Vulnerability and Protector Control: Cellular Automata Approach

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Abstract. In this work we consider the protector control problem using cellular automata approach. We give some definitions and characterizations of vulnerable zones and protector control for a cellular automaton model. We illustrate this notion through a fire forest example using a developed application with JAVA environment.

1 Introduction

Understanding of complex environmental systems which involve a large number of interacting components and exhibit important features at multiple spatial and temporal scales, remains an important issue on which several researchers are focused . In connection with the theory of complex systems, the study of the behavior of these systems will help to predict and/or control their evolution.

For the study of distributed parameters systems, the classical approach was based on Partial Differential Equations (PDEs) and a large literature has been devoted to this approach. However, the complexity of real systems ranging from biology and ecology to medicine and species abundance, leads to serious difficulties both in control theory and in the model implementation.

In this context and with the growing interest in spatial modeling, the investigation of more and more efficient modeling tools is necessary. The adopted methods should ensure the persistence of the main properties of these systems, such as their symmetries and conservation laws and their global dynamics.

Among the problems related to systems analysis, we consider the concept of vulnerability which characterizes the response of a distributed parameters system subject to an expanding spatial disturbance. For such problem, we consider an approach based on cellular automata (CA) models and investigate the possibility of protecting the region which is vulnerable to the effects of a given disturbance. The notions of vulnerability and protector control have been recently introduced and studied by means of PDEs in both linear and nonlinear cases [1,2]. Their practical use which generally needs a hard implementation of PDEs, particularly in the case of controlled processes, is still limited. The purpose of this paper is to consider CA models, which are often described as a counterpart to PDEs, as tools for modelling and simulating vulnerability and risk for some real phenomena such as pollution, flood, forest fire, etc.). The reason behind the choice of

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CA models is their capability to provide a powerful framework for investigating spatial expanding behaviours in part due to their explicit representation of space.

Despite the apparent simplicity of CA models, they are able to describe the complexity of real world systems by means of local transition rules which will be directly expressed as algorithms and then easily implemented as computer programs [3,12,15]. They will be used in this paper to illustrate a theoretical aspect of systems and control theory through a simple example. It consists of modelling and simulating a fire spread, considered as a disturbance of a system described by a vegetation dynamics. The control will be also introduced in order to protect a prescribed vulnerable area from the fire spreading. The concept of spreadability have been already studied by means of CA models [5,11] where a characterization of spreadable zones has been established. The control which is defined for distributed parameters systems as an external excitation which may steer the system to a given objective, has been also introduced for CA models [6]. For the studied example, several models for vegetation or fire dynamics can be considered among the large literature devoted to these systems [8,9,13,14]. The investigated model in this paper which has been used only to illustrate the notions of vulnerability and protector control, has to be simplified for a more general study.

This paper is organized as follows. In the next section, after recalling the basic definition of CA models, the study of vulnerability and protector control by means of CA models will be given. Section 3 is focused on the example of a forest fire dynamics. Simulation results will be given to illustrate the considered phenomenon using a developed simulator. Some concluding remarks are given in section 4.

2 Vulnerability and Protector Control with Cellular Automata Approach

In this paper we consider the protector control problem with cellular automata approach. We recall the CA principle.

2.1 Cellular Automata

Cellular automata are discrete models for systems that vary in space and time. First introduced by von Neumann in the 1950s when studying self-reproducing machines, they have been used in several settings and provided models for a wide range of complex phenomena, including fluid dynamics, traffic model, excitable media, forest fire or pattern formation. This has led to a large body of literature, e.g. [3,4,7,16,17] and the references therein.

A standard CA consists of a regular infinite uniform lattice with a discrete variable at each site, named cell. The state of each cell is updated based on the previous states of its immediate neighboring cells according to a set of local rules.

In what follows, the time variable is given in $I = \{0, 1, 2, \cdots\}$.

