***Q: Regard to the recurrence equation below:***

$$T(n) = 4T(n/2) + n$$

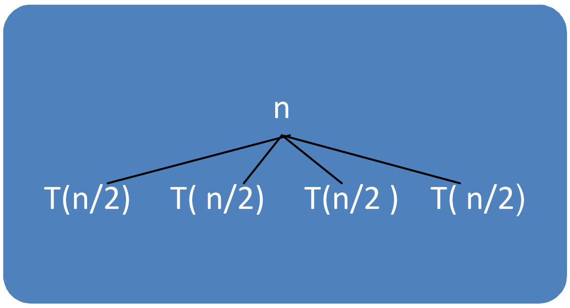
- ***a. Draw the recursion tree.***
- ***b. Determine the number of nodes and cost at each level.***
- ***c. Determine the total cost for all levels (i.e. in summation form).***

**Steps:**

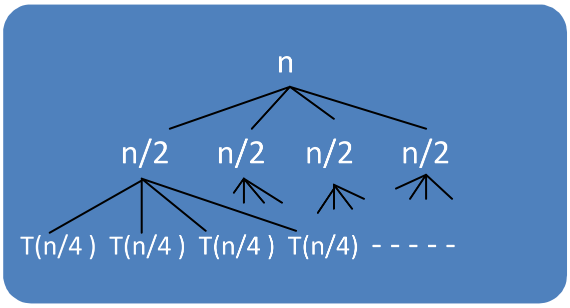
1. ***Draw the tree based on the recurrence***
2. ***From the tree determine:***
   - ***a. # of levels in the tree***
   - ***b. cost per level***
   - ***c. # of nodes in the last level***
   - ***d. cost of the last level (which is based on the number found in 2c)***
3. ***Write down the summation using ∑ notation – this summation sums up the cost of all the levels in the recursion tree***
4. ***Recognize the sum or look for a closed form solution for the summation created in 3). Use Appendix A.***
5. ***Apply that closed form solution to your summation coming up with your "guess" in terms of Big-O, or Θ, or Ω (depending on which type of asymptotic bound is being sought).***

## Solution :

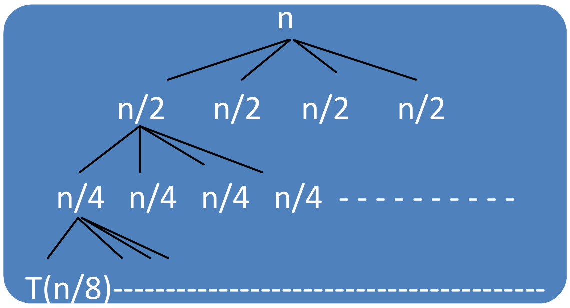
T(n) = 4T(n/2) +n




T(n/2)=4T(n/4)+n/2




T(n/4) = 4T(n/8) +(n/4)



| Level # | #of node | Recursion tree | Level Sum |
|---|---|---|---|
| **0** | 1 | n | n |
| **1** | 4 | n/2 n/2 n/2 n/2 | 4(n/2) = 2n |
| **2** | $4^2$ | n/4 n/4 n/4 n/4 - - - - - - - | $4^2$ (n/4) = 4n |
| **3** | $4^3$ | n/8 n/8 ------------- | $4^3$(n/8) = 8n |
| | . | . | . |
| | . | . | . |
| | . | . | . |
| **i** | $4^i$ | $n/2^i$ | $4^i(n/2^i) = 2^i n$ |
| | | . | |
| | | . | $4^{h-1}(n/2^{h-1}) = 2^{h-1}n$ |
| | | . | |
| **h** | $4^h$ | T(1) T(1) T(1) T(1) | $4^h$ |

$$T(n) = 4^h T(1) + \sum_{i=0}^{h-1} 2^i n$$

$$\frac{n}{2^h} = 1 \rightarrow h = log_2 n \leftarrow equation\ \#1$$

from *equation* #1

$$T(n) = 4^{log_2 n} T(1) + \sum_{i=0}^{h-1} 2^i n$$

$$T(n) = n^{log_2 4}\, T(1) + \sum_{i=0}^{h-1} 2^i n$$

$$n^{log_2 4} = n^2\ since\ log_2 4 = 2 \quad \leftarrow equation\ \#2$$

from *equation* #2

$$T(n) = n^2 T(1) + n \sum_{i=0}^{h-1} 2^i$$

$$T(n) = n^2 T(1) + n\,[1 + 2 + 2^2 + \cdots .. + 2^{h-1}]$$

$$Increasing\ Geometric\ Series \qquad r = 2 \quad a = 1$$

$$S = a + ar + ar^2 + \ \ldots + ar^{k-1} = \frac{a(r^k - 1)}{r - 1} \qquad r > 1$$

$$Since \quad r = 2 \quad a = 1$$

$$n\,[1 + 2 + 2^2 + \cdots + 2^{h-1}] = \frac{1(2^h - 1)}{2 - 1} = 2^h - 1$$

$$from\ Equation\ \#1$$

$$= 2^{log_2 n} - 1$$

$$= n^{log_2 2} - 1 \quad = n - 1 \qquad Since\ \ log_2 2 = 1$$

$$T(n) = n^2 \ \ T(1) + n(n-1)$$

$$T(n) \quad = \theta(n^2)$$