Adaptive Fuzzy Backstepping Tracking Control for Strict-Feedback Systems with Input Delay

Hongyi Li, Lijie Wang, Haiping Du and Abdesselem Boulkroune

***Abstract*—This paper investigates the problem of adaptive fuzzy tracking control for nonlinear strict-feedback systems with input delay and output constraint. Input delay is handled based on the information of Pade approximation and output constraint problem is solved by barrier Lypaunov function. Some adaptive parameters of the controller need to be updated online through considering the norm of membership function vector instead of all sub-vectors. A novel adaptive fuzzy tracking control scheme is developed to guarantee all variables of the closed-loop systems are semi-globally uniformly ultimately bounded (SGUUB), and the tracking error can be adjusted around the origin with a small neighborhood. The stability of the closed-loop systems is proved and simulation results are given to demonstrate the effectiveness of the proposed control approach.**

***Index Terms*—Adaptive fuzzy control; Backstepping design technique; Input delay; Output constraint.**

I. INTRODUCTION

It is well known the nonlinear systems play an important role in the realistic industrial process. To tackle nonlinear characteristics existing in practical systems, various advanced control approaches have been proposed, such as adaptive backstepping control [1]–[3], sliding mode control [4]–[9] and fuzzy-model based control [10]–[18]. To mention a few, based on Takagi–Sugeno (T–S) multiple models, a new indirect adaptive switching fuzzy control method was proposed for fuzzy dynamical systems in [11]. The authors in [19] studied the problem of event-triggered fault detection for T-S fuzzy systems. In [15], the authors considered the robust adaptive multiple models based fuzzy control problem for a category of unknown nonlinear systems. Among these advanced methods, adaptive control approach, which can regulate the adaptive parameters via online learning, has received considerable attention [1], [20], [21]. With the favorite approximation property, fuzzy logic systems (FLSs) and neural network (NN) have been used to handle unknown nonlinear systems. Combining FLSs/NN with adaptive backstepping control scheme, dramatic results have been reported [22]–[31]. In [23], the authors designed a novel adaptive NN controller for multiple-input and multiple-output (MIMO) nonlinear systems appearing input saturation. The authors in [27] investigated the adaptive fuzzy identification and control problems for a category of MIMO nonlinear systems in pure-feedback frame and with dead-zone input. In [29], the authors developed an adaptive fuzzy optimal control scheme for unknown nonlinear discrete-time systems.

In real engineering applications, delay often occurs and is a source of degrading system performance. Therefore, it is necessary to consider it in the system analysis process. The authors in [32] investigated the issue of adaptive NN control for MIMO systems with delay. Considering input delay, the authors in [33] addressed the problem of adaptive tracking control for nonlinear systems with unknown parameters. The authors in [34] proposed a new fuzzy adaptive output feedback control approach for uncertain MIMO nonlinear systems based on designing a serial-parallel estimation model. The proposed control schemes in [34] can achieve better fuzzy identification effect and improve control performance. Pade approximation technique was used to handle network-induced delay in [35].

On the other hand, many systems have constraints on system output. A barrier Lyapunov function was introduced to prevent constraint violation in [36], and then a controller was designed for nonlinear single-input single-output (SISO) system. The authors in [37] proposed an adaptive control approach for an uncertain n-link robot system, wherein the full-state constraints were considered. In practical systems, input and output may suffer many problems. We hope that input delay and output constraint problems may be addressed in a unified framework, which motivates our study.

This paper designed an adaptive fuzzy backstepping tracking controller for uncertain nonlinear systems subject to input delay and output constraint. FLSs are employed to identify the unknown nonlinear terms existing in practical systems. Input delay is tackled by introducing Pade approximation technique, which facilitates reducing the analysis complexity of delayed systems. The key contributions of this paper can be summarized as follows: 1. Based on adaptive fuzzy control, input delay and output constraint are taken into account simultaneously in the design process, which facilitates applying the proposed control scheme to practical engineering. 2. In the process of designing adaptive fuzzy controller, a new coordinate transform z_n is introduced, which is used to eliminate variable γ containing input delay in the nth subsystem. 3. A novel adaptive fuzzy tracking control scheme is developed to guarantee all variables of the closed-loop systems are semi-

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globally uniformly ultimately bounded (SGUUB), and the tracking error can be adjusted around the origin with a small neighborhood. Finally, some simulation results are given to show the effectiveness of the proposed control scheme.

Notation: The following notations will be adopted throughout this paper. $\mathbf{R}^n$ denotes the real n-dimensional space. $\mathbf{R}_+$ denotes the set of all non-negative real numbers. $\|\kappa\|$ denotes the Euclidean norm of vector κ. C^1 denotes the set of all functions with continuous first-order partial derivatives. The superscript "T" and "-1" represent matrix transposition and matrix inverse, respectively.

II. Problem Formulation

Consider the following nonlinear system with input delay and output constraint,

$$\begin{aligned} \dot{x}_i &= x_{i+1} + f_i(\tilde{x}_i) + d_i(\tilde{x}_n, t), \quad i = 1, 2, \ldots, n-1, \\ \dot{x}_n &= f_n(\tilde{x}_n) + u(t-\tau) + d_n(\tilde{x}_n, t), \\ y &= x_1, \end{aligned} \tag{1}$$

where $\tilde{x}_i = [x_1, x_2, \ldots, x_i]^T \in \mathbf{R}^i$, $i = 1, 2, \ldots, n$, $x \in \mathbf{R}^n$ and $y \in \mathbf{R}$ are state vector and output vector, respectively. $u \in \mathbf{R}$ is the input vector. $f_i(\tilde{x}_i)$ is unknown but smooth nonlinear function, and the output y should remain in the set $|y| \leq k_{c1}$, with k_{c1} being a positive constant. $d_i(\tilde{x}_n, t)$ represents the disturbance and τd denotes the input delay. To tackle the problem of input delay and follow the method in [35], Pade approximation approach is introduced. Then, one can have

$$\begin{aligned} \ell\{u(t-\tau)\} &= \exp(-\tau v)\ell\{u(t)\} = \frac{\exp(-\tau v/2)}{\exp(\tau v/2)}\ell\{u(t)\} \\ &\approx \frac{(1-\tau v/2)}{(1+\tau v/2)}\ell\{u(t)\}, \end{aligned}$$

where $\ell\{u(t)\}$ is the Laplace transform of $u(t)$, and v is the Laplace variable. For further investigation, another variable x_{n+1} is introduced and satisfies the following equation,

$$\frac{1-\tau v/2}{1+\tau v/2}\ell\{u(t)\} = \ell\{x_{n+1}(t)\} - \ell\{u(t)\},$$

which yields

$$u - \frac{\tau \dot{u}}{2} = x_{n+1} + \frac{\tau \dot{x}_{n+1}}{2} - u - \frac{\tau \dot{u}}{2}.$$

Then, one can obtain

$$\dot{x}_{n+1} = -\gamma x_{n+1} + 2\gamma u, \tag{2}$$

where $\gamma = \frac{2}{\tau}$.

Remark 1: Even the introduced variable is named as x_{n+1}, it is not a system state in real sense. It can be regarded as an error variable.