Definition 1. *A cellular automaton is usually specified by the quadruple* $\mathcal{A} = (\mathcal{L}, \mathcal{S}, N, f)$ *where :*

- $\mathcal{L}$ *is a lattice defined by a regular grid of cells denoted by* $c_{i_1 i_2 \cdots i_d}$*, on a d-dimensional domain* Ω*. In a two-dimensional domain, a cell will be denoted by* $c_{i,j}$*,*
- $\mathcal{S}$ *a finite set of states given as a commutative ring* $\mathcal{S} = \{0, 1, \cdots, k-1\}$ *in which modular arithmetic will be used,*
- *a finite neighborhood* $\mathbf{N}(c)$ *of size* n *which summarizes the effects of the surrounding cells on the state evolution of cell* c*,*
- *a set of local transition rules which update the state of cell* c *depending on the neighborhood of cell* c $(N(c))$*. It can be given by a transition function :*

$$f : \begin{array}{ll} \mathcal{S}^{N(c)} & \longrightarrow \mathcal{S} \\ s_t(N(c)) & \longrightarrow s_{t+1}(c) \end{array}$$

In this section we consider the concept of vulnerability and protector control as it was introduced in [1,2,10] using PDEs, through cellular automata approach.

2.2 Vulnerability and Protector Control: Cellular Automata Approach

Let us consider a dynamical system which evolves in a domain $\Omega \subset \mathbb{R}^n$.

We will consider in what follows, a disturbed controlled system described by a cellular automaton. Denote by $\mathcal{A} = (\mathcal{L}, \mathcal{S}, N, f)$, $\mathcal{A}' = (\mathcal{L}, \mathcal{S}', N, f')$ and $\mathcal{A}'' = (\mathcal{L}, \mathcal{S}'', N, f'')$ the considered cellular automata in the autonomous, disturbed not controlled and disturbed-controlled cases respectively.

Spreadability and Vulnerability

Let $\mathcal{P}$ be a given property specifying the spatial disturbance defined by:

$$\mathcal{P}s'_{t_i}(c) \Leftrightarrow s'_{t_i}(c) = s' \tag{1}$$

where $s' \in \mathcal{S}'$ is a given state and $s'_{t_i}(c)$ denotes the state of cell c in the disturbed case, at time $t_i \in I$ where I is a given time interval. Consider the following set :

$$\omega_{t_i} = \left\{c \in \mathcal{L} |\ \mathcal{P}s'_{t_i}(c)\right\} = \left\{c \in \mathcal{L} | s'_{t_i}(c) = s'\right\} \tag{2}$$

Let σ be a nonempty subset of $\mathcal{L}$ consisting of n_σ cells.

Definition 2. – *We say that the property* $\mathcal{P}$ *is spreadable from* ω_{t_0} *if :*

$$\omega_{t_i} \subset \omega_{t_{i+1}} \quad \forall i \in I \tag{3}$$

- *We say that a given region* σ *is* $\mathcal{P}$*-vulnerable if there exist an instant* $t \in]t_0, T[$ *such that :*

$$\sigma \cap \omega_t \neq \emptyset \tag{4}$$

Considering $\tau_{t_N} = \underset{t_s \in]t_0, t_N[}{\cup} \omega_{t_s}$ where $\omega_{t_s} = \left\{c \in \mathcal{L} | \mathcal{P} s'_{t_s}(c)\right\}$, the trajectory of the property $\mathcal{P}$. We have the characterization of the vulnerable zones.

Proposition 1. *The zone σ is vulnerable if and only if :* $\quad \sigma \cap \tau_{t_N} \neq \emptyset$

The fact that $\sigma \cap \tau_{t_N} \neq \emptyset$ means that σ is in the trajectory of the property $\mathcal{P}$ and consequently the property $\mathcal{P}$ will reach σ.

Protector control

Let us now consider the protector control problem using a CA approach.

For a given area $\sigma \subset \mathcal{L}$, denote by $S^\sigma = \{s : \sigma \to S\}$ the set of mappings consisting of the restriction to σ of a CA configuration. Assume that σ is vulnerable, so there exists a time t_i such that : $s'_{t_i} \neq s_{t_i}$

Consider now a dynamical control denoted by u which aims to protect the zone σ. The control domain is assumed to be variable and it is given at time t by $D_t = \{c \in \mathcal{L} |\ u_t(c) \neq 0\}$ where u_t denotes the control variable. The problem consists in finding the family of domains D_t such that $\sigma \cap \tau^u_{t_N} = \emptyset$ where $\tau^u_{t_N}$ is the new trajectory of $\mathcal{P}$ for the controlled system.

