Cost after iteration 99: 0.693141
Cost after iteration 100: 0.693141
Cost after iteration 101: 0.693140
Cost after iteration 102: 0.693140
Cost after iteration 103: 0.693140
Cost after iteration 104: 0.693140
Cost after iteration 105: 0.693140
Cost after iteration 106: 0.693140
Cost after iteration 107: 0.693139
Cost after iteration 108: 0.693139
Cost after iteration 109: 0.693139
Cost after iteration 110: 0.693139
Cost after iteration 111: 0.693139
Cost after iteration 112: 0.693139
Cost after iteration 113: 0.693138
Cost after iteration 114: 0.693138
Cost after iteration 115: 0.693138
Cost after iteration 116: 0.693138
Cost after iteration 117: 0.693138
```

```
Cost after iteration 118: 0.693137
Cost after iteration 119: 0.693137
Cost after iteration 120: 0.693137
Cost after iteration 121: 0.693137
Cost after iteration 122: 0.693137
Cost after iteration 123: 0.693136
Cost after iteration 124: 0.693136
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Cost after iteration 126: 0.693136
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Cost after iteration 130: 0.693135
Cost after iteration 131: 0.693134
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Cost after iteration 134: 0.693133
Cost after iteration 135: 0.693133
Cost after iteration 136: 0.693133
Cost after iteration 137: 0.693132
Cost after iteration 138: 0.693132
Cost after iteration 139: 0.693132
Cost after iteration 140: 0.693131
Cost after iteration 141: 0.693131
Cost after iteration 142: 0.693131
Cost after iteration 143: 0.693130
Cost after iteration 144: 0.693130
Cost after iteration 145: 0.693130
Cost after iteration 146: 0.693129
Cost after iteration 147: 0.693129
Cost after iteration 148: 0.693128
Cost after iteration 149: 0.693128
Cost after iteration 150: 0.693128
Cost after iteration 151: 0.693127
Cost after iteration 152: 0.693127
Cost after iteration 153: 0.693126
Cost after iteration 154: 0.693126
Cost after iteration 155: 0.693125
Cost after iteration 156: 0.693125
Cost after iteration 157: 0.693124
Cost after iteration 158: 0.693124
Cost after iteration 159: 0.693123
Cost after iteration 160: 0.693123
Cost after iteration 161: 0.693122
Cost after iteration 162: 0.693121
Cost after iteration 163: 0.693121
Cost after iteration 164: 0.693120
Cost after iteration 165: 0.693119
Cost after iteration 166: 0.693119
```

```
Cost after iteration 167: 0.693118
Cost after iteration 168: 0.693117
Cost after iteration 169: 0.693117
Cost after iteration 170: 0.693116
Cost after iteration 171: 0.693115
Cost after iteration 172: 0.693114
Cost after iteration 173: 0.693113
Cost after iteration 174: 0.693113
Cost after iteration 175: 0.693112
Cost after iteration 176: 0.693111
Cost after iteration 177: 0.693110
Cost after iteration 178: 0.693109
Cost after iteration 179: 0.693108
Cost after iteration 180: 0.693107
Cost after iteration 181: 0.693106
Cost after iteration 182: 0.693105
Cost after iteration 183: 0.693103
Cost after iteration 184: 0.693102
Cost after iteration 185: 0.693101
Cost after iteration 186: 0.693100
Cost after iteration 187: 0.693098
Cost after iteration 188: 0.693097
Cost after iteration 189: 0.693095
Cost after iteration 190: 0.693094
Cost after iteration 191: 0.693092
Cost after iteration 192: 0.693091
Cost after iteration 193: 0.693089
Cost after iteration 194: 0.693087
Cost after iteration 195: 0.693085
Cost after iteration 196: 0.693083
Cost after iteration 197: 0.693081
Cost after iteration 198: 0.693079
Cost after iteration 199: 0.693077
Cost after iteration 200: 0.693075
Cost after iteration 201: 0.693072
Cost after iteration 202: 0.693069
Cost after iteration 203: 0.693067
Cost after iteration 204: 0.693064
Cost after iteration 205: 0.693061
Cost after iteration 206: 0.693057
Cost after iteration 207: 0.693054
Cost after iteration 208: 0.693050
Cost after iteration 209: 0.693047
Cost after iteration 210: 0.693043
Cost after iteration 211: 0.693038
Cost after iteration 212: 0.693034
Cost after iteration 213: 0.693029
Cost after iteration 214: 0.693023
Cost after iteration 215: 0.693018
```

```
Cost after iteration 216: 0.693012
Cost after iteration 217: 0.693005
Cost after iteration 218: 0.692998
Cost after iteration 219: 0.692991
Cost after iteration 220: 0.692983
Cost after iteration 221: 0.692974
Cost after iteration 222: 0.692965
Cost after iteration 223: 0.692955
Cost after iteration 224: 0.692943
Cost after iteration 225: 0.692931
Cost after iteration 226: 0.692918
Cost after iteration 227: 0.692904
Cost after iteration 228: 0.692888
Cost after iteration 229: 0.692870
Cost after iteration 230: 0.692851
Cost after iteration 231: 0.692829
Cost after iteration 232: 0.692805
Cost after iteration 233: 0.692778
Cost after iteration 234: 0.692749
Cost after iteration 235: 0.692715
Cost after iteration 236: 0.692677
Cost after iteration 237: 0.692635
Cost after iteration 238: 0.692587
Cost after iteration 239: 0.692532
Cost after iteration 240: 0.692470
Cost after iteration 241: 0.692398
Cost after iteration 242: 0.692316
Cost after iteration 243: 0.692221
Cost after iteration 244: 0.692112
Cost after iteration 245: 0.691985
Cost after iteration 246: 0.691837
Cost after iteration 247: 0.691665
Cost after iteration 248: 0.691463
Cost after iteration 249: 0.691227
Cost after iteration 250: 0.690949
Cost after iteration 251: 0.690623
Cost after iteration 252: 0.690240
Cost after iteration 253: 0.689791
Cost after iteration 254: 0.689264
Cost after iteration 255: 0.688648
Cost after iteration 256: 0.687929
Cost after iteration 257: 0.687096
Cost after iteration 258: 0.686136
Cost after iteration 259: 0.685036
Cost after iteration 260: 0.683786
Cost after iteration 261: 0.682379
Cost after iteration 262: 0.680809
Cost after iteration 263: 0.679075
Cost after iteration 264: 0.677181
```

```
Cost after iteration 265: 0.675134
Cost after iteration 266: 0.672943
Cost after iteration 267: 0.670625
Cost after iteration 268: 0.668194
Cost after iteration 269: 0.665670
Cost after iteration 270: 0.663073
Cost after iteration 271: 0.660421
Cost after iteration 272: 0.657733
Cost after iteration 273: 0.655027
Cost after iteration 274: 0.652318
Cost after iteration 275: 0.649621
Cost after iteration 276: 0.646947
Cost after iteration 277: 0.644307
Cost after iteration 278: 0.641710
Cost after iteration 279: 0.639161
Cost after iteration 280: 0.636665
Cost after iteration 281: 0.634225
Cost after iteration 282: 0.631842
Cost after iteration 283: 0.629517
Cost after iteration 284: 0.627249
Cost after iteration 285: 0.625036
Cost after iteration 286: 0.622874
Cost after iteration 287: 0.620759
Cost after iteration 288: 0.618687
Cost after iteration 289: 0.616652
Cost after iteration 290: 0.614645
Cost after iteration 291: 0.612659
Cost after iteration 292: 0.610684
Cost after iteration 293: 0.608708
Cost after iteration 294: 0.606716
Cost after iteration 295: 0.604689
Cost after iteration 296: 0.602606
Cost after iteration 297: 0.600438
Cost after iteration 298: 0.598153
Cost after iteration 299: 0.595716
Cost after iteration 300: 0.593086
Cost after iteration 301: 0.590224
Cost after iteration 302: 0.587092
Cost after iteration 303: 0.583654
Cost after iteration 304: 0.579882
Cost after iteration 305: 0.575759
Cost after iteration 306: 0.571277
Cost after iteration 307: 0.566444
Cost after iteration 308: 0.561279
Cost after iteration 309: 0.555812
Cost after iteration 310: 0.550082
Cost after iteration 311: 0.544130
Cost after iteration 312: 0.538003
Cost after iteration 313: 0.531746
```

```
Cost after iteration 314: 0.525401
Cost after iteration 315: 0.519007
Cost after iteration 316: 0.512599
Cost after iteration 317: 0.506208
Cost after iteration 318: 0.499859
Cost after iteration 319: 0.493576
Cost after iteration 320: 0.487375
Cost after iteration 321: 0.481273
Cost after iteration 322: 0.475281
Cost after iteration 323: 0.469408
Cost after iteration 324: 0.463661
Cost after iteration 325: 0.458046
Cost after iteration 326: 0.452567
Cost after iteration 327: 0.447224
Cost after iteration 328: 0.442021
Cost after iteration 329: 0.436956
Cost after iteration 330: 0.432028
Cost after iteration 331: 0.427238
Cost after iteration 332: 0.422582
Cost after iteration 333: 0.418059
Cost after iteration 334: 0.413665
Cost after iteration 335: 0.409398
Cost after iteration 336: 0.405255
Cost after iteration 337: 0.401232
Cost after iteration 338: 0.397327
Cost after iteration 339: 0.393536
Cost after iteration 340: 0.389855
Cost after iteration 341: 0.386281
Cost after iteration 342: 0.382810
Cost after iteration 343: 0.379439
Cost after iteration 344: 0.376166
Cost after iteration 345: 0.372986
Cost after iteration 346: 0.369896
Cost after iteration 347: 0.366893
Cost after iteration 348: 0.363974
Cost after iteration 349: 0.361137
Cost after iteration 350: 0.358377
Cost after iteration 351: 0.355694
Cost after iteration 352: 0.353082
Cost after iteration 353: 0.350541
Cost after iteration 354: 0.348068
Cost after iteration 355: 0.345659
Cost after iteration 356: 0.343314
Cost after iteration 357: 0.341029
Cost after iteration 358: 0.338802
Cost after iteration 359: 0.336631
Cost after iteration 360: 0.334515
Cost after iteration 361: 0.332450
Cost after iteration 362: 0.330437
```

```
Cost after iteration 363: 0.328471
Cost after iteration 364: 0.326552
Cost after iteration 365: 0.324679
Cost after iteration 366: 0.322849
Cost after iteration 367: 0.321060
Cost after iteration 368: 0.319313
Cost after iteration 369: 0.317604
Cost after iteration 370: 0.315933
Cost after iteration 371: 0.314298
Cost after iteration 372: 0.312699
Cost after iteration 373: 0.311133
Cost after iteration 374: 0.309600
Cost after iteration 375: 0.308098
Cost after iteration 376: 0.306627
Cost after iteration 377: 0.305186
Cost after iteration 378: 0.303772
Cost after iteration 379: 0.302387
Cost after iteration 380: 0.301027
Cost after iteration 381: 0.299694
Cost after iteration 382: 0.298385
Cost after iteration 383: 0.297100
Cost after iteration 384: 0.295839
Cost after iteration 385: 0.294600
Cost after iteration 386: 0.293382
Cost after iteration 387: 0.292186
Cost after iteration 388: 0.291009
Cost after iteration 389: 0.289853
Cost after iteration 390: 0.288716
Cost after iteration 391: 0.287596
Cost after iteration 392: 0.286495
Cost after iteration 393: 0.285411
Cost after iteration 394: 0.284344
Cost after iteration 395: 0.283293
Cost after iteration 396: 0.282258
Cost after iteration 397: 0.281238
Cost after iteration 398: 0.280233
Cost after iteration 399: 0.279242
Cost after iteration 400: 0.278265
Cost after iteration 401: 0.277302
Cost after iteration 402: 0.276352
Cost after iteration 403: 0.275414
Cost after iteration 404: 0.274489
Cost after iteration 405: 0.273575
Cost after iteration 406: 0.272674
Cost after iteration 407: 0.271784
Cost after iteration 408: 0.270904
Cost after iteration 409: 0.270036
Cost after iteration 410: 0.269177
Cost after iteration 411: 0.268329
```

```
Cost after iteration 412: 0.267491
Cost after iteration 413: 0.266662
Cost after iteration 414: 0.265843
Cost after iteration 415: 0.265033
Cost after iteration 416: 0.264231
Cost after iteration 417: 0.263439
Cost after iteration 418: 0.262654
Cost after iteration 419: 0.261878
Cost after iteration 420: 0.261110
Cost after iteration 421: 0.260350
Cost after iteration 422: 0.259597
Cost after iteration 423: 0.258852
Cost after iteration 424: 0.258114
Cost after iteration 425: 0.257383
Cost after iteration 426: 0.256659
Cost after iteration 427: 0.255942
Cost after iteration 428: 0.255232
Cost after iteration 429: 0.254528
Cost after iteration 430: 0.253831
Cost after iteration 431: 0.253139
Cost after iteration 432: 0.252455
Cost after iteration 433: 0.251776
Cost after iteration 434: 0.251103
Cost after iteration 435: 0.250436
Cost after iteration 436: 0.249774
Cost after iteration 437: 0.249119
Cost after iteration 438: 0.248469
Cost after iteration 439: 0.247824
Cost after iteration 440: 0.247185
Cost after iteration 441: 0.246551
Cost after iteration 442: 0.245922
Cost after iteration 443: 0.245299
Cost after iteration 444: 0.244681
Cost after iteration 445: 0.244067
Cost after iteration 446: 0.243459
Cost after iteration 447: 0.242856
Cost after iteration 448: 0.242257
Cost after iteration 449: 0.241664
Cost after iteration 450: 0.241075
Cost after iteration 451: 0.240491
Cost after iteration 452: 0.239912
Cost after iteration 453: 0.239337
Cost after iteration 454: 0.238767
Cost after iteration 455: 0.238202
Cost after iteration 456: 0.237641
Cost after iteration 457: 0.237085
Cost after iteration 458: 0.236533
Cost after iteration 459: 0.235986
Cost after iteration 460: 0.235443
```

```
Cost after iteration 461: 0.234905
Cost after iteration 462: 0.234371
Cost after iteration 463: 0.233842
Cost after iteration 464: 0.233317
Cost after iteration 465: 0.232796
Cost after iteration 466: 0.232280
Cost after iteration 467: 0.231768
Cost after iteration 468: 0.231260
Cost after iteration 469: 0.230757
Cost after iteration 470: 0.230258
Cost after iteration 471: 0.229763
Cost after iteration 472: 0.229273
Cost after iteration 473: 0.228787
Cost after iteration 474: 0.228305
Cost after iteration 475: 0.227827
Cost after iteration 476: 0.227354
Cost after iteration 477: 0.226885
Cost after iteration 478: 0.226420
Cost after iteration 479: 0.225959
Cost after iteration 480: 0.225502
Cost after iteration 481: 0.225050
Cost after iteration 482: 0.224602
Cost after iteration 483: 0.224158
Cost after iteration 484: 0.223718
Cost after iteration 485: 0.223282
Cost after iteration 486: 0.222850
Cost after iteration 487: 0.222422
Cost after iteration 488: 0.221999
Cost after iteration 489: 0.221579
Cost after iteration 490: 0.221164
Cost after iteration 491: 0.220753
Cost after iteration 492: 0.220345
Cost after iteration 493: 0.219942
Cost after iteration 494: 0.219542
Cost after iteration 495: 0.219147
Cost after iteration 496: 0.218755
Cost after iteration 497: 0.218367
Cost after iteration 498: 0.217984
Cost after iteration 499: 0.217604
Cost after iteration 500: 0.217228
Cost after iteration 501: 0.216856
Cost after iteration 502: 0.216487
Cost after iteration 503: 0.216123
Cost after iteration 504: 0.215762
Cost after iteration 505: 0.215405
Cost after iteration 506: 0.215051
Cost after iteration 507: 0.214701
Cost after iteration 508: 0.214355
Cost after iteration 509: 0.214013
Cost after iteration 510: 0.213674
```

