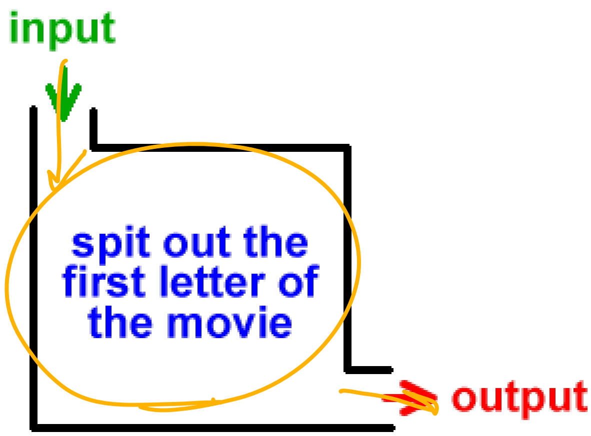
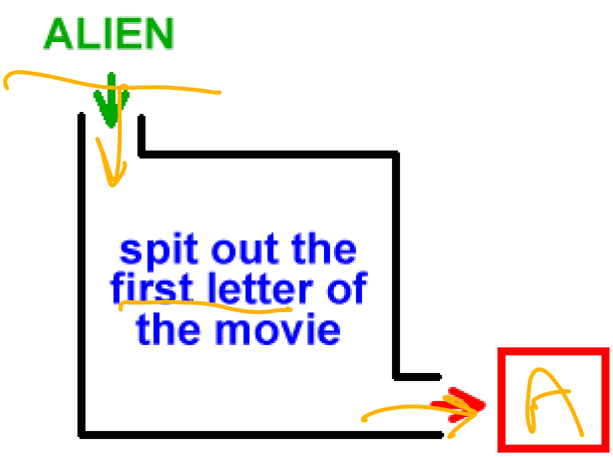
What's a Function? (Intro Domain and Range)

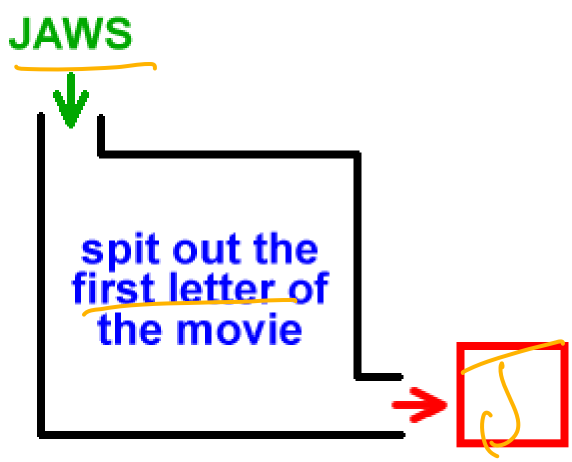
You can think of a function as being a box with a special rule... stuff goes in the box... and stuff comes out of the box.

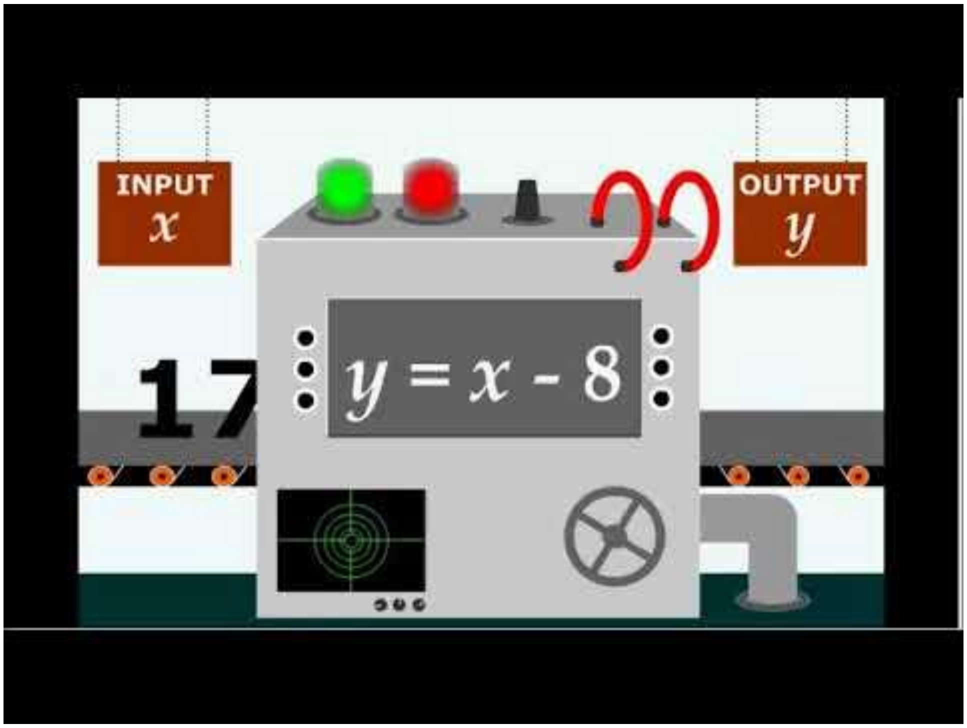
Let's start with a movie title box:

THE RULE: **Spit out the first letter of the movie title.**
(Only movie titles can go in.)









Here are some official math terms:

'x'

The stuff that goes IN the box (the INPUT)
is called the Domain**.**

'y'

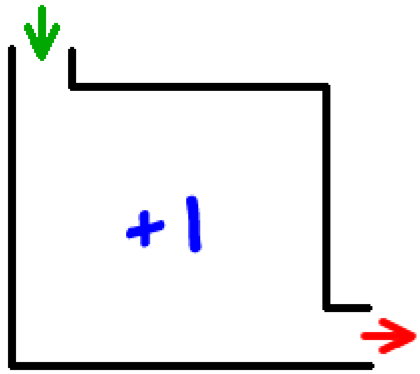
The stuff that spits OUT of the box (the OUTPUT)
is called the Range**.**

Domain guys go in... Range guys spit out.

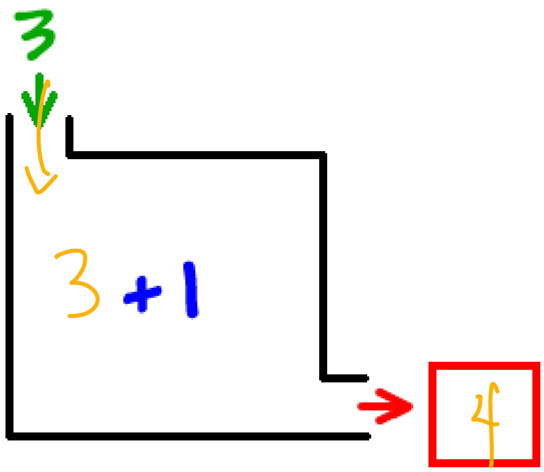
(If you forget the order, it's alphabetical D → R.)

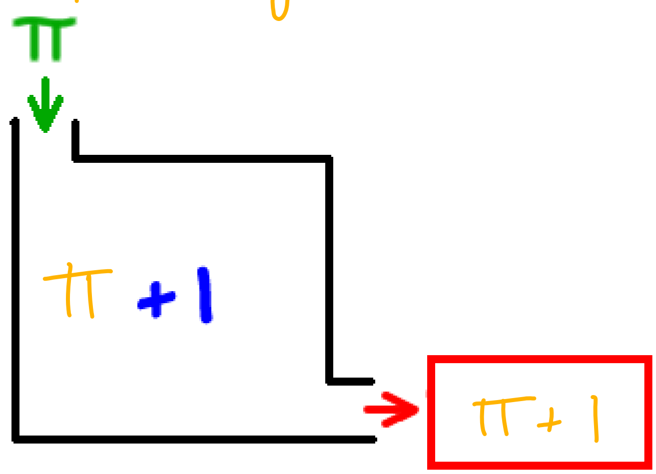
Example

THE RULE: Add 1



What can we put in this box? Anything





We can put anything in this box -- even goofy irrational numbers like π! So...

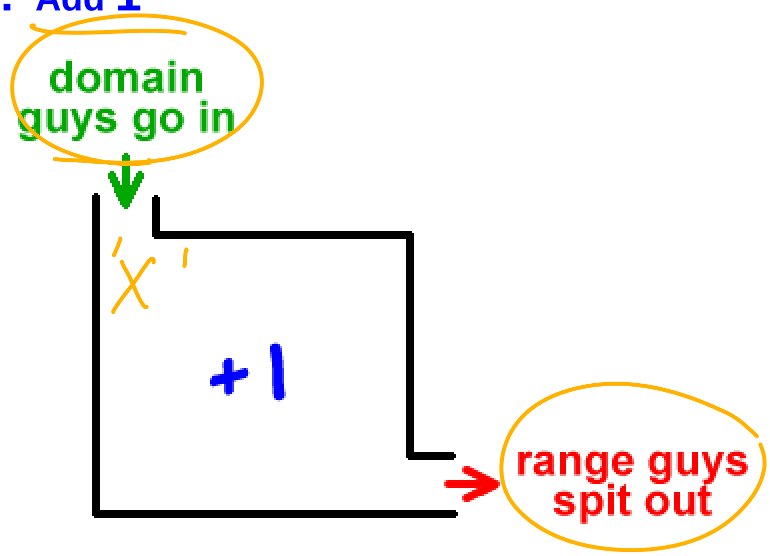
domain = all real numbers

range = all real numbers

Function Notation

Instead of drawing boxes all the time, we need a way to talk about functions with math symbols.

