Week 8 Activities

● Graded

Student

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Total Points

100 / 100 pts

Question 1

Activity 1

30 / 30 pts

– 0 pts Correct

✔ **– 0 pts** Good showing work!

– 0 pts See notes

Question 2

Activity 2

30 / 30 pts

✔ **– 0 pts** Correct

– 0 pts Beautiful! Superbly done.

– 3 pts Missed one or two - check your answers below.

– 5 pts a) 1 b) 12. c) 5 d) 11/16

– 5 pts e) 34. f) -1/4 g) 20 h) 4 I) - 2/9

– 4 pts J) -130

– 3 pts k) not enough info.

– 3 pts L) about 7.1

– 15 pts Did you misunderstand the instructions? Looking for the DERIVATIVE.

– 0 pts Click here to replace this description.

– 0 pts Click here to replace this description.

Question 3

Activity 4

40 / 40 pts

✔ **– 0 pts** Correct

– 10 pts Need more here.

– 0 pts Not complete

– 0 pts Good!

Question assigned to the following page: 1

Activity 1

(a)

$$3x^5 - 10x^2 + 8$$

Apply the power rule to each term.

apply the power rule:

$$3x^5 = 3 \cdot 5x^{5-1} = 15x^4$$

apply the power rule:

$$(-10x^2) = -10 \cdot 2x^{2-1} = -20x$$

For the constant 8, the derivative is:

$$8 = 0$$

combining all, the derivative is:

$$f'(x) = 15x^4 - 20x$$

(b)

$$(5x^{12} + 2)(\pi - \pi^2 x^4)$$

Apply the product rule:

$$f'(x)g(x) + f(x)g'(x)$$

Let:

$$f(x) = 5x^{12} + 2$$

$$g(x) = \pi - \pi^2 x^4$$

apply the power rule:

$$5x^{12} = 5 \cdot 12x^{12-1} = 60x^{11}$$

$$2 = 0$$

So,

$$f'(x) = 60x^{11}$$

Question assigned to the following page: 1

apply the power rule:

$$-\pi^2x^4 = -\pi^2 \cdot 4x^{4-1} = -4\pi^2x^3$$

So,

$$g'(x) = -4\pi^2x^3$$

combining all, the derivative is:

$$f'(x) = 60x^{11}(\pi - \pi^2x^4) + (5x^{12} + 2)(-4\pi^2x^3)$$

(c)

$$\sqrt{u} - \frac{3}{u^3} + 2u^7$$

Rewrite the terms using exponents:

$$(\sqrt{u} = u^{1/2})$$

$$(-\frac{3}{u^3} = -3u^{-3})$$

apply the power rule:

$$u^{1/2} = \frac{1}{2}u^{1/2-1} = \frac{1}{2}u^{-1/2}$$

apply the power rule:

$$-3u^{-3} = -3 \cdot (-3)u^{-3-1} = 9u^{-4}$$

apply the power rule:

$$2u^7 = 2 \cdot 7u^{7-1} = 14u^6$$

combining all, the derivative is:

$$f'(u) = \frac{1}{2}u^{-1/2} + 9u^{-4} + 14u^6$$

(d)

$$mx + b$$

This is a linear function.

For (mx), the derivative is:

Question assigned to the following page: 1

$$mx = m$$

For (b), the derivative is:

$$b = 0$$

Now, combining all, the derivative is:

$$f'(x) = m$$

(e)

$$0.5 \sin x + \sqrt[3]{x} + \pi^2$$

Apply the power and chain rules

the derivative is:

$$0.5 \cdot \cos x$$

apply the power rule:

$$x^{1/3} = \frac{1}{3} x^{1/3-1} = \frac{1}{3} x^{-2/3}$$

the derivative is:

$$\pi^2 = 0$$

combining all, the derivative is:

$$f'(x) = 0.5 \cos x + \frac{1}{3} x^{-2/3}$$

(f)

$$\frac{\pi - \pi^2 x^4}{5x^{12} + 2}$$

Apply the quotient rule:

$$\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Let:

$$f(x) = \pi - \pi^2 x^4$$

$$g(x) = 5x^{12} + 2$$

the derivative is:

Question assigned to the following page: 1

$$-\pi^2 \cdot 4x^3 = -4\pi^2 x^3$$

the derivative is:

$$5 \cdot 12x^{12-1} = 60x^{11}$$

applying the quotient rule:

$$f'(x) = \frac{(-4\pi^2x^3)(5x^{12}+2) - (\pi - \pi^2x^4)(60x^{11})}{(5x^{12}+2)^2}$$

(g)

$$2\sqrt{x} - \frac{1}{\sqrt{x}}$$

Rewrite the terms using exponents:

$$(\sqrt{x} = x^{1/2})$$

$$(\frac{1}{\sqrt{x}} = x^{-1/2})$$

Now, differentiate each term:

apply the power rule:

$$2x^{1/2} = 2 \cdot \frac{1}{2}x^{1/2-1} = x^{-1/2}$$

apply the power rule:

$$x^{-1/2} = -\frac{1}{2}x^{-3/2}$$

combining all, the derivative is:

$$f'(x) = x^{-1/2} + \frac{1}{2}x^{-3/2}$$

(h)

$$\tan z(\sin z - 5)$$

This is a product, use the product rule:

$$f'(z)g(z) + f(z)g'(z)$$

Let:

$$f(z) = \tan z$$

Question assigned to the following page: 1

$$g(z) = \sin z - 5$$

the derivative is:

$$\sec^2 z$$

the derivative is:

$$\cos z$$

applying the product rule:

$$f'(z) = \sec^2 z(\sin z - 5) + \tan z \cdot \cos z$$

(i)

$$\frac{\sin x}{x^2}$$

Apply the quotient rule:

$$\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Let:

$$f(x) = \sin x$$

$$g(x) = x^2$$

the derivative is:

$$\cos x$$

the derivative is:

$$2x$$

Now, applying the quotient rule:

$$f'(x) = \frac{\cos x \cdot x^2 - \sin x \cdot 2x}{x^4}$$

Simplifying:

$$f'(x) = \frac{x^2 \cos x - 2x \sin x}{x^4} = \frac{\cos x}{x} - \frac{2 \sin x}{x^3}$$

(j)

Question assigned to the following page: 1

$$x^2e^x$$

apply the product rule:

$$f'(x)g(x) + f(x)g'(x)$$

Let:

$$f(x) = x^2$$

$$g(x) = e^x$$

the derivative is:

$$2x$$

the derivative is:

$$e^x$$

Now, applying the product rule:

$$f'(x) = 2x \cdot e^x + x^2 \cdot e^x$$

Factoring:

$$f'(x) = e^x(2x + x^2)$$

(k)

$$\cos x + e^x$$

Differentiate each term separately.

the derivative is:

$$-\sin x$$

the derivative is:

$$e^x$$

Now, combining all, the derivative is:

$$f'(x) = -\sin x + e^x$$

(l)

$$\frac{\sin x}{\cos x}$$

use the derivative of tangent.

Question assigned to the following page: 1

the derivative is:

$$\sec^2 x$$

Thus, the derivative is:

$$f'(x) = \sec^2 x$$

(m)

$$\frac{e^x \ln x}{2x}$$

apply the quotient rule:

$$\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Let:

$$f(x) = e^x \ln x$$

$$g(x) = 2x$$

First, differentiate (f(x)). Using the product rule for (e^x \ln x):

- For

  $$(e^x)$$

  derivative is

  $$e^x$$

- For

  $$\ln x$$

  derivative is

  $$\frac{1}{x}$$

So, applying the product rule:

$$f'(x) = e^x \cdot \ln x + e^x \cdot \frac{1}{x}$$

differentiate:

$$g'(x) = 2$$

Applying the quotient rule:

Question assigned to the following page: 1

$$f'(x) = \frac{\left(e^x \ln x + \frac{e^x}{x}\right) \cdot 2x - e^x \ln x \cdot 2}{(2x)^2}$$

(n)

$$\frac{2^x}{10 + \sin x}$$

This is a quotient, apply the quotient rule:

$$f'(x)g(x) - f(x)g'(x) \text{ over } [g(x)]^2$$

Let:

$$f(x) = 2^x$$

$$g(x) = 10 + \sin x$$

the derivative is:

$$2^x \ln 2$$

the derivative is:

$$\cos x$$

applying the quotient rule:

$$f'(x) = \frac{(2^x \ln 2)(10 + \sin x) - (2^x)(\cos x)}{(10 + \sin x)^2}$$

(o)

$$\sin(e^x \cos x)$$

This is a composite function, apply the chain rule.

Let:

$$f(x) = e^x \cos x$$

the derivative is:

$$\cos(f(x)) \cdot f'(x)$$

Use the product rule for

$$e^x \cos x$$

:

Question assigned to the following page: 1

- For (e^x), the derivative is

$$e^x$$

- For (\cos x), the derivative is

$$-\sin x$$

Applying the product rule:

$$f'(x) = e^x \cos x - e^x \sin x$$

combining everything:

$$f'(x) = \cos(e^x \cos x) \cdot (e^x \cos x - e^x \sin x)$$

(p)

$$6e^{\cos t}/5\sqrt[3]{t}$$

This is a quotient, apply the quotient rule.

Let:

$$f(t) = 6e^{\cos t}$$

$$g(t) = 5\sqrt[3]{t} = 5t^{1/3}$$

using the chain rule:

- For

$$(e^{\cos t})$$

  , the derivative is

$$(e^{\cos t} \cdot (-\sin t))$$

  (since the derivative of

$$(\cos t)$$

  is

$$(-\sin t))$$

- Multiply by 6:

$$f'(t) = 6e^{\cos t} \cdot (-\sin t) = -6e^{\cos t} \sin t$$

Differentiate (g(t) = 5t^{1/3}):

$$g'(t) = 5 \cdot \frac{1}{3} t^{-2/3} = \frac{5}{3} t^{-2/3}$$

Question assigned to the following page: 1

applying the quotient rule:

$$f'(t) = \frac{(-6e^{\cos t}\sin t)\cdot 5t^{1/3} - 6e^{\cos t}\cdot \frac{5}{3}t^{-2/3}}{(5t^{1/3})^2}$$

(q)

$$\ln(x^2 + xe^x)$$

This is a composite function, apply the chain rule.

Let:

$$f(x) = x^2 + xe^x$$

The derivative is:

$$\frac{f'(x)}{f(x)}$$

Now, differentiate

$$(f(x) = x^2 + xe^x)$$

the derivative is:

$$2x$$

apply the product rule:

- The derivative of (x) is

$$(1)$$

- The derivative of (e^x) is

$$(e^x)$$

So, the derivative is:

$$xe^x + e^x$$

combining the derivatives:

$$f'(x) = 2x + xe^x + e^x$$

applying the chain rule:

$$f'(x) = \frac{2x + xe^x + e^x}{x^2 + xe^x}$$

(r)

Questions assigned to the following page: 2 and 1

$$\frac{5x^2 + \ln x}{7\sqrt{x} + 5}$$

This is a quotient, apply the quotient rule.