```
Cost after iteration 511: 0.213338
Cost after iteration 512: 0.213007
Cost after iteration 513: 0.212678
Cost after iteration 514: 0.212353
Cost after iteration 515: 0.212032
Cost after iteration 516: 0.211714
Cost after iteration 517: 0.211400
Cost after iteration 518: 0.211088
Cost after iteration 519: 0.210780
Cost after iteration 520: 0.210476
Cost after iteration 521: 0.210175
Cost after iteration 522: 0.209877
Cost after iteration 523: 0.209584
Cost after iteration 524: 0.209301
Cost after iteration 525: 0.209047
Cost after iteration 526: 0.208904
Cost after iteration 527: 0.209223
Cost after iteration 528: 0.211509
Cost after iteration 529: 0.222599
Cost after iteration 530: 0.276923
Cost after iteration 531: 0.544826
Cost after iteration 532: 0.813378
Cost after iteration 533: 1.350587
Cost after iteration 534: 0.233226
Cost after iteration 535: 0.229575
Cost after iteration 536: 0.227303
Cost after iteration 537: 0.225588
Cost after iteration 538: 0.224099
Cost after iteration 539: 0.222793
Cost after iteration 540: 0.221618
Cost after iteration 541: 0.220560
Cost after iteration 542: 0.219595
Cost after iteration 543: 0.218714
Cost after iteration 544: 0.217901
Cost after iteration 545: 0.217149
Cost after iteration 546: 0.216448
Cost after iteration 547: 0.215792
Cost after iteration 548: 0.215175
Cost after iteration 549: 0.214594
Cost after iteration 550: 0.214041
Cost after iteration 551: 0.213518
Cost after iteration 552: 0.213017
Cost after iteration 553: 0.212542
Cost after iteration 554: 0.212087
Cost after iteration 555: 0.211656
Cost after iteration 556: 0.211245
Cost after iteration 557: 0.210864
Cost after iteration 558: 0.210506
Cost after iteration 559: 0.210198
```

```
Cost after iteration 560: 0.209922
Cost after iteration 561: 0.209746
Cost after iteration 562: 0.209625
Cost after iteration 563: 0.209739
Cost after iteration 564: 0.209953
Cost after iteration 565: 0.210792
Cost after iteration 566: 0.211774
Cost after iteration 567: 0.214601
Cost after iteration 568: 0.217277
Cost after iteration 569: 0.225712
Cost after iteration 570: 0.230394
Cost after iteration 571: 0.251461
Cost after iteration 572: 0.249485
Cost after iteration 573: 0.284327
Cost after iteration 574: 0.261340
Cost after iteration 575: 0.292942
Cost after iteration 576: 0.260804
Cost after iteration 577: 0.281966
Cost after iteration 578: 0.277206
Cost after iteration 579: 0.316482
Cost after iteration 580: 0.345965
Cost after iteration 581: 0.412882
Cost after iteration 582: 0.382526
Cost after iteration 583: 0.361773
Cost after iteration 584: 0.288284
Cost after iteration 585: 0.250980
Cost after iteration 586: 0.230284
Cost after iteration 587: 0.221691
Cost after iteration 588: 0.216954
Cost after iteration 589: 0.215185
Cost after iteration 590: 0.213366
Cost after iteration 591: 0.212398
Cost after iteration 592: 0.211415
Cost after iteration 593: 0.210743
Cost after iteration 594: 0.210085
Cost after iteration 595: 0.209554
Cost after iteration 596: 0.209037
Cost after iteration 597: 0.208579
Cost after iteration 598: 0.208131
Cost after iteration 599: 0.207714
Cost after iteration 600: 0.207304
Cost after iteration 601: 0.206914
Cost after iteration 602: 0.206529
Cost after iteration 603: 0.206157
Cost after iteration 604: 0.205791
Cost after iteration 605: 0.205435
Cost after iteration 606: 0.205083
Cost after iteration 607: 0.204742
Cost after iteration 608: 0.204405
```

```
Cost after iteration 609: 0.204078
Cost after iteration 610: 0.203755
Cost after iteration 611: 0.203443
Cost after iteration 612: 0.203136
Cost after iteration 613: 0.202842
Cost after iteration 614: 0.202554
Cost after iteration 615: 0.202285
Cost after iteration 616: 0.202023
Cost after iteration 617: 0.201790
Cost after iteration 618: 0.201568
Cost after iteration 619: 0.201398
Cost after iteration 620: 0.201247
Cost after iteration 621: 0.201201
Cost after iteration 622: 0.201189
Cost after iteration 623: 0.201413
Cost after iteration 624: 0.201700
Cost after iteration 625: 0.202580
Cost after iteration 626: 0.203559
Cost after iteration 627: 0.206196
Cost after iteration 628: 0.208818
Cost after iteration 629: 0.216452
Cost after iteration 630: 0.222304
Cost after iteration 631: 0.242895
Cost after iteration 632: 0.249521
Cost after iteration 633: 0.295589
Cost after iteration 634: 0.303329
Cost after iteration 635: 0.416171
Cost after iteration 636: 0.467718
Cost after iteration 637: 0.664723
Cost after iteration 638: 0.364939
Cost after iteration 639: 0.332104
Cost after iteration 640: 0.251638
Cost after iteration 641: 0.225930
Cost after iteration 642: 0.215132
Cost after iteration 643: 0.211271
Cost after iteration 644: 0.209008
Cost after iteration 645: 0.207871
Cost after iteration 646: 0.207000
Cost after iteration 647: 0.206368
Cost after iteration 648: 0.205821
Cost after iteration 649: 0.205341
Cost after iteration 650: 0.204894
Cost after iteration 651: 0.204473
Cost after iteration 652: 0.204069
Cost after iteration 653: 0.203679
Cost after iteration 654: 0.203301
Cost after iteration 655: 0.202933
Cost after iteration 656: 0.202574
Cost after iteration 657: 0.202223
```

```
Cost after iteration 658: 0.201880
Cost after iteration 659: 0.201544
Cost after iteration 660: 0.201215
Cost after iteration 661: 0.200893
Cost after iteration 662: 0.200577
Cost after iteration 663: 0.200268
Cost after iteration 664: 0.199965
Cost after iteration 665: 0.199668
Cost after iteration 666: 0.199377
Cost after iteration 667: 0.199091
Cost after iteration 668: 0.198812
Cost after iteration 669: 0.198537
Cost after iteration 670: 0.198269
Cost after iteration 671: 0.198005
Cost after iteration 672: 0.197747
Cost after iteration 673: 0.197494
Cost after iteration 674: 0.197246
Cost after iteration 675: 0.197004
Cost after iteration 676: 0.196766
Cost after iteration 677: 0.196533
Cost after iteration 678: 0.196305
Cost after iteration 679: 0.196082
Cost after iteration 680: 0.195864
Cost after iteration 681: 0.195650
Cost after iteration 682: 0.195441
Cost after iteration 683: 0.195237
Cost after iteration 684: 0.195037
Cost after iteration 685: 0.194842
Cost after iteration 686: 0.194652
Cost after iteration 687: 0.194468
Cost after iteration 688: 0.194289
Cost after iteration 689: 0.194118
Cost after iteration 690: 0.193956
Cost after iteration 691: 0.193808
Cost after iteration 692: 0.193679
Cost after iteration 693: 0.193586
Cost after iteration 694: 0.193548
Cost after iteration 695: 0.193623
Cost after iteration 696: 0.193888
Cost after iteration 697: 0.194581
Cost after iteration 698: 0.195994
Cost after iteration 699: 0.199265
Cost after iteration 700: 0.205704
Cost after iteration 701: 0.222046
Cost after iteration 702: 0.257552
Cost after iteration 703: 0.372036
Cost after iteration 704: 0.549190
Cost after iteration 705: 1.146287
Cost after iteration 706: 0.342378
```

```
Cost after iteration 707: 0.376645
Cost after iteration 708: 0.215812
Cost after iteration 709: 0.210265
Cost after iteration 710: 0.207333
Cost after iteration 711: 0.205969
Cost after iteration 712: 0.205022
Cost after iteration 713: 0.204266
Cost after iteration 714: 0.203597
Cost after iteration 715: 0.202982
Cost after iteration 716: 0.202407
Cost after iteration 717: 0.201867
Cost after iteration 718: 0.201355
Cost after iteration 719: 0.200871
Cost after iteration 720: 0.200411
Cost after iteration 721: 0.199973
Cost after iteration 722: 0.199555
Cost after iteration 723: 0.199157
Cost after iteration 724: 0.198776
Cost after iteration 725: 0.198411
Cost after iteration 726: 0.198061
Cost after iteration 727: 0.197726
Cost after iteration 728: 0.197403
Cost after iteration 729: 0.197093
Cost after iteration 730: 0.196794
Cost after iteration 731: 0.196505
Cost after iteration 732: 0.196227
Cost after iteration 733: 0.195959
Cost after iteration 734: 0.195699
Cost after iteration 735: 0.195448
Cost after iteration 736: 0.195204
Cost after iteration 737: 0.194969
Cost after iteration 738: 0.194741
Cost after iteration 739: 0.194519
Cost after iteration 740: 0.194304
Cost after iteration 741: 0.194096
Cost after iteration 742: 0.193893
Cost after iteration 743: 0.193696
Cost after iteration 744: 0.193505
Cost after iteration 745: 0.193319
Cost after iteration 746: 0.193138
Cost after iteration 747: 0.192961
Cost after iteration 748: 0.192790
Cost after iteration 749: 0.192623
Cost after iteration 750: 0.192460
Cost after iteration 751: 0.192302
Cost after iteration 752: 0.192147
Cost after iteration 753: 0.191997
Cost after iteration 754: 0.191850
Cost after iteration 755: 0.191707
```

```
Cost after iteration 756: 0.191568
Cost after iteration 757: 0.191431
Cost after iteration 758: 0.191298
Cost after iteration 759: 0.191169
Cost after iteration 760: 0.191042
Cost after iteration 761: 0.190919
Cost after iteration 762: 0.190798
Cost after iteration 763: 0.190680
Cost after iteration 764: 0.190566
Cost after iteration 765: 0.190455
Cost after iteration 766: 0.190347
Cost after iteration 767: 0.190246
Cost after iteration 768: 0.190154
Cost after iteration 769: 0.190078
Cost after iteration 770: 0.190036
Cost after iteration 771: 0.190067
Cost after iteration 772: 0.190253
Cost after iteration 773: 0.190807
Cost after iteration 774: 0.192154
Cost after iteration 775: 0.195618
Cost after iteration 776: 0.203744
Cost after iteration 777: 0.227671
Cost after iteration 778: 0.291121
Cost after iteration 779: 0.490895
Cost after iteration 780: 0.544962
Cost after iteration 781: 0.680301
Cost after iteration 782: 0.328559
Cost after iteration 783: 0.410514
Cost after iteration 784: 0.270860
Cost after iteration 785: 0.280541
Cost after iteration 786: 0.218667
Cost after iteration 787: 0.215118
Cost after iteration 788: 0.206518
Cost after iteration 789: 0.204167
Cost after iteration 790: 0.201559
Cost after iteration 791: 0.200314
Cost after iteration 792: 0.199230
Cost after iteration 793: 0.198526
Cost after iteration 794: 0.197920
Cost after iteration 795: 0.197436
Cost after iteration 796: 0.197001
Cost after iteration 797: 0.196614
Cost after iteration 798: 0.196253
Cost after iteration 799: 0.195916
Cost after iteration 800: 0.195597
Cost after iteration 801: 0.195293
Cost after iteration 802: 0.195002
Cost after iteration 803: 0.194723
Cost after iteration 804: 0.194454
```

```
Cost after iteration 805: 0.194196
Cost after iteration 806: 0.193946
Cost after iteration 807: 0.193705
Cost after iteration 808: 0.193472
Cost after iteration 809: 0.193246
Cost after iteration 810: 0.193028
Cost after iteration 811: 0.192816
Cost after iteration 812: 0.192612
Cost after iteration 813: 0.192413
Cost after iteration 814: 0.192221
Cost after iteration 815: 0.192035
Cost after iteration 816: 0.191854
Cost after iteration 817: 0.191679
Cost after iteration 818: 0.191509
Cost after iteration 819: 0.191344
Cost after iteration 820: 0.191184
Cost after iteration 821: 0.191028
Cost after iteration 822: 0.190878
Cost after iteration 823: 0.190732
Cost after iteration 824: 0.190590
Cost after iteration 825: 0.190454
Cost after iteration 826: 0.190321
Cost after iteration 827: 0.190194
Cost after iteration 828: 0.190072
Cost after iteration 829: 0.189955
Cost after iteration 830: 0.189845
Cost after iteration 831: 0.189744
Cost after iteration 832: 0.189652
Cost after iteration 833: 0.189576
Cost after iteration 834: 0.189517
Cost after iteration 835: 0.189494
Cost after iteration 836: 0.189508
Cost after iteration 837: 0.189614
Cost after iteration 838: 0.189807
Cost after iteration 839: 0.190265
Cost after iteration 840: 0.190946
Cost after iteration 841: 0.192475
Cost after iteration 842: 0.194604
Cost after iteration 843: 0.199792
Cost after iteration 844: 0.206609
Cost after iteration 845: 0.225598
Cost after iteration 846: 0.246202
Cost after iteration 847: 0.319083
Cost after iteration 848: 0.373238
Cost after iteration 849: 0.673669
Cost after iteration 850: 0.547436
Cost after iteration 851: 0.757178
Cost after iteration 852: 0.285885
Cost after iteration 853: 0.239398
```

```
Cost after iteration 854: 0.211292
Cost after iteration 855: 0.204282
Cost after iteration 856: 0.201738
Cost after iteration 857: 0.200646
Cost after iteration 858: 0.199945
Cost after iteration 859: 0.199397
Cost after iteration 860: 0.198912
Cost after iteration 861: 0.198463
Cost after iteration 862: 0.198038
Cost after iteration 863: 0.197634
Cost after iteration 864: 0.197248
Cost after iteration 865: 0.196879
Cost after iteration 866: 0.196525
Cost after iteration 867: 0.196185
Cost after iteration 868: 0.195859
Cost after iteration 869: 0.195546
Cost after iteration 870: 0.195245
Cost after iteration 871: 0.194955
Cost after iteration 872: 0.194676
Cost after iteration 873: 0.194407
Cost after iteration 874: 0.194148
Cost after iteration 875: 0.193898
Cost after iteration 876: 0.193657
Cost after iteration 877: 0.193424
Cost after iteration 878: 0.193199
Cost after iteration 879: 0.192981
Cost after iteration 880: 0.192771
Cost after iteration 881: 0.192567
Cost after iteration 882: 0.192370
Cost after iteration 883: 0.192180
Cost after iteration 884: 0.191995
Cost after iteration 885: 0.191816
Cost after iteration 886: 0.191642
Cost after iteration 887: 0.191474
Cost after iteration 888: 0.191311
Cost after iteration 889: 0.191153
Cost after iteration 890: 0.191000
Cost after iteration 891: 0.190851
Cost after iteration 892: 0.190706
Cost after iteration 893: 0.190566
Cost after iteration 894: 0.190429
Cost after iteration 895: 0.190297
Cost after iteration 896: 0.190168
Cost after iteration 897: 0.190043
Cost after iteration 898: 0.189921
Cost after iteration 899: 0.189803
Cost after iteration 900: 0.189688
Cost after iteration 901: 0.189576
Cost after iteration 902: 0.189467
```

