Week One Activities

● Graded

Student

Harper Chen

Total Points

96 / 100 pts

Question 1

Math Reading Activity

10 / 10 pts

✔ **– 0 pts** Correct

Question 2

CODING - Python code for SIR

20 / 20 pts

✔ **– 0 pts** Correct

Question 3

1.1 #1-#6 Reading Graphs

20 / 20 pts

✔ **– 0 pts** Correct

– 0 pts Good!

– 0 pts Excellent visuals!

Question 4

1.1 #7 The Simple Model

10 / 10 pts

✔ **– 0 pts** Correct

– 0 pts Excellent analysis! (fyi - more than required but superb work. Worthy of note.)

– 2 pts a) about 15000

– 2 pts b) about 42 or 43 days

– 2 pts c) about 3 days, ~ 21,000

– 2 pts d) 21 or 22 days

Question 5

1.1 #19. Quarantine

10 / 10 pts

✔ **– 0 pts** Correct

– 0 pts Excellent explanation.

– 2 pts a) .000005

– 2 pts b) about 14,200

– 2 pts c) 454,000

– 2 pts d) 1/14 * 45400

– 2 pts about .16

Question 6

1.1 #20. A new Model

6 / 10 pts

– 0 pts Correct

– 0 pts Good explanations

– 0 pts Great use of code for this.

– 2 pts a) about 12 or 13

– 2 pts b) about 4000

– 2 pts c) 8

✔ **– 2 pts** d) S' = 30. Careful here. The number of new case is expressed as S'. I' includes new cases AND subtracts those who recover.

✔ **– 2 pts** e) From c) and d), S' = -30, I = 100, so S = 15,000

– 10 pts no submission

Question 7

1.1 #21. another model

10 / 10 pts

✔ **– 0 pts** Correct

– 5 pts a) a = .0005 (.003*.1667), b = .25

– 5 pts b) <= 500

– 10 pts No submission

Question 8

1.1 #22 and another!

10 / 10 pts

✔ **– 0 pts** Correct

– 10 pts no submission

Question assigned to the following page: 1

Week One Activities

Question 1

After I watched the video, I recognized that the key to solving this problem lay in effectively manipulating the equations to express Q in terms of the other variables. I started by isolating g from the first equation, which allowed me to substitute it into the second equation. This substitution was a critical step because it reduced the number of variables in the equation for Q, making it easier to understand how Q changes based on x, y, and z.

After performing the substitution, I simplified the resulting equation and arrived at $Q = 183 - 24x - 9y - 4z$. This expression clearly showed that Q would decrease as x, y, and z increased. However, I also had to consider that the values of these variables should keep g non-negative to make the solution realistic. This realization led me to carefully consider different values for x, y, and z to find the combination that minimized Q while maintaining a valid value for g.

In exploring different combinations of x, y, and z, I discovered that setting these variables to zero yielded $Q = 183$, but I recognized that this might not be the true minimum. Exploring non-zero values could potentially lower Q even further, but would require more in-depth analysis or optimization techniques.

This activity reinforced my understanding of algebraic manipulation and optimization. It highlighted the importance of carefully considering constraints when minimizing a variable. I found the experience to be a valuable exercise in problem-solving and mathematical reasoning, both of which are crucial skills in my studies and future work. Overall, this task was not only challenging but also deeply satisfying, as it allowed me to apply theoretical concepts in a practical context.

Question assigned to the following page: 2

Question 1.3

```
# Initial conditions
t = 0
S = 45400
I = 2100
R = 2500
deltat = 1

# Print the initial state
print(f"t = {t}, S = {S}, I = {I}, R = {R}")

# Time loop for 3 steps
for k in range(1, 4):
    # Step I calculations
    Sprime = -0.00001 * S * I
    Iprime = 0.00001 * S * I - I / 14
    Rprime = I / 14

    # Calculate the changes
    deltaS = Sprime * deltat
    deltaI = Iprime * deltat
    deltaR = Rprime * deltat

    # Update the values
    t = t + deltat
    S = S + deltaS
    I = I + deltaI
    R = R + deltaR

    # Print the current state
    print(f"t = {t}, S = {S:.0f}, I = {I:.0f}, R = {R:.0f}")
```

Question assigned to the following page: 3

Question 1.1 1 - 6

1. The infection hits its peak around day 13. At that time, approximately 13,000 people are infected.

2. Initially, there are about more than 40,000 susceptible people. It takes around 20 days for the susceptible population to be reduced to about 20,000, which is half of the initial number.

