

Useful Fact Sheet Module 2 – Alg.Trig. I

Transformations		
Vertical Shift $y = f(x) + b$ $y = f(x) - b$	Raise the graph of f by b units Lower the graph of f by b units	b is added to $f(x)$ b is subtracted from $f(x)$
Horizontal Shift $y = f(x + d)$ $y = f(x - d)$	Shift the graph of f to the left by d units Shift the graph of f to the right by d units	x is replaced with $x+d$ x is replaced with $x-d$
Reflections: x -axis $y = -f(x)$	Reflect the graph of f about the x -axis	$f(x)$ is multiplied by -1
Reflections: y -axis $y = f(-x)$	Reflect the graph of f about the y -axis	x is replaced with $-x$
Vertical Stretching $y = af(x); a > 1$	Multiply each y-coordinate of $y = f(x)$ by a	$f(x)$ is multiplied by c ; $c > 1$
Vertical Shrinking $y = af(x); 0 < a < 1$	Multiply each y-coordinate of $y = f(x)$ by a	$f(x)$ is multiplied by c ; $0 < c < 1$
Horizontal Shrinking $y = f(cx); c > 1$	Divide each x-coordinate of $y = f(x)$ by c	x is replaced by cx ; $c > 1$
Horizontal Stretching $y = f(cx); 0 < c < 1$	Divide each x-coordinate of $y = f(x)$ by c	x is replaced by cx ; $0 < c < 1$

Horizontal Line Test – If it is possible for a horizontal line to intersect the graph of a function more than once, then the function is **not** 1-1 and its inverse is **not** a function.

Vertical line test which states that if any vertical line intersects the graph of a relation in more than one point, then the relation graphed is not a function.

Definition of the Inverse function: $f(f^{-1}(x)) = x$ and for every x in the domain of f^{-1} and $f^{-1}(f(x)) = x$ for every x in the domain of f

Obtaining an inverse relation: interchange the first and second coordinates the relation obtained is an inverse.

Determinate of a 2x2 Matrix :
$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

(second order determinate)

Determinate of a 3x3 Matrix :
$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - a_3b_2c_1 - b_3c_2a_1 - c_3a_2b_1$$

Solving Three Equations in Three Variables Using Determinants

Cramer's Rule

If

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$

then

$$x = \frac{D_x}{D}, y = \frac{D_y}{D}, \text{ and } z = \frac{D_z}{D}.$$

These four third-order determinants are given by

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \text{These are the coefficients of the variables } x, y \text{ and } z. D \neq 0$$

$$D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} \quad \text{Replace } x\text{-coefficients in } D \text{ with the constants on the right of the three equations.}$$

$$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \quad \text{Replace } y\text{-coefficients in } D \text{ with the constants on the right of the three equations.}$$

$$D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} \quad \text{Replace } z\text{-coefficients in } D \text{ with the constants on the right of the three equations.}$$

central angle, θ , that intercepts the arc is $\theta = \frac{s}{r}$ radians

Solving a Linear System in Two Variables Using Determinants

Cramer's Rule

If

$$\begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2 \end{aligned}$$

then

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad \text{and} \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

where

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0.$$

$\sin(t) = \frac{y}{r}$	$\csc(t) = \frac{r}{y}, y \neq 0$
$\cos(t) = \frac{x}{r}$	$\sec(t) = \frac{r}{x}, x \neq 0$
$\tan(t) = \frac{y}{x}, x \neq 0$	$\cot(t) = \frac{x}{y}, y \neq 0$

Coterminal Angles – Increasing or decreasing the degree of an angle in standard position by an integer multiply of 360° or 2π results in a coterminal angle.

linear speed is $v=s/t$, where s is the **arc length** given by $s = r\theta$, and its **angular speed** is $w=\theta/t$.

We can write linear speed in terms of angular speed $v = \frac{s}{t} = \frac{r\theta}{t} = r \cdot \frac{\theta}{t} = rw$

Even and Odd Trigonometric Functions	
$\cos(\theta) = \cos(-\theta)$ Even	$-\sin(\theta) = \sin(-\theta)$ Odd
$\sec(\theta) = \sec(-\theta)$ Even	$-\csc(\theta) = \csc(-\theta)$ Odd
$-\tan(\theta) = \tan(-\theta)$ Odd	$-\cot(\theta) = \cot(-\theta)$ Odd

Pythagorean Identities –

$$\begin{aligned} \sin^2(\theta) + \cos^2(\theta) &= 1 & 1 + \tan^2(\theta) &= \sec^2(\theta) \\ 1 + \cot^2(\theta) &= \csc^2(\theta) \end{aligned}$$

Periodic Properties of the Sine and Cosine Functions

$$\sin(\theta + 2\pi) = \sin(\theta) \quad \text{and} \quad \cos(\theta + 2\pi) = \cos(\theta)$$

Periodic Properties of the Tangent & Cotangent Functions

$$\tan(\theta + \pi) = \tan(\theta) \quad \text{and} \quad \cot(\theta + \pi) = \cot(\theta)$$

Pythagorean Theorem

$$\begin{aligned} \sin\theta &= \frac{\text{opp}}{\text{hyp}} & \cos\theta &= \frac{\text{adj}}{\text{hyp}} & \tan\theta &= \frac{\text{opp}}{\text{adj}} \\ \csc\theta &= \frac{\text{hyp}}{\text{opp}} & \sec\theta &= \frac{\text{hyp}}{\text{adj}} & \cot\theta &= \frac{\text{adj}}{\text{opp}} \end{aligned}$$

Reciprocal Identities	
$\sin(\theta) = \frac{1}{\csc(\theta)}$	$\csc(\theta) = \frac{1}{\sin(\theta)}$
$\cos(\theta) = \frac{1}{\sec(\theta)}$	$\sec(\theta) = \frac{1}{\cos(\theta)}$
$\tan(\theta) = \frac{1}{\cot(\theta)}$	$\cot(\theta) = \frac{1}{\tan(\theta)}$
Quotient Identities	
$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$	$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$