The new obtained transition rule for the controlled CA is defined as follows :

$$s''_{t_{i+1}}(c) = f''\left(s''_{t_i}(N(c)), u_{t_i}(c)\,\chi_\sigma\right) \tag{5}$$

where $f''(s''_t(N(c)), u) = s_{t+1}$. It means that for cells in σ, the state value s_t evolves normally without disturbance when a control u is applied. Otherwise, $f''(s''_t(N(c)), 0) = s'_t$ and the disturbance is expanding in the absence of control.

We give the following definitions.

Definition 3. *If a given zone σ is vulnerable then:*

- *The zone σ is said to be exactly protectable from time t_1 if there exists a control u such that at every time $t_i \geq t_1$:*

$$s_{t_i} = s''_{t_i} \tag{6}$$

where $s_{t_i} \in S^\sigma$ and $s''_{t_i} \in S''^\sigma$ are the configuration at the time t_i of the autonomous and the disturbed-controlled system respectively .
In this case u is said to be an exact protector control.

- *The zone σ is said to be weakly protectable if there exists a control u such that $\forall \varepsilon > 0$:*

$$\frac{Card\left(\sigma \cap \tau^u_{t_N}\right)}{Card\,(\sigma)} \leq \varepsilon \tag{7}$$

where $\dfrac{Card\left(\sigma \cap \tau^u_{t_N}\right)}{Card\,(\sigma)}$ represents the proportion of cells in the region σ that are reached by the property P at a given time. We say that u is a weak protector control.

Proposition 2. *The zone σ is protectable, if there exists a protector control u such that:*

$$\tau^u_{t_N} \cap \sigma = \emptyset \tag{8}$$

where $\tau^u_{t_N}$ is the controlled trajectory of the disturbance.

To illustrate our approach, we consider the following example.

3 Forest fire Dynamics Example

3.1 Model Description

Consider a two dimensional CA defined on a regular lattice $\mathcal{L}$ where cells are denoted by c_{ij} and $N(c_{ij})$ a Von Neumann neighborhood of radius 1. The fundamental dynamical properties of a CA model are the definition of the cell state and the local rules that update this state from one time step to the next. We consider a CA model in three situation: autonomous, disturbed not controlled and disturbed-controlled systems, defined on the same lattice L with the same type of neighborhood but each CA has its own set of states and transition function.

Autonomous case: CA for a vegetation dynamics

The states set $\mathcal{S}$ $= \{0, 1, 2, 3, 4\}$ where values 1 to 4 describe successive vegetation stages from a seed to adult plant. Value 0 is for a vacant cell.

State 0 : vacant cell ,  *or*  **State 1 : seed**

 *or*  **State 2 : germination** ,  *or*  **State 3 : young plant**

 *or*  **State 4 : adult plant** ,

The transition rules f. Defined by the following updating rules :

- A vacant cell will be occupied by a seed if at least one of its neighbours is an adult plant.

$$\text{if } \; s_{t_i}(c_{ij}) = 0 \; \text{ then } \; s_{t_{i+1}}(c_{ij}) = \begin{cases} 1 & \text{if } \exists k, l \mid c_{kl} \in N(c_{ij}) \text{ and } s_{t_i}(c_{kl}) = 4 \\ 0 & \text{otherwise} \end{cases}$$

- A seed will evolve to a germination state at the next time step.

$$\text{if } \; s_{t_i}(c_{ij}) = 1 \; \text{ then } \; s_{t_{i+1}}(c_{ij}) = 2$$

- The transition from germination to young vegetation is given by :

$$\text{if } \; s_{t_i}(c_{ij}) = 2 \; \text{ then } \; s_{t_{i+1}}(c_{ij}) = \begin{cases} 0 & \text{if } \forall k, l | c_{kl} \in N(c_{ij}) | s_{t_i}(c_{kl}) = 4 \\ 3 & \text{otherwise} \end{cases}$$

