```
(*Blade Arnold TTH at 2:00*)
(*1 (10 points).Use the function StreamPlot to reproduce the given computer-
  generated direction field between
[-5,5] including letting Mathematica sketch the approximate
   solution curve(in red)that passes through the
point [illegible]([illegible] 4)=[illegible]8.*)
```

In[∘]:=
```
p1 = StreamPlot[{1, Sinh[x] Cosh[y]}, {x, -5, 5}, {y, -5, 5}];
DSolve[{y'[x] == Sinh[x] Cosh[y[x]], y[π / 4] == π / 8}, y[x], x]
```

Out[∘]=

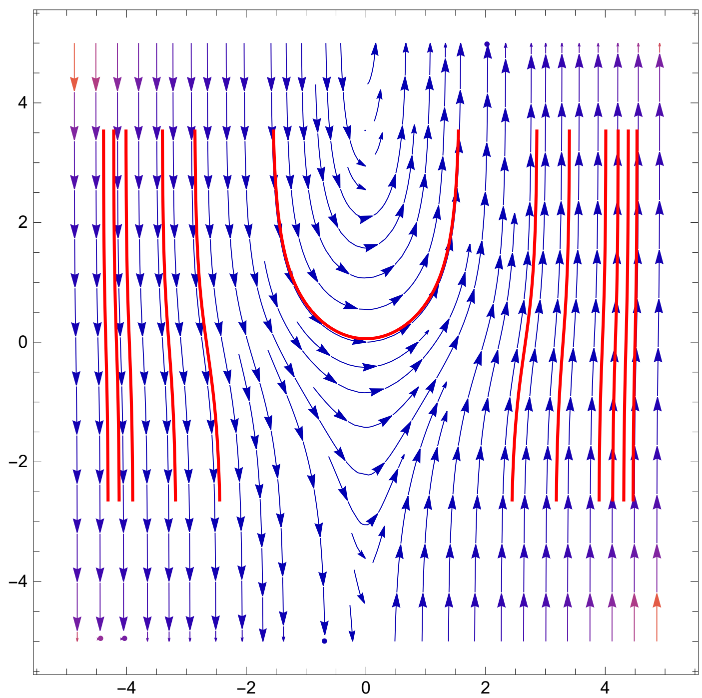
$$\left\{\left\{y[x] \to 2\,\text{ArcTanh}\left[\text{Tan}\left[\frac{1}{4}\left(2\left(2\,\text{ArcTan}\left[\text{Tanh}\left[\frac{\pi}{16}\right]\right] - \text{Cosh}\left[\frac{\pi}{4}\right]\right) + 2\,\text{Cosh}[x]\right)\right]\right]\right\}\right\}$$

In[∘]:=

$$\text{p2} = \text{Plot}\left[2\,\text{ArcTanh}\left[\text{Tan}\left[\frac{1}{4}\left(2\left(2\,\text{ArcTan}\left[\text{Tanh}\left[\frac{\pi}{16}\right]\right] - \text{Cosh}\left[\frac{\pi}{4}\right]\right) + 2\,\text{Cosh}[x]\right)\right]\right],\ \{x, -5, 5\},\ \text{PlotStyle} \to \text{Red}\right];$$

In[∘]:=
```
Show[p1, p2]
```

Out[∘]=



In[∘]:=
```
(*2. (10 points).Solve the following differential equation.Use//
 Expand after the command to reduce.*)
Clear

DSolve[{x^3 y'''[x] + 3 x^2 y''[x] + 6 x y'[x] + y[x] == ln (x) - 47 / 8}, y[x], x] // Expand
```

Out[∘]= Clear

Out[∘]=

$$\{\{y[x] \to x^{-0.198\ldots} \mathbb{c}_1 + x^{0.0992\ldots-2.24\ldots i} \mathbb{c}_2 + \cdots 202 \cdots + (\ln x\ (0.0992\ldots - 2.24\ldots i)\ (0.0992\ldots + 2.24\ldots i)^5) / (((-0.198\ldots) - (0.0992\ldots - 2.24\ldots i))\ (-1 + (0.0992\ldots - 2.24\ldots i)) \cdots 5 \cdots (-5 + (0.0992\ldots - 2.24\ldots i)^2 + 2\ (-0.198\ldots)\ (0.0992\ldots + 2.24\ldots i)))\}\}$$

large output | show less | show more | show all | set size limit...

```
(*3. Solve the following differential equation.Use//
 Simplify after the command to reduce.*)
```