```
Cost after iteration 903: 0.189361
Cost after iteration 904: 0.189258
Cost after iteration 905: 0.189158
Cost after iteration 906: 0.189060
Cost after iteration 907: 0.188965
Cost after iteration 908: 0.188872
Cost after iteration 909: 0.188782
Cost after iteration 910: 0.188694
Cost after iteration 911: 0.188608
Cost after iteration 912: 0.188524
Cost after iteration 913: 0.188442
Cost after iteration 914: 0.188363
Cost after iteration 915: 0.188285
Cost after iteration 916: 0.188209
Cost after iteration 917: 0.188135
Cost after iteration 918: 0.188062
Cost after iteration 919: 0.187992
Cost after iteration 920: 0.187923
Cost after iteration 921: 0.187855
Cost after iteration 922: 0.187789
Cost after iteration 923: 0.187725
Cost after iteration 924: 0.187661
Cost after iteration 925: 0.187600
Cost after iteration 926: 0.187539
Cost after iteration 927: 0.187480
Cost after iteration 928: 0.187422
Cost after iteration 929: 0.187366
Cost after iteration 930: 0.187310
Cost after iteration 931: 0.187256
Cost after iteration 932: 0.187202
Cost after iteration 933: 0.187150
Cost after iteration 934: 0.187099
Cost after iteration 935: 0.187049
Cost after iteration 936: 0.187000
Cost after iteration 937: 0.186952
Cost after iteration 938: 0.186905
Cost after iteration 939: 0.186860
Cost after iteration 940: 0.186816
Cost after iteration 941: 0.186777
Cost after iteration 942: 0.186743
Cost after iteration 943: 0.186722
Cost after iteration 944: 0.186724
Cost after iteration 945: 0.186775
Cost after iteration 946: 0.186926
Cost after iteration 947: 0.187301
Cost after iteration 948: 0.188119
Cost after iteration 949: 0.190031
Cost after iteration 950: 0.194018
Cost after iteration 951: 0.204175
```

```
Cost after iteration 952: 0.226098
Cost after iteration 953: 0.300687
Cost after iteration 954: 0.446805
Cost after iteration 955: 0.895187
Cost after iteration 956: 0.464465
Cost after iteration 957: 0.715489
Cost after iteration 958: 0.257962
Cost after iteration 959: 0.230311
Cost after iteration 960: 0.211772
Cost after iteration 961: 0.203147
Cost after iteration 962: 0.199885
Cost after iteration 963: 0.198312
Cost after iteration 964: 0.197462
Cost after iteration 965: 0.196847
Cost after iteration 966: 0.196330
Cost after iteration 967: 0.195862
Cost after iteration 968: 0.195428
Cost after iteration 969: 0.195021
Cost after iteration 970: 0.194638
Cost after iteration 971: 0.194275
Cost after iteration 972: 0.193932
Cost after iteration 973: 0.193607
Cost after iteration 974: 0.193299
Cost after iteration 975: 0.193007
Cost after iteration 976: 0.192729
Cost after iteration 977: 0.192465
Cost after iteration 978: 0.192214
Cost after iteration 979: 0.191974
Cost after iteration 980: 0.191747
Cost after iteration 981: 0.191529
Cost after iteration 982: 0.191322
Cost after iteration 983: 0.191124
Cost after iteration 984: 0.190935
Cost after iteration 985: 0.190754
Cost after iteration 986: 0.190581
Cost after iteration 987: 0.190415
Cost after iteration 988: 0.190256
Cost after iteration 989: 0.190104
Cost after iteration 990: 0.189958
Cost after iteration 991: 0.189817
Cost after iteration 992: 0.189682
Cost after iteration 993: 0.189553
Cost after iteration 994: 0.189428
Cost after iteration 995: 0.189308
Cost after iteration 996: 0.189192
Cost after iteration 997: 0.189080
Cost after iteration 998: 0.188972
Cost after iteration 999: 0.188868
Cost after iteration 1000: 0.188768
```

```
Cost after iteration 1001: 0.188670
Cost after iteration 1002: 0.188577
Cost after iteration 1003: 0.188486
Cost after iteration 1004: 0.188398
Cost after iteration 1005: 0.188312
Cost after iteration 1006: 0.188230
Cost after iteration 1007: 0.188150
Cost after iteration 1008: 0.188072
Cost after iteration 1009: 0.187996
Cost after iteration 1010: 0.187923
Cost after iteration 1011: 0.187852
Cost after iteration 1012: 0.187782
Cost after iteration 1013: 0.187715
Cost after iteration 1014: 0.187650
Cost after iteration 1015: 0.187586
Cost after iteration 1016: 0.187523
Cost after iteration 1017: 0.187463
Cost after iteration 1018: 0.187404
Cost after iteration 1019: 0.187346
Cost after iteration 1020: 0.187290
Cost after iteration 1021: 0.187235
Cost after iteration 1022: 0.187181
Cost after iteration 1023: 0.187129
Cost after iteration 1024: 0.187078
Cost after iteration 1025: 0.187028
Cost after iteration 1026: 0.186979
Cost after iteration 1027: 0.186931
Cost after iteration 1028: 0.186884
Cost after iteration 1029: 0.186839
Cost after iteration 1030: 0.186794
Cost after iteration 1031: 0.186751
Cost after iteration 1032: 0.186709
Cost after iteration 1033: 0.186669
Cost after iteration 1034: 0.186632
Cost after iteration 1035: 0.186599
Cost after iteration 1036: 0.186574
Cost after iteration 1037: 0.186562
Cost after iteration 1038: 0.186574
Cost after iteration 1039: 0.186632
Cost after iteration 1040: 0.186773
Cost after iteration 1041: 0.187086
Cost after iteration 1042: 0.187700
Cost after iteration 1043: 0.189002
Cost after iteration 1044: 0.191454
Cost after iteration 1045: 0.197002
Cost after iteration 1046: 0.207567
Cost after iteration 1047: 0.237559
Cost after iteration 1048: 0.298774
Cost after iteration 1049: 0.492242
Cost after iteration 1050: 0.516488
```

```
Cost after iteration 1051: 0.792485
Cost after iteration 1052: 0.387529
Cost after iteration 1053: 0.491179
Cost after iteration 1054: 0.231826
Cost after iteration 1055: 0.216220
Cost after iteration 1056: 0.203005
Cost after iteration 1057: 0.199571
Cost after iteration 1058: 0.197733
Cost after iteration 1059: 0.196822
Cost after iteration 1060: 0.196198
Cost after iteration 1061: 0.195709
Cost after iteration 1062: 0.195281
Cost after iteration 1063: 0.194889
Cost after iteration 1064: 0.194522
Cost after iteration 1065: 0.194176
Cost after iteration 1066: 0.193847
Cost after iteration 1067: 0.193535
Cost after iteration 1068: 0.193238
Cost after iteration 1069: 0.192955
Cost after iteration 1070: 0.192685
Cost after iteration 1071: 0.192428
Cost after iteration 1072: 0.192182
Cost after iteration 1073: 0.191947
Cost after iteration 1074: 0.191723
Cost after iteration 1075: 0.191508
Cost after iteration 1076: 0.191303
Cost after iteration 1077: 0.191106
Cost after iteration 1078: 0.190918
Cost after iteration 1079: 0.190738
Cost after iteration 1080: 0.190565
Cost after iteration 1081: 0.190399
Cost after iteration 1082: 0.190240
Cost after iteration 1083: 0.190087
Cost after iteration 1084: 0.189940
Cost after iteration 1085: 0.189798
Cost after iteration 1086: 0.189662
Cost after iteration 1087: 0.189531
Cost after iteration 1088: 0.189405
Cost after iteration 1089: 0.189284
Cost after iteration 1090: 0.189167
Cost after iteration 1091: 0.189054
Cost after iteration 1092: 0.188946
Cost after iteration 1093: 0.188841
Cost after iteration 1094: 0.188739
Cost after iteration 1095: 0.188641
Cost after iteration 1096: 0.188547
Cost after iteration 1097: 0.188455
Cost after iteration 1098: 0.188367
Cost after iteration 1099: 0.188281
```

```
Cost after iteration 1100: 0.188198
Cost after iteration 1101: 0.188118
Cost after iteration 1102: 0.188040
Cost after iteration 1103: 0.187965
Cost after iteration 1104: 0.187891
Cost after iteration 1105: 0.187820
Cost after iteration 1106: 0.187752
Cost after iteration 1107: 0.187685
Cost after iteration 1108: 0.187620
Cost after iteration 1109: 0.187556
Cost after iteration 1110: 0.187495
Cost after iteration 1111: 0.187435
Cost after iteration 1112: 0.187377
Cost after iteration 1113: 0.187321
Cost after iteration 1114: 0.187265
Cost after iteration 1115: 0.187212
Cost after iteration 1116: 0.187159
Cost after iteration 1117: 0.187108
Cost after iteration 1118: 0.187059
Cost after iteration 1119: 0.187010
Cost after iteration 1120: 0.186963
Cost after iteration 1121: 0.186917
Cost after iteration 1122: 0.186871
Cost after iteration 1123: 0.186827
Cost after iteration 1124: 0.186784
Cost after iteration 1125: 0.186742
Cost after iteration 1126: 0.186701
Cost after iteration 1127: 0.186660
Cost after iteration 1128: 0.186621
Cost after iteration 1129: 0.186582
Cost after iteration 1130: 0.186544
Cost after iteration 1131: 0.186507
Cost after iteration 1132: 0.186471
Cost after iteration 1133: 0.186435
Cost after iteration 1134: 0.186400
Cost after iteration 1135: 0.186366
Cost after iteration 1136: 0.186332
Cost after iteration 1137: 0.186299
Cost after iteration 1138: 0.186267
Cost after iteration 1139: 0.186235
Cost after iteration 1140: 0.186205
Cost after iteration 1141: 0.186175
Cost after iteration 1142: 0.186147
Cost after iteration 1143: 0.186121
Cost after iteration 1144: 0.186099
Cost after iteration 1145: 0.186083
Cost after iteration 1146: 0.186079
Cost after iteration 1147: 0.186097
Cost after iteration 1148: 0.186155
```

```
Cost after iteration 1149: 0.186298
Cost after iteration 1150: 0.186584
Cost after iteration 1151: 0.187221
Cost after iteration 1152: 0.188379
Cost after iteration 1153: 0.191129
Cost after iteration 1154: 0.195859
Cost after iteration 1155: 0.208863
Cost after iteration 1156: 0.229156
Cost after iteration 1157: 0.296234
Cost after iteration 1158: 0.350359
Cost after iteration 1159: 0.693791
Cost after iteration 1160: 0.645185
Cost after iteration 1161: 1.194999
Cost after iteration 1162: 0.222068
Cost after iteration 1163: 0.207425
Cost after iteration 1164: 0.203660
Cost after iteration 1165: 0.202409
Cost after iteration 1166: 0.201514
Cost after iteration 1167: 0.200725
Cost after iteration 1168: 0.199994
Cost after iteration 1169: 0.199312
Cost after iteration 1170: 0.198674
Cost after iteration 1171: 0.198076
Cost after iteration 1172: 0.197515
Cost after iteration 1173: 0.196987
Cost after iteration 1174: 0.196491
Cost after iteration 1175: 0.196024
Cost after iteration 1176: 0.195584
Cost after iteration 1177: 0.195169
Cost after iteration 1178: 0.194777
Cost after iteration 1179: 0.194406
Cost after iteration 1180: 0.194056
Cost after iteration 1181: 0.193725
Cost after iteration 1182: 0.193411
Cost after iteration 1183: 0.193114
Cost after iteration 1184: 0.192832
Cost after iteration 1185: 0.192565
Cost after iteration 1186: 0.192311
Cost after iteration 1187: 0.192070
Cost after iteration 1188: 0.191840
Cost after iteration 1189: 0.191622
Cost after iteration 1190: 0.191415
Cost after iteration 1191: 0.191217
Cost after iteration 1192: 0.191028
Cost after iteration 1193: 0.190849
Cost after iteration 1194: 0.190677
Cost after iteration 1195: 0.190513
Cost after iteration 1196: 0.190357
Cost after iteration 1197: 0.190207
```

```
Cost after iteration 1198: 0.190063
Cost after iteration 1199: 0.189926
Cost after iteration 1200: 0.189795
Cost after iteration 1201: 0.189669
Cost after iteration 1202: 0.189548
Cost after iteration 1203: 0.189431
Cost after iteration 1204: 0.189320
Cost after iteration 1205: 0.189212
Cost after iteration 1206: 0.189109
Cost after iteration 1207: 0.189010
Cost after iteration 1208: 0.188914
Cost after iteration 1209: 0.188822
Cost after iteration 1210: 0.188733
Cost after iteration 1211: 0.188647
Cost after iteration 1212: 0.188564
Cost after iteration 1213: 0.188484
Cost after iteration 1214: 0.188406
Cost after iteration 1215: 0.188331
Cost after iteration 1216: 0.188259
Cost after iteration 1217: 0.188188
Cost after iteration 1218: 0.188120
Cost after iteration 1219: 0.188054
Cost after iteration 1220: 0.187990
Cost after iteration 1221: 0.187928
Cost after iteration 1222: 0.187867
Cost after iteration 1223: 0.187808
Cost after iteration 1224: 0.187751
Cost after iteration 1225: 0.187696
Cost after iteration 1226: 0.187641
Cost after iteration 1227: 0.187589
Cost after iteration 1228: 0.187537
Cost after iteration 1229: 0.187487
Cost after iteration 1230: 0.187438
Cost after iteration 1231: 0.187390
Cost after iteration 1232: 0.187344
Cost after iteration 1233: 0.187298
Cost after iteration 1234: 0.187254
Cost after iteration 1235: 0.187211
Cost after iteration 1236: 0.187168
Cost after iteration 1237: 0.187126
Cost after iteration 1238: 0.187086
Cost after iteration 1239: 0.187046
Cost after iteration 1240: 0.187007
Cost after iteration 1241: 0.186969
Cost after iteration 1242: 0.186931
Cost after iteration 1243: 0.186894
Cost after iteration 1244: 0.186858
Cost after iteration 1245: 0.186823
Cost after iteration 1246: 0.186788
```

```
Cost after iteration 1247: 0.186754
Cost after iteration 1248: 0.186721
Cost after iteration 1249: 0.186688
Cost after iteration 1250: 0.186656
Cost after iteration 1251: 0.186624
Cost after iteration 1252: 0.186593
Cost after iteration 1253: 0.186562
Cost after iteration 1254: 0.186532
Cost after iteration 1255: 0.186502
Cost after iteration 1256: 0.186473
Cost after iteration 1257: 0.186444
Cost after iteration 1258: 0.186415
Cost after iteration 1259: 0.186388
Cost after iteration 1260: 0.186360
Cost after iteration 1261: 0.186333
Cost after iteration 1262: 0.186306
Cost after iteration 1263: 0.186280
Cost after iteration 1264: 0.186254
Cost after iteration 1265: 0.186228
Cost after iteration 1266: 0.186203
Cost after iteration 1267: 0.186178
Cost after iteration 1268: 0.186153
Cost after iteration 1269: 0.186129
Cost after iteration 1270: 0.186105
Cost after iteration 1271: 0.186081
Cost after iteration 1272: 0.186058
Cost after iteration 1273: 0.186034
Cost after iteration 1274: 0.186012
Cost after iteration 1275: 0.185989
Cost after iteration 1276: 0.185967
Cost after iteration 1277: 0.185945
Cost after iteration 1278: 0.185923
Cost after iteration 1279: 0.185901
Cost after iteration 1280: 0.185880
Cost after iteration 1281: 0.185859
Cost after iteration 1282: 0.185838
Cost after iteration 1283: 0.185817
Cost after iteration 1284: 0.185797
Cost after iteration 1285: 0.185777
Cost after iteration 1286: 0.185757
Cost after iteration 1287: 0.185737
Cost after iteration 1288: 0.185717
Cost after iteration 1289: 0.185698
Cost after iteration 1290: 0.185679
Cost after iteration 1291: 0.185660
Cost after iteration 1292: 0.185641
Cost after iteration 1293: 0.185622
Cost after iteration 1294: 0.185604
Cost after iteration 1295: 0.185585
```