– The young state goes to the adult state in the next time step

$$\text{if } s_{t_i}(c_{ij}) = 3 \text{ then } s_{t_{i+1}}(c_{ij}) = 4$$

– In adult state, the growth of the vegetation slowed down. However the plant can die (in an arbitrary way) before reaching the maximal age at time T_{dead} (where T_{dead} is a random selection for every cell).

$$\text{if } s_{t_i}(c_{ij}) = 4 \text{ then } s_{t_{i+1}}(c_{ij}) = \begin{cases} 4 & \text{if } \begin{array}{l} s_{t_i - T_{dead}}(c_{ij}) = s_{t_i - T_{dead}+1}(c_{ij}) \\ = \cdots = s_{t_i}(c_{ij}) = 4 \end{array} \\ 0 & \text{if } t_i = T_{dead} \end{cases}$$

Disturbed system: CA with Fire

The set of states $\mathcal{S}'$ $= \{0, 1, 2, 3, 4\}$ where :

 *or*  *or*  **State 0 : No fire** *(just vegetation)* ,

 **State 1 : Excitement by fire** ,  **State 2 : Fire**

 **State 3 : Ash**

State 4 : Empty cell *(nothing at all)*

The transition rules f':

– The distribution of fire is made in a plant cover. We give then the following transition rules :

$$\text{if } s'_{t_i}(c_{ij}) = 0 \text{ then } s'_{t_{i+1}}(c_{ij}) = \begin{cases} 1 & \text{if } \begin{array}{l} s_{t_i}(c_{ij}) \in \{2, 3, 4\} \quad \text{and} \\ \exists kl \mid c_{kl} \in N(c_{ij}) \text{ and } s'_{t_i}(c_{kl}) = 1 \end{array} \\ 0 & \text{otherwise} \end{cases}$$

– $\text{if } s'_{t_i}(c_{ij}) = 1 \text{ then } s'_{t_{i+1}}(c_{ij}) = 2$

– In the burned-out state, the fire remains stable during a period of combustion T_{fire}. This parameter which depends on the vegetation stage, environment, wind, etc, will be in this study, randomly selected for every cell. After this time, the fire goes out and the cells will be occupied by ash. That is the transition from fire to ash.

$$\text{if } s'_{t_i}(c_{ij}) = 2 \text{ then } s'_{t_{i+1}}(c_{ij}) = \begin{cases} 2 & \text{if } \begin{array}{l} s'_{t_i - T_{fire}}(c_{ij}) = s'_{t_i - T_{fire}+1}(c_{ij}) \\ = \cdots = s'_{t_i}(c_{ij}) = 2 \end{array} \\ 3 & \text{if } t_i = T_{fire} \end{cases}$$

– The total dispersion of ash: cells become empty.

$$\text{if } s'_{t_i}(c_{ij}) = 3 \text{ then } s'_{t_{i+1}}(c_{ij}) = 4$$

- When the cell becomes empty, it always remains empty and obviously it can be occupied by a new seed if we start again the simulation of vegetation.

$$\text{if } \; s'_{t_i}(c_{ij}) = 4 \;\text{ then }\; s'_{t_{i+1}}(c_{ij}) = 4$$

Remark 1. (i) There are several factors that may affect the spread of fire. Slope which has a considerable influence on the rate of spread, especially in the initial stages of fire, wind power and direction, type of vegetation, humidity, etc. In the CA rules, these parameters can play an important role and have to be taken into account.

(ii) It should be noted that we are not dealing in this paper with modelling of vegetation dynamics or fire. These are just examples of dynamics considered in order to illustrate the notions of vulnerability and protector control. A more rigourous study is should be done in this direction for which a good formalism of these aspects have to be defined.

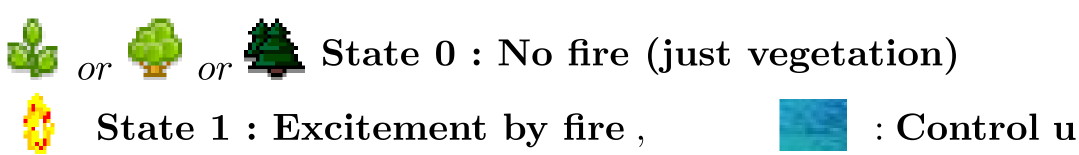
Controlled CA

We introduce the protector control in order to protect a chosen (vulnerable) zone σ. The effect of the control consists in changing the transition rule of fire spread introducing a control term u.