In[∘]:=
```
DSolve[
  {y''[x] - 4 y''[x] + 8 y[x] == (2 x^2 - 3 x) e^2 x Cos (2 x) + (10 x^2 - x - 1) e^2 x Sin (2 x),
   y[0] == π / 2, y[3] == π}, y[x], x] // Simplify
```

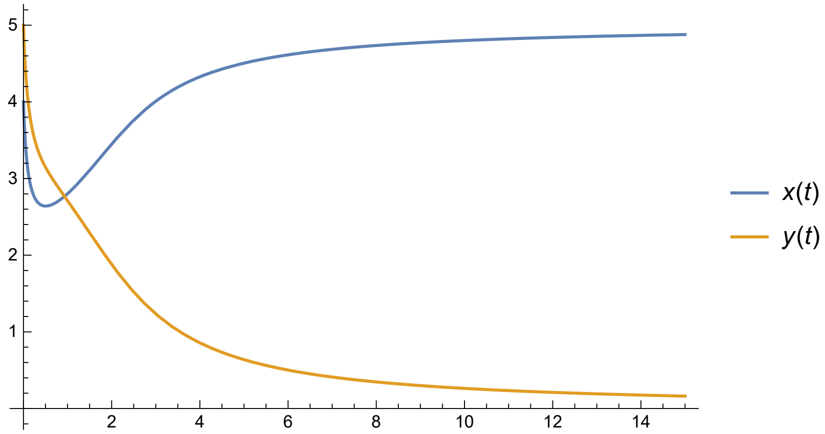
Out[∘]=

$$\Big\{\Big\{y[x] \to \frac{1}{16\left(-1 + e^{4\sqrt{6}}\right)} e^{-2\sqrt{\frac{2}{3}}x} \Big(e^{2\sqrt{\frac{2}{3}}(3+2x)}\left(16\pi - 4821 e^2 \mathrm{Sin}\right) + 4 e^{4\sqrt{6}}\left(2\pi - 33 e^2 \mathrm{Sin}\right) + e^{4\sqrt{\frac{2}{3}}x}\left(-8\pi + 132 e^2 \mathrm{Sin}\right) +$$
$$e^{2\sqrt{6}}\left(-16\pi + 4821 e^2 \mathrm{Sin}\right) - e^2 e^{2\sqrt{\frac{2}{3}}x} \mathrm{Sin}\left(132 - 9x + 176x^2 - 4x^3 + 40x^4\right) +$$
$$e^2 e^{2\sqrt{\frac{2}{3}}(6+x)} \mathrm{Sin}\left(132 - 9x + 176x^2 - 4x^3 + 40x^4\right) -$$
$$\mathrm{Cos}\, e^2 \Big(-594 e^{2\sqrt{6}} + 27 e^{4\sqrt{6}} - 27 e^{4\sqrt{\frac{2}{3}}x} + 594 e^{2\sqrt{\frac{2}{3}}(3+2x)} +$$
$$e^{2\sqrt{\frac{2}{3}}(6+x)}\left(-27 + 27x - 36x^2 + 12x^3 - 8x^4\right) + e^{2\sqrt{\frac{2}{3}}x}\left(27 - 27x + 36x^2 - 12x^3 + 8x^4\right)\Big)\Big)\Big\}\Big\}$$

In[∘]:=
```
(*4. (30 points).A competition model defined by*)
S = NDSolve[{x'[t] == x[t] (x[t] (2 - 2 / 5 x[t] - 3 / 10 y[t])),
    y'[t] == y[t] (y[t] (1 - 1 / 10 y[t] - 3 / 10 x[t])),
    x[0] == 4, y[0] == 5} , {x[t], y[t]}, {t, 0, 300}];
```

In[ ]:= `Plot[Evaluate[{x[t], y[t]} /. S], {t, 0, 15}, PlotLegends → {x[t], y[t]}]`