```
Cost after iteration 1296: 0.185567
Cost after iteration 1297: 0.185549
Cost after iteration 1298: 0.185531
Cost after iteration 1299: 0.185514
Cost after iteration 1300: 0.185497
Cost after iteration 1301: 0.185480
Cost after iteration 1302: 0.185465
Cost after iteration 1303: 0.185451
Cost after iteration 1304: 0.185443
Cost after iteration 1305: 0.185445
Cost after iteration 1306: 0.185470
Cost after iteration 1307: 0.185547
Cost after iteration 1308: 0.185748
Cost after iteration 1309: 0.186209
Cost after iteration 1310: 0.187378
Cost after iteration 1311: 0.189879
Cost after iteration 1312: 0.196918
Cost after iteration 1313: 0.211194
Cost after iteration 1314: 0.258430
Cost after iteration 1315: 0.319802
Cost after iteration 1316: 0.634904
Cost after iteration 1317: 0.652085
Cost after iteration 1318: 1.505023
Cost after iteration 1319: 0.415242
Cost after iteration 1320: 0.228158
Cost after iteration 1321: 0.213070
Cost after iteration 1322: 0.208571
Cost after iteration 1323: 0.206342
Cost after iteration 1324: 0.204869
Cost after iteration 1325: 0.203631
Cost after iteration 1326: 0.202525
Cost after iteration 1327: 0.201518
Cost after iteration 1328: 0.200594
Cost after iteration 1329: 0.199743
Cost after iteration 1330: 0.198957
Cost after iteration 1331: 0.198229
Cost after iteration 1332: 0.197554
Cost after iteration 1333: 0.196927
Cost after iteration 1334: 0.196344
Cost after iteration 1335: 0.195800
Cost after iteration 1336: 0.195293
Cost after iteration 1337: 0.194819
Cost after iteration 1338: 0.194376
Cost after iteration 1339: 0.193961
Cost after iteration 1340: 0.193573
Cost after iteration 1341: 0.193208
Cost after iteration 1342: 0.192866
Cost after iteration 1343: 0.192545
Cost after iteration 1344: 0.192243
```

```
Cost after iteration 1345: 0.191959
Cost after iteration 1346: 0.191691
Cost after iteration 1347: 0.191439
Cost after iteration 1348: 0.191202
Cost after iteration 1349: 0.190977
Cost after iteration 1350: 0.190766
Cost after iteration 1351: 0.190566
Cost after iteration 1352: 0.190377
Cost after iteration 1353: 0.190198
Cost after iteration 1354: 0.190028
Cost after iteration 1355: 0.189868
Cost after iteration 1356: 0.189716
Cost after iteration 1357: 0.189571
Cost after iteration 1358: 0.189434
Cost after iteration 1359: 0.189304
Cost after iteration 1360: 0.189180
Cost after iteration 1361: 0.189062
Cost after iteration 1362: 0.188950
Cost after iteration 1363: 0.188842
Cost after iteration 1364: 0.188740
Cost after iteration 1365: 0.188643
Cost after iteration 1366: 0.188549
Cost after iteration 1367: 0.188460
Cost after iteration 1368: 0.188375
Cost after iteration 1369: 0.188293
Cost after iteration 1370: 0.188214
Cost after iteration 1371: 0.188139
Cost after iteration 1372: 0.188067
Cost after iteration 1373: 0.187997
Cost after iteration 1374: 0.187930
Cost after iteration 1375: 0.187866
Cost after iteration 1376: 0.187803
Cost after iteration 1377: 0.187744
Cost after iteration 1378: 0.187686
Cost after iteration 1379: 0.187630
Cost after iteration 1380: 0.187576
Cost after iteration 1381: 0.187523
Cost after iteration 1382: 0.187472
Cost after iteration 1383: 0.187423
Cost after iteration 1384: 0.187376
Cost after iteration 1385: 0.187329
Cost after iteration 1386: 0.187284
Cost after iteration 1387: 0.187241
Cost after iteration 1388: 0.187198
Cost after iteration 1389: 0.187157
Cost after iteration 1390: 0.187116
Cost after iteration 1391: 0.187077
Cost after iteration 1392: 0.187039
Cost after iteration 1393: 0.187001
```

```
Cost after iteration 1394: 0.186964
Cost after iteration 1395: 0.186929
Cost after iteration 1396: 0.186894
Cost after iteration 1397: 0.186860
Cost after iteration 1398: 0.186826
Cost after iteration 1399: 0.186793
Cost after iteration 1400: 0.186761
Cost after iteration 1401: 0.186730
Cost after iteration 1402: 0.186699
Cost after iteration 1403: 0.186668
Cost after iteration 1404: 0.186639
Cost after iteration 1405: 0.186610
Cost after iteration 1406: 0.186581
Cost after iteration 1407: 0.186553
Cost after iteration 1408: 0.186525
Cost after iteration 1409: 0.186498
Cost after iteration 1410: 0.186471
Cost after iteration 1411: 0.186444
Cost after iteration 1412: 0.186418
Cost after iteration 1413: 0.186393
Cost after iteration 1414: 0.186367
Cost after iteration 1415: 0.186343
Cost after iteration 1416: 0.186318
Cost after iteration 1417: 0.186294
Cost after iteration 1418: 0.186270
Cost after iteration 1419: 0.186247
Cost after iteration 1420: 0.186223
Cost after iteration 1421: 0.186201
Cost after iteration 1422: 0.186178
Cost after iteration 1423: 0.186156
Cost after iteration 1424: 0.186134
Cost after iteration 1425: 0.186112
Cost after iteration 1426: 0.186090
Cost after iteration 1427: 0.186069
Cost after iteration 1428: 0.186048
Cost after iteration 1429: 0.186027
Cost after iteration 1430: 0.186007
Cost after iteration 1431: 0.185987
Cost after iteration 1432: 0.185967
Cost after iteration 1433: 0.185947
Cost after iteration 1434: 0.185928
Cost after iteration 1435: 0.185909
Cost after iteration 1436: 0.185891
Cost after iteration 1437: 0.185874
Cost after iteration 1438: 0.185859
Cost after iteration 1439: 0.185848
Cost after iteration 1440: 0.185844
Cost after iteration 1441: 0.185855
Cost after iteration 1442: 0.185890
```

```
Cost after iteration 1443: 0.185977
Cost after iteration 1444: 0.186160
Cost after iteration 1445: 0.186541
Cost after iteration 1446: 0.187281
Cost after iteration 1447: 0.188833
Cost after iteration 1448: 0.191761
Cost after iteration 1449: 0.198368
Cost after iteration 1450: 0.210736
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Cost after iteration 1453: 0.557842
Cost after iteration 1454: 0.546646
Cost after iteration 1455: 1.050335
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Cost after iteration 1457: 0.258693
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Cost after iteration 1459: 0.200956
Cost after iteration 1460: 0.198129
Cost after iteration 1461: 0.196844
Cost after iteration 1462: 0.196030
Cost after iteration 1463: 0.195438
Cost after iteration 1464: 0.194941
Cost after iteration 1465: 0.194499
Cost after iteration 1466: 0.194092
Cost after iteration 1467: 0.193712
Cost after iteration 1468: 0.193355
Cost after iteration 1469: 0.193018
Cost after iteration 1470: 0.192700
Cost after iteration 1471: 0.192399
Cost after iteration 1472: 0.192114
Cost after iteration 1473: 0.191844
Cost after iteration 1474: 0.191588
Cost after iteration 1475: 0.191345
Cost after iteration 1476: 0.191115
Cost after iteration 1477: 0.190896
Cost after iteration 1478: 0.190688
Cost after iteration 1479: 0.190491
Cost after iteration 1480: 0.190303
Cost after iteration 1481: 0.190124
Cost after iteration 1482: 0.189954
Cost after iteration 1483: 0.189792
Cost after iteration 1484: 0.189637
Cost after iteration 1485: 0.189490
Cost after iteration 1486: 0.189350
Cost after iteration 1487: 0.189216
Cost after iteration 1488: 0.189088
Cost after iteration 1489: 0.188966
Cost after iteration 1490: 0.188849
Cost after iteration 1491: 0.188737
```

```
Cost after iteration 1492: 0.188631
Cost after iteration 1493: 0.188528
Cost after iteration 1494: 0.188430
Cost after iteration 1495: 0.188336
Cost after iteration 1496: 0.188246
Cost after iteration 1497: 0.188160
Cost after iteration 1498: 0.188077
Cost after iteration 1499: 0.187998
Cost after iteration 1500: 0.187921
Cost after iteration 1501: 0.187848
Cost after iteration 1502: 0.187777
Cost after iteration 1503: 0.187709
Cost after iteration 1504: 0.187643
Cost after iteration 1505: 0.187580
Cost after iteration 1506: 0.187519
Cost after iteration 1507: 0.187460
Cost after iteration 1508: 0.187403
Cost after iteration 1509: 0.187348
Cost after iteration 1510: 0.187295
Cost after iteration 1511: 0.187244
Cost after iteration 1512: 0.187195
Cost after iteration 1513: 0.187147
Cost after iteration 1514: 0.187100
Cost after iteration 1515: 0.187055
Cost after iteration 1516: 0.187011
Cost after iteration 1517: 0.186969
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Cost after iteration 1519: 0.186888
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Cost after iteration 1521: 0.186811
Cost after iteration 1522: 0.186774
Cost after iteration 1523: 0.186738
Cost after iteration 1524: 0.186704
Cost after iteration 1525: 0.186670
Cost after iteration 1526: 0.186637
Cost after iteration 1527: 0.186604
Cost after iteration 1528: 0.186573
Cost after iteration 1529: 0.186542
Cost after iteration 1530: 0.186512
Cost after iteration 1531: 0.186483
Cost after iteration 1532: 0.186454
Cost after iteration 1533: 0.186426
Cost after iteration 1534: 0.186398
Cost after iteration 1535: 0.186371
Cost after iteration 1536: 0.186345
Cost after iteration 1537: 0.186319
Cost after iteration 1538: 0.186294
Cost after iteration 1539: 0.186269
Cost after iteration 1540: 0.186245
```

```
Cost after iteration 1541: 0.186221
Cost after iteration 1542: 0.186198
Cost after iteration 1543: 0.186175
Cost after iteration 1544: 0.186153
Cost after iteration 1545: 0.186132
Cost after iteration 1546: 0.186112
Cost after iteration 1547: 0.186095
Cost after iteration 1548: 0.186080
Cost after iteration 1549: 0.186072
Cost after iteration 1550: 0.186072
Cost after iteration 1551: 0.186093
Cost after iteration 1552: 0.186143
Cost after iteration 1553: 0.186263
Cost after iteration 1554: 0.186480
Cost after iteration 1555: 0.186967
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Cost after iteration 1566: 0.389328
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Cost after iteration 1568: 0.333014
Cost after iteration 1569: 0.351028
Cost after iteration 1570: 0.253918
Cost after iteration 1571: 0.229664
Cost after iteration 1572: 0.208035
Cost after iteration 1573: 0.199153
Cost after iteration 1574: 0.195457
Cost after iteration 1575: 0.194011
Cost after iteration 1576: 0.193314
Cost after iteration 1577: 0.192889
Cost after iteration 1578: 0.192560
Cost after iteration 1579: 0.192276
Cost after iteration 1580: 0.192017
Cost after iteration 1581: 0.191774
Cost after iteration 1582: 0.191545
Cost after iteration 1583: 0.191328
Cost after iteration 1584: 0.191121
Cost after iteration 1585: 0.190925
Cost after iteration 1586: 0.190737
Cost after iteration 1587: 0.190558
Cost after iteration 1588: 0.190388
Cost after iteration 1589: 0.190225
Cost after iteration 1590: 0.190070
```

```
Cost after iteration 1591: 0.189921
Cost after iteration 1592: 0.189779
Cost after iteration 1593: 0.189643
Cost after iteration 1594: 0.189514
Cost after iteration 1595: 0.189389
Cost after iteration 1596: 0.189270
Cost after iteration 1597: 0.189156
Cost after iteration 1598: 0.189046
Cost after iteration 1599: 0.188941
Cost after iteration 1600: 0.188840
Cost after iteration 1601: 0.188743
Cost after iteration 1602: 0.188650
Cost after iteration 1603: 0.188561
Cost after iteration 1604: 0.188475
Cost after iteration 1605: 0.188392
Cost after iteration 1606: 0.188312
Cost after iteration 1607: 0.188236
Cost after iteration 1608: 0.188162
Cost after iteration 1609: 0.188090
Cost after iteration 1610: 0.188021
Cost after iteration 1611: 0.187955
Cost after iteration 1612: 0.187891
Cost after iteration 1613: 0.187828
Cost after iteration 1614: 0.187768
Cost after iteration 1615: 0.187710
Cost after iteration 1616: 0.187654
Cost after iteration 1617: 0.187600
Cost after iteration 1618: 0.187547
Cost after iteration 1619: 0.187496
Cost after iteration 1620: 0.187446
Cost after iteration 1621: 0.187398
Cost after iteration 1622: 0.187351
Cost after iteration 1623: 0.187306
Cost after iteration 1624: 0.187261
Cost after iteration 1625: 0.187218
Cost after iteration 1626: 0.187177
Cost after iteration 1627: 0.187136
Cost after iteration 1628: 0.187096
Cost after iteration 1629: 0.187058
Cost after iteration 1630: 0.187020
Cost after iteration 1631: 0.186984
Cost after iteration 1632: 0.186948
Cost after iteration 1633: 0.186913
Cost after iteration 1634: 0.186879
Cost after iteration 1635: 0.186845
Cost after iteration 1636: 0.186813
Cost after iteration 1637: 0.186781
Cost after iteration 1638: 0.186750
Cost after iteration 1639: 0.186719
```