Based on the above transformation, system (1) can be rewritten as follows:

$$\begin{aligned} \dot{x}_i &= x_{i+1} + f_i(\tilde{x}_i) + d_i(\tilde{x}_n, t), \quad i = 1, 2, \ldots, n-1, \\ \dot{x}_n &= f_n(\tilde{x}_n) + x_{n+1} - u + d_n(\tilde{x}_n, t), \\ \dot{x}_{n+1} &= -\gamma x_{n+1} + 2\gamma u, \\ y &= x_1. \end{aligned} \tag{3}$$

Remark 2: Due to the increasing complexity of the engineering environment, input delay may bring some deviations, which leads to a uncertain γ. In the analysis process, the variable x_{n+1} is introduced in the nth subsystem. When deviation in γ occurs, x_{n+1} will be uncertain (i.e., the time-varying case). Since the controller u to be designed can be used to eliminate such item, it is robust to the uncertain input delay to some degree. This will be proved in simulation results.

Remark 3: In this paper, Pade approximation method is introduced to cope with small delay. Since Pade approximation has some limitations in handling delay, the proposed scheme can not work in long-delay case. Relaxed control design for systems with long delay and output constraint deserves further investigation.

To facilitate the design of control system, the following common assumptions are given.

Assumption 1: The disturbance should satisfy $|d_i(\tilde{x}_n, t)| < d_{iM}$.

Assumption 2: [38] The reference signal y_d and $y_d^{(k)}(t)$ are smooth and bounded, in which $y_d^{(k)}(t)$ means $k-$order derivatives of y_d in time t, $1 \leq k \leq n$. Therefore, for simplicity, we suppose that there exist some positive constants $\Upsilon_0, \underline{\Upsilon}_0, \bar{\Upsilon}_0, \Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n$ such that $\max\{\underline{\Upsilon}_0, \bar{\Upsilon}_0\} \leq \Upsilon_0$, $-\underline{\Upsilon}_0 \leq y_d(t) \leq \bar{\Upsilon}_0$, $|\dot{y}_d(t)| \leq \Upsilon_1$, $|\ddot{y}_d(t)| \leq \Upsilon_2, \ldots,$ $\left|y_d^{(n)}(t)\right| \leq \Upsilon_n$, $\forall\, t \geq 0$.

Control Objective: The control objectives of this paper can be summarized as follows:

1) All the variables involved in the resulting closed-loop system are SGUUB.
2) The trajectory of given tracking signal y_d can be tracked by the system output y with a small tracking error.

To tackle the unknown nonlinear function, the FLSs are introduced. Details are given as follows:

Rule j: IF x_1 is N_{j1}, and x_2 is N_{j2} and, …, and x_n is N_{jn}, THEN

$$y \text{ is } M_j, \quad j = 1, 2, \ldots, \Delta,$$

where $x = [x_1, x_2, \ldots, x_n]^T$ is the system input, y represents the output of the system, N_{jl} and M_j stand for fuzzy sets, Δ stands for the number of fuzzy rules.

With the common techniques including singleton, center average defuzzification and product inference [25], [34], the final output can be described as follows:

$$y(x) = \frac{\sum_{j=1}^{\Delta} \tilde{y}_j \prod_{l=1}^{n} u_{N_{jl}}(x_l)}{\sum_{j=1}^{\Delta} \prod_{l=1}^{n} u_{N_{jl}}(x_l)}, \tag{4}$$

where $\tilde{y}_j = \max_{y \in \mathbf{R}} u_{M_j}(y)$, $u_{N_{jl}}(x_l)$ and $u_{M_j}(y)$ denote membership functions with respect to fuzzy sets N_{jl} and M_j, respectively.

Define $\phi(x) = \frac{\prod_{l=1}^{n} u_{N_{jl}}(x_l)}{\sum_{j=1}^{\Delta}\prod_{l=1}^{n} u_{N_{jl}}(x_l)}$. The ideal constant weight vector is defined as $W = [\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_\Delta]^T = [w_1, w_2, \ldots, w_\Delta]^T$, and $\phi(x) = [\phi_1(x), \phi_2(x),$

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$\ldots, \phi_\Delta(x)]^T$ is the basis function vector, and $\phi_i(x)$ is chosen as Gaussian function, i.e., for $i = 1, 2, \ldots, \Delta$,

$$\phi_i(x) = \exp\left[\frac{-(x-u_i)^T(x-u_i)}{\eta_i^2}\right],$$

where $u_i = [u_{i1}, u_{i2}, \ldots, u_{in}]^T$ is the center vector, and η_i is the width of the Gaussian function.

Then, the final output in (4) is expressed by the following form

$$y(x) = W^T\phi(x). \tag{5}$$

The following lemma is employed to set relationship between the unknown nonlinear function and the FLSs.

Lemma 1: [38] For any given continuous function $f(x)$, which is defined on the compact set Ω_x, there exists a constant $\zeta > 0$, FLSs (5) have the following property:

$$\sup_{x\in\Omega_x}\left|f(x) - W^T\phi(x)\right| \le \zeta.$$

Definition 1: [38] A barrier Lyapunov function $V(x)$ $(V(x) > 0)$ is a scalar continuous function, which is defined to the system $\dot{x} = f(x)$ in an open set D including the origin. Based on the property of $V(x)$, the continuous first-order partial derivatives of each point in the region D exist for $V(x)$. When x is close to the boundary of D, $V(x)$ will tend to be infinity, and there is a positive constant b such that $\forall t \ge 0$, $V(x(t)) \le b$, along with the solution of a differential equation $\dot{x} = f(x)$ for $x(0) \in D$.

The barrier Lyapunov function candidate is used to be defined as

$$\bar{V} = \frac{1}{2}\log\frac{k_{d1}^2}{k_{d1}^2 - z_1^2},$$

where $\log(\cdot)$ represents the natural logarithm of $(\cdot)$, z_1 satisfies $|z_1| < k_{d1}$. It is obvious that the barrier Lyapunov function escapes to infinity if $|z_1| = k_{d1}$ from above equation. It can be seen that $\bar{V}$ is positive definite and C^1 is continuous in the region $|z_1| < k_{d1}$, therefore, there exists a Lyapunov function in the set $|z_1| < k_{d1}$.

Lemma 2: [38] For any positive constant k_{d1}, define open sets $\Phi_1 := \{z_1 \in \mathbf{R} : |z_1| < k_{d1}\} \subset \mathbf{R}$ and $\Theta := \mathbf{R}^l \times \Phi_1 \subset \mathbf{R}^{l+1}$.

Consider the following system

$$\dot{\varkappa} = h(t, \varkappa),$$

where $\varkappa := [\kappa, z_1]^T \in \Theta$ stands for the state, $h : \mathbf{R}_+ \times \Theta \to \mathbf{R}^{l+1}$ is piecewise continuous about t and locally Lipschitz in z_1, uniformly in t, on $\mathbf{R}_+ \times \Theta$. Assume that there are continuously differentiable and positive definite functions $W : \mathbf{R}^l \to \mathbf{R}_+$ and $V_1 : \Phi_1 \to \mathbf{R}_+$, $i = 1, \ldots, n$, such that

$$\begin{aligned} V_1 &\to \infty \text{ as } |z_1| \to k_{d1}, \\ \beta_1(\|\kappa\|) &\le W(\kappa) \le \beta_2(\|\kappa\|), \end{aligned}$$

where β_1 and β_2 represent class $K_\infty-$ functions. Define $V(\varkappa) := V_1(z_1) + W(\kappa)$, and $z_1(0) \in Z_1$. If the following inequality is established:

$$\dot{V} = \frac{\partial V}{\partial \varkappa}h \le -\mu V + \epsilon,$$

in the set $\varkappa \in \Theta$ and μ, ϵ are positive constants, then κ remains bounded and $z_1(0) \in \Phi_1$, $\forall\ t \in [0, \infty)$.

