How to Tell if a Function Has an Inverse Function (One-to-One)

Here it is:

A function, $f(x)$, has an inverse function if $f(x)$ is one-to-one.

OK, one-to-one...

There's an easy way to look at it, then there's a more technical way. (The technical way will really get us off track, so I'm leaving it out for now.)

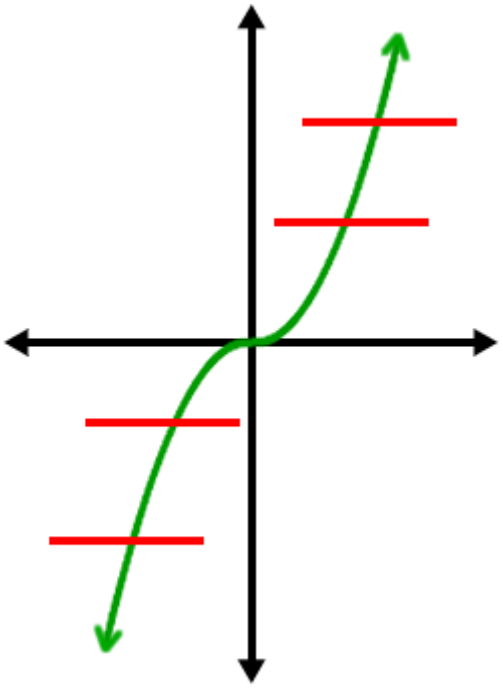
Here's the easy way:

The Horizontal Line Test:

If you can draw a horizontal line so that it hits the graph in more than one spot, then it is NOT one-to-one.

Check it out:

Is $f(x) = x^3$ one-to-one?

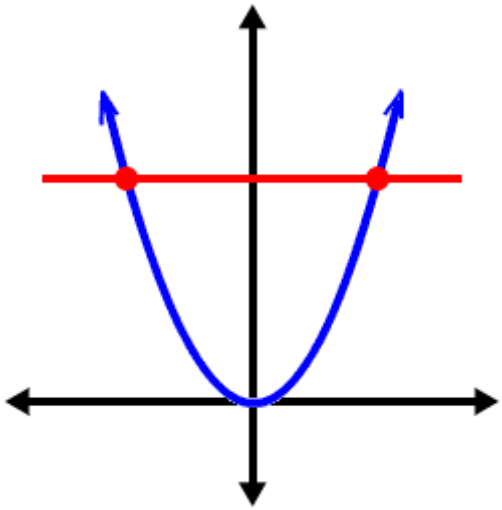


Yep!

So, $f(x) = x^3$ HAS an inverse function.

Yep it is 1-1

Is $f(x) = x^2$ one-to-one?



No! It fails the horizontal line test.

So $f(x) = x^2$ does NOT have an inverse function.

No not 1-1

Did I just say that $f(x) = x^2$ does NOT have an inverse function?

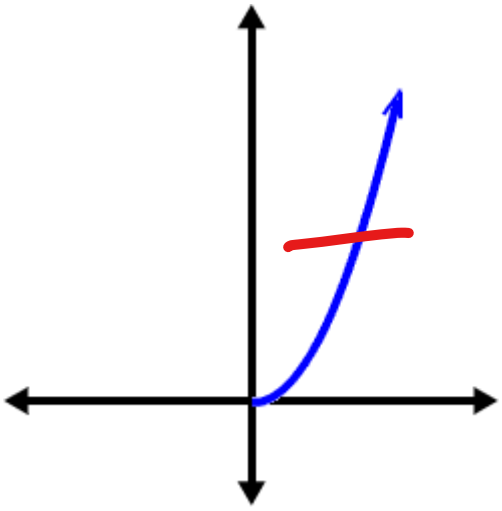
Yep, that's what I said.

But, didn't I say that the inverse of $f(x) = x^2$ was $g(x) = \sqrt{x}$?

Nope! Not quite!

Check back... Remember that I said we had to restrict it to $x \geq 0$?
Well, here's the graph of

$f(x) = x^2$ for $x \geq 0$:



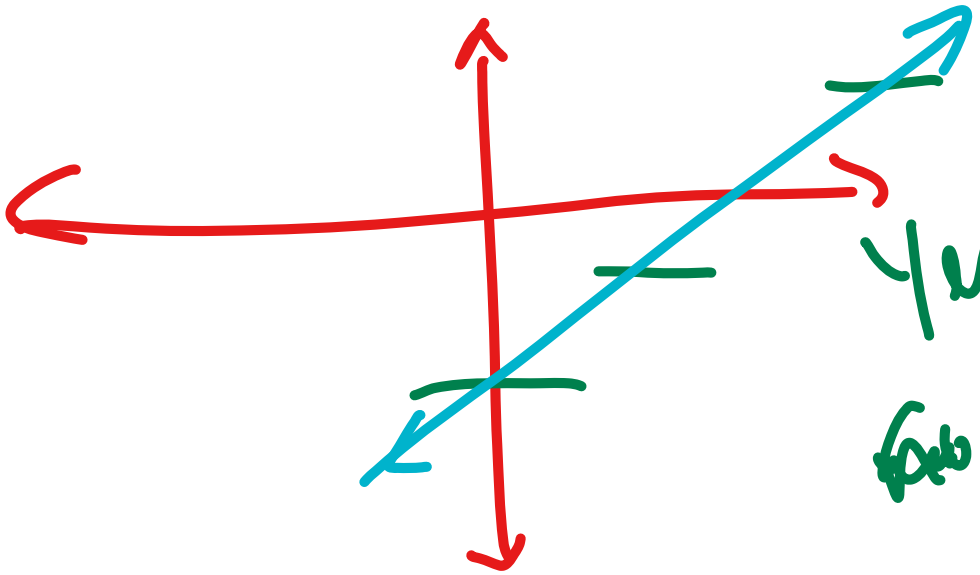
Oh!

THIS guy IS one-to-one!

YOUR TURN:

Does $f(x) = \frac{2}{3}x - 5$ have an inverse function?

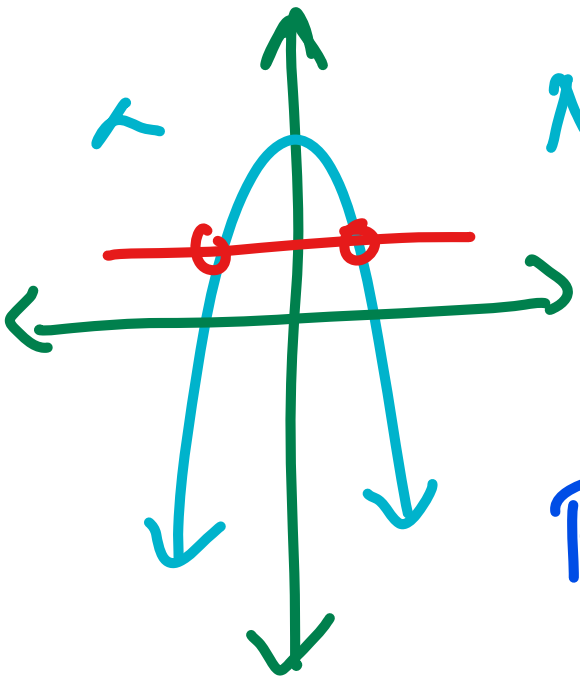
*** Just graph it and see if it's one to one.**



Yes has an inverse

Does $f(x) = 4 - x^2$ have an inverse function?

If it doesn't, fix it so that it does.



No $f(x) = 4 - x^2$ does not have an inverse fails horizontal line test.

Restrict $x \geq 0$