How to Tell If Two Functions Are Inverses

Well, we learned before that we can look at the graphs. Remember, if the two graphs are symmetric with respect to the line $y = x$ (mirror images over $y = x$), then they are inverse functions.

But, we need a way to check without the graphs, because we won't always know what the graphs look like!

If you're got two functions, $f(x)$ and $g(x)$, and

$$(f \circ g)(x) = (g \circ f)(x)$$

then $f(x)$ and $g(x)$ are inverse functions.

Let's try this on an easy one that we know will work:

$$f(x) = x + 3 \quad \text{and} \quad g(x) = x - 3$$

$\boxed{1}$ $(f \circ g)(x) = f(g(x))$

1. $f(\text{blob}) = (\text{blob}) + 3$
2. $f(g(x)) = (g(x)) + 3$
3. $f(x - 3) = (x - 3) + 3 = x$

$\boxed{2}$ $(g \circ f)(x) = g(f(x))$

① $g(\text{blob}) = (\text{blob}) - 3$

② $g(f(x)) = (f(x)) - 3$

③ $g(x+3) = (x+3) - 3 = x$

So, $(f \circ g)(x) = x$ **and** $(g \circ f)(x) = x$

$$(f \circ g)(x) = (g \circ f)(x)$$

Yep, they are inverses, just like we thought!

If $(f \circ g)(x) = x$ or $(g \circ f)(x) = x$,

then $f(x)$ and $g(x)$ are inverse functions.

So, you really only need to check one of them!

Check it out:

Are these inverse functions?

$$f(x) = 2x - 5 \text{ and } g(x) = \frac{1}{2}x + \frac{5}{2}$$

$$(f \circ g)(x) = f(g(x))$$

① $f(\text{blob}) = 2(\text{blob}) - 5$

② $f(g(x)) = 2(g(x)) - 5$

③ $f\left(\frac{1}{2}x + \frac{5}{2}\right) = 2\left(\frac{1}{2}x + \frac{5}{2}\right) - 5$

$$= x + 5 - 5 = x$$ Yep!

You can double-check it by crunching $(g \circ f)(x)$.

YOUR TURN:

Are these inverse functions?

$$f(x) = x^2 - 3 \text{ and } g(x) = \sqrt{x+3}$$

$$f(g(x)) = (\text{blob})^2 - 3$$
$$= (g(x))^2 - 3$$
$$= \left(\cancel{\sqrt{x+3}}\right)^{\cancel{2}} - 3$$
$$= x + 3 - 3 = x$$

Yes!

they are inverses because we got x as the answer

How to Find the Inverse of a Function

Let's just do one, then I'll write out the list of steps for you.

Find the inverse of $f(x) = -\frac{1}{3}x + 1$

STEP 1: Stick a "y" in for the "f(x)" guy:

$$y = -\frac{1}{3}x + 1$$

STEP 2: Switch the x and y
(because every (x, y) has a (y, x) partner!):

$$x = -\frac{1}{3}y + 1$$

STEP 3: Solve for y:

$$3(x) = \left(-\frac{1}{3}y + 1\right)3$$

multiply by 3 to ditch the fraction

$$3x = -y + 3$$
$$-3 \qquad -3$$

ditch the +3

$$3x - 3 = -y$$

multiply by -1

$$-3x + 3 = y \longrightarrow \boxed{y = -3x + 3}$$

STEP 4: Stick in the inverse notation, $f^{-1}(x)$

$$f^{-1}(x) = -3x + 3$$

Remember, you've got two ways you can double-check this answer to see if it's right:

1 **Graph** $f(x)$ **and** $f^{-1}(x)$ **on the same graph and see if they're mirror images over the line** $y = x$**.**
(Easy -- since these are both lines.)

2 **Find either** $(f \circ f^{-1})(x)$ **or** $(f^{-1} \circ f)(x)$
(or both for practice!)
***Note: This is just like** $(f \circ g)(x)$**, but with different notation.**

OK, here's the list of steps:

How to find the inverse of a function:

STEP 1: Stick a "y" in for the "$f(x)$."

STEP 2: Switch the x and y.

STEP 3: Solve for y.

STEP 4: Stick $f^{-1}(x)$ in for the "y."

THEN, CHECK IT!

YOUR TURN:

Find the inverse of $f(x) = -\frac{2}{3}x + 5$

1. $y = \frac{-2}{3}x + 5$

2. $x = \frac{-2}{3}y + 5$

3. $3(x) = \left(\frac{-2}{3}y + 5\right)3$

$3x = -2y + 15$

$-15 \qquad -15$

$\frac{3x}{-2} - \frac{15}{-2}$

$\frac{3x - 15}{-2} = \frac{-2y}{-2}$

$-\frac{3}{2}x + \frac{15}{2} = y$

4. $f^{-1}(x) = \frac{-3}{2}x + \frac{15}{2}$