```
Cost after iteration 1640: 0.186689
Cost after iteration 1641: 0.186660
Cost after iteration 1642: 0.186632
Cost after iteration 1643: 0.186603
Cost after iteration 1644: 0.186576
Cost after iteration 1645: 0.186549
Cost after iteration 1646: 0.186522
Cost after iteration 1647: 0.186496
Cost after iteration 1648: 0.186471
Cost after iteration 1649: 0.186446
Cost after iteration 1650: 0.186421
Cost after iteration 1651: 0.186397
Cost after iteration 1652: 0.186373
Cost after iteration 1653: 0.186350
Cost after iteration 1654: 0.186327
Cost after iteration 1655: 0.186304
Cost after iteration 1656: 0.186282
Cost after iteration 1657: 0.186261
Cost after iteration 1658: 0.186240
Cost after iteration 1659: 0.186220
Cost after iteration 1660: 0.186201
Cost after iteration 1661: 0.186183
Cost after iteration 1662: 0.186169
Cost after iteration 1663: 0.186159
Cost after iteration 1664: 0.186157
Cost after iteration 1665: 0.186171
Cost after iteration 1666: 0.186208
Cost after iteration 1667: 0.186300
Cost after iteration 1668: 0.186467
Cost after iteration 1669: 0.186834
Cost after iteration 1670: 0.187438
Cost after iteration 1671: 0.188864
Cost after iteration 1672: 0.191033
Cost after iteration 1673: 0.196987
Cost after iteration 1674: 0.205341
Cost after iteration 1675: 0.232050
Cost after iteration 1676: 0.251492
Cost after iteration 1677: 0.352629
Cost after iteration 1678: 0.312912
Cost after iteration 1679: 0.452288
Cost after iteration 1680: 0.322346
Cost after iteration 1681: 0.453834
Cost after iteration 1682: 0.414140
Cost after iteration 1683: 0.473850
Cost after iteration 1684: 0.280472
Cost after iteration 1685: 0.259887
Cost after iteration 1686: 0.218325
Cost after iteration 1687: 0.203710
Cost after iteration 1688: 0.197041
```

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Cost after iteration 1689: 0.194558
Cost after iteration 1690: 0.193504
Cost after iteration 1691: 0.192973
Cost after iteration 1692: 0.192607
Cost after iteration 1693: 0.192314
Cost after iteration 1694: 0.192053
Cost after iteration 1695: 0.191814
Cost after iteration 1696: 0.191589
Cost after iteration 1697: 0.191376
Cost after iteration 1698: 0.191173
Cost after iteration 1699: 0.190981
Cost after iteration 1700: 0.190797
Cost after iteration 1701: 0.190622
Cost after iteration 1702: 0.190455
Cost after iteration 1703: 0.190295
Cost after iteration 1704: 0.190142
Cost after iteration 1705: 0.189996
Cost after iteration 1706: 0.189857
Cost after iteration 1707: 0.189723
Cost after iteration 1708: 0.189595
Cost after iteration 1709: 0.189472
Cost after iteration 1710: 0.189354
Cost after iteration 1711: 0.189242
Cost after iteration 1712: 0.189133
Cost after iteration 1713: 0.189029
Cost after iteration 1714: 0.188929
Cost after iteration 1715: 0.188834
Cost after iteration 1716: 0.188741
Cost after iteration 1717: 0.188653
Cost after iteration 1718: 0.188567
Cost after iteration 1719: 0.188485
Cost after iteration 1720: 0.188406
Cost after iteration 1721: 0.188329
Cost after iteration 1722: 0.188256
Cost after iteration 1723: 0.188185
Cost after iteration 1724: 0.188116
Cost after iteration 1725: 0.188050
Cost after iteration 1726: 0.187986
Cost after iteration 1727: 0.187924
Cost after iteration 1728: 0.187864
Cost after iteration 1729: 0.187806
Cost after iteration 1730: 0.187749
Cost after iteration 1731: 0.187695
Cost after iteration 1732: 0.187642
Cost after iteration 1733: 0.187591
Cost after iteration 1734: 0.187541
Cost after iteration 1735: 0.187493
Cost after iteration 1736: 0.187446
Cost after iteration 1737: 0.187401
```

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Cost after iteration 1738: 0.187357
Cost after iteration 1739: 0.187314
Cost after iteration 1740: 0.187272
Cost after iteration 1741: 0.187231
Cost after iteration 1742: 0.187191
Cost after iteration 1743: 0.187153
Cost after iteration 1744: 0.187115
Cost after iteration 1745: 0.187078
Cost after iteration 1746: 0.187042
Cost after iteration 1747: 0.187007
Cost after iteration 1748: 0.186973
Cost after iteration 1749: 0.186940
Cost after iteration 1750: 0.186907
Cost after iteration 1751: 0.186875
Cost after iteration 1752: 0.186844
Cost after iteration 1753: 0.186813
Cost after iteration 1754: 0.186783
Cost after iteration 1755: 0.186754
Cost after iteration 1756: 0.186725
Cost after iteration 1757: 0.186697
Cost after iteration 1758: 0.186669
Cost after iteration 1759: 0.186642
Cost after iteration 1760: 0.186616
Cost after iteration 1761: 0.186590
Cost after iteration 1762: 0.186564
Cost after iteration 1763: 0.186539
Cost after iteration 1764: 0.186514
Cost after iteration 1765: 0.186490
Cost after iteration 1766: 0.186466
Cost after iteration 1767: 0.186443
Cost after iteration 1768: 0.186420
Cost after iteration 1769: 0.186398
Cost after iteration 1770: 0.186376
Cost after iteration 1771: 0.186355
Cost after iteration 1772: 0.186335
Cost after iteration 1773: 0.186316
Cost after iteration 1774: 0.186299
Cost after iteration 1775: 0.186286
Cost after iteration 1776: 0.186278
Cost after iteration 1777: 0.186281
Cost after iteration 1778: 0.186300
Cost after iteration 1779: 0.186355
Cost after iteration 1780: 0.186460
Cost after iteration 1781: 0.186692
Cost after iteration 1782: 0.187079
Cost after iteration 1783: 0.187968
Cost after iteration 1784: 0.189328
Cost after iteration 1785: 0.192889
Cost after iteration 1786: 0.197960
```

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Cost after iteration 1787: 0.213608
Cost after iteration 1788: 0.230229
Cost after iteration 1789: 0.296097
Cost after iteration 1790: 0.290911
Cost after iteration 1791: 0.426852
Cost after iteration 1792: 0.285298
Cost after iteration 1793: 0.337574
Cost after iteration 1794: 0.296505
Cost after iteration 1795: 0.403276
Cost after iteration 1796: 0.376783
Cost after iteration 1797: 0.430272
Cost after iteration 1798: 0.284064
Cost after iteration 1799: 0.267367
Cost after iteration 1800: 0.219959
Cost after iteration 1801: 0.204246
Cost after iteration 1802: 0.196927
Cost after iteration 1803: 0.194273
Cost after iteration 1804: 0.193177
Cost after iteration 1805: 0.192647
Cost after iteration 1806: 0.192293
Cost after iteration 1807: 0.192014
Cost after iteration 1808: 0.191771
Cost after iteration 1809: 0.191548
Cost after iteration 1810: 0.191339
Cost after iteration 1811: 0.191143
Cost after iteration 1812: 0.190956
Cost after iteration 1813: 0.190778
Cost after iteration 1814: 0.190608
Cost after iteration 1815: 0.190446
Cost after iteration 1816: 0.190291
Cost after iteration 1817: 0.190142
Cost after iteration 1818: 0.190001
Cost after iteration 1819: 0.189865
Cost after iteration 1820: 0.189735
Cost after iteration 1821: 0.189610
Cost after iteration 1822: 0.189491
Cost after iteration 1823: 0.189376
Cost after iteration 1824: 0.189266
Cost after iteration 1825: 0.189160
Cost after iteration 1826: 0.189059
Cost after iteration 1827: 0.188961
Cost after iteration 1828: 0.188867
Cost after iteration 1829: 0.188777
Cost after iteration 1830: 0.188690
Cost after iteration 1831: 0.188607
Cost after iteration 1832: 0.188526
Cost after iteration 1833: 0.188448
Cost after iteration 1834: 0.188373
Cost after iteration 1835: 0.188301
```

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Cost after iteration 1836: 0.188231
Cost after iteration 1837: 0.188164
Cost after iteration 1838: 0.188099
Cost after iteration 1839: 0.188036
Cost after iteration 1840: 0.187975
Cost after iteration 1841: 0.187916
Cost after iteration 1842: 0.187858
Cost after iteration 1843: 0.187803
Cost after iteration 1844: 0.187749
Cost after iteration 1845: 0.187697
Cost after iteration 1846: 0.187647
Cost after iteration 1847: 0.187598
Cost after iteration 1848: 0.187550
Cost after iteration 1849: 0.187504
Cost after iteration 1850: 0.187459
Cost after iteration 1851: 0.187415
Cost after iteration 1852: 0.187373
Cost after iteration 1853: 0.187332
Cost after iteration 1854: 0.187291
Cost after iteration 1855: 0.187252
Cost after iteration 1856: 0.187214
Cost after iteration 1857: 0.187176
Cost after iteration 1858: 0.187140
Cost after iteration 1859: 0.187104
Cost after iteration 1860: 0.187070
Cost after iteration 1861: 0.187036
Cost after iteration 1862: 0.187003
Cost after iteration 1863: 0.186970
Cost after iteration 1864: 0.186939
Cost after iteration 1865: 0.186908
Cost after iteration 1866: 0.186878
Cost after iteration 1867: 0.186848
Cost after iteration 1868: 0.186819
Cost after iteration 1869: 0.186791
Cost after iteration 1870: 0.186763
Cost after iteration 1871: 0.186735
Cost after iteration 1872: 0.186709
Cost after iteration 1873: 0.186682
Cost after iteration 1874: 0.186657
Cost after iteration 1875: 0.186632
Cost after iteration 1876: 0.186607
Cost after iteration 1877: 0.186583
Cost after iteration 1878: 0.186560
Cost after iteration 1879: 0.186538
Cost after iteration 1880: 0.186517
Cost after iteration 1881: 0.186497
Cost after iteration 1882: 0.186481
Cost after iteration 1883: 0.186469
Cost after iteration 1884: 0.186463
```

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Cost after iteration 1885: 0.186472
Cost after iteration 1886: 0.186499
Cost after iteration 1887: 0.186568
Cost after iteration 1888: 0.186692
Cost after iteration 1889: 0.186963
Cost after iteration 1890: 0.187390
Cost after iteration 1891: 0.188374
Cost after iteration 1892: 0.189800
Cost after iteration 1893: 0.193573
Cost after iteration 1894: 0.198653
Cost after iteration 1895: 0.214460
Cost after iteration 1896: 0.229680
Cost after iteration 1897: 0.291834
Cost after iteration 1898: 0.282692
Cost after iteration 1899: 0.401925
Cost after iteration 1900: 0.272057
Cost after iteration 1901: 0.307912
Cost after iteration 1902: 0.250121
Cost after iteration 1903: 0.286740
Cost after iteration 1904: 0.299703
Cost after iteration 1905: 0.389568
Cost after iteration 1906: 0.333911
Cost after iteration 1907: 0.363672
Cost after iteration 1908: 0.262326
Cost after iteration 1909: 0.242693
Cost after iteration 1910: 0.211910
Cost after iteration 1911: 0.200243
Cost after iteration 1912: 0.195055
Cost after iteration 1913: 0.193208
Cost after iteration 1914: 0.192393
Cost after iteration 1915: 0.191969
Cost after iteration 1916: 0.191670
Cost after iteration 1917: 0.191430
Cost after iteration 1918: 0.191219
Cost after iteration 1919: 0.191026
Cost after iteration 1920: 0.190845
Cost after iteration 1921: 0.190675
Cost after iteration 1922: 0.190513
Cost after iteration 1923: 0.190359
Cost after iteration 1924: 0.190211
Cost after iteration 1925: 0.190070
Cost after iteration 1926: 0.189935
Cost after iteration 1927: 0.189806
Cost after iteration 1928: 0.189683
Cost after iteration 1929: 0.189564
Cost after iteration 1930: 0.189450
Cost after iteration 1931: 0.189341
Cost after iteration 1932: 0.189236
Cost after iteration 1933: 0.189135
```

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Cost after iteration 1934: 0.189038
Cost after iteration 1935: 0.188945
Cost after iteration 1936: 0.188855
Cost after iteration 1937: 0.188768
Cost after iteration 1938: 0.188685
Cost after iteration 1939: 0.188605
Cost after iteration 1940: 0.188528
Cost after iteration 1941: 0.188453
Cost after iteration 1942: 0.188381
Cost after iteration 1943: 0.188311
Cost after iteration 1944: 0.188244
Cost after iteration 1945: 0.188179
Cost after iteration 1946: 0.188116
Cost after iteration 1947: 0.188056
Cost after iteration 1948: 0.187997
Cost after iteration 1949: 0.187940
Cost after iteration 1950: 0.187884
Cost after iteration 1951: 0.187831
Cost after iteration 1952: 0.187779
Cost after iteration 1953: 0.187729
Cost after iteration 1954: 0.187680
Cost after iteration 1955: 0.187632
Cost after iteration 1956: 0.187586
Cost after iteration 1957: 0.187541
Cost after iteration 1958: 0.187497
Cost after iteration 1959: 0.187455
Cost after iteration 1960: 0.187413
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Cost after iteration 1962: 0.187334
Cost after iteration 1963: 0.187296
Cost after iteration 1964: 0.187258
Cost after iteration 1965: 0.187222
Cost after iteration 1966: 0.187187
Cost after iteration 1967: 0.187152
Cost after iteration 1968: 0.187118
Cost after iteration 1969: 0.187085
Cost after iteration 1970: 0.187053
Cost after iteration 1971: 0.187021
Cost after iteration 1972: 0.186990
Cost after iteration 1973: 0.186960
Cost after iteration 1974: 0.186930
Cost after iteration 1975: 0.186902
Cost after iteration 1976: 0.186873
Cost after iteration 1977: 0.186846
Cost after iteration 1978: 0.186819
Cost after iteration 1979: 0.186793
Cost after iteration 1980: 0.186768
Cost after iteration 1981: 0.186744
Cost after iteration 1982: 0.186721
```

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Cost after iteration 1983: 0.186701
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Cost after iteration 1985: 0.186671
Cost after iteration 1986: 0.186666
Cost after iteration 1987: 0.186674
Cost after iteration 1988: 0.186700
Cost after iteration 1989: 0.186766
Cost after iteration 1990: 0.186878
Cost after iteration 1991: 0.187122
Cost after iteration 1992: 0.187485
Cost after iteration 1993: 0.188307
Cost after iteration 1994: 0.189435
Cost after iteration 1995: 0.192342
Cost after iteration 1996: 0.196047
Cost after iteration 1997: 0.207292
Cost after iteration 1998: 0.218876
Cost after iteration 1999: 0.261419
Cost after iteration 2000: 0.262678
Cost after iteration 2001: 0.360249
Cost after iteration 2002: 0.273468
Cost after iteration 2003: 0.326402
Cost after iteration 2004: 0.238628
Cost after iteration 2005: 0.249038
Cost after iteration 2006: 0.238699
Cost after iteration 2007: 0.265228
Cost after iteration 2008: 0.272905
Cost after iteration 2009: 0.334248
Cost after iteration 2010: 0.304056
Cost after iteration 2011: 0.340634
Cost after iteration 2012: 0.263585
Cost after iteration 2013: 0.253105
Cost after iteration 2014: 0.217542
Cost after iteration 2015: 0.203890
Cost after iteration 2016: 0.196234
Cost after iteration 2017: 0.193323
Cost after iteration 2018: 0.192061
Cost after iteration 2019: 0.191504
Cost after iteration 2020: 0.191161
Cost after iteration 2021: 0.190919
Cost after iteration 2022: 0.190719
Cost after iteration 2023: 0.190544
Cost after iteration 2024: 0.190383
Cost after iteration 2025: 0.190233
Cost after iteration 2026: 0.190092
Cost after iteration 2027: 0.189958
Cost after iteration 2028: 0.189830
Cost after iteration 2029: 0.189708
Cost after iteration 2030: 0.189591
Cost after iteration 2031: 0.189478
```