Lemma 3: [38] For any positive constant k_{d1}, we can known that the following inequality is established, when z_1 remains in the interval $|z_1| < k_{d1}$,

$$\log\frac{k_{d1}^2}{k_{d1}^2 - z_1^2} < \frac{z_1^2}{k_{d1}^2 - z_1^2}.$$

III. MAIN RESULTS

To obtain the main results, the following change of coordinates is given.

$$z_1 = x_1 - y_d, \tag{6}$$

$$z_i = x_i - \alpha_{i-1}, \quad i = 2, 3, \ldots, n-1, \tag{7}$$

$$z_n = x_n - \alpha_{n-1} + \frac{1}{\gamma}x_{n+1}. \tag{8}$$

Remark 4: It can be seen from the change (8) that $\frac{1}{\gamma}x_{n+1}$ is considered to eliminate the introduced variable x_{n+1}.

To achieve the desired system performance, system (1) is regarded as a series of subsystems. With the adaptive backstepping control method, the following virtual signals, adaptive laws and the desired controller are designed, respectively.

The first virtual control signal α_1 and adaptive law $\dot{\theta}_1$ are chosen as follows:

$$\begin{aligned} \alpha_1 &= -\frac{b_1 z_1}{2a_1^2(k_{d1}^2 - z_1^2)}\theta_1\phi_1^T(Z_1)\phi_1(Z_1) \\ &\quad -\frac{b_1 z_1}{2r_1^2(k_{d1}^2 - z_1^2)} - c_1 z_1, \end{aligned} \tag{9}$$

$$\dot{\theta}_1 = \frac{\eta_1 b_1 z_1^2}{2a_1^2(k_{d1}^2 - z_1^2)^2}\phi_1^T(Z_1)\phi_1(Z_1) - \sigma_1\theta_1, \tag{10}$$

where $a_1, b_1, c_1, r_1, \eta_1$ and σ_1 are positive constants to be designed. θ_1 is the adaptive parameter. $Z_1 = [x_1^T, \theta_1^T, y_d^T, \dot{y}_d^T]^T \in \mathbf{R}^4$.

Define the ith virtual control signal α_i and the adaptive law $\dot{\theta}_i$ (i= $2, 3, \ldots, n-1$) as

$$\alpha_i = -c_i z_i - \frac{b_i z_i}{2a_i^2}\theta_i\phi_i^T(Z_i)\phi_i(Z_i) - \frac{b_i z_i}{2r_i^2}, \tag{11}$$

$$\dot{\theta}_i = \frac{b_i\eta_i z_i^2}{2a_i^2}\phi_i^T(Z_i)\phi_i(Z_i) - \sigma_i\theta_i, \tag{12}$$

where $a_{i,}$ b_i, c_i, r_i, η_i and σ_i are positive parameters to be designed. θ_i is the adaptive parameter. $Z_i = [\tilde{x}_i^T, \bar{\theta}_i^T, (\bar{y}_d^{(i)})^T]^T \in \mathbf{R}^{3i+1}$, $\bar{\theta}_i = [\theta_1, \theta_2, \ldots, \theta_i]^T \in \mathbf{R}^i$, $\bar{y}_d^{(i)} = [y_d, \dot{y}_d, \ddot{y}_d, \ldots, y_d^{(i)}]^T \in \mathbf{R}^{i+1}$, $i = 2, 3, \ldots, n-1$.

In the final step, design the actual control signal u and the adaptive law $\dot{\theta}_n$ as

$$u = -c_n z_n - \frac{b_n z_n}{2a_n^2}\theta_n\phi_n^T(Z_n)\phi_n(Z_n) - \frac{b_n z_n}{2r_n^2}, \tag{13}$$

$$\dot{\theta}_n = \frac{b_n\eta_n z_n^2}{2a_n^2}\phi_n^T(Z_n)\phi_n(Z_n) - \sigma_n\theta_n, \tag{14}$$

where the unknown constants $a_{n,}b_n, c_n, r_{n,}$ η_n and σ_n are positive design parameters. θ_n is the adaptive parameter. $Z_n = [x^T, \bar{\theta}_n^T, (\bar{y}_d^{(n)})^T]^T \in \mathbf{R}^{3n+1}$, $\bar{\theta}_n = [\theta_1, \theta_2, \ldots, \theta_n]^T \in \mathbf{R}^n$, $\bar{y}_d^{(n)} = [y_d, \dot{y}_d, \ddot{y}_d, \ldots, y_d^{(n)}]^T \in \mathbf{R}^{n+1}$.

With the above design, system (1) can be guaranteed to realize the control objective. In the following part, more details and stability analysis are provided.

Theorem 1: Considering the virtual control signal α_i (9) and (11), adaptive law $\dot{\theta}_i$ (10), (12) and (14), and controller u (13), all signals in resulting closed-loop system (1) can be ensured to be SGUUB and the system output y can track the given reference signal y_d with a small tracking error.

Proof: ***Step 1***. According to (3) and (6), the derivative of z_1 is calculated as follows:

$$\begin{aligned}\dot{z}_1 &= \dot{x}_1 - \dot{y}_d \\ &= x_2 + f_1(x_1) + d_1(\tilde{x}_n, t) - \dot{y}_d.\end{aligned} \tag{15}$$

The Lyapunov function is defined as follows:

$$V_1 = \bar{V} + \frac{\tilde{\theta}_1^2}{2\eta_1},$$

where $\eta_1 > 0$, $\tilde{\theta}_1 = \theta_1^* - \theta_1$ is the estimation error and θ_1 is the estimation of θ_1^*.

Then, one can have

$$\begin{aligned}\dot{V}_1 &= \dot{\bar{V}} - \frac{\tilde{\theta}_1\dot{\theta}_1}{\eta_1} \\ &= \frac{z_1\dot{z}_1}{k_{d1}^2 - z_1^2} - \frac{\tilde{\theta}_1\dot{\theta}_1}{\eta_1} \\ &= \frac{z_1}{k_{d1}^2 - z_1^2}\left(x_2 + f_1(x_1)\right. \\ &\quad \left. + d_1(\tilde{x}_n, t) - \dot{y}_d\right) - \frac{\tilde{\theta}_1\dot{\theta}_1}{\eta_1}.\end{aligned} \tag{16}$$

According to Young's inequality and Assumption 1, one can get

$$\frac{z_1}{k_{d1}^2 - z_1^2} d_1(\tilde{x}_n, t) \leq \frac{z_1^2}{2(k_{d1}^2 - z_1^2)^2} + \frac{d_{1M}^2}{2}. \tag{17}$$

Substituting (7) and (17) into (16), (16) can be rewritten as

$$\begin{aligned}\dot{V}_1 &\leq \frac{z_1}{k_{d1}^2 - z_1^2}\left(z_2 + \alpha_1 + \frac{z_1}{2(k_{d1}^2 - z_1^2)}\right. \\ &\quad \left. + f_1(x_1) - \dot{y}_d\right) + \frac{d_{1M}^2}{2} - \frac{\tilde{\theta}_1\dot{\theta}_1}{\eta_1}.\end{aligned} \tag{18}$$

According to Young's inequality, one can have

$$\frac{z_1 z_2}{k_{d1}^2 - z_1^2} \leq \frac{z_1^2}{2(k_{d1}^2 - z_1^2)^2} + \frac{z_2^2}{2}. \tag{19}$$