THE RULE: Add 1



name

$'y' = f(x)$

input

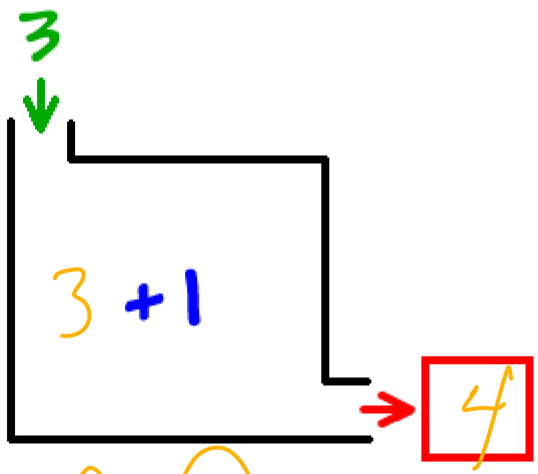
If we use X to represent our domain guys, here's the official notation:

$$y = f(x) = \underbrace{x + 1}_{\text{output}}$$

input

(You read f(x), as "f of X.")

X guys go in... and X + 1 guys spit out.



Is officially written as $f(x) = x + 1$

$$f(3) = 3 + 1 = \boxed{4}$$

$$f(3) = 4$$

domain guy

range guy

Example

$f(x) = x + 1$

(x, y)

What if our input is -5?

$x = -5$

$$f(-5) = -5 + 1 = -4$$

$(-5, -4)$

$$f(-5) = -4$$

$f(x) = x + 1$

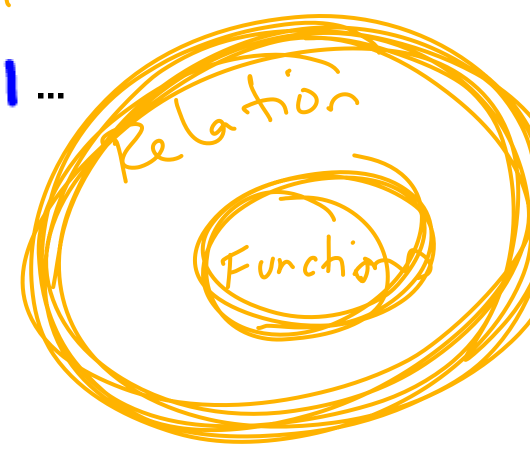
x

y

Here's another way to look at it:

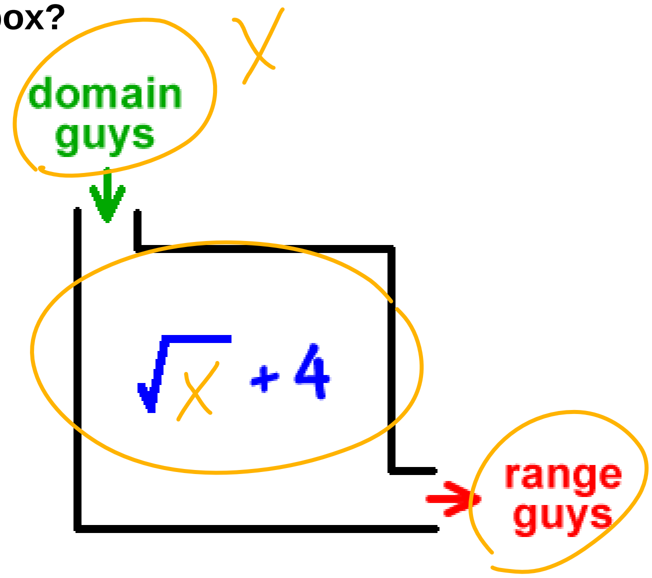
We started with $f(x) = x + 1$ ...

$f(-5)$ is like saying "let $x = -5$."