-Transition rules f'':

A given cell c_{ij}, with some vegetation, can be burned if there is a fire in its neighborhood. So to protect the zone σ we have to determine a domain of the control variable u_{t_i} denoted by D, such that for each $c_{ij} \in D$, and $t_i \in I$ $u_{t_i}(c_{ij}) = u$. The effect of the control u is to stop the fire spread toward the region σ.

$$\text{if } \; s''_{t_i}(c_{ij}) = 0 \;\text{ then }\; s''_{t_{i+1}}(c_{ij}) = \begin{cases} 1 & \text{if} \quad \begin{matrix} s_{t_i} \in \{2,3,4\} \text{ and} \\ \exists kl \mid c_{kl} \in N(c_{ij}) \text{ and } s''_{t_i}(c_{kl}) = 1 \end{matrix} \\ u & \text{if} \quad c_{ij} \in D \\ 0 & \text{otherwise} \end{cases}$$



3.2 Simulation Results

The given simulations for the considered CA were performed using a developed application with JAVA environment. The CA is based on a regular grid of cells representing the domain Ω. The obtained lattice is $L = \{c_{ij} \mid 1 \leq i \leq M,\ 1 \leq j \leq M\}$ with $M \times M$ representing the total total number of cells in L.

The states of cells on the boundaries of the grid are set to zero value which means the choice of adiabatic boundary conditions.

Autonomous CA

The simulation starts with an initial state consisting of empty cells except for some isolated seeds.

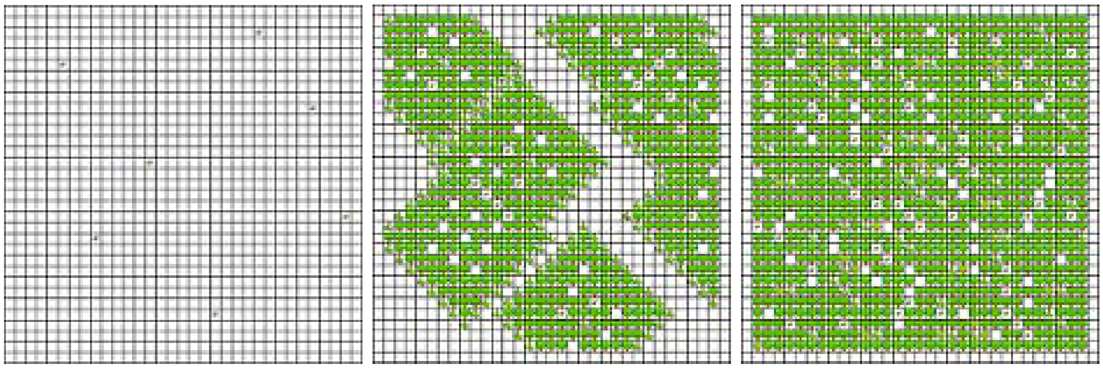


Fig.1. Evolution of a plant cover with M=33

Disturbed System with Fire

The simulation starts with an initial condition at $t' = 1$ when the fire starts at the cell $c_{5,15}$.

