Finding the Domain Without the Graph

OK, so suppose we don't have the graph of a function to look at

Can we still find the domain and range?

Domains: Yes (as long as the algebra doesn't get too hairy... and it won't for us.)

Ranges: Not really (you usually need the picture -- unless it's something really basic.)

Asking for the domain of a function is the same as asking

"What are all the possible X guys that I can stick into this thing?"

Sometimes, what you'll really be looking for is

"Is there anything I CAN'T stick in?"

Check it out:

Let's find the domain of $f(x) = \frac{2}{x-3}$

Do you see any X guys that would cause a problem here?

What about $x = 3$ **?**

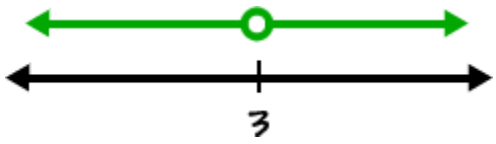
$$f(3) = \frac{2}{3-3} = \frac{2}{0}$$ ← ouch!

So, $x = 3$ **is a bad guy! Everyone else is OK, though.**

The domain is all real numbers except 3.

What would the interval notation be?

When in doubt, graph it on a number line:



Do the interval notation in two pieces:

domain = $(-\infty, 3) \cup (3, \infty)$

EXAMPLE

Find the domain of $f(x) = \frac{5}{x+7}$

Variable in denominator

Step 1: Denominator = 0

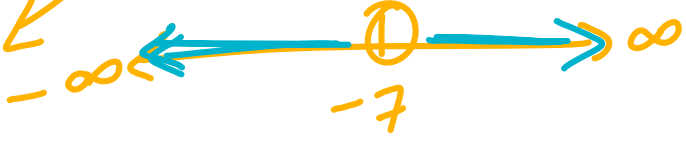
$x + 7 = 0$

Step 2: Solve for x

$x + 7 = 0$

$x = -7$

Bad guy not in domain



$(-\infty, -7) \cup (-7, \infty)$

Sometimes, you can't find the domain with a quick look.

Check it out:

Let's find the domain of $f(x) = \frac{1}{3-2x}$

Hmm... It's not so obvious!

BUT, we are still looking for the same thing:

$f(x) = \frac{1}{3-2x}$ ← **The bad X that makes the denominator 0!**

How do we find it? Easy!

Set the denominator = 0 and solve!

$$3 - 2x = 0$$
$$-3 \qquad -3$$
$$-2x = -3$$
$$x = \frac{-3}{-2} = \frac{3}{2}$$

The domain is $= (-\infty, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$

EXAMPLE

Find the domain of $f(x) = \frac{6}{5x+3}$ ← variable in denominator

Step 1: Denominator = 0

$5x + 3 = 0$

Step 2: Solve for x

$5x + 3 = 0$

$5x = -3$

$x = -3/5$ ← Bad guy not in Domain

$(-\infty, -3/5) \cup (-3/5, \infty)$

Where are the bad guys with this one?

$$f(x) = \sqrt{x+5}$$

Square roots -- what do we know about square roots?

$\sqrt{16} = 4$ **... So, 16 is OK to put in.**

$\sqrt{0} = 0$ **... So, 0 is OK.**

$\sqrt{3.2} \approx 1.788$ **... Yuck! But, 3.2 is OK.**

$\sqrt{-25} = ?$ **... Nope! Can't do it!**

***We only want real numbers!**

$$\sqrt{\text{inside}}$$

The inside of a radical cannot be negative if we want real answers only (no i guys). So, the inside of a radical has to be 0 or a positive number.

Set inside $\geq$ 0 and solve it!

Now, let's find the domain of

$$f(x) = \sqrt{x+5}$$

$$x + 5 \geq 0$$

$$x \geq -5$$

So, the domain of $f(x) = \sqrt{x+5}$ **is** $[-5, \infty)$**.**