What about this box?

x



$y = f(x)$

The official notation would be

$$\underbrace{f(x)}_{\text{x guys go in}} = \underbrace{\sqrt{x} + 4}_{\text{and spits this out}}$$

x guys go in

and spits this out

Example

$x = 9$

What if our $\boxed{\text{input is } 9}$?

So, $f(9) = \sqrt{9} + 4 = 3 + 4 = 7$

like letting $x = 9$

(x, y) $\quad f(x) = \sqrt{x} + 4$

$(9, 7)$

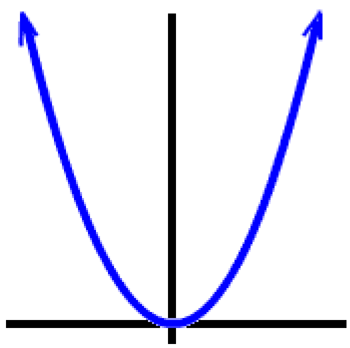
If we've got the picture of a graph?
How do can I tell if it is a function or not?

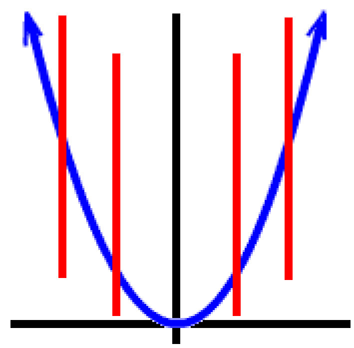
It's called **THE VERTICAL LINE TEST:**

> **If you can draw a vertical line anywhere on a graph so that it hits the graph in more than one spot, then the graph is NOT a function.**

Function?

YES **NO**



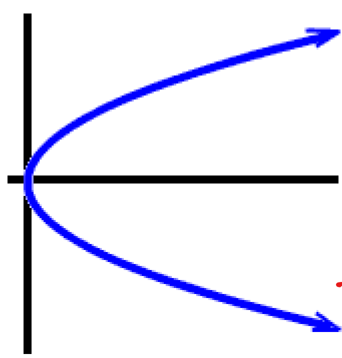


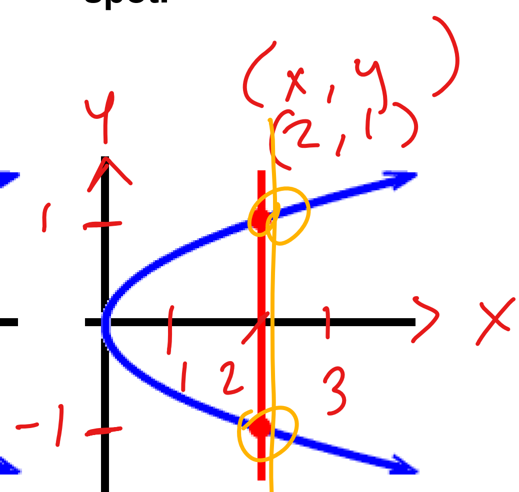
No matter where we drop a **vertical line**, it only hits the parabola in <u>one</u> spot.

What about this graph?

Function?

YES **NO**





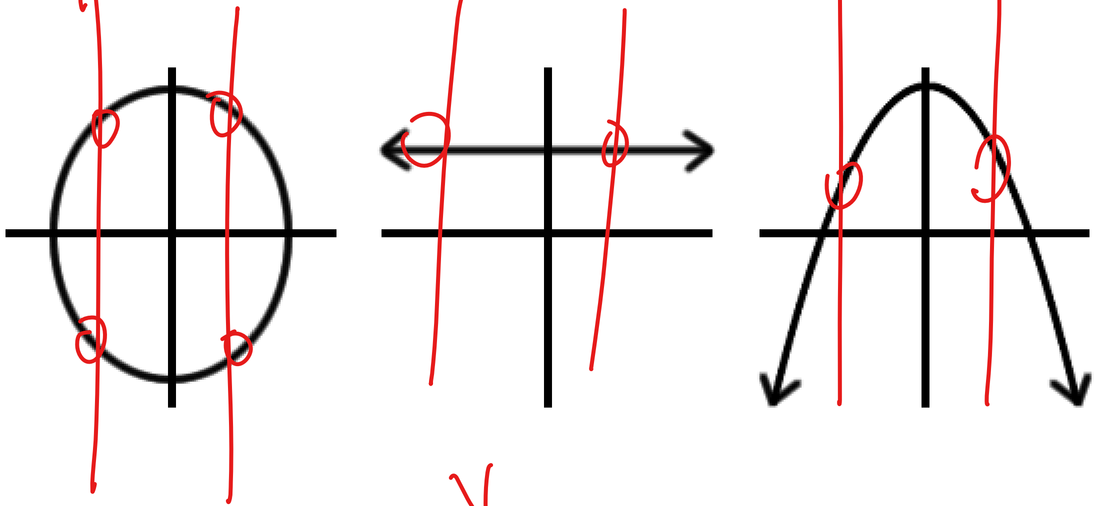
Ouch! This guy hits in **two spots**!