Out[ ]=



(*It will take about 1 year for the x[t] population to surpass the other*)

(*5.(10 points).Solve the given differential equation by using Mathematica.*)

In[ ]:= `DSolve[{5.23 y'''[x] + 4.76 y''[x] + 8.39 y'[x] + 0.625 y[x] == 0}, y[x], x]`

Out[ ]= $\left\{\left\{y[x] \to e^{-0.0776201\,x}\, c_3 + e^{-0.416257\,x}\, c_2 \operatorname{Cos}[1.1689\,x] + e^{-0.416257\,x}\, c_1 \operatorname{Sin}[1.1689\,x]\right\}\right\}$

(*6. (10 points).Use the reduction of order formula
to find the second solution to the following differential
equation.*)

In[ ]:= `y[x_] := u[x] * 3 / x^(7 / 5);`

In[ ]:= `y'[x]`

Out[ ]= $-\frac{21\,u[x]}{5\,x^{12/5}} + \frac{3\,u'[x]}{x^{7/5}}$

In[ ]:= `y''[x]`

Out[ ]= $\frac{252\,u[x]}{25\,x^{17/5}} - \frac{42\,u'[x]}{5\,x^{12/5}} + \frac{3\,u''[x]}{x^{7/5}}$

In[ ]:= `a = Simplify[5 x^2 y''[x] + 17 x y'[x] + 7 y[x]]`

Out[ ]= $\frac{3\left(7 - 7\,u[x] + 3\,x\,u'[x] + 5\,x^2\,u''[x]\right)}{x^{7/5}}$

In[ ]:= `DSolveValue[a == 0, u[x], x]`

Out[ ]= $1 + \frac{c_1}{x} + x^{7/5}\, c_2$

(*Second solution will be 3/x + 3*)

(*7.(10 points).Find the Laplace Transform of the following:*)

In[•]:=
```
LaplaceTransform[10 Exp[7 t] + 5 t Exp[2 t] + Cosh[3 t], t, s]
```

Out[•]= $\frac{10}{-7+s}+\frac{5}{(-2+s)^2}+\frac{s}{-9+s^2}$

In[•]:=
```
LaplaceTransform[Sin[2 t] + 500 t^4 + Exp[t] Cos[5 t], t, s]
```

Out[•]= $\frac{-1+s}{25+(-1+s)^2}+\frac{12000}{s^5}+\frac{2}{4+s^2}$

In[•]:=
```
LaplaceTransform[t^6 e Exp[4 t] + t δ (t - 2), t, s]
```

Out[•]= $\frac{720\,e}{(-4+s)^7}+\frac{2\,\delta}{s^3}-\frac{2\,\delta}{s^2}$

In[•]:=
```
LaplaceTransform[(3 t - 2) ⋃ (t - 4), t, s]
```

Out[•]= $\frac{4}{s^2}-\frac{6}{s}$

```
(*8. (10 points).Find the Inverse Laplace Transform of the following:*)
```

In[•]:=
```
InverseLaplaceTransform[s / ((s - 1)^2 (s^2 + 1) (s^2 - 1)), s, t]
```

Out[•]= $\frac{e^{-t}}{16}-\frac{e^{t}}{16}-\frac{e^{t}\,t}{8}+\frac{e^{t}\,t^2}{8}+\frac{\mathrm{Sin}[t]}{4}$

In[•]:=
```
InverseLaplaceTransform[Exp[-2 s] / (s^2 + 4 s + 5), s, t]
```

Out[•]= $-\frac{1}{2}\,i\,e^{(-2-i)\,(-2+t)}\left(-1+e^{2\,i\,(-2+t)}\right)\mathrm{HeavisideTheta}[-2+t]$

In[•]:=
```
InverseLaplaceTransform[240 s / (s - 9)^7 + 24 s / (s^2 + 9)^2, s, t]
```

Out[•]= $e^{9\,t}\,t^5\,(2+3\,t)+4\,t\,\mathrm{Sin}[3\,t]$

In[•]:=
```
InverseLaplaceTransform[1, s, t]
```

Out[•]= DiracDelta[t]