Substituting (19) into (18), the following inequality can be obtained

$$\begin{aligned}\dot{V}_1 &\leq \frac{z_1}{k_{d1}^2 - z_1^2}\left(\alpha_1 + f_1(x_1) + \frac{z_1}{k_{d1}^2 - z_1^2}\right. \\ &\quad \left. - \dot{y}_d\right) + \frac{z_2^2}{2} + \frac{d_{1M}^2}{2} - \frac{\tilde{\theta}_1\dot{\theta}_1}{\eta_1} \\ &= \frac{z_1}{k_{d1}^2 - z_1^2}\left(\alpha_1 + F_1(Z_1)\right) + \frac{z_2^2}{2} \\ &\quad + \frac{d_{1M}^2}{2} - \frac{\tilde{\theta}_1\dot{\theta}_1}{\eta_1},\end{aligned} \tag{20}$$

where

$$F_1(Z_1) = f_1(x_1) + \frac{z_1}{k_{d1}^2 - z_1^2} - \dot{y}_d.$$

According to Lemma 1, $F_1(Z_1)$ is rewritten as follows:

$$F_1(Z_1) = W_1^T\phi_1(Z_1) + \delta_1(Z_1).$$

Using Young's inequality, we have

$$\begin{aligned}&\frac{z_1}{k_{d1}^2 - z_1^2} F_1(Z_1) \\ &= \frac{z_1}{k_{d1}^2 - z_1^2}\frac{W_1^T\|W_1\|\phi_1(Z_1)}{\|W_1\|} + \frac{z_1}{k_{d1}^2 - z_1^2}\delta_1(Z_1) \\ &\leq \frac{z_1^2}{2a_1^2(k_{d1}^2 - z_1^2)^2}\|W_1\|^2\phi_1^T(Z_1)\phi_1(Z_1) \\ &\quad + \frac{a_1^2 W_1^T W_1}{2\|W_1\|^2} + \frac{b_1 z_1^2}{2r_1^2(k_{d1}^2 - z_1^2)^2} + \frac{r_1^2\varepsilon_1^2}{2b_1} \\ &= \frac{b_1 z_1^2}{2a_1^2(k_{d1}^2 - z_1^2)^2}\theta_1^*\phi_1^T(Z_1)\phi_1(Z_1) + \frac{a_1^2}{2} \\ &\quad + \frac{b_1 z_1^2}{2r_1^2(k_{d1}^2 - z_1^2)^2} + \frac{r_1^2\varepsilon_1^2}{2b_1},\end{aligned} \tag{21}$$

where $\theta_1^* = \frac{\|W_1\|^2}{b_1}$.

Then, one can obtain

$$\begin{aligned}\dot{V}_1 &\leq \frac{z_1}{k_{d1}^2 - z_1^2}\left(\alpha_1 + \frac{b_1 z_1}{2a_1^2(k_{d1}^2 - z_1^2)}\theta_1^*\phi_1^T(Z_1)\phi_1(Z_1)\right. \\ &\quad \left. + \frac{b_1 z_1}{2r_1^2(k_{d1}^2 - z_1^2)}\right) + \frac{z_2^2}{2} + \frac{d_{1M}^2}{2} + \frac{a_1^2}{2} + \frac{r_1^2\varepsilon_1^2}{2b_1} - \frac{\tilde{\theta}_1\dot{\theta}_1}{\eta_1}.\end{aligned}$$

Based on the designed virtual control signal (9) and adaptive law (10), one can get

$$\dot{V}_1 \leq -\frac{c_1 z_1^2}{k_{d1}^2 - z_1^2} + \frac{\sigma_1\tilde{\theta}_1\theta_1}{\eta_1} + \frac{a_1^2}{2} + \frac{r_1^2\varepsilon_1^2}{2b_1} + \frac{z_2^2}{2} + \frac{d_{1M}^2}{2}. \tag{22}$$

It is obvious that

$$\tilde{\theta}_1\theta_1 = \tilde{\theta}_1(\theta_1^* - \tilde{\theta}_1) \leq \frac{\theta_1^{*2}}{2} - \frac{\tilde{\theta}_1^2}{2}. \tag{23}$$

Then, (22) is rewritten as follows:

$$\begin{aligned}\dot{V}_1 &\leq -\frac{c_1 z_1^2}{k_{d1}^2 - z_1^2} + \frac{\sigma_1\theta_1^{*2}}{2\eta_1} - \frac{\sigma_1\tilde{\theta}_1^2}{2\eta_1} \\ &\quad + \frac{z_2^2}{2} + \frac{a_1^2}{2} + \frac{r_1^2\varepsilon_1^2}{2b_1} + \frac{d_{1M}^2}{2} \\ &= -\frac{c_1 z_1^2}{k_{d1}^2 - z_1^2} - \frac{\sigma_1\tilde{\theta}_1^2}{2\eta_1} + \frac{z_2^2}{2} + d_1,\end{aligned}$$

where $d_1 = \frac{a_1^2}{2} + \frac{r_1^2\varepsilon_1^2}{2b_1} + \frac{d_{1M}^2}{2} + \frac{\sigma_1\theta_1^{*2}}{2\eta_1}$.

Step i (i=2,3,...n-1). Consider the following Lyapunov function

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{\tilde{\theta}_i^2}{2\eta_i},$$

where $\eta_i > 0$, $\tilde{\theta}_i = \theta_i^* - \theta_i$ is the estimation error and θ_i is the estimation of θ_i^*.

The derivative of V_i is

$$\dot{V}_i = \dot{V}_{i-1} + z_i\dot{z}_i - \frac{\tilde{\theta}_i\dot{\theta}_i}{\eta_i}. \tag{24}$$

According to (7), the derivative of z_i is

$$\dot{z}_i = x_{i+1} + f_i(\tilde{x}_i) + d_i(\tilde{x}_n, t) - \dot{\alpha}_{i-1}. \tag{25}$$

The derivative of α_{i-1} is as follows:

$$\begin{aligned}\dot{\alpha}_{i-1} &= \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(k)}}y_d^{(k+1)}+\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\left(f_k(\tilde{x}_k)\right.\\ &\quad \left.+x_{k+1}+d_k(\tilde{x}_n,t)\right)+\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\theta_k}\dot{\theta}_k. \quad (26)\end{aligned}$$

According to Young's inequality and Assumption 1, one can get

$$\begin{aligned}z_i d_i(\tilde{x}_n,t) &\leq \frac{z_i^2}{2}+\frac{d_{iM}^2}{2},\\ z_i\frac{\partial\alpha_{i-1}}{\partial x_k}d_k(\tilde{x}_n,t) &\leq \frac{z_i^2}{2}\left(\frac{\partial\alpha_{i-1}}{\partial x_k}\right)^2+\frac{d_{kM}^2}{2}. \quad (27)\end{aligned}$$

Based on the recursive computation, $\dot{V}_{i-1}$ can be derived as

$$\dot{V}_{i-1}\leq-\frac{c_1 z_1^2}{k_{d1}^2-z_1^2}-\sum_{k=2}^{i-1}c_k z_k^2+\sum_{k=1}^{i-1}d_k-\sum_{k=1}^{i-1}\frac{\sigma_k\tilde{\theta}_k^2}{2\eta_k}+z_{i-1}z_i, \quad (28)$$

where $d_k=\frac{a_k^2}{2}+\frac{r_k^2\varepsilon_k^2}{2b_k}+\sum_{l=1}^{k}\frac{d_{lM}^2}{2}+\frac{\sigma_k\theta_k^{*2}}{2\eta_k}$.