(2, 1), (2, -1)

(x, y)
(2, -1)

Example

Which of these are functions?



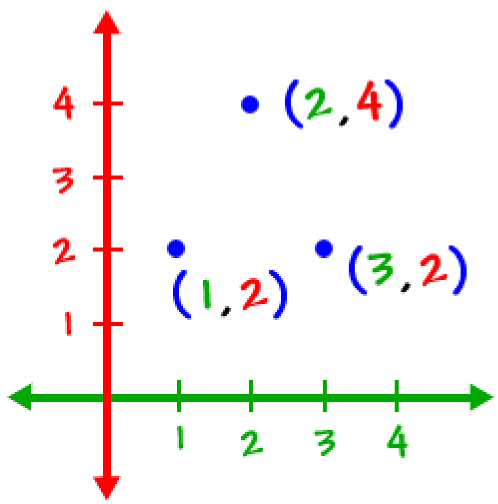
No Yes Yes

Functions with Sets

Rule $= \{(1,2),(2,4),(3,2)\}$

These guys in the set are just like (x , y) points on a graph...

So, we can actually graph this:



*** Don't connect the dots!**
These are isolated points

Just like before, the X guys are our input guys -- the domain:

$R = \{(1,2),(2,4),(3,2)\}$

domain guy **domain guy** **domain guy**

$D = \{1, 2, 3\}$

And remember that y guys are really f(x) guys -- range guys:

$R = \{(1,2),(2,4),(3,2)\}$

range guy **range guy** **range guy**

$R = \{2, 4\}$

OK, so how can I tell if this guy is a function or not?

Just ask yourself do the X's repeat

Here's the official definition:

A rule is a function if x
each element in the domain
goes to
exactly one element in the range.
y

Example

Repeated

R = {(1, 2), (2, 3), (3, 4), (1, 4)}

Function?

YES **NO**

Repeated

R = {(a, 1), (b, 1), (c, 2), (b, 3)}

Function?

YES **NO**

R = { (1, a), (2, a), (3, a) }

Yes this function

Functions with Sets

Domain: Read from LEFT to RIGHT

Range: Read from BOTTOM to TOP

Desmos Activity Builder: Domain and Range

Difference Quotient

The Difference Quotient:

$$\frac{f(x+h) - f(x)}{h}$$

So, you're going to be given a function like

$$f(x) = x + 2$$

and you'll need to work out that big mess, it's not that bad trust me

① $f(x+h)$ **Which we can now do!**

② $- f(x)$ **Easy - just remember to use ( ) !**

③ $\frac{\qquad}{h}$ **Piece of cake!**

Let's start working this thing out. Here's the original function:

$$f(x) = x + 2$$

Change x to x+h

Here are the pieces:

① $f(x+h)$:

So $f(x+h) = (x+h) + 2$

$f(x+h) = \boxed{x+h+2}$ **we don't need the ( ) here**

② $-f(x)$:

This is just subtracting the original function

$-(x+2)$ **( ) is important!**

③ **Just stick an h under everything and clean up**

$$\frac{f(x+h)-f(x)}{h} = \frac{\overbrace{[x+h+2]}^{f(x+h)} - \overbrace{(x+2)}^{f(x)}}{h}$$

clean it up!

$$= \frac{\cancel{x}+h+\cancel{2}-\cancel{x}-\cancel{2}}{h} = \frac{\cancel{h}}{\cancel{h}} = 1$$

Let's do another one:

Given $f(x) = 2x^2 + 3$ **find the difference quotient:**

$f(x+h) \qquad f(x)$

$$\frac{f(x+h) - f(x)}{h} = \frac{[2(x+h)^2+3] - (2x^2+3)}{h}$$

$(x+h)(x+h)$

$$= \frac{2(x^2+2xh+h^2)+3-2x^2-3}{h}$$

at this point, everything without an h should subtract out!

$$= \frac{\cancel{2x^2}+4xh+2h^2\cancel{+3}\cancel{-2x^2}\cancel{-3}}{h}$$

$$= \frac{4xh+2h^2}{h} = \frac{\cancel{h}(4x+2h)}{\cancel{h}} = 4x+2h$$

*** If you do these correctly,**
this h will reduce out at the end!