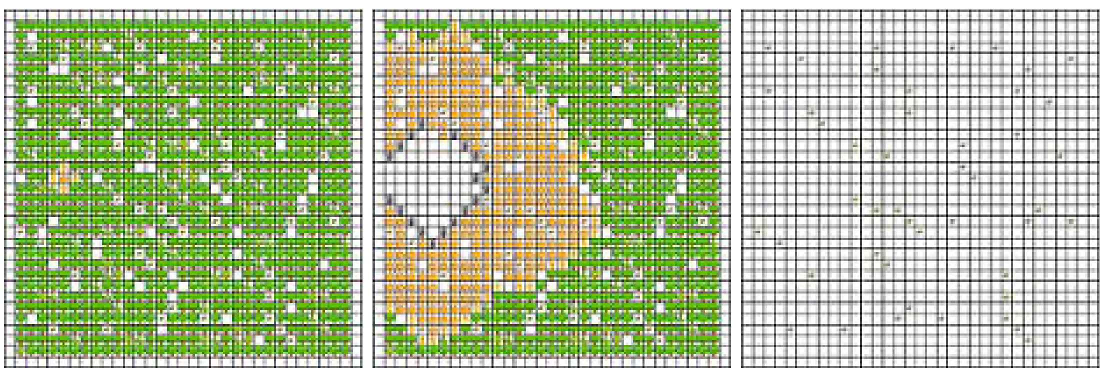


Fig.2. Spreadable fire and vulnerable area

Remark 2. The fire is spreading over the whole area. We can then deduce the vulnerable area which has to be protected.

Controlled Case

According to the simulation of fire spreading, the vulnerable zone is the area located in the right of cell $c_{5,15}$ which is excited with fire. Therefore, the region σ to be protected, must be set in this area.

Let $\sigma \subset \mathcal{L}$ be a region defined by: $\sigma = \{c_{ij} \mid 18 \leq i \leq 25\ ,\ 12 \leq j \leq 22\}$

Example 1. In this particular case, we can notice that the region σ can be protected from fire spreading when the control domain consists of a rectangular zone $D = \{c_{ij} \mid 16 \leq i \leq 27\ ,\ 10 \leq j \leq 24\}$.

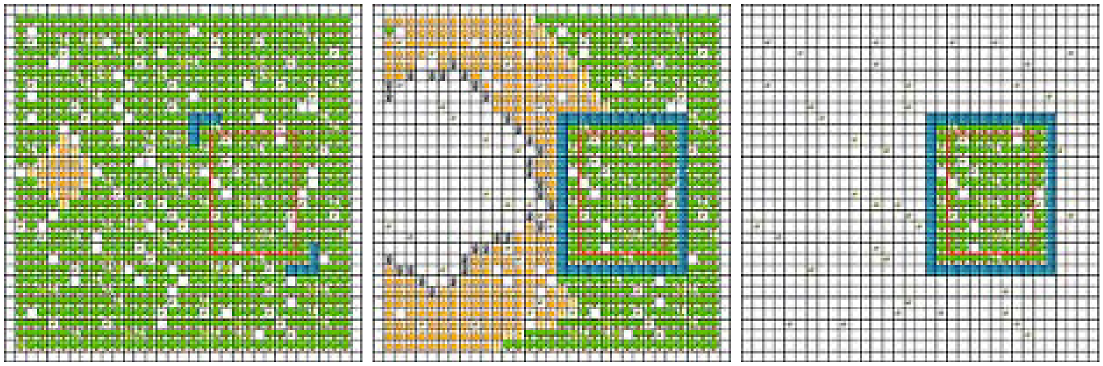


Fig.3. The zone σ is protected

Example 2. Let us consider another region σ with an increased area which has to be protected. Let D be the following new domain of the control action :

$$D = \{c_{ij} \mid 14 \leq i \leq 28 \ , \ 8 \leq j \leq 26\}$$

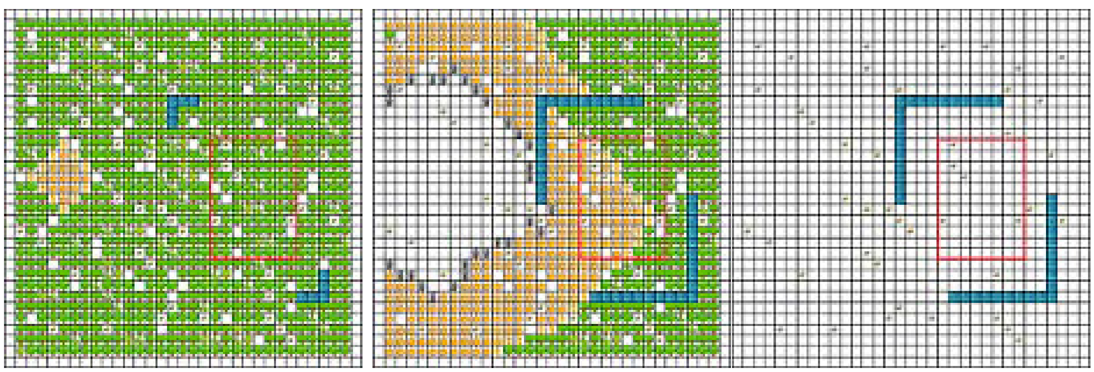


Fig.4. The zone σ is not protected

In the two examples, we illustrate the possibility of protecting a given region against the fire spread with a 'good' choice of the domain D in example 1, Fig. 3. As the control value u has been fixed for all controlled cells, the size and location of the domain D remains for the moment, the only way to control the propagation of fire toward the vulnerable area.