Substituting (25), (26), (27) and (28) into (24), one can have

$$\begin{aligned}\dot{V}_i &\leq -\frac{c_1 z_1^2}{k_{d1}^2-z_1^2}-\sum_{k=2}^{i-1}c_k z_k^2+\sum_{k=1}^{i-1}d_k-\sum_{k=1}^{i-1}\frac{\sigma_k\tilde{\theta}_k^2}{2\eta_k}\\ &\quad -\frac{\tilde{\theta}_i\dot{\theta}_i}{\eta_i}+\sum_{l=1}^{i}\frac{d_{lM}^2}{2}+z_i\left(z_{i-1}+z_{i+1}+f_i\left(\tilde{x}_i\right)\right.\\ &\quad +\alpha_i+\frac{z_i}{2}+\frac{z_i}{2}\sum_{k=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_k}\right)^2-\sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(k)}}y_d^{(k+1)}\\ &\quad \left.-\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\theta_k}\dot{\theta}_k-\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\left(f_k(\tilde{x}_k)+x_{k+1}\right)\right)\\ &= -\frac{c_1 z_1^2}{k_{d1}^2-z_1^2}-\sum_{k=2}^{i-1}c_k z_k^2+\sum_{k=1}^{i-1}d_k-\sum_{k=1}^{i-1}\frac{\sigma_k\tilde{\theta}_k^2}{2\eta_k}-\frac{\tilde{\theta}_i\dot{\theta}_i}{\eta_i}\\ &\quad +z_i\left(z_{i+1}+\alpha_i+F_i(Z_i)\right)+\sum_{l=1}^{i}\frac{d_{lM}^2}{2}, \quad (29)\end{aligned}$$

where

$$\begin{aligned}F_i(Z_i) &= z_{i-1}+f_i\left(\tilde{x}_i\right)+\frac{z_i}{2}+\frac{z_i}{2}\sum_{k=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_k}\right)^2\\ &\quad -\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\theta_k}\dot{\theta}_k-\sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(k)}}y_d^{(k+1)}\\ &\quad -\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\left(f_k(\tilde{x}_k)+x_{k+1}\right).\end{aligned}$$

According to Lemma 1 and Young's inequality, it follows that

$$z_i F_i(Z_i)\leq\frac{b_i z_i^2}{2a_i^2}\theta_i^*\phi_i^T(Z_i)\phi_i(Z_i)+\frac{a_i^2}{2}+\frac{b_i z_i^2}{2r_i^2}+\frac{r_i^2\varepsilon_i^2}{2b_i}. \quad (30)$$

Substituting (30) into (29), one can get

$$\begin{aligned}\dot{V}_i &\leq -\frac{c_1 z_1^2}{k_{d1}^2-z_1^2}-\sum_{k=2}^{i-1}c_k z_k^2+\sum_{k=1}^{i-1}d_k-\sum_{k=1}^{i-1}\frac{\sigma_k\tilde{\theta}_k^2}{2\eta_k}\\ &\quad +z_i(z_{i+1}+\alpha_i)+\frac{a_i^2}{2}+\frac{b_i z_i^2}{2a_i^2}\theta_i^*\phi_i^T(Z_i)\phi_i(Z_i)\\ &\quad -\frac{\tilde{\theta}_i\dot{\theta}_i}{\eta_i}+\frac{b_i z_i^2}{2r_i^2}+\frac{r_i^2\varepsilon_i^2}{2b_i}+\sum_{l=1}^{i}\frac{d_{lM}^2}{2},\end{aligned}$$

where $\theta_i^*=\frac{\|W_i\|^2}{b_i}$.

Based on the designed virtual control input α_i (11) and adaptive law $\dot{\theta}_i$ (12), one can obtain

$$\begin{aligned}\dot{V}_i &\leq -\frac{c_1 z_1^2}{k_{d1}^2-z_1^2}-\sum_{k=2}^{i-1}c_k z_k^2+\sum_{k=1}^{i-1}d_k\\ &\quad -\sum_{k=1}^{i-1}\frac{\sigma_k\tilde{\theta}_k^2}{2\eta_k}+z_i z_{i+1}+\frac{\sigma_i\tilde{\theta}_i\theta_i}{\eta_i}-c_i z_i^2\\ &\quad +\frac{a_i^2}{2}+\frac{r_i^2\varepsilon_i^2}{2b_i}+\sum_{l=1}^{i}\frac{d_{lM}^2}{2}. \quad (31)\end{aligned}$$

Clearly, the following inequality holds

$$\tilde{\theta}_i\theta_i=\tilde{\theta}_i(\theta_i^*-\tilde{\theta}_i)\leq\frac{\theta_i^{*2}}{2}-\frac{\tilde{\theta}_i^2}{2}. \quad (32)$$

Substituting (32) into (31), one obtains

$$\dot{V}_i\leq-\frac{c_1 z_1^2}{k_{d1}^2-z_1^2}-\sum_{k=2}^{i}c_k z_k^2+\sum_{k=1}^{i}d_k-\sum_{k=1}^{i}\frac{\sigma_k\tilde{\theta}_k^2}{2\eta_k}+z_i z_{i+1}, \quad (33)$$

where

$$d_i=\frac{a_i^2}{2}+\frac{r_i^2\varepsilon_i^2}{2b_i}+\sum_{l=1}^{i}\frac{d_{lM}^2}{2}+\frac{\sigma_i\theta_i^{*2}}{2\eta_i}.$$

Step n. Define the Lyapunov function as follows:

$$V_n=V_{n-1}+\frac{z_n^2}{2}+\frac{\tilde{\theta}_n^2}{2\eta_n},$$

where $\eta_n>0$, $\tilde{\theta}_n=\theta_n^*-\theta_n$ is the estimation error and θ_n is the estimation of θ_n^*.

The derivative of V_n is

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1}+z_n\dot{z}_n-\frac{\tilde{\theta}_n\dot{\theta}_n}{\eta_n}\\ &= \dot{V}_{n-1}+z_n(\dot{x}_n-\dot{\alpha}_{n-1}+\frac{1}{\gamma}\dot{x}_{n+1})-\frac{\tilde{\theta}_n\dot{\theta}_n}{\eta_n}\\ &= \dot{V}_{n-1}+z_n\left(f_n(x)+x_{n+1}-u+d_n(\tilde{x}_n,t)\right.\\ &\quad \left.-\dot{\alpha}_{n-1}+\frac{1}{\gamma}(-\gamma x_{n+1}+2\gamma u)\right)-\frac{\tilde{\theta}_n\dot{\theta}_n}{\eta_n}\\ &= \dot{V}_{n-1}+z_n\left(f_n(x)+u+d_n(\tilde{x}_n,t)\right.\\ &\quad \left.-\dot{\alpha}_{n-1}\right)-\frac{\tilde{\theta}_n\dot{\theta}_n}{\eta_n}, \quad (34)\end{aligned}$$

where

$$\begin{aligned}\dot{\alpha}_{n-1} &= \sum_{k=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(k)}}y_d^{(k+1)}+\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}\left(f_k(\tilde{x}_k)\right.\\ &\quad \left.+x_{k+1}+d_k(\tilde{x}_n,t)\right)+\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\theta_k}\dot{\theta}_k.\end{aligned}$$

