

Useful Fact Sheet Module 2 – Alg.Trig. I

Transformations		
Vertical Shift $y = f(x) + b$ $y = f(x) - b$	Raise the graph of f by b units Lower the graph of f by b units	b is added to f(x) b is subtracted from f(x)
Horizontal Shift $y = f(x + d)$ $y = f(x - d)$	Shift the graph of f to the left by d units Shift the graph of f to the right by d units	x is replaced with x+d x is replaced with x-d
Reflections: x-axis $y = -f(x)$	Reflect the graph of f about the x-axis	f(x) is multiplied by -1
Reflections: y-axis $y = f(-x)$	Reflect the graph of f about the y-axis	x is replaced with -x
Vertical Stretching $y = af(x); a > 1$ Vertical Shrinking $y = af(x); 0 < a < 1$	Multiply each y-coordinate of $y = f(x)$ by a Multiply each y-coordinate of $y = f(x)$ by a	f(x) is multiplied by c; $c > 1$ f(x) is multiplied by c; $0 < c < 1$
Horizontal Shrinking $y = f(cx); c > 1$ Horizontal Stretching $y = f(cx); 0 < c < 1$	Divide each x-coordinate of $y = f(x)$ by c Divide each x-coordinate of $y = f(x)$ by c	x is replaced by cx; $c > 1$ x is replaced by cx; $0 < c < 1$

Horizontal Line Test – If it is possible for a horizontal line to intersect the graph of a function more than once, then the function is **not** 1-1 and its inverse is **not** a function.

Vertical line test which states that if any vertical line intersects the graph of a relation in more than one point, then the relation graphed is not a function.

Definition of the Inverse function: $f(f^{-1}(x)) = x$ and for every x in the domain of f^{-1} and $f^{-1}(f(x)) = x$ for every x in the domain of f

Obtaining an inverse relation: interchange the first and second coordinates the relation obtained is an inverse

Obtaining a formula for an inverse:

Step 1: Replace f(x) with y	Step 2: Interchange x and y
Step 3: Solve for y	Step 4: Replace y with f(x)

Determinate of a 2x2 Matrix :
(second order determinate)

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

Determinate of a 3x3 Matrix :
(third order determinate)

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 b_2 c_3 + b_1 c_2 a_3 + c_1 a_2 b_3 - a_3 b_2 c_1 - b_3 c_2 a_1 - c_3 a_2 b_1$$

Solving Three Equations in Three Variables Using Determinants

Cramer's Rule

If

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

then

$$x = \frac{D_x}{D}, y = \frac{D_y}{D}, \text{ and } z = \frac{D_z}{D}.$$

These four third-order determinants are given by

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \text{These are the coefficients of the variables } x, y \text{ and } z. D \neq 0$$

$$D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} \quad \text{Replace } x\text{-coefficients in } D \text{ with the constants on the right of the three equations.}$$

$$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \quad \text{Replace } y\text{-coefficients in } D \text{ with the constants on the right of the three equations.}$$

$$D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} \quad \text{Replace } z\text{-coefficients in } D \text{ with the constants on the right of the three equations.}$$

Solving a Linear System in Two Variables Using Determinants

Cramer's Rule

If

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

then

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad \text{and} \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

where

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0.$$

central angle, θ , that intercepts the arc is $\theta = \frac{s}{r}$ radians

Coterminal Angles – Increasing or decreasing the degree of an angle in standard position by an integer multiply of 360° or 2π results in a coterminal angle.

linear speed is $v=s/t$, where s is the **arc length** given by $s = r\theta$, and its **angular speed** is $w=\theta/t$.

We can write linear speed in terms of angular speed $v = \frac{s}{t} = \frac{r\theta}{t} = r \cdot \frac{\theta}{t} = rw$

Even and Odd Trigonometric Functions

$$\cos(\theta) = \cos(-\theta) \text{ Even} \quad -\sin(\theta) = \sin(-\theta) \text{ Odd}$$

$$\sec(\theta) = \sec(-\theta) \text{ Even} \quad -\csc(\theta) = \csc(-\theta) \text{ Odd}$$

$$-\tan(\theta) = \tan(-\theta) \text{ Odd} \quad -\cot(\theta) = \cot(-\theta) \text{ Odd}$$

Pythagorean Identities –

$$\sin^2(\theta) + \cos^2(\theta) = 1 \quad 1 + \tan^2(\theta) = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

Periodic Properties of the Sine and Cosine Functions

$$\sin(\theta + 2\pi) = \sin(\theta) \quad \text{and} \quad \cos(\theta + 2\pi) = \cos(\theta)$$

Periodic Properties of the Tangent & Cotangent Functions

$$\tan(\theta + \pi) = \tan(\theta) \quad \text{and} \quad \cot(\theta + \pi) = \cot(\theta)$$

Pythagorean Theorem

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \quad \cos\theta = \frac{\text{adj}}{\text{hyp}} \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \quad \sec\theta = \frac{\text{hyp}}{\text{adj}} \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Reciprocal Identities

$$\sin(\theta) = \frac{1}{\csc(\theta)} \quad \csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\cos(\theta) = \frac{1}{\sec(\theta)} \quad \sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\tan(\theta) = \frac{1}{\cot(\theta)} \quad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Quotient Identities

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \quad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$