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Cost after iteration 2032: 0.189371
Cost after iteration 2033: 0.189268
Cost after iteration 2034: 0.189169
Cost after iteration 2035: 0.189073
Cost after iteration 2036: 0.188982
Cost after iteration 2037: 0.188893
Cost after iteration 2038: 0.188808
Cost after iteration 2039: 0.188727
Cost after iteration 2040: 0.188648
Cost after iteration 2041: 0.188572
Cost after iteration 2042: 0.188498
Cost after iteration 2043: 0.188427
Cost after iteration 2044: 0.188359
Cost after iteration 2045: 0.188293
Cost after iteration 2046: 0.188229
Cost after iteration 2047: 0.188167
Cost after iteration 2048: 0.188107
Cost after iteration 2049: 0.188049
Cost after iteration 2050: 0.187993
Cost after iteration 2051: 0.187939
Cost after iteration 2052: 0.187886
Cost after iteration 2053: 0.187835
Cost after iteration 2054: 0.187785
Cost after iteration 2055: 0.187737
Cost after iteration 2056: 0.187690
Cost after iteration 2057: 0.187644
Cost after iteration 2058: 0.187600
Cost after iteration 2059: 0.187557
Cost after iteration 2060: 0.187515
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Cost after iteration 2065: 0.187321
Cost after iteration 2066: 0.187285
Cost after iteration 2067: 0.187250
Cost after iteration 2068: 0.187215
Cost after iteration 2069: 0.187182
Cost after iteration 2070: 0.187149
Cost after iteration 2071: 0.187118
Cost after iteration 2072: 0.187087
Cost after iteration 2073: 0.187056
Cost after iteration 2074: 0.187027
Cost after iteration 2075: 0.186998
Cost after iteration 2076: 0.186971
Cost after iteration 2077: 0.186944
Cost after iteration 2078: 0.186920
Cost after iteration 2079: 0.186897
Cost after iteration 2080: 0.186876
```

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Cost after iteration 2081: 0.186860
Cost after iteration 2082: 0.186849
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Cost after iteration 2085: 0.186894
Cost after iteration 2086: 0.186958
Cost after iteration 2087: 0.187098
Cost after iteration 2088: 0.187302
Cost after iteration 2089: 0.187743
Cost after iteration 2090: 0.188329
Cost after iteration 2091: 0.189726
Cost after iteration 2092: 0.191440
Cost after iteration 2093: 0.196195
Cost after iteration 2094: 0.201579
Cost after iteration 2095: 0.218902
Cost after iteration 2096: 0.230399
Cost after iteration 2097: 0.285613
Cost after iteration 2098: 0.267197
Cost after iteration 2099: 0.351648
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Cost after iteration 2101: 0.279515
Cost after iteration 2102: 0.226021
Cost after iteration 2103: 0.231968
Cost after iteration 2104: 0.225564
Cost after iteration 2105: 0.238038
Cost after iteration 2106: 0.240736
Cost after iteration 2107: 0.270224
Cost after iteration 2108: 0.267857
Cost after iteration 2109: 0.309453
Cost after iteration 2110: 0.272275
Cost after iteration 2111: 0.287382
Cost after iteration 2112: 0.239461
Cost after iteration 2113: 0.227175
Cost after iteration 2114: 0.207787
Cost after iteration 2115: 0.199021
Cost after iteration 2116: 0.194110
Cost after iteration 2117: 0.192160
Cost after iteration 2118: 0.191206
Cost after iteration 2119: 0.190746
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Cost after iteration 2121: 0.190236
Cost after iteration 2122: 0.190062
Cost after iteration 2123: 0.189913
Cost after iteration 2124: 0.189777
Cost after iteration 2125: 0.189653
Cost after iteration 2126: 0.189535
Cost after iteration 2127: 0.189424
Cost after iteration 2128: 0.189319
Cost after iteration 2129: 0.189218
Cost after iteration 2130: 0.189122
```

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Cost after iteration 2131: 0.189029
Cost after iteration 2132: 0.188941
Cost after iteration 2133: 0.188855
Cost after iteration 2134: 0.188773
Cost after iteration 2135: 0.188694
Cost after iteration 2136: 0.188618
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Cost after iteration 2139: 0.188406
Cost after iteration 2140: 0.188339
Cost after iteration 2141: 0.188276
Cost after iteration 2142: 0.188214
Cost after iteration 2143: 0.188154
Cost after iteration 2144: 0.188096
Cost after iteration 2145: 0.188040
Cost after iteration 2146: 0.187986
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Cost after iteration 2148: 0.187882
Cost after iteration 2149: 0.187833
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Cost after iteration 2153: 0.187648
Cost after iteration 2154: 0.187605
Cost after iteration 2155: 0.187564
Cost after iteration 2156: 0.187523
Cost after iteration 2157: 0.187483
Cost after iteration 2158: 0.187445
Cost after iteration 2159: 0.187407
Cost after iteration 2160: 0.187371
Cost after iteration 2161: 0.187336
Cost after iteration 2162: 0.187301
Cost after iteration 2163: 0.187268
Cost after iteration 2164: 0.187235
Cost after iteration 2165: 0.187204
Cost after iteration 2166: 0.187174
Cost after iteration 2167: 0.187146
Cost after iteration 2168: 0.187119
Cost after iteration 2169: 0.187095
Cost after iteration 2170: 0.187074
Cost after iteration 2171: 0.187058
Cost after iteration 2172: 0.187047
Cost after iteration 2173: 0.187046
Cost after iteration 2174: 0.187055
Cost after iteration 2175: 0.187091
Cost after iteration 2176: 0.187146
Cost after iteration 2177: 0.187269
Cost after iteration 2178: 0.187432
Cost after iteration 2179: 0.187785
```

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Cost after iteration 2180: 0.188213
Cost after iteration 2181: 0.189213
Cost after iteration 2182: 0.190340
Cost after iteration 2183: 0.193353
Cost after iteration 2184: 0.196514
Cost after iteration 2185: 0.206353
Cost after iteration 2186: 0.214558
Cost after iteration 2187: 0.245570
Cost after iteration 2188: 0.245156
Cost after iteration 2189: 0.309089
Cost after iteration 2190: 0.258523
Cost after iteration 2191: 0.305484
Cost after iteration 2192: 0.233892
Cost after iteration 2193: 0.241681
Cost after iteration 2194: 0.218602
Cost after iteration 2195: 0.222489
Cost after iteration 2196: 0.217962
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Cost after iteration 2198: 0.224431
Cost after iteration 2199: 0.237259
Cost after iteration 2200: 0.237129
Cost after iteration 2201: 0.257777
Cost after iteration 2202: 0.249213
Cost after iteration 2203: 0.268917
Cost after iteration 2204: 0.245074
Cost after iteration 2205: 0.248383
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Cost after iteration 2209: 0.197166
Cost after iteration 2210: 0.193309
Cost after iteration 2211: 0.191607
Cost after iteration 2212: 0.190649
Cost after iteration 2213: 0.190169
Cost after iteration 2214: 0.189848
Cost after iteration 2215: 0.189640
Cost after iteration 2216: 0.189470
Cost after iteration 2217: 0.189335
Cost after iteration 2218: 0.189214
Cost after iteration 2219: 0.189107
Cost after iteration 2220: 0.189008
Cost after iteration 2221: 0.188916
Cost after iteration 2222: 0.188828
Cost after iteration 2223: 0.188746
Cost after iteration 2224: 0.188667
Cost after iteration 2225: 0.188592
Cost after iteration 2226: 0.188519
Cost after iteration 2227: 0.188450
Cost after iteration 2228: 0.188383
```

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Cost after iteration 2229: 0.188319
Cost after iteration 2230: 0.188256
Cost after iteration 2231: 0.188196
Cost after iteration 2232: 0.188138
Cost after iteration 2233: 0.188082
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Cost after iteration 2237: 0.187875
Cost after iteration 2238: 0.187827
Cost after iteration 2239: 0.187781
Cost after iteration 2240: 0.187736
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Cost after iteration 2244: 0.187569
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Cost after iteration 2248: 0.187423
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Cost after iteration 2251: 0.187331
Cost after iteration 2252: 0.187305
Cost after iteration 2253: 0.187283
Cost after iteration 2254: 0.187263
Cost after iteration 2255: 0.187252
Cost after iteration 2256: 0.187244
Cost after iteration 2257: 0.187252
Cost after iteration 2258: 0.187267
Cost after iteration 2259: 0.187316
Cost after iteration 2260: 0.187376
Cost after iteration 2261: 0.187512
Cost after iteration 2262: 0.187667
Cost after iteration 2263: 0.188012
Cost after iteration 2264: 0.188375
Cost after iteration 2265: 0.189245
Cost after iteration 2266: 0.190099
Cost after iteration 2267: 0.192413
Cost after iteration 2268: 0.194541
Cost after iteration 2269: 0.201184
Cost after iteration 2270: 0.206487
Cost after iteration 2271: 0.225731
Cost after iteration 2272: 0.230056
Cost after iteration 2273: 0.273383
Cost after iteration 2274: 0.248198
Cost after iteration 2275: 0.294963
Cost after iteration 2276: 0.238042
Cost after iteration 2277: 0.253873
```

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Cost after iteration 2278: 0.219742
Cost after iteration 2279: 0.223726
Cost after iteration 2280: 0.214462
Cost after iteration 2281: 0.217819
Cost after iteration 2282: 0.214599
Cost after iteration 2283: 0.219572
Cost after iteration 2284: 0.218518
Cost after iteration 2285: 0.226388
Cost after iteration 2286: 0.225208
Cost after iteration 2287: 0.236101
Cost after iteration 2288: 0.231726
Cost after iteration 2289: 0.242600
Cost after iteration 2290: 0.232101
Cost after iteration 2291: 0.236569
Cost after iteration 2292: 0.222478
Cost after iteration 2293: 0.218481
Cost after iteration 2294: 0.207408
Cost after iteration 2295: 0.201652
Cost after iteration 2296: 0.196189
Cost after iteration 2297: 0.193434
Cost after iteration 2298: 0.191536
Cost after iteration 2299: 0.190589
Cost after iteration 2300: 0.189944
Cost after iteration 2301: 0.189583
Cost after iteration 2302: 0.189306
Cost after iteration 2303: 0.189123
Cost after iteration 2304: 0.188965
Cost after iteration 2305: 0.188845
Cost after iteration 2306: 0.188734
Cost after iteration 2307: 0.188642
Cost after iteration 2308: 0.188554
Cost after iteration 2309: 0.188476
Cost after iteration 2310: 0.188401
Cost after iteration 2311: 0.188333
Cost after iteration 2312: 0.188266
Cost after iteration 2313: 0.188205
Cost after iteration 2314: 0.188145
Cost after iteration 2315: 0.188088
Cost after iteration 2316: 0.188033
Cost after iteration 2317: 0.187982
Cost after iteration 2318: 0.187931
Cost after iteration 2319: 0.187883
Cost after iteration 2320: 0.187836
Cost after iteration 2321: 0.187792
Cost after iteration 2322: 0.187749
Cost after iteration 2323: 0.187709
Cost after iteration 2324: 0.187669
Cost after iteration 2325: 0.187633
Cost after iteration 2326: 0.187598
```

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Cost after iteration 2327: 0.187566
Cost after iteration 2328: 0.187535
Cost after iteration 2329: 0.187510
Cost after iteration 2330: 0.187485
Cost after iteration 2331: 0.187469
Cost after iteration 2332: 0.187453
Cost after iteration 2333: 0.187451
Cost after iteration 2334: 0.187450
Cost after iteration 2335: 0.187473
Cost after iteration 2336: 0.187496
Cost after iteration 2337: 0.187567
Cost after iteration 2338: 0.187634
Cost after iteration 2339: 0.187798
Cost after iteration 2340: 0.187949
Cost after iteration 2341: 0.188313
Cost after iteration 2342: 0.188629
Cost after iteration 2343: 0.189445
Cost after iteration 2344: 0.190116
Cost after iteration 2345: 0.192040
Cost after iteration 2346: 0.193536
Cost after iteration 2347: 0.198395
Cost after iteration 2348: 0.201806
Cost after iteration 2349: 0.214464
Cost after iteration 2350: 0.218979
Cost after iteration 2351: 0.247400
Cost after iteration 2352: 0.236114
Cost after iteration 2353: 0.273771
Cost after iteration 2354: 0.236465
Cost after iteration 2355: 0.258242
Cost after iteration 2356: 0.223194
Cost after iteration 2357: 0.230212
Cost after iteration 2358: 0.216108
Cost after iteration 2359: 0.220046
Cost after iteration 2360: 0.214413
Cost after iteration 2361: 0.218571
Cost after iteration 2362: 0.216175
Cost after iteration 2363: 0.222119
Cost after iteration 2364: 0.220891
Cost after iteration 2365: 0.229554
Cost after iteration 2366: 0.227335
Cost after iteration 2367: 0.238014
Cost after iteration 2368: 0.231762
Cost after iteration 2369: 0.240269
Cost after iteration 2370: 0.228583
Cost after iteration 2371: 0.229555
Cost after iteration 2372: 0.216675
Cost after iteration 2373: 0.211378
Cost after iteration 2374: 0.202640
Cost after iteration 2375: 0.198043
```

```
Cost after iteration 2376: 0.194183
Cost after iteration 2377: 0.192277
Cost after iteration 2378: 0.190931
Cost after iteration 2379: 0.190249
Cost after iteration 2380: 0.189736
Cost after iteration 2381: 0.189445
Cost after iteration 2382: 0.189195
Cost after iteration 2383: 0.189033
Cost after iteration 2384: 0.188880
Cost after iteration 2385: 0.188768
Cost after iteration 2386: 0.188658
Cost after iteration 2387: 0.188570
Cost after iteration 2388: 0.188482
Cost after iteration 2389: 0.188408
Cost after iteration 2390: 0.188334
Cost after iteration 2391: 0.188269
Cost after iteration 2392: 0.188204
Cost after iteration 2393: 0.188146
Cost after iteration 2394: 0.188088
Cost after iteration 2395: 0.188035
Cost after iteration 2396: 0.187983
Cost after iteration 2397: 0.187935
Cost after iteration 2398: 0.187887
Cost after iteration 2399: 0.187844
Cost after iteration 2400: 0.187801
Cost after iteration 2401: 0.187763
Cost after iteration 2402: 0.187724
Cost after iteration 2403: 0.187692
Cost after iteration 2404: 0.187658
Cost after iteration 2405: 0.187632
Cost after iteration 2406: 0.187606
Cost after iteration 2407: 0.187590
Cost after iteration 2408: 0.187572
Cost after iteration 2409: 0.187571
Cost after iteration 2410: 0.187566
Cost after iteration 2411: 0.187590
Cost after iteration 2412: 0.187607
Cost after iteration 2413: 0.187675
Cost after iteration 2414: 0.187730
Cost after iteration 2415: 0.187881
Cost after iteration 2416: 0.188004
Cost after iteration 2417: 0.188322
Cost after iteration 2418: 0.188575
Cost after iteration 2419: 0.189251
Cost after iteration 2420: 0.189765
Cost after iteration 2421: 0.191267
Cost after iteration 2422: 0.192353
Cost after iteration 2423: 0.195906
Cost after iteration 2424: 0.198304
```

```
Cost after iteration 2425: 0.207138
Cost after iteration 2426: 0.211340
Cost after iteration 2427: 0.231816
Cost after iteration 2428: 0.228764
Cost after iteration 2429: 0.261810
Cost after iteration 2430: 0.235451
Cost after iteration 2431: 0.261879
Cost after iteration 2432: 0.226197
Cost after iteration 2433: 0.236497
Cost after iteration 2434: 0.216992
Cost after iteration 2435: 0.221381
Cost after iteration 2436: 0.213299
Cost after iteration 2437: 0.216587
Cost after iteration 2438: 0.212767
Cost after iteration 2439: 0.216564
Cost after iteration 2440: 0.214644
Cost after iteration 2441: 0.219798
Cost after iteration 2442: 0.218270
Cost after iteration 2443: 0.224888
Cost after iteration 2444: 0.222148
Cost after iteration 2445: 0.228987
Cost after iteration 2446: 0.223452
Cost after iteration 2447: 0.227619
Cost after iteration 2448: 0.219245
Cost after iteration 2449: 0.218416
Cost after iteration 2450: 0.209785
Cost after iteration 2451: 0.205773
Cost after iteration 2452: 0.199767
Cost after iteration 2453: 0.196535
Cost after iteration 2454: 0.193592
Cost after iteration 2455: 0.192047
Cost after iteration 2456: 0.190829
Cost after iteration 2457: 0.190174
Cost after iteration 2458: 0.189645
Cost after iteration 2459: 0.189339
Cost after iteration 2460: 0.189069
Cost after iteration 2461: 0.188898
Cost after iteration 2462: 0.188734
Cost after iteration 2463: 0.188621
Cost after iteration 2464: 0.188507
Cost after iteration 2465: 0.188422
Cost after iteration 2466: 0.188334
Cost after iteration 2467: 0.188265
Cost after iteration 2468: 0.188193
Cost after iteration 2469: 0.188134
Cost after iteration 2470: 0.188072
Cost after iteration 2471: 0.188022
Cost after iteration 2472: 0.187968
Cost after iteration 2473: 0.187924
```