According to Young's inequality and Assumption 1, one can get

$$\begin{aligned}z_nd_n(\tilde{x}_n,t) &\leq \frac{z_n^2}{2}+\frac{d_{nM}^2}{2},\\ z_n\frac{\partial\alpha_{n-1}}{\partial x_k}d_k(\tilde{x}_n,t) &\leq \frac{z_n^2}{2}\left(\frac{\partial\alpha_{n-1}}{\partial x_k}\right)^2+\frac{d_{kM}^2}{2}.\end{aligned}\tag{35}$$

Substituting (33) and (35) into (34), we have

$$\begin{aligned}\dot{V}_n &\leq -\frac{c_1z_1^2}{k_{d1}^2-z_1^2}-\sum_{k=2}^{n-1}c_kz_k^2+\sum_{k=1}^{n-1}d_k\\ &\quad -\sum_{k=1}^{n-1}\frac{\sigma_k\tilde{\theta}_k^2}{2\eta_k}+z_n\Big(z_{n-1}+f_n(x)+u+\frac{z_n}{2}\\ &\quad +\frac{z_n}{2}\sum_{k=1}^{n-1}\left(\frac{\partial\alpha_{n-1}}{\partial x_k}\right)^2-\sum_{k=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(k)}}y_d^{(k+1)}\\ &\quad -\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}\left(f_k(\tilde{x}_k)+x_{k+1}\right)\\ &\quad -\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\theta_k}\dot{\theta}_k\Big)-\frac{\tilde{\theta}_n\dot{\theta}_n}{\eta_n}+\sum_{l=1}^{n}\frac{d_{lM}^2}{2}\\ &= -\frac{c_1z_1^2}{k_{d1}^2-z_1^2}-\sum_{k=2}^{n-1}c_kz_k^2+\sum_{k=1}^{n-1}d_k-\sum_{k=1}^{n-1}\frac{\sigma_k\tilde{\theta}_k^2}{2\eta_k}\\ &\quad +\sum_{l=1}^{n}\frac{d_{lM}^2}{2}+z_n\left(u+F_n(Z_n)\right)-\frac{\tilde{\theta}_n\dot{\theta}_n}{\eta_n},\end{aligned}\tag{36}$$

where

$$\begin{aligned}F_n(Z_n) &= z_{n-1}+f_n(x)+\frac{z_n}{2}+\frac{z_n}{2}\sum_{k=1}^{n-1}(\frac{\partial\alpha_{n-1}}{\partial x_k})^2\\ &\quad -\sum_{k=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(k)}}y_d^{(k+1)}-\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\theta_k}\dot{\theta}_k\\ &\quad -\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}\left(f_k(\tilde{x}_k)+x_{k+1}\right).\end{aligned}$$

Similar to the step i, (36) can be rewritten as follows:

$$\begin{aligned}\dot{V}_n &\leq -\frac{c_1z_1^2}{k_{d1}^2-z_1^2}-\sum_{k=2}^{n-1}c_kz_k^2+\sum_{k=1}^{n-1}d_k-\sum_{k=1}^{n-1}\frac{\sigma_k\tilde{\theta}_k^2}{2\eta_k}\\ &\quad -c_nz_n^2+\frac{\sigma_n\tilde{\theta}_n\theta_n}{\eta_n}+\frac{a_n^2}{2}+\frac{r_n^2\varepsilon_n^2}{2b_n}+\sum_{l=1}^{n}\frac{d_{lM}^2}{2}\\ &\leq -\frac{c_1z_1^2}{k_{d1}^2-z_1^2}-\sum_{k=2}^{n}c_kz_k^2+\sum_{k=1}^{n}d_k-\sum_{k=1}^{n}\frac{\sigma_k\tilde{\theta}_k^2}{2\eta_k},\end{aligned}$$

where

$$d_n=\frac{a_n^2}{2}+\frac{r_n^2\varepsilon_n^2}{2b_n}+\sum_{l=1}^{n}\frac{d_{lM}^2}{2}+\frac{\sigma_n\theta_n^{*2}}{2\eta_n}.$$

According to Lemma 3, one can obtain

$$-\frac{c_1z_1^2}{k_{d1}^2-z_1^2}\leq -c_1\log\frac{k_{d1}^2}{k_{d1}^2-z_1^2}.$$

Finally, based on the above discussions, we can get the following inequality

$$\dot{V}_n\leq -cV_n+\lambda,\tag{37}$$

where $c=\min\{2c_k,\sigma_k,\ k=1,2,\ldots,n\},\ \lambda=\sum_{k=1}^{n}d_k$.

According to (37) and Lemma 2, we can obtain that signals of the resulting closed-loop system (1), i.e., $x_i(t),z_i(t),\theta_i(t)$ $i=1,2,\ldots,n$, and $u(t)$ are bounded and $|z_1|\leq k_{d1}$. Based on Assumption 2, it is clear that $y(t)=y_d(t)+z_1(t)$ and $|y_d(t)|\leq\Upsilon_0$, we get $|y(t)|\leq|y_d(t)|+|z_1(t)|<\Upsilon_0+k_{d1}=k_{c1}$, $\forall\ t>0$. Therefore, all signals, which exist in the closed-loop system, are semi-globally uniformly ultimately bounded. ■

Remark 5: As can be seen from (37), increasing parameter c can decrease the tracking error. However, when c is designed enough big, the control energy is large. Therefore, in practical engineering, the parameters should be designed suitably to obtain better transient performance and control action.

Remark 6: In the design process, the derivative of virtual control signal α_1 has been computed repeatedly, leading to the so-called issue of "explosion of complexity". To avoid such problem, dynamic surface control technique can be utilized in many results, for example, [39], [40] and references therein. In our future work, dynamic surface technique can be exploited to avoid the problem of "explosion of complexity".

Remark 7: The control design for a class of nonlinear systems with input delay have been studied in some literatures. However, the method used in this paper is different from those existing results. In our work, Pade approximation approach is used to separate delay from the input control signal, which simplifies the analysis process. In addition, the proposed method is robust to uncertain input delay, which will be proved in the simulation results.

Based on the above analysis, a detailed algorithm is given as follows.

Algorithm 1.

Step 1. Determine the number of fuzzy rules and provide approximate membership functions.

Step 2. Based on Step 1, determine the fuzzy logic systems (4) to identify the nonlinear characteristics existing in the considered system.

Step 3. Choose approximate parameters $a_i>0,b_i>0,c_i>0$, $\eta_i>0$, $r_i>0$, $\sigma_i>0$ $(i=1,2,\ldots,n-1)$ and update the adaptive law $\dot{\theta}_1$ (10) and $\dot{\theta}_i$ (12) $(i=2,3,\ldots,n-1)$ and virtual control signal α_1 (9) and α_i (11) $(i=2,3,\ldots,n-1)$.

Step 4. Provide approximate parameters $a_n>0$, $b_n>0$, $c_n>0$, $\eta_n>0$, $r_n>0$, $\sigma_n>0$, and update the adaptive law $\dot{\theta}_n$ (14) and the designed controller u (13) to control the dynamic system (1).

IV. Simulation Results

This part provides two examples to demonstrate that our proposed method is effective. First, a numerical example is considered.