Let:

$$f(x) = 5x^2 + \ln x$$

$$g(x) = 7\sqrt{x} + 5 = 7x^{1/2} + 5$$

Differentiate (f(x) = 5x^2 + \ln x):

- For (5x^2), the derivative is (10x)
- For (\ln x), the derivative is (\frac{1}{x} $$

Thus,

$$f'(x) = 10x + \frac{1}{x}$$

Differentiate (g(x) = 7x^{1/2} + 5):

- For (7x^{1/2}), the derivative is (7 \cdot \frac{1}{2} x^{-1/2} = \frac{7}{2} x^{-1/2} $$

Thus,

$$g'(x) = \frac{7}{2}x^{-1/2}$$

applying the quotient rule:

$$f'(x) = \frac{(10x + \frac{1}{x}) \cdot (7x^{1/2} + 5) - (5x^2 + \ln x) \cdot \frac{7}{2}x^{-1/2}}{(7x^{1/2} + 5)^2}$$

Activity 2

(a)

$$f(t) + g(t)$$

The derivative of a sum is the sum of the derivatives:

$$(f(t) + g(t))' = f'(t) + g'(t)$$

At (t = 2):

$$f'(2) + g'(2) = 2 + (-1) = 1$$

(b)

Question assigned to the following page: 2

$$5f(t) - 2g(t)$$

The derivative of a constant multiple of a function is the constant times the derivative of the function:

$$(5f(t) - 2g(t))' = 5f'(t) - 2g'(t)$$

At (t = 2):

$$5f'(2) - 2g'(2) = 5(2) - 2(-1) = 10 + 2 = 12$$

(c)

$$f(t)g(t)$$

This is a product, apply the product rule:

$$(f(t)g(t))' = f'(t)g(t) + f(t)g'(t)$$

At (t = 2):

$$f'(2)g(2) + f(2)g'(2) = (2)(4) + (3)(-1) = 8 - 3 = 5$$

(d)

$$\frac{f(t)}{g(t)}$$

This is a quotient, apply the quotient rule:

$$\left(\frac{f(t)}{g(t)}\right)' = \frac{f'(t)g(t) - f(t)g'(t)}{(g(t))^2}$$

At (t = 2):

$$\frac{(2)(4) - (3)(-1)}{(4)^2} = \frac{8+3}{16} = \frac{11}{16}$$

(e)

$$g(f(t))$$

This is a composite function, apply the chain rule:

$$(g(f(t)))' = g'(f(t)) \cdot f'(t)$$

At (t = 2), we first find (f(2) = 3), so:

$$g'(f(2)) = g'(3) = 17$$

Question assigned to the following page: 2

applying the chain rule:

$$g'(f(2)) \cdot f'(2) = 17 \cdot 2 = 34$$

(f)

$$\sqrt{g(t)}$$

This is a square root function, apply the chain rule:

$$\left(\sqrt{g(t)}\right)' = \frac{1}{2\sqrt{g(t)}} \cdot g'(t)$$

At (t = 2):

$$\frac{1}{2\sqrt{4}} \cdot (-1) = \frac{1}{4} \cdot (-1) = -\frac{1}{4}$$

(g)

$$t^2 f(t)$$

This is a product apply the product rule:

$$(t^2 f(t))' = 2tf(t) + t^2 f'(t)$$

At (t = 2):

$$2(2)(3) + (2^2)(2) = 12 + 8 = 20$$

(h)

$$(f(t))^2 + (g(t))^2$$

differentiate each term separately and apply the chain rule:

$$\frac{d}{dt}\left((f(t))^2 + (g(t))^2\right) = 2f(t)f'(t) + 2g(t)g'(t)$$

At (t = 2):

$$2(3)(2) + 2(4)(-1) = 12 - 8 = 4$$

(i)

Question assigned to the following page: 2

$$\frac{1}{f(t)}$$

This is a reciprocal, apply the chain rule:

$$\left(\frac{1}{f(t)}\right)' = -\frac{f'(t)}{(f(t))^2}$$

At (t = 2):

$$-\frac{2}{(3)^2} = -\frac{2}{9}$$

(j)

$$f(3t - (g(1+t))^2)$$

This is a composite function, And need use the chain rule multiple times.