```
Cost after iteration 2474: 0.187876
Cost after iteration 2475: 0.187838
Cost after iteration 2476: 0.187797
Cost after iteration 2477: 0.187766
Cost after iteration 2478: 0.187731
Cost after iteration 2479: 0.187708
Cost after iteration 2480: 0.187679
Cost after iteration 2481: 0.187667
Cost after iteration 2482: 0.187648
Cost after iteration 2483: 0.187652
Cost after iteration 2484: 0.187646
Cost after iteration 2485: 0.187674
Cost after iteration 2486: 0.187689
Cost after iteration 2487: 0.187759
Cost after iteration 2488: 0.187806
Cost after iteration 2489: 0.187953
Cost after iteration 2490: 0.188057
Cost after iteration 2491: 0.188349
Cost after iteration 2492: 0.188554
Cost after iteration 2493: 0.189139
Cost after iteration 2494: 0.189540
Cost after iteration 2495: 0.190757
Cost after iteration 2496: 0.191559
Cost after iteration 2497: 0.194243
Cost after iteration 2498: 0.195927
Cost after iteration 2499: 0.202188
Cost after iteration 2500: 0.205431
Cost after iteration 2501: 0.219824
Cost after iteration 2502: 0.221127
Cost after iteration 2503: 0.247605
Cost after iteration 2504: 0.232168
Cost after iteration 2505: 0.260123
Cost after iteration 2506: 0.229305
Cost after iteration 2507: 0.244390
Cost after iteration 2508: 0.220905
Cost after iteration 2509: 0.227714
Cost after iteration 2510: 0.217225
Cost after iteration 2511: 0.222301
Cost after iteration 2512: 0.217600
Cost after iteration 2513: 0.223679
Cost after iteration 2514: 0.221705
Cost after iteration 2515: 0.230873
Cost after iteration 2516: 0.229563
Cost after iteration 2517: 0.243212
Cost after iteration 2518: 0.238688
Cost after iteration 2519: 0.254023
Cost after iteration 2520: 0.240826
Cost after iteration 2521: 0.248459
Cost after iteration 2522: 0.229515
```

```
Cost after iteration 2523: 0.225209
Cost after iteration 2524: 0.210897
Cost after iteration 2525: 0.203805
Cost after iteration 2526: 0.197195
Cost after iteration 2527: 0.194068
Cost after iteration 2528: 0.191936
Cost after iteration 2529: 0.190961
Cost after iteration 2530: 0.190252
Cost after iteration 2531: 0.189885
Cost after iteration 2532: 0.189570
Cost after iteration 2533: 0.189376
Cost after iteration 2534: 0.189192
Cost after iteration 2535: 0.189061
Cost after iteration 2536: 0.188933
Cost after iteration 2537: 0.188832
Cost after iteration 2538: 0.188731
Cost after iteration 2539: 0.188646
Cost after iteration 2540: 0.188562
Cost after iteration 2541: 0.188488
Cost after iteration 2542: 0.188415
Cost after iteration 2543: 0.188349
Cost after iteration 2544: 0.188283
Cost after iteration 2545: 0.188223
Cost after iteration 2546: 0.188163
Cost after iteration 2547: 0.188108
Cost after iteration 2548: 0.188054
Cost after iteration 2549: 0.188003
Cost after iteration 2550: 0.187953
Cost after iteration 2551: 0.187907
Cost after iteration 2552: 0.187861
Cost after iteration 2553: 0.187819
Cost after iteration 2554: 0.187777
Cost after iteration 2555: 0.187740
Cost after iteration 2556: 0.187702
Cost after iteration 2557: 0.187671
Cost after iteration 2558: 0.187639
Cost after iteration 2559: 0.187614
Cost after iteration 2560: 0.187590
Cost after iteration 2561: 0.187576
Cost after iteration 2562: 0.187561
Cost after iteration 2563: 0.187565
Cost after iteration 2564: 0.187566
Cost after iteration 2565: 0.187599
Cost after iteration 2566: 0.187627
Cost after iteration 2567: 0.187715
Cost after iteration 2568: 0.187792
Cost after iteration 2569: 0.187988
Cost after iteration 2570: 0.188155
Cost after iteration 2571: 0.188578
```

```
Cost after iteration 2572: 0.188922
Cost after iteration 2573: 0.189854
Cost after iteration 2574: 0.190572
Cost after iteration 2575: 0.192734
Cost after iteration 2576: 0.194312
Cost after iteration 2577: 0.199667
Cost after iteration 2578: 0.203120
Cost after iteration 2579: 0.216591
Cost after iteration 2580: 0.220105
Cost after iteration 2581: 0.248485
Cost after iteration 2582: 0.234919
Cost after iteration 2583: 0.269240
Cost after iteration 2584: 0.232956
Cost after iteration 2585: 0.251333
Cost after iteration 2586: 0.220232
Cost after iteration 2587: 0.226082
Cost after iteration 2588: 0.213106
Cost after iteration 2589: 0.215874
Cost after iteration 2590: 0.210016
Cost after iteration 2591: 0.212208
Cost after iteration 2592: 0.209255
Cost after iteration 2593: 0.211784
Cost after iteration 2594: 0.210080
Cost after iteration 2595: 0.213207
Cost after iteration 2596: 0.211626
Cost after iteration 2597: 0.215017
Cost after iteration 2598: 0.212696
Cost after iteration 2599: 0.215473
Cost after iteration 2600: 0.211936
Cost after iteration 2601: 0.213012
Cost after iteration 2602: 0.208495
Cost after iteration 2603: 0.207484
Cost after iteration 2604: 0.202988
Cost after iteration 2605: 0.200864
Cost after iteration 2606: 0.197440
Cost after iteration 2607: 0.195552
Cost after iteration 2608: 0.193466
Cost after iteration 2609: 0.192273
Cost after iteration 2610: 0.191145
Cost after iteration 2611: 0.190481
Cost after iteration 2612: 0.189878
Cost after iteration 2613: 0.189510
Cost after iteration 2614: 0.189170
Cost after iteration 2615: 0.188955
Cost after iteration 2616: 0.188745
Cost after iteration 2617: 0.188609
Cost after iteration 2618: 0.188468
Cost after iteration 2619: 0.188374
Cost after iteration 2620: 0.188272
```

```
Cost after iteration 2621: 0.188203
Cost after iteration 2622: 0.188125
Cost after iteration 2623: 0.188073
Cost after iteration 2624: 0.188010
Cost after iteration 2625: 0.187971
Cost after iteration 2626: 0.187920
Cost after iteration 2627: 0.187892
Cost after iteration 2628: 0.187851
Cost after iteration 2629: 0.187835
Cost after iteration 2630: 0.187805
Cost after iteration 2631: 0.187804
Cost after iteration 2632: 0.187785
Cost after iteration 2633: 0.187805
Cost after iteration 2634: 0.187802
Cost after iteration 2635: 0.187853
Cost after iteration 2636: 0.187871
Cost after iteration 2637: 0.187972
Cost after iteration 2638: 0.188024
Cost after iteration 2639: 0.188211
Cost after iteration 2640: 0.188319
Cost after iteration 2641: 0.188661
Cost after iteration 2642: 0.188863
Cost after iteration 2643: 0.189502
Cost after iteration 2644: 0.189878
Cost after iteration 2645: 0.191120
Cost after iteration 2646: 0.191833
Cost after iteration 2647: 0.194385
Cost after iteration 2648: 0.195791
Cost after iteration 2649: 0.201316
Cost after iteration 2650: 0.203891
Cost after iteration 2651: 0.215809
Cost after iteration 2652: 0.217458
Cost after iteration 2653: 0.239131
Cost after iteration 2654: 0.229062
Cost after iteration 2655: 0.254307
Cost after iteration 2656: 0.230375
Cost after iteration 2657: 0.247463
Cost after iteration 2658: 0.226258
Cost after iteration 2659: 0.236624
Cost after iteration 2660: 0.226356
Cost after iteration 2661: 0.236968
Cost after iteration 2662: 0.233452
Cost after iteration 2663: 0.250719
Cost after iteration 2664: 0.249774
Cost after iteration 2665: 0.279533
Cost after iteration 2666: 0.267245
Cost after iteration 2667: 0.296147
Cost after iteration 2668: 0.259031
Cost after iteration 2669: 0.262515
Cost after iteration 2670: 0.229491
```

```
Cost after iteration 2671: 0.217700
Cost after iteration 2672: 0.203901
Cost after iteration 2673: 0.197802
Cost after iteration 2674: 0.193870
Cost after iteration 2675: 0.192322
Cost after iteration 2676: 0.191242
Cost after iteration 2677: 0.190737
Cost after iteration 2678: 0.190312
Cost after iteration 2679: 0.190059
Cost after iteration 2680: 0.189832
Cost after iteration 2681: 0.189667
Cost after iteration 2682: 0.189515
Cost after iteration 2683: 0.189390
Cost after iteration 2684: 0.189272
Cost after iteration 2685: 0.189168
Cost after iteration 2686: 0.189069
Cost after iteration 2687: 0.188978
Cost after iteration 2688: 0.188891
Cost after iteration 2689: 0.188809
Cost after iteration 2690: 0.188730
Cost after iteration 2691: 0.188654
Cost after iteration 2692: 0.188582
Cost after iteration 2693: 0.188512
Cost after iteration 2694: 0.188445
Cost after iteration 2695: 0.188381
Cost after iteration 2696: 0.188318
Cost after iteration 2697: 0.188258
Cost after iteration 2698: 0.188200
Cost after iteration 2699: 0.188144
Cost after iteration 2700: 0.188089
Cost after iteration 2701: 0.188036
Cost after iteration 2702: 0.187985
Cost after iteration 2703: 0.187935
Cost after iteration 2704: 0.187887
Cost after iteration 2705: 0.187841
Cost after iteration 2706: 0.187796
Cost after iteration 2707: 0.187752
Cost after iteration 2708: 0.187710
Cost after iteration 2709: 0.187669
Cost after iteration 2710: 0.187630
Cost after iteration 2711: 0.187592
Cost after iteration 2712: 0.187556
Cost after iteration 2713: 0.187522
Cost after iteration 2714: 0.187489
Cost after iteration 2715: 0.187459
Cost after iteration 2716: 0.187431
Cost after iteration 2717: 0.187408
Cost after iteration 2718: 0.187387
Cost after iteration 2719: 0.187373
```

```
Cost after iteration 2720: 0.187364
Cost after iteration 2721: 0.187369
Cost after iteration 2722: 0.187379
Cost after iteration 2723: 0.187421
Cost after iteration 2724: 0.187470
Cost after iteration 2725: 0.187587
Cost after iteration 2726: 0.187715
Cost after iteration 2727: 0.188003
Cost after iteration 2728: 0.188294
Cost after iteration 2729: 0.188991
Cost after iteration 2730: 0.189648
Cost after iteration 2731: 0.191406
Cost after iteration 2732: 0.192956
Cost after iteration 2733: 0.197712
Cost after iteration 2734: 0.201532
Cost after iteration 2735: 0.214934
Cost after iteration 2736: 0.220819
Cost after iteration 2737: 0.253604
Cost after iteration 2738: 0.241154
Cost after iteration 2739: 0.286561
Cost after iteration 2740: 0.240521
Cost after iteration 2741: 0.264157
Cost after iteration 2742: 0.222245
Cost after iteration 2743: 0.227477
Cost after iteration 2744: 0.213015
Cost after iteration 2745: 0.215152
Cost after iteration 2746: 0.209608
Cost after iteration 2747: 0.211529
Cost after iteration 2748: 0.209019
Cost after iteration 2749: 0.211571
Cost after iteration 2750: 0.210147
Cost after iteration 2751: 0.213397
Cost after iteration 2752: 0.211833
Cost after iteration 2753: 0.215231
Cost after iteration 2754: 0.212693
Cost after iteration 2755: 0.215203
Cost after iteration 2756: 0.211365
Cost after iteration 2757: 0.211930
Cost after iteration 2758: 0.207268
Cost after iteration 2759: 0.205814
Cost after iteration 2760: 0.201473
Cost after iteration 2761: 0.199258
Cost after iteration 2762: 0.196172
Cost after iteration 2763: 0.194422
Cost after iteration 2764: 0.192636
Cost after iteration 2765: 0.191589
Cost after iteration 2766: 0.190648
Cost after iteration 2767: 0.190075
Cost after iteration 2768: 0.189576
```

```
Cost after iteration 2769: 0.189257
Cost after iteration 2770: 0.188974
Cost after iteration 2771: 0.188783
Cost after iteration 2772: 0.188607
Cost after iteration 2773: 0.188482
Cost after iteration 2774: 0.188362
Cost after iteration 2775: 0.188273
Cost after iteration 2776: 0.188184
Cost after iteration 2777: 0.188116
Cost after iteration 2778: 0.188046
Cost after iteration 2779: 0.187992
Cost after iteration 2780: 0.187934
Cost after iteration 2781: 0.187890
Cost after iteration 2782: 0.187842
Cost after iteration 2783: 0.187807
Cost after iteration 2784: 0.187766
Cost after iteration 2785: 0.187739
Cost after iteration 2786: 0.187706
Cost after iteration 2787: 0.187688
Cost after iteration 2788: 0.187663
Cost after iteration 2789: 0.187657
Cost after iteration 2790: 0.187642
Cost after iteration 2791: 0.187654
Cost after iteration 2792: 0.187653
Cost after iteration 2793: 0.187691
Cost after iteration 2794: 0.187711
Cost after iteration 2795: 0.187794
Cost after iteration 2796: 0.187847
Cost after iteration 2797: 0.188007
Cost after iteration 2798: 0.188116
Cost after iteration 2799: 0.188420
Cost after iteration 2800: 0.188626
Cost after iteration 2801: 0.189211
Cost after iteration 2802: 0.189597
Cost after iteration 2803: 0.190768
Cost after iteration 2804: 0.191513
Cost after iteration 2805: 0.193991
Cost after iteration 2806: 0.195497
Cost after iteration 2807: 0.201051
Cost after iteration 2808: 0.203947
Cost after iteration 2809: 0.216424
Cost after iteration 2810: 0.218654
Cost after iteration 2811: 0.242258
Cost after iteration 2812: 0.231476
Cost after iteration 2813: 0.259376
Cost after iteration 2814: 0.232923
Cost after iteration 2815: 0.251417
Cost after iteration 2816: 0.228534
Cost after iteration 2817: 0.239981
```