Example 1: We study a second-order system as follows.

$$\begin{aligned}\dot{x}_1 &= x_2 + \frac{-x_1}{1+x_1^4} + 0.1\sin(x_1),\\ \dot{x}_2 &= -x_2 e^{-x_1^2} x_1^2 + u(t-\tau) + 0.1\cos(x_2),\\ y &= x_1.\end{aligned}$$

To address the unknown nonlinear functions, the fuzzy basis functions are given as follows:

$$u_{M_i^j}(x_i) = \exp\left[-\frac{(x_i - 3 + j)^2}{4}\right],\ i = 1,2,\ j = 1,\ldots,5.$$

The sampling time is 0.01. Suppose input delay $\tau = 0.0043$, tracking signal $y_d = \sin(t)$ and the parameters to be designed $a_1 = 2,\ a_2 = 3,\ c_1 = 50,\ c_2 = 54,\ b_1 = 0.08,\ b_2 = 0.09,\ \sigma_1 = 1,\ \sigma_2 = 2,\ r_1 = 1,\ r_2 = 2,\ k_{d1} = 0.12,\ \eta_1 = 0.4,\ \eta_2 = 0.2$. The initial conditions are given as $[x_1(0) \quad x_2(0)]^T = [0.07 \quad 0.2]^T$ and $[\theta_1(0) \quad \theta_2(0)]^T = [0 \quad 0]^T$, respectively.

Figs. 1-6 show the simulation results. Fig. 1 plots the responses of output y, the desired signal y_d and output bound k_{c1}, respectively. This figure shows that the trajectory of the system output y can track the given tracking signal y_d with a small tracking error, and output y evolves strictly within the bound as $|y| < k_{c1} = 1.12$. Fig. 2 plots the trajectories of states x_1 and x_2 and it shows the closed-loop system is stable. Fig. 3 plots the responses of adaptive parameters θ_1 and θ_2. Fig. 4 plots the response of control input u and its two zoomed-in versions. Fig. 5 shows the trajectory of the introduced variable x_3 and its two zoomed-in versions. Fig. 6 plots the trajectory of error z_1. From these figures, it is clear that all variables of the considered system are bounded with input delay and output constraint.

Example 2: Assume there exists a bounded deviation in γ. To demonstrate the robustness of the proposed scheme, consider the following practical pendulum system with input delay.

$$\begin{aligned}\dot{x}_1 &= x_2 - 0.2x_1,\\ \dot{x}_2 &= 0.8\sin(x_1) + u(t-\tau),\\ y &= x_1.\end{aligned}$$

The membership functions are defined as follows:

$$\begin{aligned}u_{M_i^j}(x_i) &= \exp\left[-\frac{(x_i + 2 - 0.5j)^2}{2}\right],\\ i &= 1,2,\ j = 1,2,\ldots,7.\end{aligned}$$

Different from Example 1, input delay is uncertain, i.e., $\tau = 0.0051 + 0.0005\cos(t)$. Parameters are given as $a_1 = a_2 = 2,\ b_1 = b_2 = 0.05,\ \lambda_1 = \lambda_2 = 40,\ \sigma_1 = \sigma_2 = 1,\ r_1 = r_2 = 2,\ k_{d1} = 0.12,\ \eta_1 = \eta_2 = 0.1$. The initial conditions are $[x_1(0) \quad x_2(0)]^T = [0.1 \quad 0.3]^T$ and $[\theta_1(0) \quad \theta_2(0)]^T = [0 \quad 0]^T$ and tracking signal y_d is defined as same as that in Example 1. Figs. 7-12 demonstrate the simulation results. Fig. 7 shows the trajectories of the output signal y, the reference signal y_d and output bound k_{c1}. It can be observed that the output tracking is achieved with a small error. Fig. 8 plots the responses of state variables x_1 and x_2, it shows the closed-loop system is stable. Fig. 9 plots the responses of adaptive parameters θ_1 and θ_2. Fig. 10 plots the trajectory of controller u. Fig. 11 plots the trajectory of the introduced variable x_3 and Fig. 12 plots the response of error z_1. According to simulation results in both Example 1 and Example 2, the effectiveness of the proposed control scheme is demonstrated.

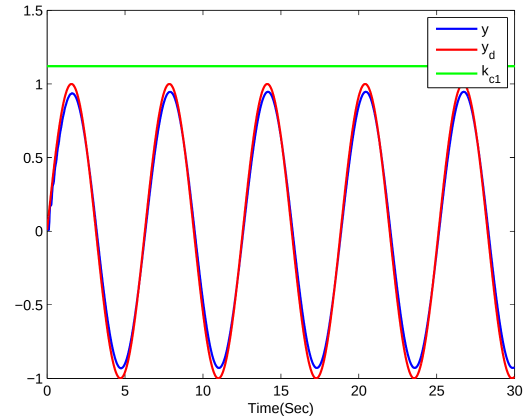

Fig. 1. Trajectories of output y, the reference signal y_d and output bound k_{c_1}.

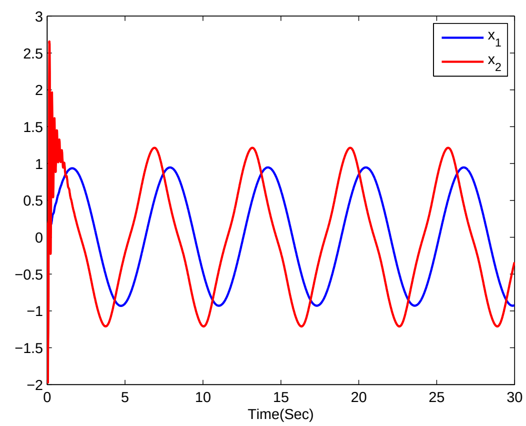

Fig. 2. Trajectories of states x_1 and x_2.

V. Conclusions

A tracking control strategy has been developed for a category of strict-feedback systems in the framework of fuzzy adaptive backstepping control. Input delay and output constraint have been considered in the design process. Using FLSs, the unknown nonlinear functions existing in systems have been identified. By using Pade approximation technique and barrier Lyapunov function, input delay and output constraint have been handled, respectively. Furthermore, on the basis of the adaptive backstepping control, a novel adaptive fuzzy tracking controller was derived to ensure that all signals in the closed-loop systems are SGUUB, and the given reference output signal can be tracked by the system output

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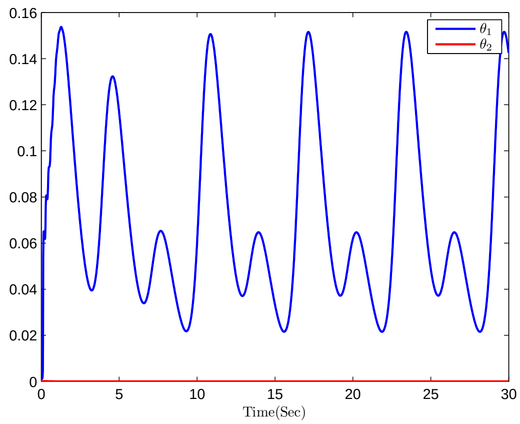

Fig. 3. Trajectories of adaptive parameters θ_1 and θ_2.

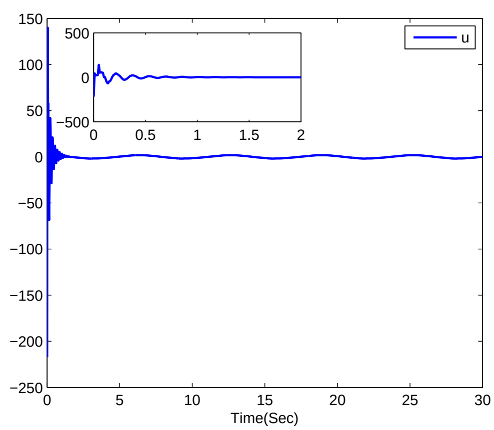

Fig. 4. Trajectory of true control input u.