4 Conclusion

In this paper we attempt to investigate the protector control problem using cellular automata approach. The protector control consists to protect a given zone faced to a spreadable disturbance. We consider the case of a control with dynamic support and we illustrate this aspect through a simple example of forest fire.

The modelling of the vegetation and fire dynamics is not the main objective of this work. These dynamics have been considered only to illustrate the protector control concept and vulnerability through cellular automata approach. Furthermore, the concept of protector control has to be rigourously defined and characterized in relation with vulnerability and spreadability concepts. Such problems are under investigation.

References

1. Bernoussi, A.: Spreadability and vulnerability of distributed parameter systems. International Journal of Systems Science 38(4), 305–317 (2007)
2. Bernoussi, A.: Spreadability, vulnerability and protector control. Mathematical Modelling of Natural Phenomena 5(7), 145–150 (2010)
3. Chopard, B., Droz, M.: Cellular automata Modelling of Physical Systems. Cambridge University Press, Cambridge (1998)
4. Deutsch, A., Dormann, S.: Cellular automaton modeling of biological pattern formation (2005)
5. El Yacoubi, S., El Jai, A.: Cellular automata and spreadability. International Journal of Mathematical and Computer Modelling 36, 1059–1074 (2002)
6. El Yacoubi, S.: A Mathematical method for control problems on Cellular Automata models. International Journal of Systems Sciences 39(5), 529–538 (2008)
7. Greenberg, M.J., Hastings, S.P.: Spatial patterns for discrete models of diffusion in excitable media. SIAM Journal on Applied Mathematics 34(3), 515–523 (1978)
8. Karafyllidis, I., Thanailakis, A.: A model for predicting forest fire spreading using cellular automata. Ecological Modelling 99, 87–97 (1997)
9. Progias, P., Sirakoulis, G.C.: An FPGA Processor for Modelling Wildfire Spread. Mathematical and Computer Modeling 57(5-6), 1436–1452 (2013)
10. Qaraai, Y., Bernoussi, A.: Protector control: Extension to a class of nonlinear distributed systems. International Journal of Applied Mathematics and Computer Science 20(3), 427–443 (2010)
11. Slimi, R., El Yacoubi, S.: Spreadable Cellular Automata: Modelling and Simulation. International Journal of Systems Sciences 40(5), 507–520 (2009)
12. Toffoli, T.: Cellular automata as an alternative to (rather than an approximation of) differential equations in modeling physics. Physica D: Nonlinear Phenomena 10(1), 117–127 (1984)
13. Trunfio, G.A.: Predicting Wildfire Spreading Through a Hexagonal Cellular Automata Model. In: Sloot, P.M.A., Chopard, B., Hoekstra, A.G. (eds.) ACRI 2004. LNCS, vol. 3305, pp. 385–394. Springer, Heidelberg (2004)
14. Trunfio, G.A., D'Ambrosio, D., Rongo, R., Spataro, W., Di Gregorio, S.: A New Algorithm for Simulating Wildfire Spread Through Cellular Automata. ACM Transactions on Modeling and Computer Simulation 22, 6:1–6:26 (2011) ISSN: 1049-3301
15. Vichniac, G.Y.: Simulating physics with cellular automata. Physica D: Nonlinear Phenomena 10(1), 96–116 (1984)
16. Wolfram, S.: Cellular automata and complexity: collected papers, vol. 1. Addison-Wesley, Reading (1994)
17. Zapp, M.L., Green, M.R.: Sequence-specific RNA binding by the HIV-1 Rev protein. Nature 342(6250), 714–716 (1989)