Let:

- (h(t) = 3t - (g(1 + t))^2)
- Then we need

$$f'(h(t)) \cdot h'(t)$$

know that (f'(h(t)) = f'(h(2))), but we need to calculate (h(2)) and (h'(2)).

1. (g(1 + t)): At (t = 2), (g(3) = 2), so:

$$g(1+2) = g(3) = 2$$

2. ((g(1 + t))^2 = (2)^2 = 4).

Thus, (h(2) = 3(2) - 4 = 6 - 4 = 2), so we need (f'(2)), which is given as 2.

compute (h'(t)):

$$h'(t) = 3 - 2g(1+t)g'(1+t)$$

At (t = 2), (g'(3) = 17), so:

$$h'(2) = 3 - 2(2)(17) = 3 - 68 = -65$$

Thus, the derivative is:

$$f'(h(2)) \cdot h'(2) = 2 \cdot (-65) = -130$$

(k) What additional piece of information would you need to calculate the derivative of f(g(t)) at t = 2 ?

Question assigned to the following page: 2

To calculate the derivative of the composite function f(g(t)) at t = 2, we need to use the **chain rule**:

$$(f(g(t)))' = f'(g(t)) \cdot g'(t)$$

The information we already have:

- g(2) = 4
- g'(2) = -1

But to calculate f'(g(2)) , need to know the value of f'(4) , which is not provided.

Conclusion: The additional piece of information required is f'(4) .

(l) Estimate the value of f(t) / g(t) at t = 1.95

use the **linear approximation** or **tangent line approximation** to estimate the value of f(t) / g(t) at t = 1.95.

From part (d), already know the derivative of f(t) / g(t) at (t = 2) is:

$$\left(\frac{f(t)}{g(t)}\right)' = \frac{11}{16}$$

also know:

$$\frac{f(2)}{g(2)} = \frac{3}{4} = 0.75$$

use the formula for linear approximation:

$$f(1.95) \approx f(2) + f'(2) \cdot (1.95 - 2)$$

Here:

$$\frac{f(1.95)}{g(1.95)} \approx \frac{f(2)}{g(2)} + \left(\frac{f'(t)}{g'(t)}\right) \cdot (1.95 - 2)$$

Substitute the values:

$$\frac{f(1.95)}{g(1.95)} \approx 0.75 + \left(\frac{11}{16}\right) \cdot (-0.05)$$

Now calculate:

$$\frac{f(1.95)}{g(1.95)} \approx 0.75 - 0.034375 = 0.715625$$

Thus, the estimated value is approximately **0.716**.

Activity 3

Question assigned to the following page: 3

1. Function `differentiate(expression)`

- **Input**: An algebraic expression (e.g., "3x^5 - 10x^2 + 8")
- **Output**: The derivative of the expression

```
plaintext
# 1: Parse the input expression into components (constants,
variables, operators).
components = parse_expression(expression)

# 2: Apply the basic rules of differentiation.
For each component in components:
    If the component is a constant:
        derivative = 0
    If the component is a variable (e.g., x):
        derivative = 1
    If the component is a power of x (e.g., x^n):
        Apply the power rule:
        derivative = n * x^(n-1)

# 3: Handle combinations (addition, subtraction, multiplication,
division).
    If the component involves addition/subtraction:
        derivative = differentiate(left_component) +
differentiate(right_component)
    If the component involves multiplication:
        Apply the product rule:
        derivative = differentiate(left_component) *
right_component + left_component *
differentiate(right_component)
    If the component involves division:
        Apply the quotient rule:
        numerator = differentiate(left_component) *
right_component - left_component *
differentiate(right_component)
        denominator = right_component^2
        derivative = numerator / denominator

# 4: Handle chain rule.
    If the component is a function of a function:
        outer_function = get_outer_function(component)
        inner_function = get_inner_function(component)
        derivative = differentiate(outer_function) *
differentiate(inner_function)

# 5: Return the final derivative after applying all necessary
rules.
return derivative
```