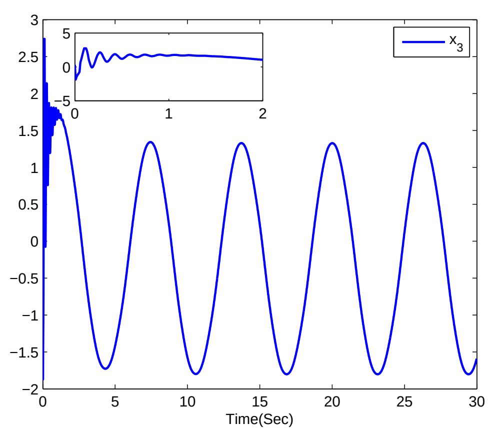

Fig. 5. Trajectory of the introduced variable x_3.

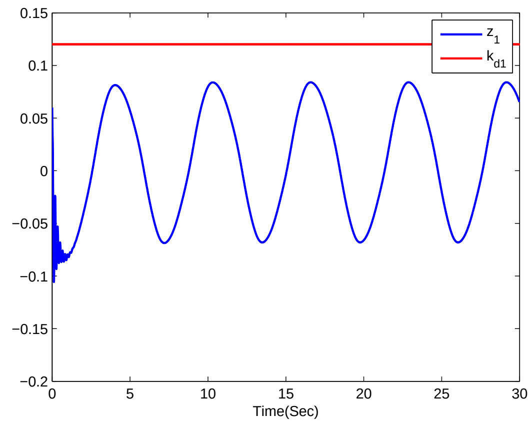

Fig. 6. Trajectory of tracking error z_1.

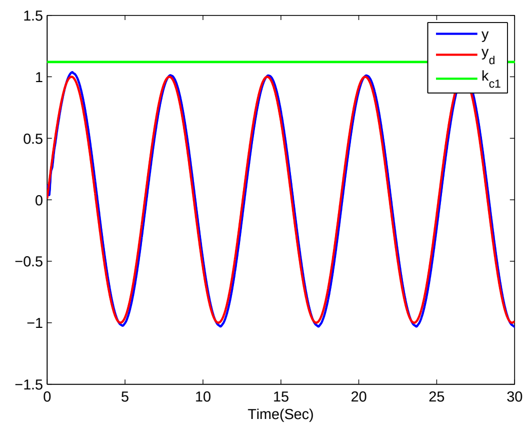

Fig. 7. Trajectories of output y, the reference signal y_d and output bound k_{c_1} of Example 2.

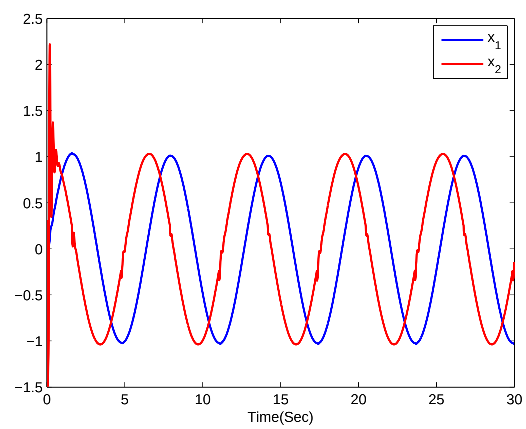

Fig. 8. Trajectories of states x_1 and x_2 of Example 2.

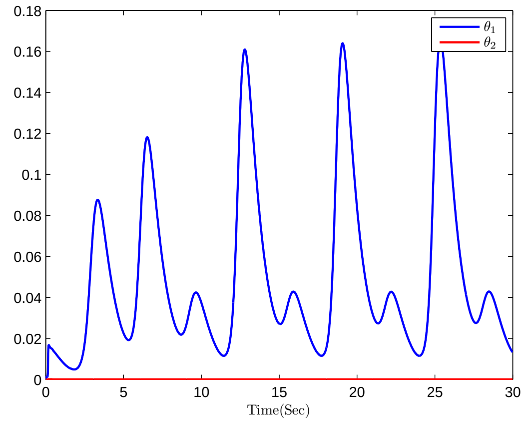

Fig. 9. Trajectories of adaptive parameters θ_1 and θ_2 of Example 2.

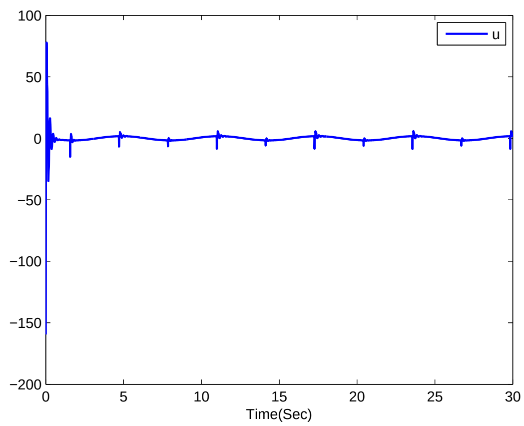

Fig. 10. Trajectory of true control input u of Example 2.

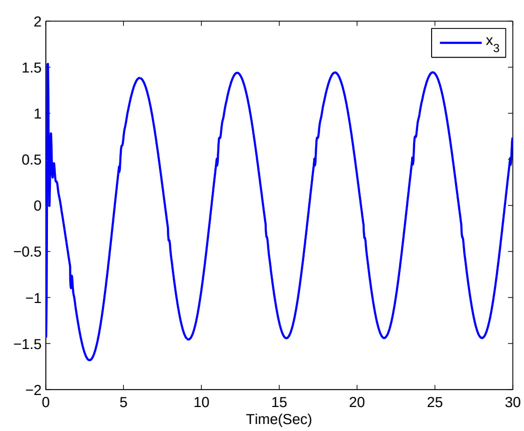

Fig. 11. Trajectory of the introduced variable x_3 of Example 2.

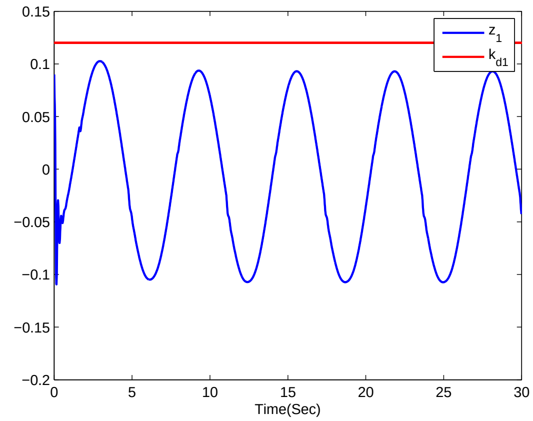

Fig. 12. Trajectory of tracking error z_1 of Example 2.

with a small tracking error. Finally, simulation results have illustrated the effectiveness of the approach proposed in this paper. In future work, we will attempt to apply the proposed control scheme and dynamic surface control technique to solve the control problem of MIMO nonlinear systems with input and output constraints.

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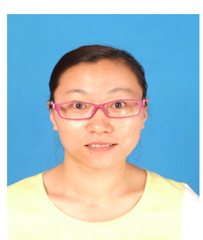

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