3. It takes approximately 35 days for the recovered population to reach 25,000 people. Eventually, around more than 40,000 people recover.

4. The size of the infected population is increasing most rapidly around day 10, and it decreases most rapidly around day 20. This is determined by the steepest slopes in the I graph.

5. Approximately 20,000 people caught the illness during the first 20 days. This is determined by the difference in the susceptible population from day 0 to day 20.

6. When the graph of S is superimposed on R, the S graph appears compressed horizontally because R increases over a longer time period while S decreases more rapidly.

Question assigned to the following page: 4

Question 1.1 7

a) How many susceptibles will be left ten days later?

$$S(t) = S_0 + S' \times t$$

$$S(10) = 20{,}000 + (-470) \times 10 = 20{,}000 - 4{,}700 = 15{,}300$$

So, 15,300 susceptibles will be left ten days later.

b) How many days will it take for the susceptible population to vanish entirely?

Set $S(t) = 0$ and solve for t:

$$0 = 20{,}000 - 470 \times t$$

$$470 \times t = 20{,}000$$

$$t = \frac{20{,}000}{470} \approx 42.55 \text{ days}$$

It will take approximately 43 days for the susceptible population to vanish entirely.

c) How many susceptibles were there on the previous Sunday?

Sunday is 3 days before Wednesday, so:

$$S(-3) = 20{,}000 + (-470) \times (-3) = 20{,}000 + 1{,}410 = 21{,}410$$

There were 21,410 susceptibles on the previous Sunday.

d) How many days before Wednesday were there 30,000 susceptibles?

Set $S(t) = 30{,}000$ and solve for t:

$$30{,}000 = 20{,}000 - 470 \times t$$

$$470 \times t = 20{,}000 - 30{,}000 = -10{,}000$$

$$t = \frac{-10{,}000}{-470} \approx 21.28 \text{ days}$$

So, 30,000 susceptibles existed approximately 21 days before Wednesday.

Question assigned to the following page: 5

Question 1.1 19

a) **New Transmission Coefficient:**

If the quarantine cuts in half the chance that a susceptible will fall ill, then the new transmission coefficient β' is:

$$\beta' = \frac{\beta}{2}$$

b) **New Threshold Level for S:**

Originally, the threshold level for S was 7143 when the transmission coefficient was β. With the new transmission coefficient $\beta' = \frac{\beta}{2}$, the new threshold level S' can be calculated using:

$$S' = \frac{S_{\text{threshold}}}{2} = \frac{7143}{2} = 3571.5$$

c) **Quarantine Impact on Epidemic:**

Starting with $S = 45{,}400$, the quarantine will reduce the transmission coefficient and hence lower the rate of new infections. However, without further specifics on β and other parameters, we can hypothesize that the quarantine may reduce the number of cases significantly but may not eliminate the epidemic completely unless the S drops below the new threshold.

d) **Smaller Transmission Coefficient:**

If the current transmission coefficient is not small enough to guarantee that I never increases, we can find a smaller value β'' that would ensure I does not rise. To guarantee I does not increase, S must stay below the new threshold:

$$\beta'' \text{ is calculated based on } S \leq 3571.5$$

e) **Largest Transmission Coefficient to Prevent Epidemic:**

For the initial susceptible population $S = 45{,}400$, the largest transmission coefficient β_{max} that can still guarantee I never goes up is determined by ensuring that S remains below the threshold that would cause I to grow:

β_{max} should be a fraction of the original transmission coefficient, such as one-third or one-fourth.