```
Cost after iteration 2818: 0.230144
Cost after iteration 2819: 0.243688
Cost after iteration 2820: 0.241514
Cost after iteration 2821: 0.266314
Cost after iteration 2822: 0.263893
Cost after iteration 2823: 0.302326
Cost after iteration 2824: 0.275130
Cost after iteration 2825: 0.297308
Cost after iteration 2826: 0.250193
Cost after iteration 2827: 0.242551
Cost after iteration 2828: 0.216895
Cost after iteration 2829: 0.205897
Cost after iteration 2830: 0.197640
Cost after iteration 2831: 0.194360
Cost after iteration 2832: 0.192310
Cost after iteration 2833: 0.191461
Cost after iteration 2834: 0.190810
Cost after iteration 2835: 0.190463
Cost after iteration 2836: 0.190163
Cost after iteration 2837: 0.189962
Cost after iteration 2838: 0.189781
Cost after iteration 2839: 0.189637
Cost after iteration 2840: 0.189504
Cost after iteration 2841: 0.189389
Cost after iteration 2842: 0.189280
Cost after iteration 2843: 0.189180
Cost after iteration 2844: 0.189085
Cost after iteration 2845: 0.188996
Cost after iteration 2846: 0.188911
Cost after iteration 2847: 0.188829
Cost after iteration 2848: 0.188751
Cost after iteration 2849: 0.188676
Cost after iteration 2850: 0.188604
Cost after iteration 2851: 0.188535
Cost after iteration 2852: 0.188467
Cost after iteration 2853: 0.188403
Cost after iteration 2854: 0.188340
Cost after iteration 2855: 0.188280
Cost after iteration 2856: 0.188221
Cost after iteration 2857: 0.188165
Cost after iteration 2858: 0.188110
Cost after iteration 2859: 0.188057
Cost after iteration 2860: 0.188005
Cost after iteration 2861: 0.187955
Cost after iteration 2862: 0.187907
Cost after iteration 2863: 0.187860
Cost after iteration 2864: 0.187814
Cost after iteration 2865: 0.187770
Cost after iteration 2866: 0.187727
```

```
Cost after iteration 2867: 0.187685
Cost after iteration 2868: 0.187645
Cost after iteration 2869: 0.187606
Cost after iteration 2870: 0.187569
Cost after iteration 2871: 0.187533
Cost after iteration 2872: 0.187498
Cost after iteration 2873: 0.187465
Cost after iteration 2874: 0.187433
Cost after iteration 2875: 0.187404
Cost after iteration 2876: 0.187377
Cost after iteration 2877: 0.187355
Cost after iteration 2878: 0.187335
Cost after iteration 2879: 0.187322
Cost after iteration 2880: 0.187313
Cost after iteration 2881: 0.187320
Cost after iteration 2882: 0.187333
Cost after iteration 2883: 0.187378
Cost after iteration 2884: 0.187434
Cost after iteration 2885: 0.187563
Cost after iteration 2886: 0.187708
Cost after iteration 2887: 0.188032
Cost after iteration 2888: 0.188371
Cost after iteration 2889: 0.189181
Cost after iteration 2890: 0.189970
Cost after iteration 2891: 0.192095
Cost after iteration 2892: 0.194027
Cost after iteration 2893: 0.200025
Cost after iteration 2894: 0.204845
Cost after iteration 2895: 0.222120
Cost after iteration 2896: 0.227231
Cost after iteration 2897: 0.267403
Cost after iteration 2898: 0.246165
Cost after iteration 2899: 0.292916
Cost after iteration 2900: 0.238356
Cost after iteration 2901: 0.255903
Cost after iteration 2902: 0.219000
Cost after iteration 2903: 0.222495
Cost after iteration 2904: 0.211668
Cost after iteration 2905: 0.213473
Cost after iteration 2906: 0.209191
Cost after iteration 2907: 0.211238
Cost after iteration 2908: 0.209303
Cost after iteration 2909: 0.212151
Cost after iteration 2910: 0.210833
Cost after iteration 2911: 0.214292
Cost after iteration 2912: 0.212464
Cost after iteration 2913: 0.215756
Cost after iteration 2914: 0.212719
Cost after iteration 2915: 0.214673
```

```
Cost after iteration 2916: 0.210365
Cost after iteration 2917: 0.210124
Cost after iteration 2918: 0.205353
Cost after iteration 2919: 0.203393
Cost after iteration 2920: 0.199390
Cost after iteration 2921: 0.197216
Cost after iteration 2922: 0.194625
Cost after iteration 2923: 0.193131
Cost after iteration 2924: 0.191716
Cost after iteration 2925: 0.190872
Cost after iteration 2926: 0.190138
Cost after iteration 2927: 0.189681
Cost after iteration 2928: 0.189287
Cost after iteration 2929: 0.189028
Cost after iteration 2930: 0.188799
Cost after iteration 2931: 0.188639
Cost after iteration 2932: 0.188492
Cost after iteration 2933: 0.188384
Cost after iteration 2934: 0.188280
Cost after iteration 2935: 0.188200
Cost after iteration 2936: 0.188120
Cost after iteration 2937: 0.188058
Cost after iteration 2938: 0.187994
Cost after iteration 2939: 0.187943
Cost after iteration 2940: 0.187889
Cost after iteration 2941: 0.187847
Cost after iteration 2942: 0.187801
Cost after iteration 2943: 0.187766
Cost after iteration 2944: 0.187727
Cost after iteration 2945: 0.187699
Cost after iteration 2946: 0.187667
Cost after iteration 2947: 0.187647
Cost after iteration 2948: 0.187622
Cost after iteration 2949: 0.187613
Cost after iteration 2950: 0.187597
Cost after iteration 2951: 0.187604
Cost after iteration 2952: 0.187601
Cost after iteration 2953: 0.187630
Cost after iteration 2954: 0.187646
Cost after iteration 2955: 0.187715
Cost after iteration 2956: 0.187762
Cost after iteration 2957: 0.187899
Cost after iteration 2958: 0.187997
Cost after iteration 2959: 0.188261
Cost after iteration 2960: 0.188448
Cost after iteration 2961: 0.188960
Cost after iteration 2962: 0.189314
Cost after iteration 2963: 0.190344
Cost after iteration 2964: 0.191027
```

```
Cost after iteration 2965: 0.193213
Cost after iteration 2966: 0.194598
Cost after iteration 2967: 0.199528
Cost after iteration 2968: 0.202304
Cost after iteration 2969: 0.213628
Cost after iteration 2970: 0.216672
Cost after iteration 2971: 0.239332
Cost after iteration 2972: 0.231035
Cost after iteration 2973: 0.260351
Cost after iteration 2974: 0.234250
Cost after iteration 2975: 0.254920
Cost after iteration 2976: 0.229474
Cost after iteration 2977: 0.241511
Cost after iteration 2978: 0.230092
Cost after iteration 2979: 0.243380
Cost after iteration 2980: 0.240714
Cost after iteration 2981: 0.264879
Cost after iteration 2982: 0.263183
Cost after iteration 2983: 0.302354
Cost after iteration 2984: 0.276760
Cost after iteration 2985: 0.301451
Cost after iteration 2986: 0.252882
Cost after iteration 2987: 0.246337
Cost after iteration 2988: 0.218725
Cost after iteration 2989: 0.207140
Cost after iteration 2990: 0.198208
Cost after iteration 2991: 0.194639
Cost after iteration 2992: 0.192443
Cost after iteration 2993: 0.191543
Cost after iteration 2994: 0.190866
Cost after iteration 2995: 0.190508
Cost after iteration 2996: 0.190202
Cost after iteration 2997: 0.189997
Cost after iteration 2998: 0.189814
Cost after iteration 2999: 0.189669
W1 = [[ 2.08801776 -1.89503493]
 [ 2.46515642 -2.07480543]]
b1 = [[-4.84976773]
 [ 6.30623075]]
W2 = [[-7.20236323  7.07357254]]
b2 = [[-3.45260284]]
```

Expected Output

Note: the actual values can be different!

```
Cost after iteration 0: 0.693148
Cost after iteration 1: 0.693147
Cost after iteration 2: 0.693147
```

```
Cost after iteration 3: 0.693147
Cost after iteration 4: 0.693147
Cost after iteration 5: 0.693147
...
Cost after iteration 2995: 0.209524
Cost after iteration 2996: 0.208025
Cost after iteration 2997: 0.210427
Cost after iteration 2998: 0.208929
Cost after iteration 2999: 0.211306
W1 = [[ 2.14274251 -1.93155541]
 [ 2.20268789 -2.1131799 ]]
b1 = [[-4.83079243]
 [ 6.2845223 ]]
W2 = [[-7.21370685  7.0898022 ]]
b2 = [[-3.48755239]]

# Note:
# Actual values are not checked here in the unit tests (due to random initialization).
w3_unittest.test_nn_model(nn_model)

 All tests passed
```

The final model parameters can be used to find the boundary line and for making predictions.

Exercise 8

Computes probabilities using forward propagation, and make classification to 0/1 using 0.5 as the threshold.

```
# GRADED FUNCTION: predict

def predict(X, parameters):
    """
    Using the learned parameters, predicts a class for each example in X

    Arguments:
    parameters -- python dictionary containing your parameters
    X -- input data of size (n_x, m)

    Returns
    predictions -- vector of predictions of our model (blue: 0 / red: 1)
    """

    ### START CODE HERE ### (≈ 2 lines of code)
    A2, cache = forward_propagation(X, parameters)
```

```
    predictions = A2 > 0.5
    ### END CODE HERE ###

    return predictions

X_pred = np.array([[2, 8, 2, 8], [2, 8, 8, 2]])
Y_pred = predict(X_pred, parameters)

print(f"Coordinates (in the columns):\n{X_pred}")
print(f"Predictions:\n{Y_pred}")
```

```
Coordinates (in the columns):
[[2 8 2 8]
 [2 8 8 2]]
Predictions:
[[ True  True False False]]
```

Expected Output

```
Coordinates (in the columns):
[[2 8 2 8]
 [2 8 8 2]]
Predictions:
[[ True  True False False]]
```

```
w3_unittest.test_predict(predict)
```

```
 All tests passed
```

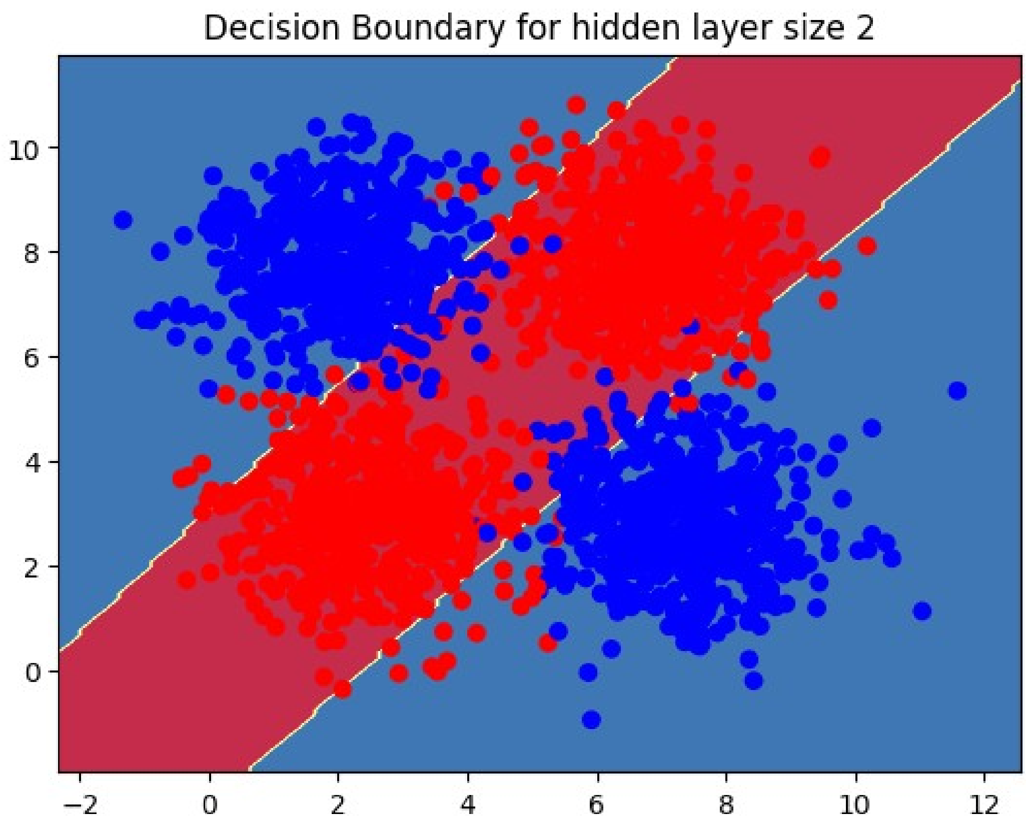
Let's visualize the boundary line. Do not worry if you don't understand the function `plot_decision_boundary` line by line - it simply makes prediction for some points on the plane and plots them as a contour plot (just two colors - blue and red).

```
def plot_decision_boundary(predict, parameters, X, Y):
    # Define bounds of the domain.
    min1, max1 = X[0, :].min()-1, X[0, :].max()+1
    min2, max2 = X[1, :].min()-1, X[1, :].max()+1
    # Define the x and y scale.
    x1grid = np.arange(min1, max1, 0.1)
    x2grid = np.arange(min2, max2, 0.1)
    # Create all of the lines and rows of the grid.
    xx, yy = np.meshgrid(x1grid, x2grid)
    # Flatten each grid to a vector.
    r1, r2 = xx.flatten(), yy.flatten()
    r1, r2 = r1.reshape((1, len(r1))), r2.reshape((1, len(r2)))
    # Vertical stack vectors to create x1,x2 input for the model.
    grid = np.vstack((r1,r2))
    # Make predictions for the grid.
    predictions = predict(grid, parameters)
    # Reshape the predictions back into a grid.
```

```
    zz = predictions.reshape(xx.shape)
    # Plot the grid of x, y and z values as a surface.
    plt.contourf(xx, yy, zz, cmap=plt.cm.Spectral.reversed())
    plt.scatter(X[0, :], X[1, :], c=Y,
cmap=colors.ListedColormap(['blue', 'red']));

# Plot the decision boundary.
plot_decision_boundary(predict, parameters, X, Y)
plt.title("Decision Boundary for hidden layer size " + str(n_h))

Text(0.5, 1.0, 'Decision Boundary for hidden layer size 2')
```



That's great, you can see that more complicated classification problems can be solved with two layer neural network!

4 - Optional: Other Dataset

Build a slightly different dataset:

```
n_samples = 2000
samples, labels = make_blobs(n_samples=n_samples,
                             centers=([2.5, 3], [6.7, 7.9], [2.1,
```

```
7.9], [7.4, 2.8]),
                              cluster_std=1.1,
                              random_state=0)
labels[(labels == 0)] = 0
labels[(labels == 1)] = 1
labels[(labels == 2) | (labels == 3)] = 1
X_2 = np.transpose(samples)
Y_2 = labels.reshape((1,n_samples))

plt.scatter(X_2[0, :], X_2[1, :], c=Y_2,
cmap=colors.ListedColormap(['blue', 'red']));
```

Notice that when building your neural network, a number of the nodes in the hidden layer could be taken as a parameter. Try to change this parameter and investigate the results:

```
# parameters_2 = nn_model(X_2, Y_2, n_h=1, num_iterations=3000,
learning_rate=1.2, print_cost=False)
parameters_2 = nn_model(X_2, Y_2, n_h=2, num_iterations=3000,
learning_rate=1.2, print_cost=False)
# parameters_2 = nn_model(X_2, Y_2, n_h=15, num_iterations=3000,
learning_rate=1.2, print_cost=False)

# This function will call predict function
plot_decision_boundary(predict, parameters_2, X_2, Y_2)
plt.title("Decision Boundary")
```

You can see that there are some misclassified points - real-world datasets are usually linearly inseparable, and there will be a small percentage of errors. More than that, you do not want to build a model that fits too closely, almost exactly to a particular set of data - it may fail to predict future observations. This problem is known as **overfitting**.

Congrats on finishing this programming assignment!

```
```