Exercices on Linear Error-correcting Codes

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Exercice 1.- In order to show that $A_q(n,d) = M$ it is enough to show that $A_q(n,d) \leq M$ and then find a specific q-ary (n,M)-code $\mathcal{C}$ for which $d(C) \geq d$, which shows that $A_q(n,d) \geq A_q(n,d(C)) \geq M$. Let $\mathcal{C}$ be a binary $(4,M,3)$-code. We assume that $\mathcal{C}$ contains the **0** = 00...00 codeword. Prove that $A_2(4,3) = 2$.

Exercice 2.- Let G be the generator matrix for the Hamming code $\mathcal{H}_2(3)$

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

How do you encode the string $x = 1101$? Give the parity-check matrix P and the parity-check equations.

Exercice 3.- Let $\mathcal{C}$ be the ternary code whose generator matrix is given by

$$G = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix}$$

Write out all the codewords for this code and give the parameters of this code.

Exercice 4.- Let $\mathcal{C}$ be the 5-repetition code ($k = 1, n = 5$). Give the parity-check matrix for this code and well as the parity-check equations. Give the codewords of $\mathcal{C}$.

Exercice 5.- Let $\mathcal{C}$ a code with generator matrix given by

$$G = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

Give a standard array for $\mathcal{C}$. We received the string $y = 1111$ for the message x being sent. How do you decode y and what is the message x ?

Exercice 6.- A binary code $\mathcal{C}$ is described by its parity-check matrix

$$P = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

- How would you encode the message 0001101 ?
- How would you decode the received vectors 0000111 and 0001110 ?
- Give the parameters n, M, d of this code.

Exercice 7.- A binary code $\mathcal{C}$ is described by its parity-check matrix

$$P = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

- Construct the syndrome table for $\mathcal{C}$.
- Decode the received vectors 1111111, 1101011, 0110111 and 0111000 ?
- The channel error probability is $p = 0.015$. Give P[correct decoding] and P[undetected error] probabilities.