Question assigned to the following page: 6

Question 1.1 20

a) **Duration of Infection:**

The rate equation $R' = 0.08I$ suggests that 8% of the infected population I recovers each day. Therefore, the average duration that someone remains infected is approximately:

$$\text{Duration} = \frac{1}{0.08} \approx 12.5 \text{ days}$$

This means that, on average, a person who catches this illness will remain infected for about 12.5 days.

b) **Threshold for Susceptible Population:**

For the number of cases to increase, I' must be positive, meaning:

$$0.00002SI > 0.08I$$

Dividing both sides by I (assuming $I > 0$):

$$S > \frac{0.08}{0.00002} = 4000$$

Therefore, the susceptible population S must be greater than 4000 for the illness to take hold and the number of cases to increase.

c) **Number of Recoveries:**

If 100 people are currently ill, the number of recoveries during the next 24 hours is given by:

$$R' = 0.08 \times 100 = 8$$

So, 8 out of the 100 infected individuals will recover during the next 24 hours.

d) **Interpretation of New Cases:**

If 30 new cases appear during the same 24 hours, this implies:

$$I' = 30$$

Since $I' = 0.00002SI - 0.08I$, this tells us that the net change in the number of infected individuals is 30.

e) **Determining the Current Susceptible Population:**

Using the information from parts (c) and (d), we know:

$$0.00002SI - 0.08I = 30$$

Substituting $I = 100$ into the equation:

$$0.00002S \times 100 - 0.08 \times 100 = 30$$

Question assigned to the following page: 6

Simplifying:

$$0.002S - 8 = 30$$

$$0.002S = 38$$

$$S = \frac{38}{0.002} = 19,000$$

Therefore, the current susceptible population S is 19,000.

Question assigned to the following page: 7

Question 1.1 21

a) **Constructing the SIR Model:**

We are given that:

- The illness lasts for 4 days.
- A typical susceptible person meets 0.3% of the infected population each day.
- The infection is transmitted in only one contact out of six.

From this, we can derive the parameters for the SIR model:

- **Recovery rate** (γ): Since the illness lasts for 4 days, the recovery rate is:

$$\gamma = \frac{1}{4} = 0.25 \text{ per day}$$

- **Transmission rate** (β): The transmission rate can be calculated by combining the contact rate and the probability of transmission. If 0.3% of the susceptible population meets an infected individual each day, and transmission occurs in one out of six contacts, then:

$$\beta = 0.003 \times \frac{1}{6} = 0.0005 \text{ per day}$$

Therefore, the SIR model can be constructed as follows:

$$S' = -0.0005 \cdot S \cdot I$$

$$I' = 0.0005 \cdot S \cdot I - 0.25 \cdot I$$

$$R' = 0.25 \cdot I$$

b) **Determining the Susceptible Population Threshold:**

To determine how small the susceptible population S needs to be for the illness to fade away without becoming an epidemic, we need to find when the number of infected individuals stops increasing. This occurs when $I' \leq 0$. Therefore:

$$0.0005 \cdot S \cdot I - 0.25 \cdot I \leq 0$$

Dividing both sides by I (assuming $I > 0$):

$$0.0005 \cdot S \leq 0.25$$

$$S \leq \frac{0.25}{0.0005} = 500$$

Therefore, the susceptible population must be 500 or fewer for the illness to fade away without becoming an epidemic.

Question assigned to the following page: 8

Question 1.1 22

a) **Threshold Level for S:**

The threshold level for S is the value below which the number of infected individuals I will only decline. This occurs when $I' \leq 0$. Therefore, we set:

$$aSI - bI \leq 0$$

Dividing by I (assuming $I > 0$):

$$aS \leq b$$

$$S \leq \frac{b}{a}$$

Thus, the threshold level for S is $\frac{b}{a}$. Below this level, the number of infected individuals will decline.

The threshold level S is derived by analyzing when the rate of change of the infected population I' becomes non-positive, which indicates that the infection is no longer spreading. This leads to the expression $S \leq \frac{b}{a}$.

b) **Comparing Two Illnesses:**

Consider two illnesses with the same transmission coefficient a but differing in the length of time someone stays ill, which affects the recovery coefficient b. The illness with the lower recovery coefficient b corresponds to a longer duration of illness. The threshold level for S is $\frac{b}{a}$, so:

$$S_{\text{threshold}} \propto b$$

Therefore, the illness with the shorter duration of illness (higher b) will have a higher threshold level for S, while the illness with the longer duration of illness (lower b) will have a lower threshold level for S.

Since the threshold level S is directly proportional to the recovery coefficient b, the illness with the longer duration (lower b) has a lower threshold level for S, making it easier for the illness to spread in a population.