

# Programming

optimisation and operations research algorithms with Julia

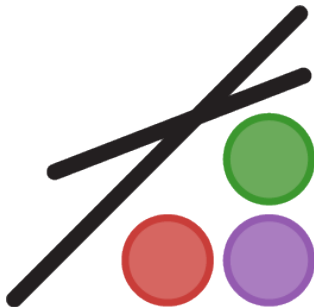
## for Business Tasks

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Lesson 3 – May-June 2022

# Optimisation JuMP (part 1)



version 1.0.0 and later

# Overview of JuMP

A modeling language for Mathematical Optimization (linear, mixed-integer, conic, semidefinite, nonlinear):

- ▶ **User friendliness:** syntax that mimics natural mathematical expressions.
- ▶ **Speed:** similar speeds to special-purpose modeling languages such as AMPL.
- ▶ **Solver independence:** JuMP uses *MathOptInterface (MOI)*, an abstraction layer designed to provide a unified interface to mathematical optimization solvers.

Currently supported solvers: Artelys Knitro, Baron, Bonmin, Cbc, Clp, Couenne, CPLEX, FICO Xpress, GLPK, Gurobi, SCIP, etc.

- ▶ **Ease of embedding:** JuMP itself is written purely in Julia. Solvers are the only binary dependencies.

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# Getting started

Install:

```
using Pkg  
Pkg.add("JuMP")
```

```
Pkg.add("GLPK")
```

Setup:

```
using JuMP
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## Writing a model (1/5)

Example:

$$\left[ \begin{array}{llllll} \max z(x) = & x_1 & + & 3x_2 & & (0) \\ s.t & x_1 & + & x_2 & \leq & 14 & (1) \\ & -2x_1 & + & 3x_2 & \leq & 12 & (2) \\ & 2x_1 & - & x_2 & \leq & 12 & (3) \\ & x_1 & , & x_2 & \geq & 0 & (4) \end{array} \right]$$

## Writing a model (2/5)

Creating a Model:

```
modelName = Model(solver)
```

```
julia> model = Model(GLPK.Optimizer)
```

Result:

```
[ ]
```

## Writing a model (3/5)

Defining Variables:

```
@variable(modName, varName definition)
```

```
julia> @variable(model, x1 >= 0)
```

```
julia> @variable(model, x2 >= 0)
```

Result:

$$\left[ \begin{array}{c} x_1, \quad x_2 \geq 0 \quad (4) \end{array} \right]$$

## Writing a model (4/5)

Defining Objective:

```
@objective(modName, min/max, objectiveFunction)
```

```
julia> @objective(model, Max, x1 + 3x2)
```

Result:

$$\left[ \begin{array}{rcll} \max z(x) = & x_1 & + & 3x_2 & (0) \\ & x_1 & , & x_2 & \geq 0 & (4) \end{array} \right]$$

## Writing a model (5/5)

### Defining Constraints:

```
@constraint(modName, cstName, cstDefinition)
```

```
julia> @constraint(model, cst1, x1 + x2 <= 14)
julia> @constraint(model, cst2, -2x1 + 3x2 <= 12)
julia> @constraint(model, cst3, 2x1 - x2 <= 12)
```

### Result:

$$\left[ \begin{array}{rcllcl} \max z(x) = & x_1 & + & 3x_2 & & (0) \\ s.t & x_1 & + & x_2 & \leq & 14 & (1) \\ & -2x_1 & + & 3x_2 & \leq & 12 & (2) \\ & 2x_1 & - & x_2 & \leq & 12 & (3) \\ & x_1 & , & x_2 & \geq & 0 & (4) \end{array} \right]$$

# Details on

The model:

- print a summary of the problem:

```
julia> @show(model)
```

- print the formulation of the model:

```
julia> print(model)
```



# Solve a model

```
optimize!(modName)
```

```
julia> optimize!(model)
```

## Querying the solution

```
julia> @show termination_status(model)
julia> @show primal_status(model)
julia> @show dual_status(model)
```

```
julia> @show objective_value(model)
julia> @show value(x1)
julia> @show value(x2)
```

```
julia> @show dual(cst1)
julia> @show dual(cst2)
julia> @show dual(cst3)
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julia> solution_summary(model)
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## Details on

termination\_status:

```
termination_status(modName)
```

### Common return values

- ▶ **OPTIMAL:**  
The algorithm found a globally optimal solution
- ▶ **INFEASIBLE:**  
The algorithm concluded that no feasible solution exists.
- ▶ **TIME\_LIMIT:**  
The algorithm stopped after a user-specified computation time.
- ▶ **NUMERICAL\_ERROR:**  
The algorithm stopped because a numerical error.
- ▶ etc.

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# Recommended workflow

For solving a model and querying the solution:

```
julia> if termination_status(model) == MOI.OPTIMAL
    zOpt = objective_value(model)
    @printf(" z=%5.2f x1=%5.2f x2=%5.2f \n",
           zOpt,
           value(x1),
           value(x2))
    @printf(" u1=%5.2f u2=%5.2f u3=%5.2f \n",
           dual(cst1),
           dual(cst2),
           dual(cst3))
elseif termination_status(model) == DUAL_INFEASIBLE
    println("problem unbounded")
elseif termination_status(model) == MOI.INFEASIBLE
    println("problem infeasible")
end
```



# Details on

## Variables:

- ▶ by default, the variables are **continuous** and **unbounded**:

```
julia> @variable(model, x)           # x is free
```

- ▶ possible to setup lower and/or upper bounds on a variable:

```
julia> @variable(model, x ≥ lb))    # x is bounded  
julia> @variable(model, x ≤ ub)  
julia> @variable(model, lb ≤ x ≤ ub)  
julia> @variable(model, x == 2)     # x is fixed
```

- ▶ possible to specify the type of a variable:

```
julia> @variable(model, x ≥ 0, Int)  #  $x \in \mathbb{N}$   
julia> @variable(model, x, Bin)     #  $x \in \{0, 1\}$ 
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# Review and exercises

(notebook)

